Simulation Exercises

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Instructions

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also also 1/lambda. For these simulations, lambda = 0.2. I will investigate the distribution of averages of 40 exponential(0.2)s and will need a thousand simulated averages of those 40 exponentials.

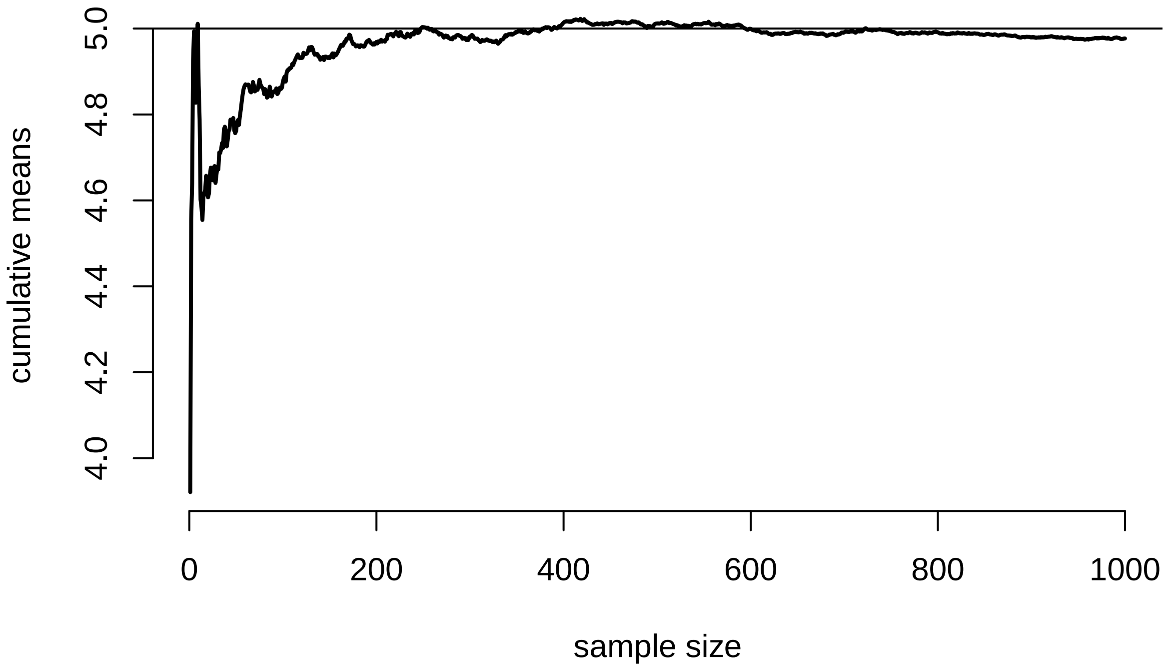
The properties of the distribution of the mean of 40 exponential(0.2)s will be illustrated via simulation and associated explanatory text. I will:

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution.
2. Show how variable it is and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.
4. Evaluate the coverage of the confidence interval for 1/lambda: X ¯ ± 1.96 S n

Simulation 1: Center of the distribution

The code below shows where the distribution is centered and compares it to the theoretical center of the distribution 1 lambda . The plot below illustrates that the distribution centers closer to the theoretical center, 1 lambda (==5), as the sample size increases.

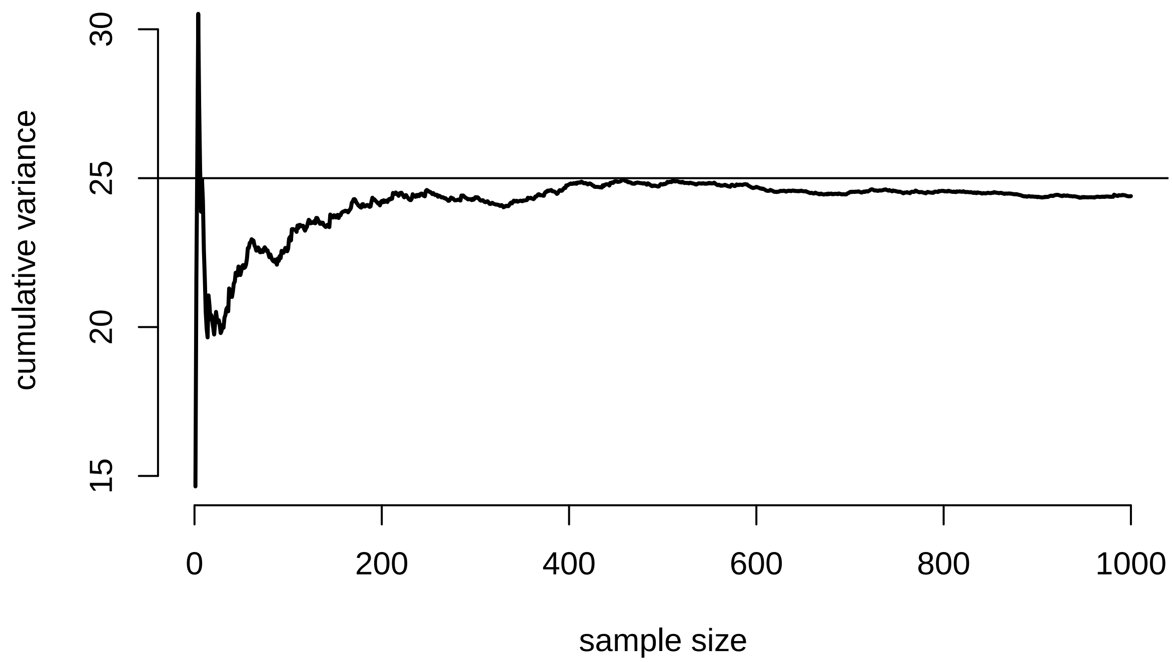
```
nosim<-1000
observations<-40
lambda<-0.2
observationDF<-replicate(nosim, rexp(observations,lambda))

means <- cumsum(colMeans(observationDF))/(1:nosim)
plot(1:nosim, means, type = "l", lwd = 2, frame = FALSE, ylab = "cumulative means",  xlab = "sample siz
abline(h = 1/lambda)
```



Simulation 2: Variance of the distribution

The code and plot below shows where the variance is centered and compares it to the theoretical variance. The theoretical standard deviation of exponential distribution is 1 lambda . Thus, the theoretical variance is 1 lambda 2 (==25). The plot below illustrates that the distribution centers closer to the theoretical center as the sample size increases.

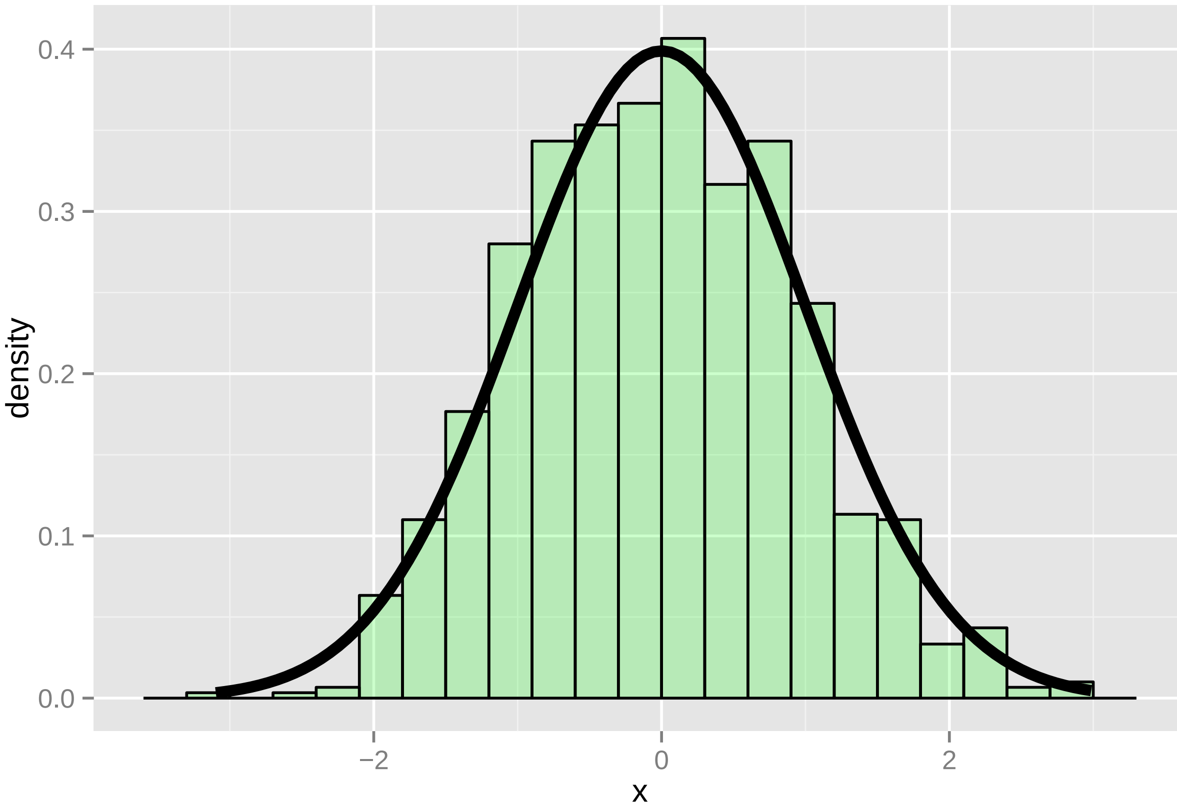
```
vars <- cumsum(apply(observationDF, 2, var))/(1:nosim)
plot(1:nosim, vars, type = "l", lwd = 2, frame = FALSE, ylab = "cumulative variance",  xlab = "sample si
abline(h = (1/lambda)^2)
```



Simulation 3: Shape of the distribution

The code and histogram below shows how the distribution is approximately normal. The the distribution is normalized by centering around zero.

```
cfunc <- function(x, n) (mean(x) - 1/lambda) / ((1/lambda)/(sqrt(n)))
dat <- data.frame(
      x = c(apply(observationDF, 2, cfunc, observations)),
      size = factor(c(observations), rep(nosim))
 )
library(ggplot2)
g <- ggplot(dat, aes(x = x)) + geom_histogram(alpha = .20, binwidth=.3, colour = "black", fill="green",
g <- g + stat_function(fun = dnorm, size = 2)
print(g)
```



Simulation 4: Evaluate the coverage of the confidence interval for 1/lambda

The 95% confidence interval is examined as X $^{-}$ ± 1.96 S n and is calculated below:

```
error<-mean(sqrt(vars))/sqrt(observations)
mean(means)+c(-1,1)*1.96*error
```

```
## [1] 3.442 6.490
```