Topics in Macro 2

Week 10 - Second Part - Part III - Exercise III

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TSE

Tuesday (17:00-18:30)

TD Second Part: Fiscal Multipliers (Weeks 6 to 10)

Exercise III: Externality in Labor Supply and the Multiplier

The Economy

Utility:

$$log(c_t) - \frac{\eta}{1+\nu}\left(\frac{n_t}{\bar{n}_t^{\theta}}\right)^{1+\nu}$$

Budget constraint:

$$c_t \leq w_t n_t - T_t + \Pi_t$$

Technology:

$$y_t = an_t$$

Profits:

$$\Pi = y_t - w_t n_t$$

Government:

$$T_t = g_t$$

Market clearing:

$$y_t = c_t + g_t$$

Question 1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)

$$\mathcal{L}(c,n,\lambda) = \log(c_t) - \frac{\eta}{1+\nu}\left(\frac{n_t}{\bar{n}_t^{\theta}}\right)^{1+\nu} - \lambda_t\left[c_t - w_t n_t - \Pi_t + T_t\right]$$

F.O.C. c_t: $\frac{1}{c_t} = \lambda_t$

n_t: $\eta\left(\frac{n_t}{\bar{n}_t^{\theta}}\right)^{\nu}\left(\frac{1}{\bar{n}_t^{\theta}}\right) = \lambda_t w_t$

$$\Longrightarrow \eta n_t^{\nu}\, \bar{n}_t^{-\theta\nu}\, \bar{n}_t^{-\theta}\, c_t = w_t$$

$$\Longrightarrow \eta n_t^{\nu}\, \bar{n}_t^{-\theta(1+\nu)} = w_t$$

Answer: $\eta c_t n_t^{\nu} \bar{n}_t^{-\theta(1+\nu)} = w_t$.

Question 2. Determine the optimality condition of the firm.

$$\max_{\{n_t\}} \; a n_t - w_t n_t$$

F.O.C. $\quad a = w_t$ (Interior Solution)

Answer: $a = w_t$.

Question 3. Determine the equilibrium output.

Definition of equilibrium:

A competitive equilibrium are quantities $c_t, n^s_t, n^d_t, g_t, T_t, y_t$ and prices w_t, such that:

1.- c_t, n^s_t solve (1)

2.- n^d_t solve (2)

3.- $g_t = T_t$

4.- $\bar{n}_t = n_t$

5.- $y_t = c_t + g_t$

Question 3.

from (1) & (2): $\eta c_t n_t^{\nu} \bar{n}_t^{-\theta(1+\nu)} = a$

from (3), (4) & (5) $\eta [y_t - g_t] n_t^{\nu-\theta(1+\nu)} = a$

from technology: $y_t = a n_t \Rightarrow n_t = y_t / a$

Then: $\eta [y_t - g_t] \left(\frac{y_t}{a}\right)^{\nu-\theta(1+\nu)} = a$

Answer: $\eta[y_t - g_t]\left(\frac{y_t}{a}\right)^{\nu-\theta(1+\nu)} = a$.

Question 4. Compute the log-linearization of equilibrium output around the deterministic steady-state (we assume that steady-state exists and is unique).

Step 1: Take logs

$$\eta\left[y_t - g_t\right]\left(\frac{y_t}{a}\right)^{\nu-\theta(1+\nu)} = a$$

$$\log(\eta) + \log(y_t - g_t) + \left[\nu - \theta(1+\nu)\right]\log(y_t) - \left[\nu - \theta(1+\nu)\right]\log(a) = \log(a)$$

Step 2. Taylor approximation

$$\log(\eta) + \log(y_t - g_t) + [\nu - \theta(1+\nu)] \log(y_t) - [\nu - \theta(1+\nu)] \log(a) = \log(a)$$

$$\cancel{\log(\eta)} + \left[\cancel{\log(\bar{y} - \bar{g})} + \frac{1}{\bar{y}-\bar{g}}[y_t - \bar{y}] - \frac{1}{\bar{y}-\bar{g}}[g_t - \bar{g}]\right] + [\nu - \theta(1+\nu)]\left[\cancel{\log(\bar{y})} + \frac{1}{\bar{y}}[y_t - \bar{y}]\right]$$
$$\cancel{- [\nu - \theta(1+\nu)] \log(a)} = \cancel{\log(a)}$$

By Steady State:

$$\frac{1}{\bar{y}-\bar{g}}[y_t - \bar{y}] - \frac{1}{\bar{y}-\bar{g}}[g_t - \bar{g}] + [\nu - \theta(1+\nu)] \frac{1}{\bar{y}-\bar{g}}[y_t - \bar{y}] = 0$$

Step 3. Deviations from the steady state

$$\frac{\bar{y}}{\bar{y}}\frac{1}{\bar{y}-\bar{g}}\left[y_t-\bar{y}\right]-\frac{\bar{g}}{\bar{g}}\frac{1}{\bar{y}-\bar{g}}\left[g_t-\bar{g}\right]+\left[\nu-\theta(1+\nu)\right]\frac{1}{\bar{y}}\left[y_t-\bar{y}\right]=0$$

Define $s_g = \bar{g}/\bar{y}$

$$\frac{1}{1-s_g}\hat{y}_t-\frac{s_g}{1-s_g}\hat{g}_t+\left[\nu-\theta(1+\nu)\right]\hat{y}_t=0$$

$$\left[\frac{1}{1-s_g}+\left[\nu-\theta(1+\nu)\right]\right]\hat{y}_t=\frac{s_g}{1-s_g}\hat{g}_t$$

$$\Rightarrow \left(1+(1-s_g)\left[\nu-\theta(1+\nu)\right]\right)\hat{y}_t=s_g\hat{g}_t$$

Answer: $\frac{\hat{y}_t}{\hat{g}_t}=\frac{s_g}{1+(1-s_g)(\nu-\theta(1+\nu))}$.

Question 5. Compute the output multiplier and discuss the value of this multiplier with respect to θ.

$$\frac{dy_t}{dg_t} = \frac{\hat{y}_t}{\hat{g}_t} \cdot \frac{\bar{y}}{\bar{g}} = \frac{\hat{y}_t}{\hat{g}_t} \cdot \frac{1}{s_g} = \frac{1}{1+(1-s_g)(\nu-\theta(1+\nu))}$$

If $\theta \nearrow \Longrightarrow \frac{dy_t}{dg_t} \nearrow$

If g_t increases, T_t increases. Individuals compensate the loss of income by working more. This increases $\bar{n}_t$ and income y_t and utility are higher.

Answer: $\frac{dy_t}{dg_t} = \frac{\hat{y}_t}{\hat{g}_t} \frac{\bar{y}}{\bar{g}} = \frac{1}{1+(1-s_g)(\nu-\theta(1+\nu))}$.

Question 6. Compute the consumption multiplier and discuss the value of this multiplier with respect to θ.

Same as (5).

Answer: $\frac{dc_t}{dg_t} = \frac{dy_t}{dg_t} - 1$.