Finance 567 Spring 2020
Financial Risk Management Neil D. Pearson

Solution to Homework 8 DCC Model and Principal Components
Due Sunday, April 12, 2020
Total 10 points

Instructions. This is a group assignment. Groups may include up to 5 people. Please submit a Word or .pdf document with your solutions via the Compass site prior to 11:59 p.m. on Sunday, April 12. Please also upload your R or Python scripts.

0. Gather the data. (0 points, but necessary for the other questions) Login in to WRDS, go to the data vendor CRSP, and find the daily stock returns data. Download the daily stock returns for the ticker symbols SPY (SPDR S&P 500 ETF Trust), EEM (iShares MSCI Emerging Markets ETF), HYG (iShares iBoxx $ High Yield Corporate Bond ETF), FXI (iShares China Large-Cap ETF), USO (United States Oil Fund, LP), and GLD (SPDR Gold Shares) for the period January 1, 2016 through December 31, 2019. (The first return should be for January 4, 2016, which was the first trading date in 2016, and the last return should be for December 31, 2019.)

1. Compute continuously compounded or log returns. (1 point) The returns you download from CRSP will be simple returns. Convert them to continuously compounded returns. What are the average continuously compounded returns for each of the six series?

Solution. The solution of this question (and all other questions) are compute in HW8.s2020.solution.R. The means of the 6 series are:

```
SPY          EEM          HYG          FXI          USO          GLD
0.0005316000 0.0004185331 0.0002943489 0.0003142455 0.0001514243 0.0003404430
```

2. Estimate a DCC model. (total 5 points) Use the six return series to estimate the DCC(1,1) model with normally distributed shocks. In doing this you should model the variance of each of the return series using a GARCH(1,1) model with normally distributed shocks, and assume that the means of the daily returns are zero.

(a) (3 points) What are the parameter estimates ω, α, and β of each of the six GARCH(1,1) models? (Your answer should consist of 18 numbers.)

Solution. The coefficient estimates of the six GARCH(1,1) models are:

```
omega        alpha1       beta1
4.259625e-06 2.006299e-01 7.374091e-01
9.250937e-06 1.629437e-01 7.702026e-01
3.197935e-07 1.800621e-01 8.027600e-01
5.869363e-06 8.384090e-02 8.802690e-01
1.156769e-05 6.457840e-02 9.060931e-01
3.920369e-07 2.173859e-02 9.702457e-01
```

(b) (2 points) What are the parameter estimates α and β of the DCC(1,1) model? (Your answer should consist of two numbers.)

Solution. The parameter estimates are

```
[Joint]dcca1 [Joint]dccb1
3.853077e-02 8.785786e-01
```

Remarks. I suggest that you use the R package rmgarch, which includes a function dccfit() that will estimate the DCC model. To use dccfit(), you must first use dccspec() to specify the model. To use dccspec(), you should first use multispec() to specify the six GARCH processes for the six return series. To use multispec(), you should first use ugarchspec() to specify the GARCH model for each of the return series. That is, you should do something like:

```
##specify a GARCH(1,1) model assuming the mean returns are zero
uspec <- ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)),
              mean.model = list(armaOrder = c(0,0), include.mean = FALSE),
              distribution.model = "norm")

## Combine the univariate specifications of the 6 GARCH models
marginspec <- multispec(replicate(6, uspec))

## Create DCC(1,1) specification
mspec <- dccspec(marginspec, dccOrder = c(1,1), model = "DCC", distribution = "mvnorm")

## Fit the DCC(1,1) model (returns are in a matrix returns[])
mod <- dccfit(mspec,returns[,1:6])
```

The package rmgarch also includes functions to simulate GARCH and DCC models, but I will not ask you to use them on this homework.

3. Principal components. (total 4 points) Now let's carry out a principal components analysis of the returns of the six ETFs. Carry out the analysis on the covariance matrix, not the correlation matrix. In computing the covariance matrix, assume that the expected returns of the stocks are zero and use the equally-weighted estimator

$$\hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T} r_t r_t',$$

where $r_t \in \Re^6$ is a vector of the returns of the six ETFs on date t.

(a) (1 point) First use the R function eigen() to compute the eigenvectors and eigenvalues. (You will first need to compute the covariance matrix of the returns.) Also use the R function prcomp() to carry out the principal components analysis of the returns. What is the standard deviation of the first principal component?

Solution. The largest eigenvalue is 0.000528736, and is the variance of the first principal component, so its square root is the standard deviation of the first principal component. Thus the standard deviation is $\sqrt{0.000528736} \approx 0.023$. This is also in the output of prcomp().

(b) (1 point) Suppose there is a one-standard deviation change in the first principal component, and the second through sixth principal components remain unchanged. In this scenario, what is the return on the FXI?

Solution. The FXI is the fourth return series, so we want the product of the 4,1 element of the matrix V, that is the 4th element of the first eigenvector, and a one standard deviation change. From part (a), a one standard deviation change in the first principal component is 0.02299426, The fourth element of the first eigenvector is -0.3953412 or 0.3953412, and the product is either -0.009090578 or 0.009090578.

(c) (1 point) What fraction of the total variance is explained by the first principal component?

Solution. The first principal component explains 61.6259% of the variance.

(d) (1 point) Use the first three principal components to estimate the covariance matrix of the returns on the six ETFs. Based on this estimate of the covariance matrix, what is the covariance of the returns between the SPY and FXI?

Solution. Use the first three principal components to compute an estimate of the covariance. Then you want the 1,4 element of the covariance matrix, which is 8.265223×10^{-5}.

Remark. To check that you are doing this correctly, note that if you use all six principal components you will recover the covariance matrix.