Topics in Macro 2

Week 10 - Second Part - Part III - Exercise II

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TSE

Tuesday (17:00-18:30)

TD Second Part: Fiscal Multipliers (Weeks 6 to 10)

Part I

- Exercise I: Habit Persistence and The Keynesian Multiplier (Week 6)
- Exercise II: A Benchmark Model (Week 7)
- Exercise III: Consumption, Labor Supply and the Multiplier (Week 7)

Part II

- Exercise I: Taxes on the Labor Input and the Multiplier (Week 8)
- Exercise II: Public Spending in Utility Function and the Multiplier (Week 8)
- Exercise III: Labor Supply, Public Spending in Utility and the Multiplier (Week 9)

Part III

- Exercise I: Endogenous Public Spending (Week 9)
- **Exercise II: Externality in Production and the Multiplier (Week 10)**
- Exercise III: Externality in Labor Supply and the Multiplier (Week 10)

Exercice II: Externality in Production and the Multiplier

The Economy

Utility:

$$log(c_t) - \frac{\eta}{1+\nu} n_t^{1+\nu}$$

Budget constraint:

$$c_t \leq w_t n_t - T_t + \Pi_t$$

Technology:

$$y_t = a n_t s_t$$

Externality:

$$s_t = \bar{n}_t^{\varphi}$$

Profits:

$$\Pi = y_t - w_t n_t$$

Government: $T_t = g_t$.
Market clearing: $y_t = c_t + g_t$.

Question 1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)

$$\mathcal{L}(c,n,\lambda) = \log(c_t) - \frac{\eta}{1+\nu} n_t^{1+\nu} - \lambda_t \left[c_t - w_t n_t - \Pi_t + T_t \right]$$

F.O.C. c_t: $\frac{1}{c_t} = \lambda_t$

n_t: $\eta n_t^{\nu} = \lambda_t w_t$

$$\Longrightarrow \underbrace{\frac{\eta n_t^{\nu}}{1/c_t}}_{MRS_{n,c}} = w_t$$

Answer: $\frac{\eta n_t^{\nu}}{1/c_t} = w_t$.

Question 2. Determine the optimality condition of the firm.

$$\max_{\{n_t\}} y_t - w_t n_t$$

$$\Rightarrow \max_{\{n_t\}} a n_t s_t - w_t n_t, \text{ where } s_t = \bar{n}_t^{\varphi}$$

F.O.C. $a s_t = w_t$

(If interior Solution)

Answer: $as_t = w_t$.

Question 3. Determine the equilibrium output.

Definition of equilibrium:

A competitive equilibrium are quantities $c_t, n_t^s, n_t^d, g_t, T_t, y_t$ and prices w_t such that:

1: c_t, n_t^s solve (1)

2- n_t^d solve (2)

3: $g_t = T_t$

4: $S_t = \bar{n}_t^{\varphi}$ where $\bar{n}_t = n_t$ because individuals are identical.

5- $y_t = c_t + g_t$

Equilibrium output.

From (1) & (2)

$$\eta n_t^{\nu} c_t = a s_t$$

$$\Rightarrow \eta n_t^{\nu}[y_t - g_t] = a n_t^{\varphi}$$

$$\Rightarrow \eta[y_t - g_t] = a n_t^{\varphi-\nu}$$

where $y_t = a n_t s_t = a n_t^{1+\varphi}$

$$\Rightarrow n_t = (y_t/a)^{\frac{1}{1+\varphi}}$$

Then: $$\eta[y_t - g_t] = a\left(\frac{y_t}{a}\right)^{\frac{\varphi-\nu}{1+\varphi}}$$

Answer: $\eta[y_t - g_t] = a\left(\frac{y_t}{a}\right)^{\frac{\varphi-\nu}{1+\varphi}}$.

Question 4. Compute the log-linearization of equilibrium output around the deterministic steady-state (we assume that steady-state exists and is unique). We know:

$$\eta[y_t - g_t] = a\left(\frac{y_t}{a}\right)^{\frac{\varphi-\nu}{1+\varphi}}$$

Step 1: Take logs

$$\log(\eta) + \log(y_t - g_t) = \log(a) + \frac{\varphi-\nu}{1+\varphi}\log(y_t) - \frac{\varphi-\nu}{1+\varphi}\log(a)$$

Step 2: Taylor approximation

$$\log(\eta) + \log(y_t - g_t) = \log(a) + \frac{\varphi - \nu}{1+\varphi}\log(y_t) - \frac{\varphi-\nu}{1+\varphi}\log(a)$$

$$\cancel{\log(\eta)} + \left[\cancel{\log(\bar{y} - \bar{g})} + \frac{1}{\bar{y}-\bar{g}}[y_t - \bar{y}] - \frac{1}{\bar{y}-\bar{g}}[g_t - \bar{g}]\right]$$
$$= \cancel{\log(a)} + \frac{\varphi-\nu}{1+\varphi}\left[\cancel{\log(\bar{y})} + \frac{1}{\bar{y}}[y_t - \bar{y}]\right]$$
$$- \cancel{\frac{\varphi+\nu}{1+\varphi}\log(a)}$$

By steady state: $$\frac{1}{\bar{y}-\bar{g}}[y_t - \bar{y}] - \frac{1}{\bar{y}-\bar{g}}[g_t - \bar{g}] = \left(\frac{\varphi-\nu}{1+\varphi}\right)\frac{1}{\bar{y}}[y_t - \bar{y}]$$

Step 3: Deviations from the steady state.

$$\frac{\bar{y}}{\bar{y}} \frac{1}{\bar{y}-\bar{g}}\left[y_t-\bar{y}\right] - \frac{\bar{g}}{\bar{g}} \frac{1}{\bar{y}-\bar{g}}\left[g_t-\bar{g}\right] = \left(\frac{\varphi-\nu}{1+\varphi}\right)\frac{1}{\bar{y}}\left[y_t-\bar{y}\right]$$

Define $s_g = \bar{g}/\bar{y}$

$$\Rightarrow \frac{1}{1-s_g}\hat{y}_t - \frac{s_g}{1-s_g}\hat{g}_t = \left(\frac{\varphi-\nu}{1+\varphi}\right)\hat{y}_t$$

$$\Rightarrow \left(\frac{1}{1-s_g} - \frac{\varphi-\nu}{1+\varphi}\right)\hat{y}_t = \frac{s_g}{1-s_g}\hat{g}_t$$

$$\Rightarrow \left(\frac{1+\varphi-(1-s_g)(\varphi-\nu)}{\cancel{(1-s_g)}(1+\varphi)}\right)\hat{y}_t = \frac{s_g}{\cancel{1-s_g}}\hat{g}_t$$

Answer: $\boxed{\frac{\hat{y}_t}{\hat{g}_t} = \frac{s_g(1+\varphi)}{1+\varphi-(1-s_g)(\varphi-\nu)}}$.

Question 5. Compute the output multiplier and discuss the value of this multiplier with respect to φ.

We know $\dfrac{\hat{y}_t}{\hat{g}_t} = \dfrac{s_g(1+\varphi)}{1+\varphi-(1-s_g)(\varphi-\nu)}$

Remember $\dfrac{dy_t}{dg_t} = \dfrac{\hat{y}_t}{\hat{g}_t} \cdot \dfrac{\bar{y}}{\bar{g}} = \dfrac{\hat{y}_t}{\hat{g}_t} \cdot \dfrac{1}{s_g} = \dfrac{1+\varphi}{1+\varphi-(1-s_g)(\varphi-\nu)}$

As φ increases, $\dfrac{dy_t}{dg_t}$ increase. If g_t increases, n_t increases this means higher s_t and y_t. φ makes the fiscal policy more effective

Answer: $\dfrac{dy_t}{dg_t} = \dfrac{\hat{y}_t}{\hat{g}_t}\dfrac{\bar{y}}{\bar{g}} = \dfrac{1+\varphi}{1+\varphi-(1-s_g)(\varphi-\nu)}$.

Question 6. Compute the consumption multiplier and discuss the value of this multiplier with respect to φ.

$$\frac{dc_t}{dg_t} = \frac{dy_t}{dg_t} - 1$$

Same interpretation as (5)

$$g_t \nearrow \Rightarrow n_t \nearrow \Rightarrow y_t \nearrow \Rightarrow c_t \nearrow$$

Answer: $\frac{d\, c_t}{d\, g_t} = \frac{y_t}{g_t} - 1.$