Finance 567 Spring 2020
Financial Risk Management Neil D. Pearson

Homework 3: Monte Carlo VaR
Due Wednesday, February 12, 2020 at 5:00 p.m.
Total 10 points

Instructions. This is a group assignment. Groups may include up to four people. Please prepare your solutions in the form of a Word or pdf file and submit your solutions via the Compass site prior to 5:00 p.m. on Wednesday, February 12, 2020. Please also submit the R script(s) that you use. Please be sure to include the names of all group members on the first page of your submission.

1. Monte Carlo VaR. (total 5 points) You have written 60 calls and 60 puts on ABC stock, and 40 calls and 40 puts on DEF stock. Each option is on 100 shares. The ABC call and puts have strike prices of $100, and the DEF calls and puts have strike prices of $150. All of the options expire in one month (0.08333 year). The prices of ABC and DEF are $101.17 and $148.97 per share, respectively. The continuously compounded interest rate is 0.01 or 1% per year, and neither ABC nor DEF will pay dividends during the next month. The implied volatility of ABC is 0.45 or 45% and the implied volatility of DEF is 0.37 or 37%.

(a) (1 point) Please use the binomial model to compute the current value of the portfolio.

(b) (3 points) The expected returns on ABC and DEF are 0.0005 (0.05%) and 0.0004 (0.04%) per day, respectively, and the standard deviations of their returns are 0.028 (2.8%) and 0.023 (2.3%) per day. The correlation between their returns is 0.40 (40%). Assume that the returns of the two stocks are normally distributed. Consider a simple VaR model in which the only market factors are the returns of the two stocks, and use the Monte Carlo method with a probability of 1% to compute the VaR of the portfolio. What is your estimate of the 1% VaR?

(c) (1 point) Plot the distribution of profits and losses on the portfolio.

2. Monte Carlo VaR of the portfolio in Question 1, but using a horizon of 21 trading days. (2 points) Consider the options portfolio from Question 1, and use the Monte Carlo method to compute the VaR with a horizon of 21 trading days (approximately one month). Use the assumptions and parameters from Question 1, but keep in mind that you will have to convert the parameters to the 21-day horizon.

3. VaR of the portfolio is Question 1, but using the bivariate *t* distribution. (total 3 points) Consider again the portfolio in Question 1, and once again assume that the VaR horizon is one trading day. The mean vector is

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0.0005 \\ 0.0004 \end{bmatrix},$$

as in Question 1. The matrix Σ is

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} (0.028)^2 & (0.028)(0.023)(0.4) \\ (0.028)(0.023)(0.4) & (0.023)^2 \end{bmatrix},$$

and the degrees of freedom is $\nu = 4$.

(a) (2 points) Use these parameters, the Monte Carlo method, and a one-day horizon to compute the 1% VaR of the portfolio.

(b) (1 point) Plot the distribution of profits and losses on the portfolio.

Remark. We will spend some time in class talking about how you can simulate random deviations from the multivariate *t* distribution using the R function `rmvt()`. Thus, it is ok if you do not yet know how to do this.

The *multivariate t distribution with v degrees of freedom* can be defined by the stochastic representation

$$\boldsymbol{X} = \boldsymbol{\mu} + \sqrt{W} A \boldsymbol{Z}, \tag{1}$$

where $W = \nu/\chi^2_\nu$ (χ^2_ν is informally used here to denote a random variable following a chi-squared distribution with $\nu > 0$ degrees of freedom) is independent of $\boldsymbol{Z}$ and all other quantities are as in (1).

By introducing the additional random factor $\sqrt{W}$, the multivariate t distribution with ν degrees of freedom (denoted by $t_\nu(\boldsymbol{\mu}, \Sigma)$) is more flexible than the multivariate normal distribution (which can be recovered by taking the limit $\nu \to \infty$) especially in the tails which are heavier for $t_\nu(\boldsymbol{\mu}, \Sigma)$ than for $\mathrm{N}_d(\boldsymbol{\mu}, \Sigma)$. The density of $\boldsymbol{X} \sim t_\nu(\boldsymbol{\mu}, \Sigma)$ is given by

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{\Gamma\left(\frac{\nu+d}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{d/2}\sqrt{\det\Sigma}} \left(1 + \frac{(\boldsymbol{x}-\boldsymbol{\mu})^\top \Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}{\nu}\right)^{-\frac{\nu+d}{2}}, \quad \boldsymbol{x} \in \mathbb{R}^d. \tag{2}$$

As for the multivariate normal distribution, the density (4) has ellipsoidal level sets and thus belongs to the class of elliptical distributions.

In the above, *d* is the dimension of the vector *x*.