Principal Component Analysis

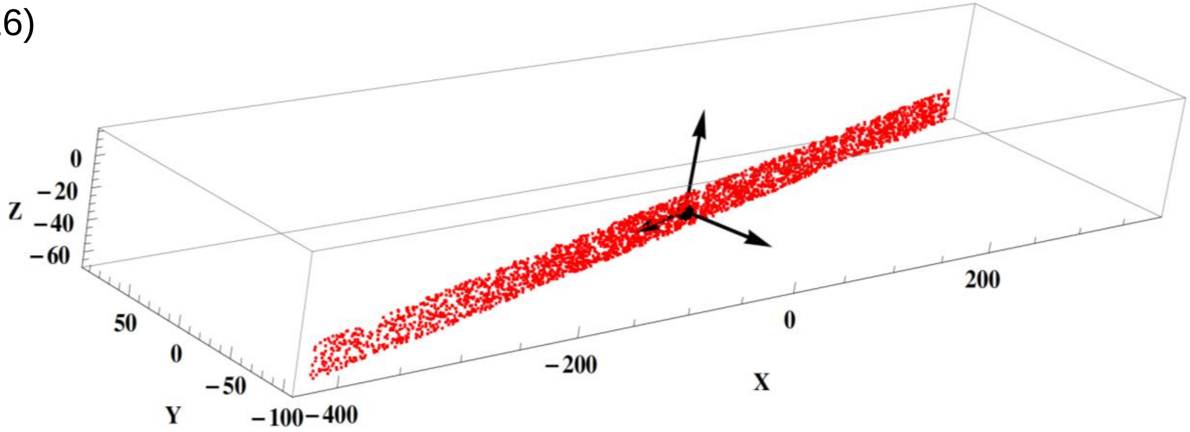
PCA: classic method for multivariate data analysis and remote sensing:

Python notebook for analysing multi-spectral satellite data using PCA:

https://docs.digitalearthafrica.org/en/latest/sandbox/notebooks/Frequently_used_code/Principal_component_analysis.html

PCA for Point Clouds

T. Hackel, J. Wegner, K. Schindler, Contour detection in Unstructured 3D Point Clouds, IEEE CVPR, (2016)



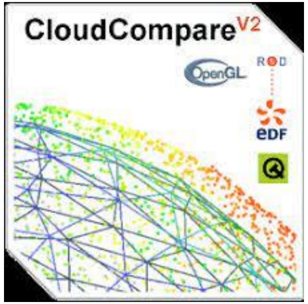
Key idea for 3D shape analysis:
determine consecutive, othogonal directions of maximal variability

=> 3 directions for 3D

Question: how many directions for nD?
Question: how many dimensions will we have?

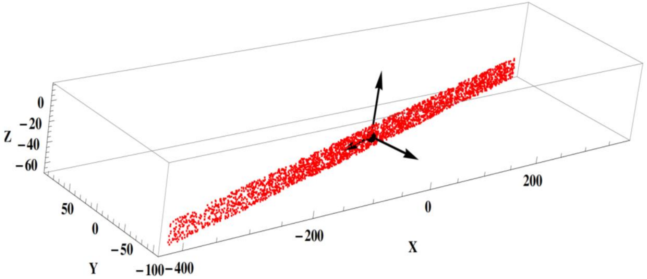
How to use PCA for point clouds?

1. Input: (*k x 3*) data matrix of k 3D points
   P
2. Determine the matrix of (k x the same) 3D mean/center of the k points:
   M_P
3. Get the centralized version of P:
   $P_c = P - M_p$ (still size k x 3)
4. Determine the data covariance matrix
   $C_P = (1/k)\, P^T_C \cdot P_C$ (size 3 x 3, T stands for transpose)
5. Output: three eigenvectors $\underline{\mathbf{e}}_1$, $\underline{\mathbf{e}}_2$, $\underline{\mathbf{e}}_3$ and corresponding ordered & positive eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of C_P



- Eigenvectors point in (remaining) max variability directions
- Eigenvalues provide size of these variability

CloudCompare: eigenvalues based shape features
(all kind of combinations of $\lambda_1, \lambda_2, \lambda_3$)



PCA loadings for point clouds

Our 3 3D eigenvectors, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, provide an alternative Cartesian coordinate system of our 3D space.

For each individual point $p \in P$, one can determine its PCA coordinates

Definition: Loadings of $p \in P$: PCA coordinates/coefficients of p

Obtaining the loadings L of P:

$$L = P_C \, . \, E$$

With E the matrix with the Eigenvectors as columns

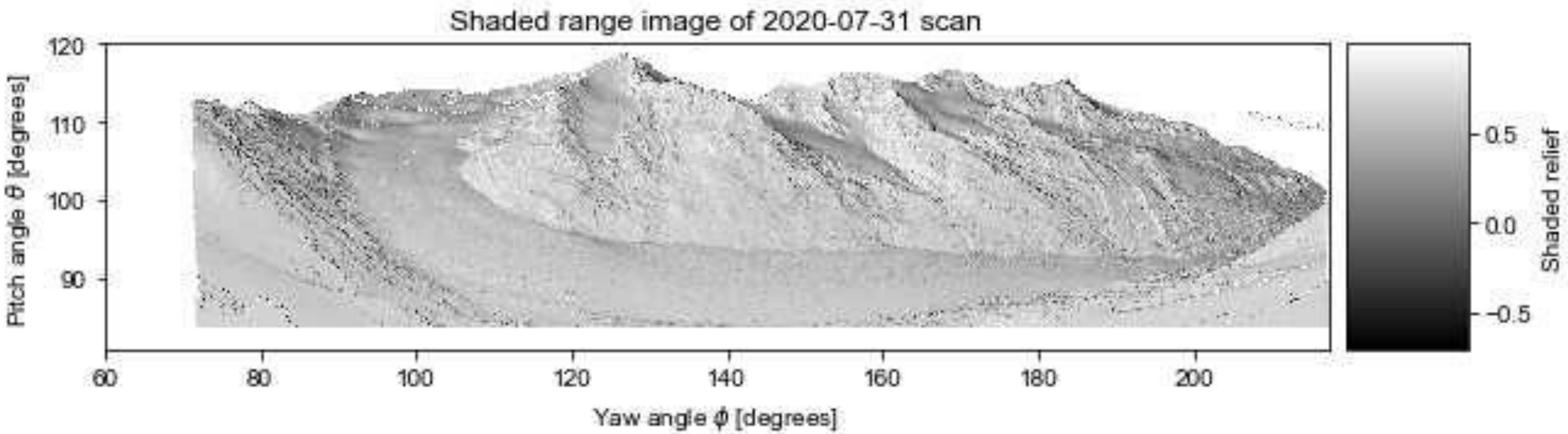
(L consists basically of the coefficients of the points in the new PCA basis)

PCA for 4D time series:

Paco Frantzen,
Identifying high Alpine geomorphological processes using permanent laser scanning time series,
MSc thesis, TU Delft (research done at Univ. Innsbruck)
http://resolver.tudelft.nl/uuid:ce98c4e3-6ca1-4966-a5cf-2120f2fa44bf

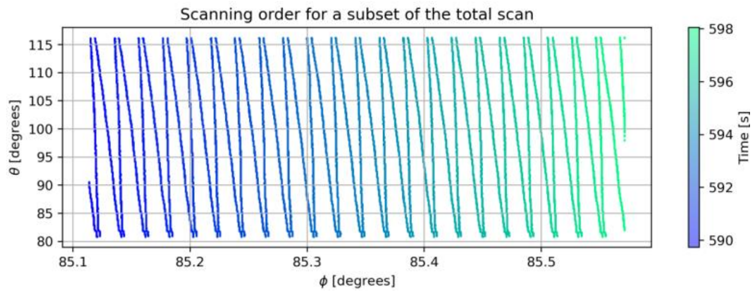
Journal version: under review

Hintereisferner, Austria

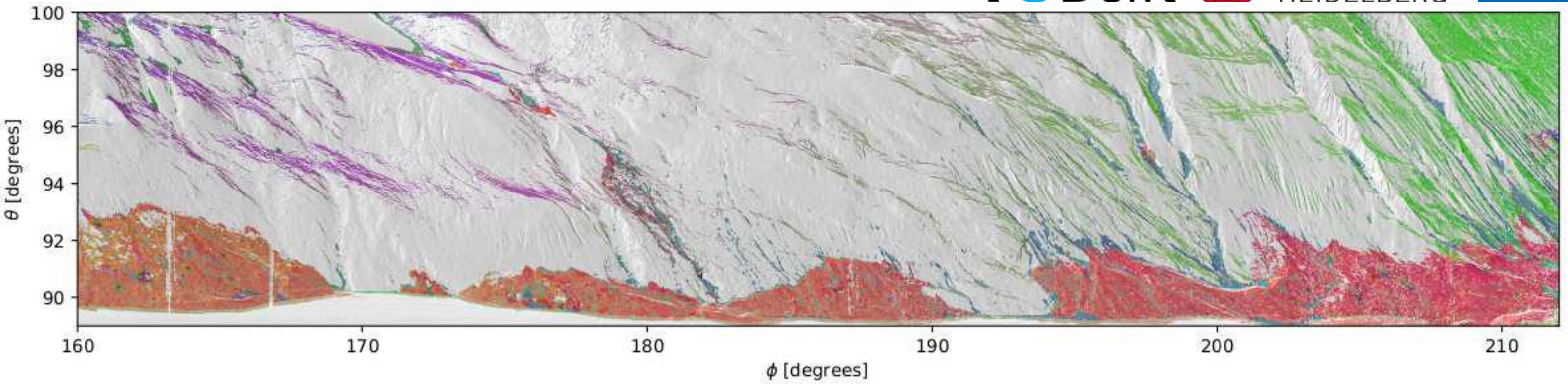


To analyze morphological change above the Hintereisferner, Paco Frantzen used

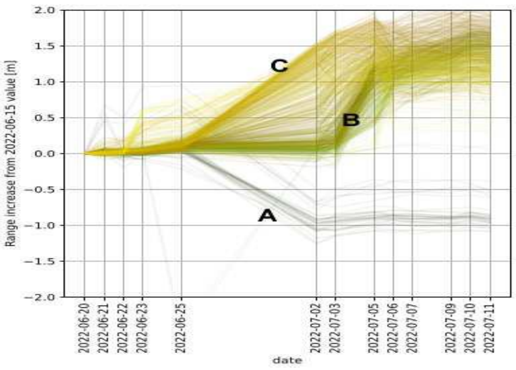
- Range images, the native scanner format
- Space time array, list of time series of deviations per range cell
- PCA: highlight principal deviation patterns

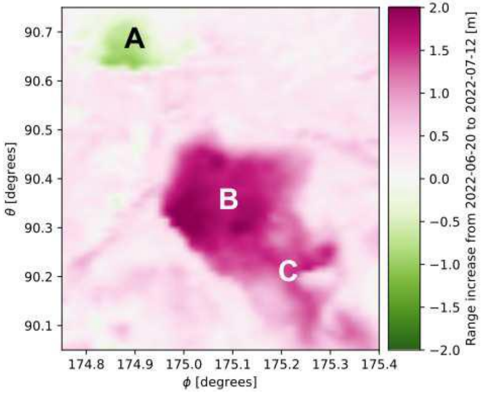


Geomorph-analysis: PCA@time series



False colour image of relative loadings per raster cell for the first three (combined) PCs of 2022 data. Red, green, and blue correspond to the first, second, and 3 and 4 combined PC, respectively.





PCA for 4D point clouds

Assume: consecutive point clouds represent change in elevation

Idea: look for patterns in the elevation deviations.

1. Input: list of k time series of elevations at m moments:
   $((x(1),y(1)), (z_1(1), z_2(1) \ldots, z_m(1)))$
   …
   $((x(k),y(k)), (z_1(k), z_2(k) \ldots, z_m(k)))$
2. Select a reference elevation, e.g. the elevation at m=1 and covert to deviations from this elevation:
   $((x(1),y(1)), (0, \Delta z_2(1) \ldots, \Delta z_m(1)))$
   …
   $((x(k),y(k)), (0, \Delta z_2(k) \ldots, \Delta z_m(k)))$
   With $\Delta z_i = z_i - z_1$

Now apply PCA to find variability in this (m-1) dimensional 'deviation space'

(forget the locations (x,y))

Thank you


Identifying high Alpine Geomorphological processes using Permanent Terrestrial Laser Scanning time series
TUDelft