Finance 567 Spring 2020
Financial Risk Management Neil D. Pearson

Homework 4: Expected shortfall and EVT
Due Sunday, February 23, 2020 at 11:59 p.m.
Total 10 points

Instructions. This is a group assignment. Groups may include up to four people. Please prepare your solutions in the form of a Word or pdf file and submit your solutions via the Compass site prior to 11:59 p.m. on Wednesday, February 23, 2020. Please also submit the R script(s) that you use. Please be sure to include the names of all group members on the first page of your submission.

1. Expected shortfall using the Normal distribution. (total 3 points) You have written 60 calls and 60 puts on ABC stock, and 40 calls and 40 puts on DEF stock. Each option is on 100 shares. The ABC call and puts have strike prices of $100, and the DEF calls and puts have strike prices of $150. All of the options expire in one month (21/252 year). The prices of ABC and DEF are $101.17 and $148.97 per share, respectively. The continuously compounded interest rate is 0.01 or 1% per year, and neither ABC nor DEF will pay dividends during the next month. The implied volatility of ABC is 0.45 or 45% and the implied volatility of DEF is 0.37 or 37%.

(a) (1 point) The expected log returns on ABC and DEF are 0.0005 (0.05%) and 0.0004 (0.04%) per day, respectively, and the standard deviations of the log returns are 0.028 (2.8%) and 0.023 (2.3%) per day. The correlation between their returns is 0.40 (40%). Assume that the log returns of the two stocks are normally distributed. Consider a simple model in which the only market factors are the returns of the two stocks, and use the Monte Carlo method with a probability of 5% to compute the VaR of the portfolio. What is your estimate of the 5% VaR?

Remark: Be sure to work with log or continuously compounded returns, not simple returns.

(b) (2 points) Use VaR(5%) denote the VaR you computed in part (a). Using the probability of 5%, what is the expected shortfall? That is, what is E[Loss | Loss ≥VaR(5%)]?

2. Expected shortfall using the bivariate *t* distribution. (total 3 points) Consider again the portfolio in Question 1, and once again assume that the VaR horizon is one trading day. The mean of the log returns is

$$\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix} = \begin{bmatrix}0.0005\\ 0.0004\end{bmatrix},$$

as in Question 1. The covariance matrix *C* is

$$C = \begin{bmatrix}\sigma_1^2 & \sigma_1\sigma_2\rho\\ \sigma_1\sigma_2\rho & \sigma_2^2\end{bmatrix} = \begin{bmatrix}(0.028)^2 & (0.028)(0.023)(0.4)\\ (0.028)(0.023)(0.4) & (0.023)^2\end{bmatrix},$$

and the degrees of freedom is $\nu = 4$.

(a) (1 point) Use these parameters, the Monte Carlo method, and a one-day horizon to compute the 5% VaR of the portfolio.

(b) (2 points) Use $\mathrm{VaR}_t(5\%)$ denote the VaR you computed in part (a). Using the probability of 5%, what is the expected shortfall? That is, what is $E[\text{Loss} \mid \text{Loss} \geq \mathrm{VaR}_t(5\%)]$?

3. EVT (total 4 points) You are interested in using Extreme Value Theory (EVT) to model the distribution of daily stock index returns above some threshold u.

(a) (2 points) You first estimated that the threshold $u = 0.02$ and you also discovered that 250 of the 8,000 observations you use exceed 0.02. You then use the method of maximum likelihood to estimate that $\xi = 0.36$, $\beta = 0.008$. Based on the estimates $u = 0.02$, $\xi = 0.36$, and $\beta = 0.008$, what is the probability that a daily return R is greater than 0.03?

(b) (2 points) Let R denote a return, and consider the mean excess function $E[R - u \mid R \geq u]$. Suppose that for a threshold of $u = 0.02$ the mean excess is $E[R - 0.02 \mid R \geq 0.02] = 0.012$, and for a threshold of $u = 0.03$ the mean excess is $E[R - 0.03 \mid R \geq 0.03] = 0.016$. If $u = 0.02$ is a reasonable choice of a threshold, then what should be the mean excess at a threshold of $u = 0.04$, that is what is $E[R - 0.04 \mid R \geq 0.04]$?

Remark: Part (b) does not use the parameter estimates from part (a).