Topics in Macro 2

Week 2 - TD 2

Oscar Fentanes
oscar.fentanes@tse-fr.eu

PhD Candidate at TSE

Tuesday (17:00-18:30)

TD First Part: 9 Exercises in 6 Weeks

What's going on?

- 1 Text: The Height of Inequality (Week 1)
- 2 Exercise: Skilled biased technological change and wage dispersion (Week 2)
- 3 Stylized facts: Top wages and incomes (Week 3)

Next Week

How do we measure it?

- 4 Exercise: Interdecile ratios of income (Week 3)
- 5 Exercise: Lorenz curve and Gini basics (Week 4)

What do we do?

- 6 Exercise: Taxes, consumption and inequality (Week 4)
- 7 Exercise: Variance based inequality indicators (Week 5-6)
- 8 Exercise: Social Welfare Function and inequality aversion (Week 5-6)
- 9 Exercise: The Atkinson inequality index (Week 5-6)

Skilled-biased technological change and wage dispersion

Objective

- Give an explanation for the increasing income inequality
- Hypothesis: Existence of a skill premia

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Trends in Inequality and Technological Change

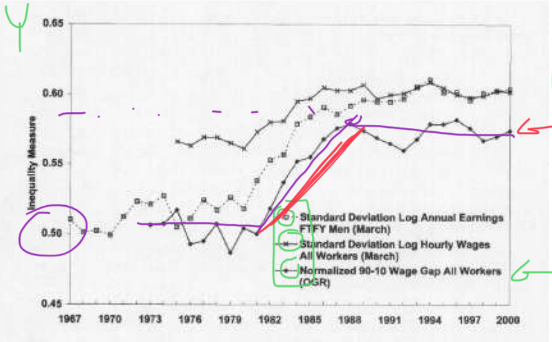


FIG. 2.—Alternative measures of aggregate wage inequality

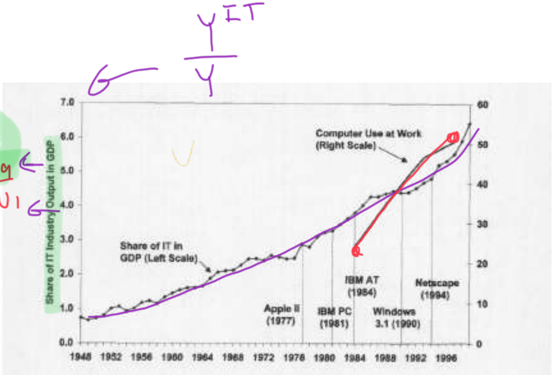
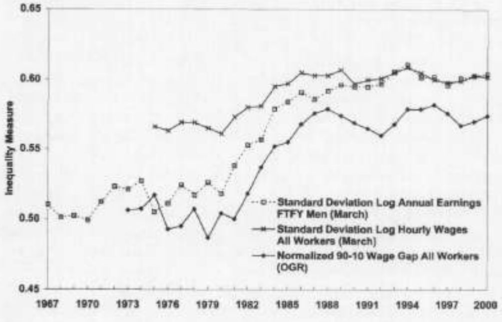


FIG. 1.—Measures of technological change

w_1 w_9

10% 90%

Trends in Inequality and Technological Change



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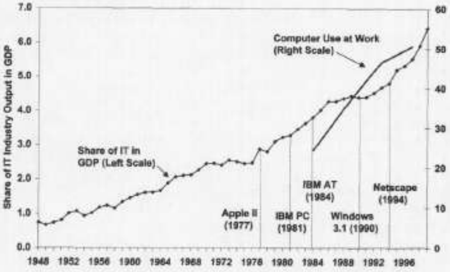


FIG. 1.—Measures of technological change

Possible explanation: **SBTC**

The CES production function

TFP: Total Factor Productivity

$$Y = \bar{A}\left[\theta(\alpha_b L_b)^{\rho} + (1-\theta)(\alpha_h L_h)^{\rho}\right]^{1/\rho}$$

bas | haut

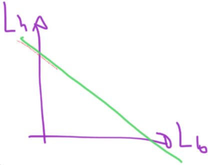
Low | High

$\nearrow MpL_b > \searrow MpL_h$

$\sigma = \frac{1}{1-\rho}$

$\boxed{\sigma > 1} \Rightarrow \frac{1}{1-\rho} > 1 \Rightarrow 1 > 1-\rho \Rightarrow \boxed{\rho > 0}$

PS



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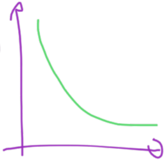


If $\rho \to 1$, then **Perfect Substitutes**. ~~If $\rho \to -\infty$, then **Perfect Complements**.~~ If $\rho \to 0$, then **Cobb-Douglas**.

Question 1

Write the profit function for the above representative firm assuming that the price of the final production good is normalized to 1.

$$\pi = \text{Prod} - \text{Costs}$$

$$= PY - w_b L_b - w_h L_h$$

$$P = 1$$

Question 2

Maximize the profit and write the first-order conditions of the firm.

$W = \frac{w_h}{w_b} = \frac{w_g}{w_1}$

Then write the first-order conditions in terms of the relative wage w.

Simplify the equation using the following notations: the ratio of higher to basic educated workers is noted $e = Lh/Lb$ and define the constant $A = \frac{1-\theta}{\theta}$.

Question 2

$\bar{A} = 1$

Maximize the profit and write the first-order conditions of the firm.

$$\pi^* = \max_{\{L_b, L_h\}} \pi = \max_{\{L_b, L_h\}} \underbrace{\left[\theta(\alpha_b L_b)^\rho + (1-\theta)(\alpha_h L_h)^\rho\right]}_{*}^{1/\rho} - w_b L_b - w_h L_h$$

F.O.C. ① L_b: $\left(\frac{1}{\rho}\right)[*]^{1/\rho - 1}\left[\theta\rho(\alpha_b L_b)^{\rho-1}\alpha_b\right] - w_b = 0$

② L_h: $\left(\frac{1}{\rho}\right)[*]^{1/\rho - 1}\left[(1-\theta)\rho(\alpha_h L_h)^{\rho-1}\alpha_h\right] - w_h = 0$

Then write the first-order conditions in terms of the relative wage w.

$$① \; L_b: \quad \left(\frac{1}{\rho}\right)[*]^{1/\rho-1}\left[\theta\rho(\alpha_b L_b)^{\rho-1}\alpha_b\right] - w_b = 0$$

$$② \; L_h: \quad \left(\frac{1}{\rho}\right)[*]^{1/\rho-1}\left[(1-\theta)\rho(\alpha_h L_h)^{\rho-1}\alpha_h\right] - w_h = 0$$

$$\frac{②}{①}: \quad \frac{\cancel{\left(\frac{1}{\rho}\right)[*]^{1/\rho-1}}\left[(1-\theta)\cancel{\rho}(\alpha_h L_h)^{\rho-1}\alpha_h\right]}{\cancel{\left(\frac{1}{\rho}\right)[*]^{1/\rho-1}}\left[\theta\cancel{\rho}(\alpha_b L_b)^{\rho-1}\alpha_b\right]} = \frac{w_h}{w_b}$$

$$\frac{(1-\theta)}{\theta}\cdot\left(\frac{\alpha_h L_h}{\alpha_b L_b}\right)^{\rho-1}\frac{\alpha_h}{\alpha_b} = \frac{w_h}{w_b}$$

Question 2

Simplify the equation using the following notations: the ratio of higher to basic educated workers is noted $e = Lh/Lb$ and define the constant $A = \frac{1-\theta}{\theta}$.

$$w \equiv \frac{w_h}{w_b}, \quad e \equiv \frac{L_h}{L_b}, \quad A \equiv \frac{1-\theta}{\theta}$$

$$\frac{(1-\theta)}{\theta} \cdot \left(\frac{\alpha_h L_h}{\alpha_b L_b}\right)^{\rho-1} \frac{\alpha_h}{\alpha_b} = \frac{w_h}{w_b}$$

$$\frac{(1-\theta)}{\theta} \left(\frac{L_h}{L_b}\right)^{\rho-1} \underbrace{\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho-1} \left(\frac{\alpha_h}{\alpha_b}\right)^{1}}_{\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho}} = \frac{w_h}{w_b} \Rightarrow \boxed{A(e)^{\rho-1}\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho} = w}$$

Hint: show that $w = A\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho} e^{\rho-1}$

Question 3

$-\frac{1}{\sigma} = \rho - 1$

$\Rightarrow \sigma = \frac{1}{1-\rho}$

Rewrite the equation in terms of the elasticity of substitution between basic educated and higher educated labor noted σ.

$$A(e)^{\rho-1}\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho} = w$$

$\sigma = \varepsilon_{e,w}$

Take Logs

$$\log(A) + (\rho-1)\log(e) + \rho\log\left(\frac{\alpha_h}{\alpha_b}\right) = \log(w)$$

$$\varepsilon = \frac{\Delta\% L_h}{\Delta\% L_b}$$

$$\frac{d\log(x)}{dx} = \frac{1}{x} \Rightarrow \boxed{d\log(x) = \frac{dx}{x}}$$

$$\varepsilon_{e,w} = \frac{\Delta\% e}{\Delta\% w} = \frac{de/e}{dw/w} = \frac{d\log(e)}{d\log(w)}$$

$$\log(e) = \left(\frac{1}{\rho-1}\right)\left[\log(w) - \log A - \rho\log\left(\frac{\alpha_h}{\alpha_b}\right)\right]$$

$$\frac{d\log(e)}{d\log(w)} = \left(\frac{1}{\rho-1}\right)$$

Hint: show that $w = A\left(\frac{\alpha_h}{\alpha_b}\right)^{1-\frac{1}{\sigma}} e^{-\frac{1}{\sigma}}$

$\sigma = |\varepsilon_{e,w}|$

Question 4

Using the relation from the last question, explain how skill-biased technological change (SBTC) has an impact on wage dispersion and wage inequality.

$$A(e)^{\rho-1}\left(\frac{\alpha_h}{\alpha_b}\right)^{\rho} = w \Rightarrow \underbrace{A e^{\frac{1}{\sigma}}\left(\frac{\alpha_h}{\alpha_b}\right)^{1-\frac{1}{\sigma}}}= w$$

$\sigma > 1 \Rightarrow 1-\frac{1}{\sigma} > 0$

$$\sigma = |\varepsilon_{e,w}| = \frac{1}{1-\rho}$$

$$\Rightarrow 1-\rho = \frac{1}{\sigma}$$

$$\Rightarrow \rho = 1-\frac{1}{\sigma}$$

SBTC: $\left(\frac{\alpha_h}{\alpha_b}\right)\nearrow$ $\leftarrow \alpha_h \nearrow$

$$\left(\frac{\alpha_h}{\alpha_b}\right)\nearrow \Rightarrow w \nearrow$$

Hint: Suppose $\sigma > 1$.

Question 5

Now use the dynamic version of the above equation and express the log growth rate of w noted $G(w)$.

$$A e^{\frac{1}{\sigma}} \left(\frac{\alpha_h}{\alpha_b}\right)^{1-\frac{1}{\sigma}} = w$$

$$A e_t^{-\frac{1}{\sigma}} \left(\frac{\alpha_{h,t}}{\alpha_{b,t}}\right)^{1-\frac{1}{\sigma}} = w_t$$

$$G(w) = \log(w_{t+1}) - \log(w_t)$$

$$G(w) = \frac{w_{t+1} - w_t}{w_t} \Rightarrow w_t G(w) = w_{t+1} - w_t$$

$$w_t G(w) + w_t = w_{t+1} \Rightarrow (1 + G(w)) w_t = w_{t+1}$$

$$\underbrace{\log(1+G(w))}_{\approx G(w)} = \log(w_{t+1}) - \log(w_t) \leftarrow \log(1+x) \approx x,\ x \to 0$$

$$\log(A) - \frac{1}{\sigma}\log(e_t) + \left(1 - \frac{1}{\sigma}\right)\log\left(\frac{\alpha_{h,t}}{\alpha_{b,t}}\right) = \log(w_t)$$

Hint: show that $G(w) = (1 - \frac{1}{\sigma}) G(\frac{\alpha_h}{\alpha_b}) - \frac{1}{\sigma} G(e)$

$$\log(w_{t+1}) - \log(w_t) = \quad t+1 \quad - \quad t$$

Question 6

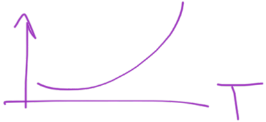
Assume the growth rate of SBTC is a constant such that $\bar{\alpha} = G(\frac{\alpha_h}{\alpha_b})$. $A e^{\frac{1}{\sigma}} \left(\frac{\alpha_h}{\alpha_b}\right)^{1-\frac{1}{\sigma}} = w$

Also assume that the growth rate of the ratio of higher to basic educated workers $G(e)$ respond to the difference between the wage premium and the cost of getting an education with an elasticity β.

Finally assume that the cost of getting an education is expressed in the following way: it is a fixed cost F over the cost of basic education and the cost of postponing earnings during the time of education by T periods, that we note $exp(rT)$.

r is the usual per period interest rate in the economy. In the end, the growth rate of the ratio of higher to basic educated workers can be written as:

$$G(e) = \beta\left[w - F - exp(rT)\right]$$



Compute w^* the long-run wage premium in the economy.

$G(w) = 0$

Question 6

Solution:
$$G(w) = \left(1-\frac{1}{\sigma}\right) G\left(\frac{\alpha_h}{\alpha_b}\right) - \frac{1}{\sigma} \underbrace{G(e)}$$

$$G(w) = \left(1-\frac{1}{\sigma}\right) (\bar{\alpha}) - \frac{1}{\sigma}\left[\beta(w - F - exp(rT))\right] \qquad \beta\left[w - F - exp(rT)\right]$$

$$e \nearrow$$

$$G(w) = 0 \implies w^*$$

$$\left(1-\frac{1}{\sigma}\right)\bar{\alpha} = \frac{1}{\sigma}\beta\left[w - F - exp(rT)\right]$$

$$\implies \frac{\sigma}{\beta}\left(1-\frac{1}{\sigma}\right)\bar{\alpha} + F + exp(rT) = w$$

$\sigma > 1$

$\beta \to 0 \implies \frac{1}{\beta} \to \infty$

$\Delta\bar{\alpha}\nearrow \implies \Delta w \nearrow$

Hint: Substitute $G(e)$. Long-run when $G(w) = 0$. Show that: $w^* = (1/\beta)\underbrace{(\sigma - 1)}_{>0}\bar{\alpha} + F + exp(rT)$.

SBTC

Question 7

Interpret the value of the long-run wage premium w^*.

- Long-run w

$$\frac{\sigma}{\beta}\left(1-\frac{1}{\sigma}\right)\bar{\alpha} + \underbrace{F + \exp(rT)} = w$$

$$\bar{\alpha} = 0$$

- If $\bar{\alpha} = 0$, wage dispersion explained by education

- β

Trends in Inequality and Technological Change

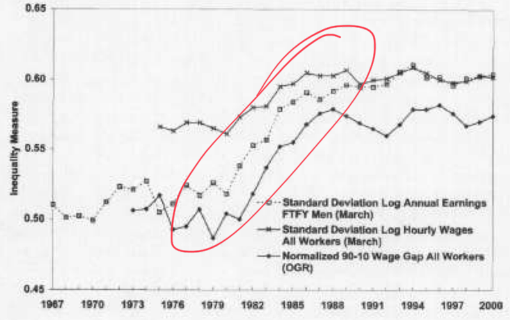


Fig. 2.—Alternative measures of aggregate wage inequality

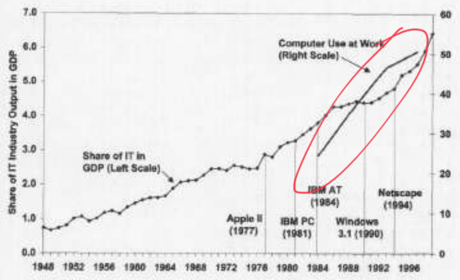


Fig. 1.—Measures of technological change

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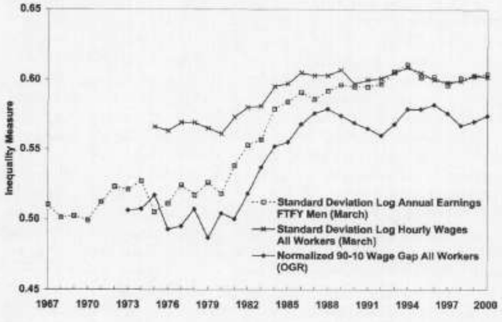

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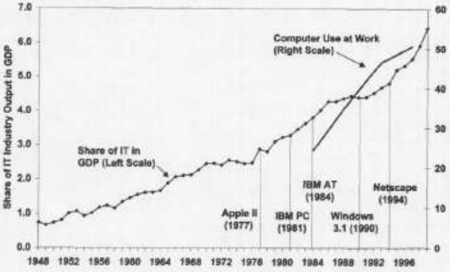

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Possible explanation: **SBTC**

Trends in Inequality and Technological Change

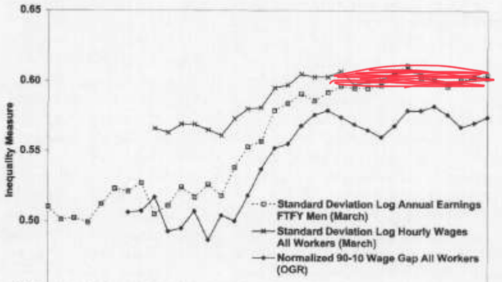

FIG. 2.—Alternative measures of aggregate wage inequality

FIG. 22.—Real minimum wage, 1973–2000

Minimum wage explains more variance of inequality.

Trends in Inequality and Technological Change

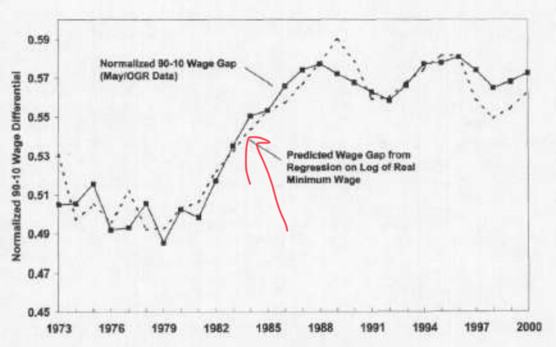

Fig. 23.—Wage inequality and the minimum wage

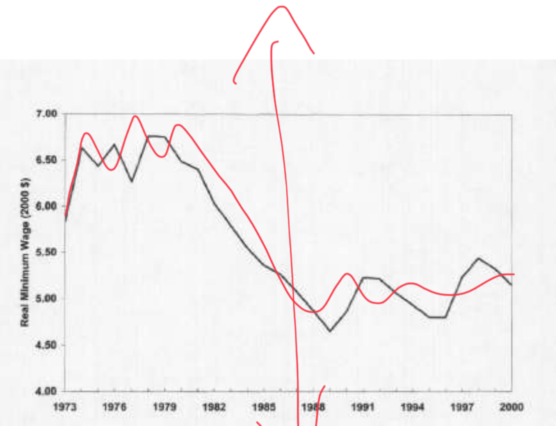

Fig. 22.—Real minimum wage, 1973–2000

Minimum wage explains more variance of inequality.

Conclusion

- SBTC might explain some of the rise in income inequality
- Other factors might play a bigger role.