Problem Set #6 - Money, inflation and interest rates

Problem 1: Quantity theory of money

1. **Quantity equation**

   (a) **In 2006, French GDP was 1800 billion euros. The price index (base of 100 in 1998) was 114, and inflation was 1.6%. The long term real growth rate was 2% and the money supply was 460 billion euros. What is the real GDP in 1998 euros?**

   Recall that:

$$Y = GDP_{real}$$

$$PY = GDP_{nominal} = 1800$$

   Here the 2006 GDP is given in billions of 2006 euros, so it is the Nominal GDP. We can therefore find the

$$GDP_{real}$$

   with the transformation $GDP_{real} = \frac{GDP_{nominal}}{P}$. However, we must be careful because we do not have in the statement a price level but a price index based on 1998. The price index is 114 in 2006. This means that 1 euro of 1998 is worth 1.14 in 2006 euros due to inflation. So to get GDP_{real} in euros of 1998, we have to divide the nominal GDP by the value of one euro of 1998 in 2006 i.e. 1.14:

$$GDP_{real} = 1800/1.14 = 1578.95$$

   We can also do the following operation which amounts to the same thing:

$$GDP_{real} = (1800/114) * 100 = 1578.95$$

   The GDP_{real} in euros for 1998 is worth 1578.95 (billions, we voluntarily omit billions for simplicity).

   (b) **What is the velocity of money?**

   To find the velocity of circulation of money, we will use the quantitative equation of money. The quantitative equation relates the number units of currency traded in the economy to the nominal GDP of the economy.

It is an accounting identity, always verified in the economy. We written as follows:

$$MV = PY$$

with M the monetary mass in circulation in the economy, V the speed of circulation of money, P the price level and Y the real GDP (and PY is the nominal GDP).

The velocity of money is a measure of the number of transactions in average that a unit of money allows to make in the economy at a time given. It is a constant. This is a stable technical parameter of an economy. According to the equation below, we deduce that:

$$V = \frac{PY}{M}$$

As PY is the nominal GDP we know that in 2006 $PY = 1800$. Moreover the statement gives us the value of the money supply, which is exactly the same thing as the monetary mass in circulation, therefore $M = 460$. deduce that:

$$V = \frac{PY}{M} = \frac{1800}{460} = 3.91$$

Each monetary unit in 2006 is used on average to make 3.91 transactions.

(c) **State the percentage rule.**

The percentage rule is explained and proven in class, you do not have to not to do it again. It makes it possible to express approximately the variations in percentage. There are two useful rules:

$$Z = X * Y \Rightarrow \%\Delta Z = \%\Delta X + \%\Delta Y$$

$$Z = X/Y \Rightarrow \%\Delta Z = \%\Delta X - \%\Delta Y$$

We note with the notation $\%\Delta$ the variation in percentage. Basically if we have a product of variables, the percent change is a sum. If we have a quotient of variables, the percentage change is a difference.

(d) **What is the growth rate of the money supply?**

To calculate the growth rate (or percentage change or relative change) in the money supply, we start from the quantitative equation and apply the rule of percentages:

$$MV = PY$$
$$\%\Delta M + \%\Delta V = \%\Delta P + \%\Delta Y$$
$$\%\Delta M = \%\Delta P + \%\Delta Y - \%\Delta DV$$

As we said above, V is a constant its variation in time is zero so that $\%\Delta V = 0$. The statement gives us the other missing information: the real long-term growth rate is 2% so $\%\Delta Y = 2\%$. In addition, the percentage change in the level price is nothing but inflation, which according to the statement gives us $\%\Delta P = 1.6\%$. Finally :

$$\%\Delta M = \%\Delta P + \%\Delta Y - \%\Delta V$$
$$= 1.6\% + 2\% - 0\% = 3.6\%$$

The growth rate of the money supply is therefore 3.6%.

2. **Financial innovations**

   (a) **Financial innovations lead agents to reduce their demand for money by 1%. What impact does this have on the velocity of money?**

   As we mentioned above, the speed of circulation of money is a technical parameter of economics that measures how many times a unit of currency is used in transactions. Financial innovations eras discussed here allow a smaller amount of money to circulate faster for an unchanged level of income. That will therefore have an impact on the speed of circulation of money because we permanently changes the financial technology of the economy. All things otherwise equal, the velocity of circulation of money will therefore increase from 1%.

   (b) **What effect does this have on the inflation rate, keeping in mind the money market equilibrium?**

   In this model, the money supply is exogenously controlled by the State or the Central Bank so that we can note the money supply in the following way:

$$M^S = \bar{M}$$

The demand for money, in its simple version, is a function of the number of transactions (or purchases) that agents intend to make. We will approximate the number of total transactions in the economy by real GDP and value each transaction by its price so that the demand for money is written as follows:

$$M^D = kPY$$

with k a proportionality parameter.
At money market equilibrium, we must satisfy the condition supply equals demand:

$$M^S = M^D \iff M = kPY$$

We can compare the equation resulting from the money market equilibrium with the accounting identity that is the quantitative equation of money (denoted $MV = PY$). We realize that as M which is the supply of exogenous money is strictly the same as the quantity of money in circulation present in the quantitative equation, by identifying the two equations, we necessarily have:

$$k = \frac{1}{V}$$

So after this identification, under the market equilibrium condition money, we have that:

$$\bar{M} = \frac{PY}{V}$$

(Attention we could not conclude what we have just written before because we must first apply the equilibrium condition on the market currency).
We apply the percentage rule to the previous equation:

$$\bar{M} = \frac{PY}{V} \iff \bar{M} = \%\Delta P + \%\Delta Y - \%\Delta V$$

In this question, it is not mentioned that the exogenous money supply is changing or there is economic growth. So $\%\Delta M = 0$ and $\%\Delta Y = 0$ by hypothesis. So we are left with:

$$\%\Delta P = \%\Delta V = 1\%$$

Financial innovations therefore lead to a price increase of 1% (inflation).

(c) **In order to minimize changes in the price level, what monetary policy would be recommended by the monetarist rule?**

Monetarists generally recommend strict mass control monetary policy to contain inflation. The economic policy that would therefore undoubtedly applied a monetarist would have been to decrease the money supply (the money supply) by 1%. In this way, we would have absorbed the variation of the speed of circulation of money without increasing inflation.

3. **Quantity theory and inflation**

(a) **Suppose that the long term production level is given, the velocity of money is 4, and the supply of money increases by 3%. According to the quantitative theory of money, what will be the percentage change in real revenue?**

We recall that real income is the same at this point as GDP real and we denote it Y. If the as the statement says the level of production term is given, so it is constant. This means that $Y = \bar{Y}$ at all periods: there can be no change in real income. So the answer to this question is basic: following a variation in the supply of currency, under the assumptions of the statement, real income cannot vary or its variation is zero.

(b) **What about the price level?**

We start from the quantitative equation of money written in variations:

$$MV = PY$$

$$\%\Delta M + \%\Delta V = \%\Delta P + \%\Delta Y$$

According to the statement, $V = 4$ so it is a constant, its variation is zero: $\%\Delta V = 0$. The real income is also fixed as we have just seen: $\%\Delta Y = 0$. The previous equation therefore becomes:

$$\%\Delta M = \%\Delta P$$

Any change in the money supply must be compensated by a change in the price level, hence inflation. So if $\%\Delta M = 3\%$ then $\%\Delta P = 3\%$, inflation increases by 3%.

(c) **What about nominal income?**

Nominal income is the same as nominal GDP at this point and we we denoted it PY. Still applying the rule of percentages we have:

$$\%\Delta(PY) = \%\Delta P + \%\Delta Y$$

But we know that the real income does not evolve in the long term ($\%\Delta Y = 0$), this which means that nominal income automatically evolves at the same speed that inflation:

$$\%\Delta(PY) = \%\Delta P = 3\%$$

Problem 2: Fisher equation

1. **Fisher equation: assume that the economy is at full employment, with an anticipated inflation rate of 7% and a nominal interest rate of 11%.**

   (a) **State the Fisher equation.**

   In its final form, the Fisher equation relates the nominal interest rate denoted i, the real interest rate denoted r and the expected inflation rate denoted π^e.

   Fisher's equation is explained in class so you don't have to explain it again. It is written:

   $$i = r + \pi^e$$

   Here is one way to interpret this equation: agents think in terms of real interest rate r because it is the only one that takes inflation into account. But when, for example, they are going to sign a mortgage contract, only the nominal interest rate i is written on the contract. So to evaluate the rate real interest, agents will form inflation expectations π^e for correct the nominal interest rate and obtain a view of the real interest rate, this is what Fisher's equation does. However, it should not be forgotten that the nominal interest rate i is linked to the activity of the Central Bank, it will therefore be likely to change as a result of monetary policy. This will not be the case in the classical model of the real interest rate which is a real variable.

   (b) **What is the real interest rate in this economy?**

   We apply Fisher's equation with the data from the statement:

   $$\begin{aligned} r &= i - \pi^e \\ &= 11\% - 7\% = 4\% \end{aligned}$$

   The real interest rate is 4%.

   (c) **Suppose that the anticipated inflation rate rises to $\pi^e =$ 9%. What is the real interest rate? The nominal interest rate?**

   If $\pi^e = 9\%$ then the real interest rate will not change it remains at 4%. It is fixed by the real variables of the economy and not by the nominal variables this is the dichotomy of the classical model. When expected inflation moves due to changes in the money supply, for example, the real interest rate is not modified. But inflation affects the nominal interest rate

$$\begin{aligned} i &= r + \pi^e \\ &= 0.04 + 0.09 \\ &= 0.13 = 13\% \end{aligned}$$

2. **Fisher equation and the quantity equation: suppose now that the money supply increases at a rate of 7%, the real interest rate is 3% and the real growth rate of GDP is 2%.**

   (a) **Write the inflation rate as a function of the growth rate of the stock of money, the velocity of money and the growth rate of real GDP. Solve for the inflation rate in this economy.**

   To answer this question, we will use the quantitative equation of money in its percentage version:

$$MV = PY$$
$$\%\Delta M + \%\Delta V = \%\Delta P + \%\Delta Y$$

   According to the statement, $\%\Delta M = 7\%$ and $\%\Delta Y = 2\%$. We also know that V is constant when no hypothesis says otherwise, so $\%\Delta V$. We conclude that:

$$\begin{aligned} \pi = \%\Delta P &= \%\Delta M + \%\Delta V - \%\Delta Y \\ &= 7\% + 0\% - 2\% = 5\% \end{aligned}$$

   (b) **According to the quantity theory of money, what should the nominal interest rate be?**

   Assuming that inflation is very stable in this economy, we can approximate that:

$$\pi^e = \pi$$

   Applying Fisher's equation now, we have:

$$i = r + \pi^e = 3\% + 5\% = 8\%$$

   The nominal interest rate is 8%.