# Performance Analysis of the Singular Value Decomposition with Block-Diagonalization Precoding in Multi-User Massive MIMO Systems

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***Abstract*—Next generation of 5G mobile communications heavily relies on exploiting the spatial degrees of freedom of the radio channels. For this reason, the idea of having base stations (BSs) with a large number of antennas is currently one of the main research topics in the wireless communications field.**

**From a radio access point of view, this calls for searching effective methods of spatially multiplexing multiple mobile stations (MSs) – otherwise termed as users – using the same time-frequency resource. This technique is known as Multi-User Multiple-Input Multiple-Output (MU-MIMO).**

**For the downlink direction, applying classical MIMO transmission methods for MU-MIMO schemes are impractical, as they force the MSs to perform a collaborative detection, which implies a considerable increase of control information traffic.**

**This paper analyzes the performances of the Block Diagonalization (BD) MU-MIMO downlink technique that proposes a precoding method which mitigates the inter-user interference and thus the need of a collaborative detection at the MS side. Moreover, in the performed simulations, the Singular Value Decomposition (SVD) method has been applied over the Single-User MIMO (SU-MIMO) links that were virtually created by the BD precoding technique. The results have been compared with the simpler MU-MIMO method: ZF beamforming.**

***Keywords—Massive MIMO, BD, Precoding, MU-MIMO***

## I. INTRODUCTION

The key requirements that characterize the next generation cellular networks (5G) focus on increasing the throughput, lowering the latency and increasing the number of simultaneously connected devices.

One of the most promising directions to achieve the 5G throughput demands is to scale the current spatial multiplexing techniques [1], especially for the MIMO use cases where multiple devices share the same time-frequency resource (MU-MIMO). However, when considering the downlink direction, enabling MU-MIMO schemes raises the challenge of finding techniques that avoid collaborative detection [2] at the user side. Most of these techniques consider performing the precoding of the transmitted symbols at the BS side in order to eliminate the inter-user interference.

Among the linear precoding techniques that facilitate the reduction of the inter-user interference is the Zero-Forcing (ZF) beamforming technique, which uses the channel matrix inverse as the precoding matrix. This technique also eliminates in theory the intra-user interference and drastically reduces the receiver complexity at the user side.

However, the ZF beamforming [3] forces equal signal-to-noise ratios for all users / all antennas, causing reduced flexibility in adjusting the received power levels for each user separately. This leads to inefficient energy distribution. In contrast, the Singular Value Decomposition MIMO technique (SVD) has very good flexibility in terms of power balancing. Also, even without applying a power allocation policy, the SVD method proves to achieve a higher sum capacity as compared to the ZF method [4]. However, the SVD method brings back the collaborative detection problem.

Compared to the ZF technique, the BD method aims to eliminate only the inter-user interference [5]-[7]. Having the inter-user interference completely eliminated means that the MU-MIMO system can be viewed as a set of independent SU-MIMO systems [8]. For each such SU-MIMO link, a convenient MIMO linear or non-linear technique can be further used.

Among the disadvantages of the BD method is its high complexity [9]. However, it has been shown that the currently available digital signal processing computation power is able to handle the numerical complexity that characterizes the BD precoding, even for Massive MIMO scenarios [10].

The present paper contributes with a set of simulations that are aimed to highlight the detection performance boost of the BD technique – when paired with the SVD method – against the ZF precoding technique. Only the downlink MU-MIMO case is studied.

The use cases analyzed in the paper are aligned to the 5G requirements. The MU-MIMO system taken into consideration has been scaled to the dimensions of a Massive MIMO system [11], where the BS consists of a large number of transmit antennas (up to 128 transmit antennas).

In the first section of this paper, an introduction to the topic is given, describing the MU-MIMO collaborative detection problem and then the BD precoding technique. In the second section, the general multi-user Massive MIMO scenario is described along with the MIMO transmission equation that has been used in the simulator. The third section explains the

978-1-5090-3748-3/16/$31.00 ©2016 IEEE

general algorithm to find the BD precoding matrix and how the SVD technique is applied on each virtual SU-MIMO link. The simulation results are presented in the last section by firstly describing the dimensions of the MIMO system that has been considered.

The operator $(\cdot)^T$ stands for the transpose of a matrix, $(\cdot)^H$ is the Hermitian operator (transpose and conjugate), $(\cdot)^{-1}$ is the inverse of a square matrix and $(\cdot)^+$ represents the Moore-Penrose pseudoinverse of a matrix.

## II. MU-MIMO SYSTEM MODEL

Figure 1 depicts the downlink MU-MIMO system used in this paper.

One particularity that differentiates the MIMO system introduced here (when compared to the classic MIMO systems) is the fact that the BS possesses a large number of transmit antennas (Massive MIMO). $n_t$ is used to denote the number of transmit antennas. In case of Massive MIMO systems, $n_t$ can be 32, 64 or 128 antennas or above.

On the receiving side, the system comprises of a set of $K$ MSs, each having $n_{r,i}$ antennas. The total number of receive antennas for all users is given by $n_r$.

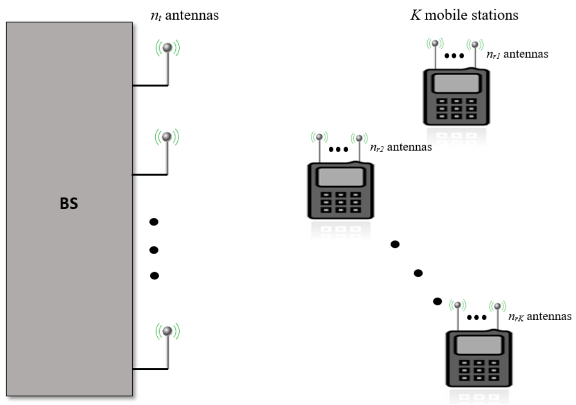


Figure 1. Representation of a multi-user MIMO system.

The MIMO system that has just been introduced is described by equation (1). Narrow band communication is considered. The generality is easily derived through extension to a multicarrier modulation technique, such as the Orthogonal Frequency-Division Multiplexing (OFDM) technique.

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{1}$$

where

$\mathbf{s}$ is the $n_t \times 1$ vector of symbols that are transmitted over the $n_t$ BS antennas.

$\mathbf{y}$ is the $n_r \times 1$ vector of symbols that are received on each antenna of each of the $K$ users; the $\mathbf{y}$ vector is structured as follows:

$$\mathbf{y} = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,n_{r,1}} & y_{2,1} & y_{2,2} & \cdots & y_{K,n_{r,K}} \end{bmatrix}^T. \tag{2}$$

where each $y_{i,j}$ element is the signal received from the $i^{\text{th}}$ MS on the $j^{\text{th}}$ receive antenna.

$\mathbf{w}$ is the $n_r \times 1$ vector that represents the AWGN noise which is captured by each antenna of each MS.

$\mathbf{H}$ is the $n_r \times n_t$ matrix representing the MIMO channel matrix; each element of $\mathbf{H}$ is a complex value that represents the propagation effect. $h_{jk}^{(i)}$ is the propagation coefficient between the $k^{\text{th}}$ transmit antenna and the $i^{\text{th}}$ receive antenna.

$$\mathbf{H} = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & . & h_{1n_t}^{(1)} \\ h_{21}^{(1)} & h_{22}^{(1)} & . & h_{2n_t}^{(1)} \\ . & . & . & . \\ h_{n_{r,1}1}^{(1)} & h_{n_{r,1}2}^{(1)} & . & h_{n_{r,1}n_t}^{(1)} \\ h_{11}^{(2)} & h_{12}^{(2)} & . & h_{1n_t}^{(2)} \\ . & . & . & . \\ h_{n_{r,K}1}^{(K)} & h_{n_{r,K}2}^{(K)} & . & h_{n_{r,K}n_t}^{(K)} \end{bmatrix}. \tag{3}$$

## III. MU-MIMO BEAMFORMING TECHNIQUES

As explained in Section 1, the problem of collaborative detection at the receiver side can be avoided if an efficient precoding technique is applied at the BS side, so that the inter-user interference is nulled out. By applying such a technique, it is assured that each user will only receive the signals that carry the information intended to it. One such technique is the ZF beamforming.

### *A. Zero Forcing beamforming technique*

The precoding matrix used in ZF beamforming is the right-sided Moore-Penrose pseudoinverse in (4).

$$\mathbf{P}_{ZF} = \mathbf{H}^+ = \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1}. \tag{4}$$

By denoting with $\mathbf{x}$ as the vector of non-precoded symbols that are to be transmitted, then, in case of applying a ZF precoding, the $\mathbf{s}$ vector becomes

$$\mathbf{s} = \mathbf{P}_{ZF}\,\mathbf{x}. \tag{5}$$

The MIMO equation can be rewritten as

$$\mathbf{y} = \mathbf{H}\mathbf{P}_{ZF}\mathbf{x} + \mathbf{w} = \mathbf{x} + \mathbf{w}. \tag{6}$$

It can be seen that the receiver design doesn't need to perform any detection. The transmitted data can be picked up directly from each of the receive antennas.

In practice, however, the available power of the BS is limited and is most likely not capable of fully compensating the propagation attenuation. Thus, a simple signal normalization will have to take place at the receiver side, especially for the situations where a higher modulation order is involved (16-QAM, 64-QAM, etc.).

## B. Block Diagonalization beamforming technique

Contrary to ZF beamforming, the BD technique aims to eliminate only the inter-user interference.

$\mathbf{P}_{BD}$ denotes the BD precoding matrix.

By considering $\mathbf{H}_e = \mathbf{H}\mathbf{P}_{BD}$ as the equivalent channel matrix – when seen from the transmit symbols space – $\mathbf{P}_{BD}$ forces zero values on the elements of the $\mathbf{H}$ matrix corresponding to inter-user interference.

Thus, it is easy to imagine that $\mathbf{H}_e$ is being formed from a set of blocks consisting of non-zero elements and positioned across the main diagonal, while all the other elements are zero. The total number of blocks is equal to the number of MSs, $K$.

$$\mathbf{H}_e = \begin{bmatrix} \mathbf{H}_{e,1} & & \mathbf{0} \\ & \mathbf{H}_{e,2} & \\ \mathbf{0} & & \ddots \\ & & & \mathbf{H}_{e,K} \end{bmatrix}. \quad (7)$$

It has been proven that $\mathbf{P}_{BD}$ can be found within the vectors that form the null spaces of a family of matrices derived from $\mathbf{H}$ (one such matrix for each user), which are denoted here as $\tilde{\mathbf{H}}_i$, where $i = 1, 2, ..., K$.

The general algorithm for finding the $\mathbf{P}_{BD}$ matrix follows the below steps:

*a) Build the* $\tilde{\mathbf{H}}_1$ *matrix from the* $\mathbf{H}$ *channel matrix, by removing the rows that correspond to the first user's receive antennas.*

*b) Compute the null-space of matrix* $\tilde{\mathbf{H}}_1$ *and select only a subset of* $n_{l,1}$ *columns, where* $n_{l,i}$ *is the number of data layers assigned for the* $i^{th}$ *user.*

*c) Update* $\mathbf{P}_{BD}$ *by adding the column vectors found at step b) as its leftmost columns.*

*d) Advance to the next user and repeat steps a)-d).*

An observation that can be made regarding the shape of the $\mathbf{H}_e$ matrix given by Eq. 7 is that each block within the $\mathbf{H}_e$ matrix can be considered as an equivalent channel matrix that characterizes the communication between the BS and a single user.

Equivalently, MU-MIMO communication has been divided into $K$ SU-MIMO systems for which classical MIMO precoding methods can be applied, such as SVD, ZF, etc.

After having the inter-user interference eliminated via BD precoding, the SVD decomposition is applied for each block that forms the $\mathbf{H}_e$ matrix, as shown in (8).

$$\mathbf{H}_{e,i} = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{V}_i^H. \quad (8)$$

where

$\mathbf{U}_i$ – A $n_{r,i} \times n_{r,i}$ unitary matrix.

$\mathbf{V}_i$ – A $n_{l,i} \times n_{l,i}$ unitary matrix.

$\mathbf{\Sigma}_i$ – A $n_{r,i} \times n_{l,i}$ diagonal matrix with the main diagonal elements equal to $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_{n_{\min}}$, real numbers that represent the singular values of the $\mathbf{H}_{e,i}$ matrix.

It is denoted:

$$\mathbf{P}_{SVD} = \begin{bmatrix} \mathbf{V}_1 & & \mathbf{0} \\ & \mathbf{V}_2 & \\ \mathbf{0} & & \ddots \\ & & & \mathbf{V}_K \end{bmatrix} \quad (9)$$

as the matrix that prepares the data symbols for SVD decomposition that is to be applied for each block within the $\mathbf{H}_e$ matrix.

The complete BD-SVD precoding can be written as

$$\mathbf{P}_{BD-SVD} = \mathbf{P}_{BD}\mathbf{P}_{SVD}. \quad (11)$$

The MIMO equation that applies for each SU-MIMO link created by the BD precoding is

$$\mathbf{y}_i = \mathbf{H}_{e,i}\mathbf{x}_i + \mathbf{w}_i. \quad (12)$$

where

$\mathbf{x}_i$ – The $n_{l,i} \times 1$ transmitted data vector intended for $i^{th}$ user.

$\mathbf{y}_i$ – The $n_{r,i} \times 1$ antenna symbols vector received by the $i^{th}$ user.

$\mathbf{w}_i$ – The $n_{r,i} \times 1$ noise vector captured by the $i^{th}$ user's antennas.

At the reception side, each user will have to apply the corresponding SVD demodulation matrix

$$\mathbf{D}_i = \mathbf{\Sigma}_i^+ \mathbf{U}_i^H. \quad (13)$$

Each MS will find the symbols transmitted to it by applying the demodulation matrix over the received vector, resulting the signal in (14).

$$\hat{\mathbf{x}}_i = \mathbf{D}_i\mathbf{y}_i = \mathbf{x}_i + \mathbf{D}_i\mathbf{w}_i. \quad (14)$$

where

$\hat{\mathbf{x}}_i$ – The $n_{l,i} \times 1$ vector which contains the detected data symbols for $i^{th}$ user.

# IV. Simulation Results

The simulations that have been performed are focused on assessing the performances of the two presented precoding techniques: ZF and BD-SVD in terms of the uncoded bit error rate averaged for all users in the system.

The MU-MIMO system has been sized according to the needs of a potential 5G network. For instance, the number of transmit antennas ($n_t$) has been set within the range of Massive

MIMO systems: 64, 128 antennas, while for the MSs, up to 16 receive antennas have been considered.

Fig. 1 and Fig. 2 show the simulation results that give the bit error rate (BER) with respect to the SNR, for the two precoding techniques. The plain SVD precoding method is also included strictly for comparison purposes, due to the fact that while it offers the best performance, applying it over the entire MIMO system is impractical, as it will require collaborative detection at the MSs side.

The authors have also considered associating the BD technique with linear ZF detection performed at each MS for the sake of providing a complete picture, being included in the performed simulations.

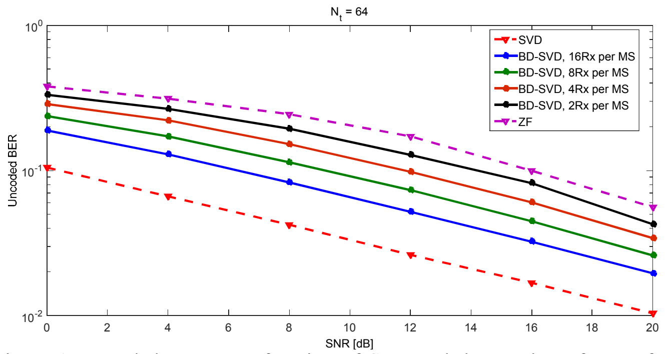


Figure 1. Uncoded BER as a function of SNR and the number of users for $n_t$=64.

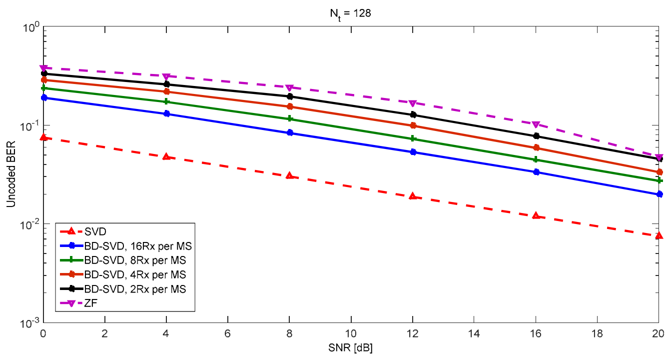


Figure 2. Uncoded BER as a function of SNR and the number of users for $n_t$=128.

Other details that characterize the MIMO system used in the simulations are given below:

- The transmission model is based on the MIMO equation given in Eq. 1
- Flat fading (single carrier) – the narrowband and time-invariant Rayleigh channel model is used to generate the **H** channel matrix
- The noise is considered to be AWGN
- The data symbols are modulated with a 64-QAM Gray encoded modulation scheme
- The transmit antenna vector is scaled such that the entire power budget is used for transmission
- No power weighting policy has been applied individually for each user, in order to maximize the entire system sum rate

When compared to the ZF precoding, it can be observed that BD precoding presents a bit error rate improvement of approximately 2 dB for the use cases where the MSs have 2 receive antennas. The improvement increases to 4 dB for 4 receive antennas and to approximately 7 dB for 8 receive antennas. The observations apply for both figures ($n_t$ = 64 and $n_t$ = 128).

## V. Conclusions

It is proven that the BD technique represents a good trade-off between the performance provided by the SVD method and the computational advantage of the ZF method, eliminating the need for a collaborative detection scheme at the user side.

The performance of BD method is shown to depend on the number of MSs in the system, with a significant performance gain in cases where the system is made up of few MSs with a large number of antennas and nearing the performance of ZF asymptotically as the number of users increases.

Also, the BD technique has the flexibility of selecting a different MIMO precoding method for each of the virtually created SU-MIMO links. For example, one can trade-off some of the performance gain in favor of a smaller computation complexity by precoding some users with ZF instead of SVD. This type of flexibility helps in lowering the computational power needed by the BS.

## References


[1] M. Min, Y.-S. Jeon and G.-H. Im, “On achievable multiplexing gain of BD in MIMO broadcast channels with limited feedback”, IEEE Trans. on Wireless Communications, vol. 15, no. 2, pp. 871-885, 2016.

[2] T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas”, IEEE Trans. on Wireless Communications, vol. 9, no. 11, pp. 3590-3600, 2010.

[3] Lee Li, Wang Jing, Wu Xiaoyun, “ZF Beamforming Performance Analysis for Multiuser Spatial Multiplexing with Imperfect Channel Feedback”, International Conference on Wireless Communications, Networking and Mobile Computing, pp. 869-872, 2007.

[4] G. Lebrun, T. Ying, M. Faulkner, “MIMO Transmission over a Time-Varying Channel Using SVD”, Global Telecommunications Conference, pp. 414-418, 2002.

[5] K. E. Dawui and D. T. M. Slock, “Multiuser-MIMO Downlink TX-RX Design Based on SVD Channel Diagonalization and Multiuser Diversity”, 39th Asilomar Conference on Signals, Systems and Computers, pp. 1493-1497, 2005.

[6] Md. H. A. Kahn, K. M. Cho, M. H. Lee and J.-G. Chung, “A simple block diagonal precoding for multi-user MIMO broadcast channels”, EURASIP Journal on Wireless Communications and Networking, vol. 95, 2014.

[7] F. Wang and M. E. Bialkowski, “Performance of block diagonalization scheme for downlink multiuser MIMO system with estimated channel state information”, Int. J. Communications, Network and System Sciences, vol. 4, pp. 82-87, 2011.

[8] Q. H. Spencer, A. L. Swindlehurst and M. Haardt, “Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels”, IEEE Trans. on Signal Processing, vol. 52, no. 2, pp. 461-471, 2004.

[9] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, Jr., B. L. Evans, “Low complexity user selection for multiuser MIMO systems with block diagonalization”, IEEE Trans. on Signal Processing, vol. 54, no. 9, pp. 3658-3663, 2006.

[10] Victor-Florin Crâşmariu, Marius-Octavian Arvinte, Andrei-Alexandru Enescu, Silviu Ciochină, “Block-Diagonalization Applied in Multi-User Massive MIMO”, Communications (COMM), 2016 11th International Conference.

[11] E. G. Larsson, O. Edfors, F. Tufvesson and T. L. Marzetta, “Massive MIMO for next generations wireless systems”, IEEE Communications Magazine, pp. 186-195, 2014.