Topics in Macro 2

Week 5 - TD 8 and TD 9

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TSE

Tuesday (17:00-18:30)

TD First Part: Inequality (6 Weeks)

What's going on?

- Exercise 1: The Height of Inequality (Week 1)
- Exercise 2: Skilled biased technological change and wage dispersion (Week 2)
- Exercise 3: Top wages and incomes (Week 3)

How do we measure it?

- Exercise 4: Interdecile ratios of income (Week 3)
- Exercise 5: Lorenz curve and Gini basics (Week 3)

What do we (have to) do?

- Exercise 6: Taxes, consumption and inequality (Week 4)
- Exercise 7: Variance based inequality indicators (Week 4)
- **Exercise 8: Social Welfare Function and inequality aversion** (Week 5)
- **Exercise 9: The Atkinson inequality index** (Week 5)

Exercise 8: Social Welfare Function and inequality aversion

We consider a Social Welfare Function (SWF) that has the 5 Assumptions seen in class. The SWF is generally defined as:

$$W = W(y_1, y_2, y_3, ..., y_n)$$

We can think of y_i, $i \in [1, n]$ as the level of income of the n individuals in the economy. The associated social utility function (or welfare index) U has the following functional form:

$$U(y_i) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$

with ε the inequality aversion parameter.

Question 1. Define what a SWF is.

A Social Welfare Function **ranks** all the possible **states** of society in the order of (this society's) preference.

What are the **states**?

Who **ranks** them?

Question 2. List the 5 Assumptions of a SWF used in class.

1. Individualistic and nondecreasing

$$W = W(y_{1A}, y_{2A}, y_{3A}, ..., y_{4A})$$
$$W = W(y_{1B}, y_{2B}, y_{3B}, ..., y_{4B})$$

If $y_{iB} \geq y_{iA}\ \forall i$, then $W_B \geq W_A$.

2. Symmetry

$$W = W(y_1, y_2, y_3, ..., y_n) = W(y_2, y_1, y_3, ..., y_n) = W(y_n, y_2, y_3, ..., y_1)$$

3. Additivity

$$W(y_1, y_2, y_3, ..., y_n) = \sum_{i=1}^{n} U_i(y_i) = U_1(y_1) + U_2(y_2) + ... + U_n(y_n)$$

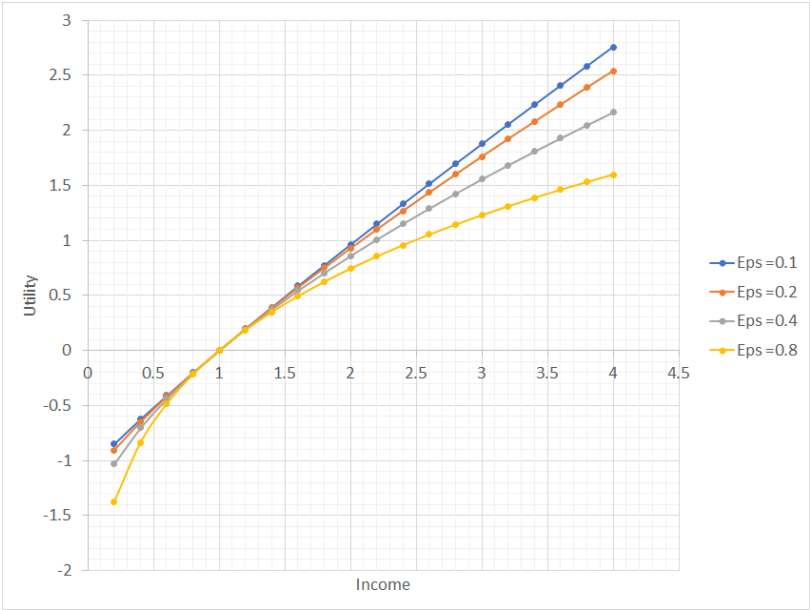
$U(y_i)$ is the **social utility** or **welfare index** of individual i.

4. Concavity

$$U'(y_i) > 0 \text{ and } U''(y_i) < 0$$

5. Constant elasticity

$$U(y_i) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$


3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Utility
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Income
Eps =0.1
Eps =0.2
Eps =0.4
Eps =0.8

Question 3. Explain why $U'(y_i)$ can be interpreted as a social weight.

Hint: Total derivative. $\frac{dW}{dt} = \sum_{i=1}^{n} \frac{\partial W}{\partial y_i} \frac{dy_i}{dt}$

We consider the following situation: there are n individuals in the economy. We know that individual k has **5 times** the income of individual j. We consider a **transfer of 1 Euro** from individual k to j. We assume that during the transfer from k to j, there are no other income variations in the economy.

Question 4. Explain why under the conditions given for our SWF, individual k would be willing to give up on her money in favor of j.

Hint: Inequality aversion. Different welfare weights.

Question 5. How much is individual k willing to give up so that individual j gets exactly 1 Euro when $\varepsilon = 0$?

Hint: 1 Euro. Same social marginal utility. No cost of tax collection.

Question 6. How much is individual k willing to give up so that individual j gets exactly 1 Euro when $\varepsilon = 1/2$. What about $\varepsilon = 1$ or $\varepsilon = 5$?

Case where $\varepsilon = 1/2$

Case where $\varepsilon = 1$

Case where $\varepsilon = 5$

Question 7. What about $\varepsilon \to \infty$?

Hint: $dy_k = -\left(\frac{y_k}{y_j}\right)^{\varepsilon}$.

Exercise 9: The Atkinson inequality index

Recall

- The Dalton inequality index
  - It measures how far the current average social utility is from potential average social utility.
  - $D_\varepsilon = 1 - \frac{\bar{U}}{U(\bar{y})}$.

- The Atkinson inequality index
  - How far the equally distributed equivalent income y_e is from the average income $\bar{y}$.
  - $y_e = U^{-1}(\bar{U})$
  - $A_\varepsilon = 1 - \frac{y_e}{\bar{y}}$.

We consider a fictitious economy with 10 individuals that each has the income yi that we report in the following table:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
Gross income	1	5	11	35	40	45	50	90	95	100

We assume that the social utility function is:

$$U(y) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$

with ε the social aversion parameter.

Question 1. Compute the Atkinson inequality index in this economy when $\varepsilon = 1.5$.

Individual	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**	**y9**	**y10**
Income	1	5	10	35	40	45	50	90	95	100
Utility	0.000	1.106	1.368	1.662	1.684	1.702	1.717	1.789	1.795	1.800

Average y	47.1000
Average U	1.4622
U(Avg y)	1.7086
Equivalent y	13.8290
Dalton Index	**0.144212**
Atkinson Index	**0.706391**

Question 2. The government consider 3 alternative transfer schemes.

- Take 1 from the agent earning y_{10} to give it to the agent earning y_8.
- The same between y_7 and y_5.
- Do the same between y_3 and y_1.

According to the Atkinson index, which policy reduces inequality the most? Comment.

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	1	5	10	35	40	45	50	91	95	99
Utility	0.000	1.106	1.368	1.662	1.684	1.702	1.717	1.790	1.795	1.799

Average y	47.1000
Average U	1.4622
U(Avg y)	1.7086
Equivalent y	13.8298
Dalton Index	**0.144203**
Atkinson Index	**0.706374**

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	1	5	10	35	41	45	49	90	95	100
Utility	0.000	1.106	1.368	1.662	1.688	1.702	1.714	1.789	1.795	1.800

Average y	47.1000
Average U	1.4623
U(Avg y)	1.7086
Equivalent y	13.8342
Dalton Index	**0.144153**
Atkinson Index	**0.706281**

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	2	5	9	35	40	45	50	90	95	100
Utility	0.586	1.106	1.333	1.662	1.684	1.702	1.717	1.789	1.795	1.800

Average y	47.1000
Average U	1.5173
U(Avg y)	1.7086
Equivalent y	17.1703
Dalton Index	**0.111929**
Atkinson Index	**0.635450**

Question 3. We consider the exact same 3 transfer policies but for an inequality aversion parameter of $\varepsilon = 3$. How does that change inequality rankings.

Atkinson Index

Epsilon	**1.5**	**3**
Initial Distribuion	0.706391	0.934562
Scheme 1	0.706374	0.934562
Scheme 2	0.706281	0.934561
Scheme 3	0.635450	0.878432