

Homework 6 GARCH(1,1) and NGARCH(1,1)

Due Sunday, March 8, 2020, at 11:59 p.m.

Total 10 points

Instructions. This is a group assignment. Groups may include up to 4 people. Please submit a Word or .pdf document with your solutions via the Compass site prior to 11:59 p.m. on Sunday, March 9. Please also upload R scripts and data files that you used.

Assignment. You have the S&P 500 index (SPX) data in the.csv file F567.s2020.HW5.data.csv. You want to use the continuously compounded returns you compute from the SPX data to estimate several GARCH models and use them to forecast volatility.

1. GARCH(1,1) estimated 4 ways. (total 4 points)

(a) (1 point) The GARCH(1,1) likelihood can be written as a function of the four unknown parameters α , β , σ , and σ_1 . Use the continuously compounded returns you compute from the SPX data to estimate the GARCH(1,1) parameters, treating all four as unknown parameters to be estimated. What are the estimated parameters α , β , σ , and σ_1 ? (Note that the question asks for the estimates of σ , and σ_1 , not σ^2 and σ_1^2 .)

Remark: You should do this using R and one of the general optimization functions such as `optim()`. To do this you will have to write your own R function to compute the negative of the log likelihood.

(b) (1 point) Using the same data, set the long-run variance σ^2 equal to the sample variance of the data used for estimation. If you do this, the GARCH(1,1) likelihood can be written as a function of the three unknown parameters α , β , and σ_1 . Use maximum likelihood to estimate the three parameters α , β , and σ_1 . What is your estimate of the long-run standard deviation σ ? What are your estimates of the parameters α , β , and σ_1 ?

(c) (1/2 point) Next set the initial variance σ_1^2 equal to the sample variance over the data period used for estimation. If you do this, the GARCH(1,1) likelihood can be written as a function of the three unknown parameters α , β , and σ . Use maximum likelihood to estimate the three parameters α , β , and σ . What is your estimate of the initial standard deviation σ_1 ? What are your estimates of the parameters α , β , and σ ?

(d) (1/2 point) Now combine the approaches in (b) and (c), that is set both the long-run variance σ^2 and the initial variance σ_1^2 equal to the sample variance of the data used for estimation. If you do this, the GARCH(1,1) likelihood can be written as a function of the two unknown parameters α and β . Use maximum likelihood to estimate the two parameters α and β . What are your estimates of the long-run standard deviation σ and the initial standard deviation σ_1 ? What are your estimates of the parameters α and β ?

(e) (1 point) Use each of the four models estimated in parts (a)-(d) to forecast the return standard deviation for February 24, 2020. What are the four forecasts of the return standard deviation?

2. Using the GARCH(1,1) model to forecast volatility (total 2 points) This question asks you to use the four sets of estimates from Question (1) to compute four forecasts of the variance

$$\text{variance} = \sum_{t=1}^{21} R_t^2$$

over the first 21 days following February 21, 2020. That is, assume that the current date and time are just after the close of trading on February 21, 2020, you have data up through the close of trading on February 21, 2020, and you have estimated the GARCH(1,1) models in Question 1. You now want to forecast the realized variance over the 21 trading days starting from February 24, 2020.

(a) (1 point) What are your four forecasts of the realized variance?

(b) (1 point) Express your four forecasts in terms of annualized volatilities. (Take the square root, and then annualize by multiplying by $\sqrt{252/21}$. Or else annualize by multiplying by 252/21, and then take the square root.)

3. GARCH(1,1) using existing R functions (total 2 points) In this question use the same continuously compounded returns you used in Question 1.

(a) (1 point) Use the R function `garch()` from the R package `tseries` to estimate the parameters of a GARCH(1,1) model. (<https://www.rdocumentation.org/packages/tseries/versions/0.10-42/topics/garch>) Does the function return estimates of all four parameters α , β , σ , and σ_1 ? If not, which parameters does it estimate? What are the estimated parameters?

Hint: Use your estimates from Question 1(a) as your initial guesses.

(b) (1 point) Now use the R function `garchFit()` from the R package `fGarch` to estimate the parameters of a GARCH(1,1) model. (See <https://cran.r-project.org/web/packages/fGarch/fGarch.pdf>) Does the function return estimates of all four parameters α , β , σ , and σ_1 ? If not, which parameters does it estimate? What are the estimated parameters?

4. NGARCH(1,1) (2 point) The NGARCH(1,1) likelihood can be written as a function of the four unknown parameters α , β , θ , σ , and σ_1 . Use the continuously compounded returns you compute from the SPX data to estimate the NGARCH(1,1) parameters, treating all five as unknown parameters to be estimated. What are the estimated parameters α , β , θ , σ , and σ_1 ?

Remark: You should do this using R and one of the general optimization functions such as `optim()`. To do this you will have to write your own R function to compute the (negative of the) log likelihood.