

Homework 1

Due Wednesday, January 29, 2019 at 5:00 p.m.

Total 10 points

Instructions. This is a group assignment. Groups may include up to 4 people. Please submit a Word or .pdf document with your solutions via the Compass site prior to 5:00 p.m. on Wednesday, January 29. Please be sure to put the names of all group members on the top of the first page of your solution.

1. (1 point) For the case of a stock index that pays a continuous dividend yield at the rate δ , the Black-Scholes-Merton formula is provided in the lecture notes. The delta is

$$\Delta = \frac{\partial C}{\partial S} = e^{-\delta(T-t)} N(d_1),$$

where d_1 is defined in the lecture notes. What is the derivative of Δ with respect to d_1 ? (That is, what is $\partial \Delta / \partial d_1$?) What is the derivative of d_1 with respect to S ? (That is, what is $\partial d_1 / \partial S$?)

2. (1 point) You are interviewing with a trading firm in Chicago that trades equity options listed on various options exchanges.

(a) (1/2 point) You are given the following hypothetical scenario:

“You have a portfolio of options on DEF stock, which is trading at \$55. Your Δ is 8,000, and your Γ is $-4,000$. Suddenly the stock price drops to \$50. How much money did you just lose?”

How should you answer? (If you answer incorrectly, the interview is over, and you will have to find a job in human resources.)

(b) (1/2 point) Please sketch a graph of the value of the portfolio as a function of the price of the underlying stock.

3. Multiple choice questions (total 2 points)

(a) (1/2 point) The gamma of a position consisting of one written (sold) ordinary put option is:

- i. greater than zero
- ii. equal to zero
- iii. less than zero
- iv. could be either less than, equal to, or greater than zero, depending upon whether the option is in or out-of-the-money

(b) (1/2 point) The vega of a long position in a stock index futures contract is:

- i. greater than zero
- ii. equal to zero
- iii. less than zero
- iv. could be either less than, equal to, or greater than zero

(c) (1/2 point) The theta of a long position in a zero-coupon bond is:

- i. greater than zero
- ii. equal to zero
- iii. less than zero
- iv. could be either less than, equal to, or greater than zero

(d) (1/2 point) Assume that the interest rate is less than the dividend yield. In this case, the theta of a long position in a stock index futures contract is:

- i. greater than zero
- ii. equal to zero
- iii. less than zero
- iv. could be either less than, equal to, or greater than zero

Hint. For (b) and (d), it might help to know that the formula for the futures price of a stock index futures contract is $F = Se^{(r-\delta)(T-t)}$, where r is the continuously compounded interest rate and δ is the continuous dividend yield.

4. GS Annual Report. (total 2 points) Use your favorite internet search engine to find Goldman Sachs' 2018 Annual Report, and go to Management's Discussion and Analysis (MD&A).

(a) (1/2 point) How does Goldman Sachs define the fair value of a financial instrument?

(b) (1/2 point) What, according to Goldman Sachs, is the fair value hierarchy under U.S. generally accepted accounting principles (U.S. GAAP)?

(c) (1/2 point) What is Goldman's definition of liquidity risk?

(d) (1/2 point) Find the place where GS reports its credit ratings. What are the credit ratings of Goldman Sachs Group long-term debt from DBRS, Fitch, Moody's, R&I, and S&P? (As of Dec. 2018)

Remark. Goldman's credit ratings are not directly related to the things we are talking about now, but we will keep them in mind for later in the course. Why are the subordinated debt ratings worse than the long-term debt ratings?

5. GS Annual Report. (total 4 points) Keep reading Goldman Sachs' 2018 MD&A.

(a) (1/2 point) Find the section “Market Risk Management,” and then the subsection “Risk Measures.” What are Goldman’s “primary risk measures”? Which risk measure or measures are used for shorter-term periods?

(b) (1/2 point) Find the section “Market Risk Management,” and then the subsection “Value-at-Risk.” What time horizon and probability (confidence level) does Goldman use in computing value-at-risk? What does Goldman think are the inherent limitations to VaR?

(c) (1/2 point) What VaR methodology does GS use?

(d) (1/2 point) How much data does GS use to construct the scenarios for its VaR calculations? Are the historical data weighted, or unweighted?

(e) (1/2 point) Within the MD&A, find the section “Market Risk Management,” and then the subsection “Metrics.” What was Goldman’s (total) VaR at the end of 2018? What was Goldman’s average daily VaR during 2018?

(f) (1/2 point) On how many days during 2018 did trading losses exceed Goldman’s 95% one-day VaR? On how many days during 2017 did trading losses exceed Goldman’s 95% one-day VaR? If Goldman’s VaR model is accurate, what is the expected number of times that the daily trading loss exceeds the VaR during a year? (Assume 252 business days per year.)

(g) (1/2 point) Consider 252 Bernoulli trials (i.e., 252 trials of a binomial random variable) where on each trial the probability of a “success” is 0.95 and the probability of a “failure” is 0.05. What is the probability that the 252 trials result in zero failures? What is the probability that the 252 trials result in two or fewer failures? (You might want to look at your statistics book to refresh your memory about the distribution of Bernoulli trials and the binomial distribution.)

Hint. You might find the R functions `dbinom()` and/or `pbinom()` to be useful. Alternatively, you might find the Excel function `binom.dist()` to be helpful.

(h) (1/2 point) What does this analysis of Bernoulli trials tell you about whether Goldman’s VaR model is or is not biased?