FIN 567: Financial Risk Management

Project: Risk Measurement System for DJIA Dispersion Trades

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1. Introduction

Dispersion trades try to figure out that sometimes implied volatilities of index options are not consistent with the implied volatilities of the options on the index components. To exploit the fact, we try to build a system to measure the risk by simultaneously writing options on DJX and buying options on the index component stocks whose weight is as same as the index. The 62 options are as follows: 1 index call option, 1 index put option, 30 component stocks call options, 30 component stock put options.

We decide to separate those options into two portfolios to study: a call portfolio with all call options, and a put portfolio with all the put options. Also, our studies are under two different conditions to calculate the risk and analyze:

- In a Vega-neutral dispersion trade, the sum of the Vegas of the index and component options is 0.
- In a theta-neutral dispersion trade, the sum of the thetas of the index and component options is 0.

Our final goals are the 5% value-at-risk of the portfolio of options and the various Greek letter risks of the portfolio. We will use historical simulation method and the data is collected from OptionMetrics and CRSP in WRDS.

2. Methodologies

2.1 Market Factors

There are three market factors we will use in this project.

- S_i: Stock prices of each of the component stocks of Dow Jones Index Average (DJIA). We get the historical stock price of the component stocks of DIJA from CRSP in WRDS
- S: DJIA index price. We derive the index price from Yahoo Finance.
- σ_i: Volatilities of each of the component stocks of Dow Jones Index Average (DJIA). From CRSP in WRDS, we obtain the historical volatilities when maturity equals to 30 days, delta of put option equals to -50 and delta of call option equals to 50.

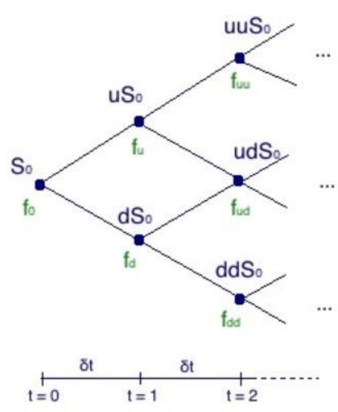
2.2 Parameter Selection

There are some other parameters we need in the project.

- Start date& End date: From 3/31/2019 to 6/30/2019. There are 63 trading days.
- Maturity: 1 month
- Risk-free rate: We get the risk-free rate from OptionMetrics in WRDS and calculate the average risk-free rate for each day
- Dividend of Stocks: We get the dividend per share from Compustat - Capital IQ in WRDS
- Dividend of Index: We get the dividend yield from OptionMetrics in WRDS
- Strike Price: We get the strike price of each option when maturity equals to 30 days, delta of put option equals to -50 and delta of call option equals to 50

2.3 Binomial Tree Model

To calculate the American option price of our portfolio, we would like to choose the binomial tree model.



Given we know the option price at time t = k+1, the option price f at t = k is

$$f_k = \frac{1}{(1+r)^{\delta t}} \{p f_{k+1}^{up} + (1-p) f_{k+1}^{down}\}$$

Therefore, starting at maturity n = T, and move backwards in time we obtain the option price at t = 0,

$$f_0 = \frac{1}{(1+r)^{\delta t}} \sum_{k=0}^{n} p^k (1-p)^{n-k} \, V\{u^k d^{n-k} S_0, K, \ldots\},$$

where $V\{u^k d^{n-k} S_0, K, \ldots\}$ is the payoff.

2.4 Portfolio Construction

We will reconstruct the index option using the options of components stocks of DJIA. We assign the equal weight, $\frac{1}{30}$, for each option. According to Vega-neutral and θ-neutral conditions,

$$vega_{portfolio} = -vega_{DJX} + \frac{1}{30} k_1 \sum_1^{30} vega_i = 0,$$

$$theta_{portfolio} = -theta_{DJX} + \frac{1}{30} k_2 \sum_1^{30} theta_i = 0,$$

where $vega_i, theta_i$ is the Vega and theta of i^{th} option. Therefore, we can obtain that the true weight of Vega and theta for i^{th} option is $\frac{k_1}{30}$ and $\frac{k_2}{30}$.

2.5 Model Selection: Historical Simulation

The model we choose in this project is historical simulation (HS). We use historical data from 03/31/2018 to 06/30/2018 to simulate the stock price and volatilities of each option.

- Compute the actual changes in the market factors.
- Use the changing rate to compute the hypothetical future values of the market factors and the market-to-market value of the portfolio.

After we obtain the hypothetical values, we apply them to the binomial tree using function CRRBinomialTreeOption(…) from package fOptions in R to calculate the price of put option and call option. However, if we want to use CRRBinomialTreeOption(…) to calculate the option, we need to recalculate the dividend yield d. From the data, we learn that a dividend of d_1 is paid at ex-dividend date and the closing price of today is S. The remaining time to expiration is 30/252 month. Then, the equivalent continuous dividend yield d can be obtained by solving

$$1-\frac{d_1}{S}=\exp\left(-\frac{30}{252}*d\right)$$

$$d=\ln\left(1-\frac{d_1}{S}\right)*(-\frac{252}{30})$$

With the parameters as the inputs, we can calculate the option price. There is one thing we need to pay attention to. The DJX index option contract is based on 1/100th (one-one-hundredth) of the current value of the Dow Jones Industrial Average. So, for example, when DJIA is at 11,000, the DJX level will be 110.

After that, we sort the value of portfolio in ascending order from most negative to largest and then 5% VaR can be calculated directly.

- Advantages of HS

  a. Does not need to make the assumption of statistical distribution of the volatility and correlation of asset returns, so it avoids the problem of estimation error
  b. It is very easy to implement
  c. It can accurately reflect the probability distribution characteristics of various risk factors

2.6 Greek Letter Risks

According to the BS formula:

$$C(S,K,\sigma,r,t,T,\delta)=Se^{-\delta(T-t)}N(d_1)-e^{-r(T-t)}KN(d_2)$$

$$P(S,K,\sigma,r,t,T,\delta)=e^{-r(T-t)}KN(-d_2)-Se^{-\delta(T-t)}N(-d_1)$$

We can easily get the Δ, γ, θ and Vega by

$$\varDelta=\frac{\partial V}{\partial S},\gamma=\frac{\partial^2 V}{\partial S^2}$$

$$\theta=\frac{\partial V}{\partial t},Vega=\frac{\partial V}{\partial \sigma}$$

We can easily obtain the Greek letters from the function GBSOption() from package fOptions in R.

Therefore, the various Greek risks of the portfolio (Δ, γ, ρ, θ, vega) can be calculated as

$$Portfoio\ Greek = \frac{k}{30}\sum_{1}^{30} stocks\ Greek_i - \ index\ Greek$$

(Where $stocks\, risk_i$ is the option risk of i^{th} stock and index risk is the risk of DJX)

Then, the risk of our portfolios can be calculated as follows

$$V(S) \approx \Delta * \mathit{\Delta} S + \frac{1}{2}\Gamma^2 * (\mathit{\Delta} S)^2 + \rho * \mathit{\Delta} r + (vega) * \mathit{\Delta}\sigma + \theta * \mathit{\Delta} t$$

$\mathit{\Delta} S, \mathit{\Delta}\sigma, \mathit{\Delta} r$ are calculated from our difference of the S, σ and r from 04/01/2019 to 06/28/2019.

$\mathit{\Delta} t$ is 1/252, since our horizon is one day.

- Vega-neutral Condition

Under this situation, the Vega of the portfolio equals to 0. Therefore, the new risk of each option can be calculated as

$$V(S) \approx \Delta * S + \frac{1}{2}\Gamma^2 * S^2 + \rho * \mathit{\Delta} r + \theta * \mathit{\Delta} t$$

- Theta-neutral condition

Under this situation, the Theta of the portfolio equals to 0. Therefore, the new risk of each option can be calculated as:

$$V(S) \approx \Delta * S + \frac{1}{2}\Gamma^2 * S^2 + \rho * r + (vega) * \sigma$$

3. Illustrative Results

3.1 VaR

We calculate the 5% VaR on 07/01/2019. Because one unit option involves 100 shares of stock. We need to multiply by 100 after calculating the VaR of per share.

	Vega-neutral	Theta-neutral
Put Option Portfolio	-23.43463	218.2687
Call Option Portfolio	-43.97643	155.8237

From the results, we learn that call and put portfolios under Vega-neutral condition has negative VaR, which means they actually get a benefit.

3.2 Greek Letter Risks

We put all the output of Greek letter risks, k and simulated P&L of the 4 portfolios into 4 csv in the folder named 'result' (from 04/02/2019 to 06/28/2019), and in this report we only show the average Greek letter risk in different portfolios as follow:

	Greek Risk	K
Theta Neutral Call	-0.37985	1.395716
Theta Neutral Put	-1.19724	1.251243
Vega Neutral Call	0.01984	2.067021
Vega Neutral Put	-0.02909	2.074305

We also need to multiply the Greek letter risk by 100 since one option has 100 shares:

	Greek Risk	K
Theta Neutral Call	-37.9851	1.395716
Theta Neutral Put	-119.724	1.251243
Vega Neutral Call	1.983958	2.067021
Vega Neutral Put	-2.9091	2.074305

Comparing to the risk in theta-neutral, risk under Vega-neutral condition is very small, which show the similar result as VaR. This can be part explained by Δ and Γ is approximately equal to 0 under Vega-neutral since those Greek letters are relative to Vega. But we can speculate from the result that Vega risk (volatility change) account for a large proportion in the total risk under Theta-neutral, by contrast, θ risk (time change) only account for a small proportion in the total risk under Vega-neutral. As for ρ, since our risk free rate change can be seemed as 0 (-8.786532e-05), so the risk caused by ρ can be ignored.

In conclusion, since volatility change really play an important role in our portfolios, it is the most important kind of risk that we should hedge, so Vega-neutral portfolio is a good choice in real world to hedge the risk as much as possible. But when quantifying the effect of each Greeks letter risk in our portfolio, it is better for us to use more risk factors. Therefore, theta-neutral would be more comprehensive and preferable than Vega-neutral in this case.

3.3 Conclusion

VaR modeling determines the potential for loss in the entity being assessed and the probability of occurrence for the defined loss. Greek letter risks are the sensitivity of the option relative to the underlying price, time, volatility and risk-free rate.

From the results above, we can learn that VaR and Greek letter risks are consistent with each other. So combining the performances of risks and VaR, the call option portfolio under Vega-neutral condition has the best performance.

There are some limitations of our project. One of them is that the we only use 62 trading days of historical data. Because the sample is not large enough, it may not include enough large losses to make VaR more accurate. Another thing we can improve is that we can use weighted historical simulation (WHS) to calculate VaR instead of HS.