Topics in Macro 2

Week 5 - TD 8 and TD 9

Oscar Fentanes
www.oscarfentanes.com

TSE

Tuesday (17:00-18:30)

TD First Part: Inequality (6 Weeks)

What's going on?

- Exercise 1: The Height of Inequality (Week 1)
- Exercise 2: Skilled biased technological change and wage dispersion (Week 2)
- Exercise 3: Top wages and incomes (Week 3)

How do we measure it?

- Exercise 4: Interdecile ratios of income (Week 3)
- Exercise 5: Lorenz curve and Gini basics (Week 3)

What do we (have to) do?

- Exercise 6: Taxes, consumption and inequality (Week 4)
- Exercise 7: Variance based inequality indicators (Week 4)
- **Exercise 8: Social Welfare Function and inequality aversion** (Week 5)
- **Exercise 9: The Atkinson inequality index** (Week 5)

Exercise 8: Social Welfare Function and inequality aversion

We consider a Social Welfare Function (SWF) that has the 5 Assumptions seen in class. The SWF is generally defined as:

$$W = W(y_1, y_2, y_3, ..., y_n)$$

We can think of y_i, $i \in [1, n]$ as the level of income of the n individuals in the economy. The associated social utility function (or welfare index) U has the following functional form:

$$U(y_i) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$

with ε the inequality aversion parameter.

Question 1. Define what a SWF is.

A Social Welfare Function **ranks** all the possible **states** of society in the order of (this society's) preference.

What are the **states**? Income, wealth, Resources

Who **ranks** them? Dem Gov
Dictator
Economist.

Question 2. List the 5 Assumptions of a SWF used in class.

1. Individualistic and nondecreasing

$$P \Rightarrow Q$$

$$W = W(y_{1A}, y_{2A}, y_{3A}, \dots, y_{4A})$$
$$W = W(y_{1B}, y_{2B}, y_{3B}, \dots, y_{4B})$$

$$\bar{Q} \Rightarrow \bar{P}$$

If $y_{iB} \geq y_{iA}\ \forall i$, then $W_B \geq W_A$. $\leftarrow$ $W_B < W_A$

$\exists i\ \ y_{iB} < y_{iA}$

2. Symmetry

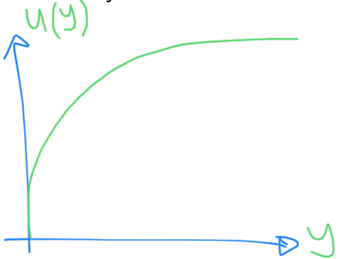
$$W = W(\underbrace{y_1, y_2, y_3}_{M}, \underbrace{\dots, y_n}_{W}) = W(\underbrace{y_2, y_1}_{\text{Low Sk.}}, \underbrace{y_3, \dots}_{\text{High Sk.}}, y_n) = W(y_n, y_2, y_3, \dots, y_1)$$

3. Additivity

$$W(y_1, y_2, y_3, \dots, y_n) = \sum_{i=1}^{n} U_i(y_i) = U_1(y_1) + U_2(y_2) + \dots + U_n(y_n)$$

$U(y_i)$ is the **social utility** or **welfare index** of individual i.

4. Concavity



$$U'(y_i) > 0 \; and \; U''(y_i) < 0$$

$$u''(y) = (-\varepsilon) y^{-\varepsilon - 1}$$

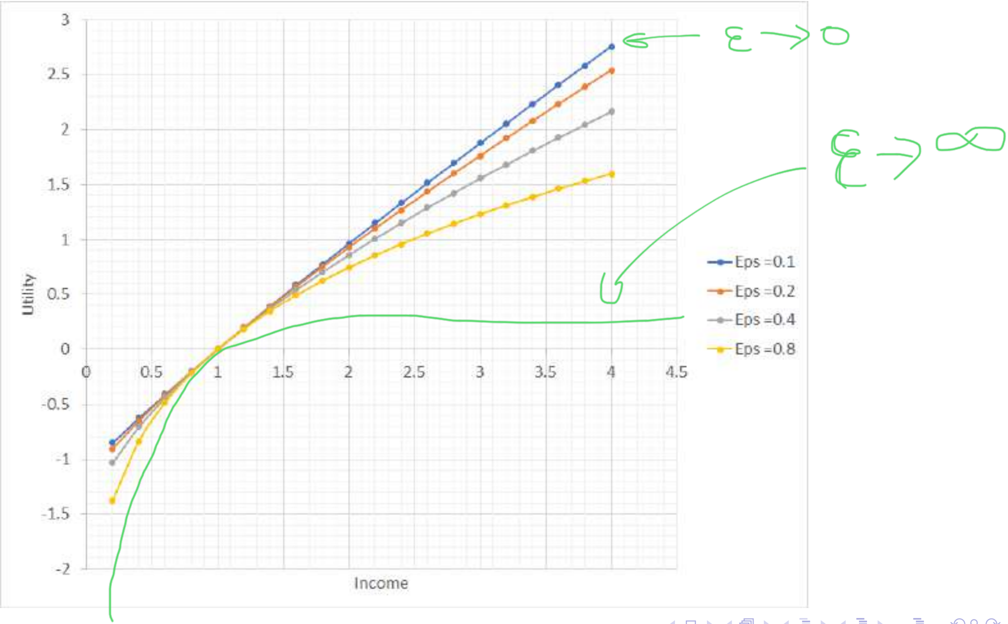
$$u'(y) = y^{-\varepsilon}$$

5. Constant elasticity

(Relative)

$$U(y_i) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$

$$\boxed{R(y) \equiv \frac{-y\,u''(y)}{u'(y)}} = \frac{-y(-\varepsilon)y^{-\varepsilon-1}}{y^{-\varepsilon}} = \frac{\varepsilon y^{-\varepsilon-1+1}}{y^{-\varepsilon}} = \varepsilon$$


$\varepsilon \to 0$
$\varepsilon \to \infty$
Utility
Income
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Eps =0.1
Eps =0.2
Eps =0.4
Eps =0.8

Question 3. Explain why $U'(y_i)$ can be interpreted as a social weight.

$$W(y_1(t), y_2(t), \dots, y_n(t))$$

$$\frac{dW}{dt} = \frac{\partial W}{\partial y_1} \cdot \frac{dy_1}{dt} + \dots + \frac{\partial W}{\partial y_n} \cdot \frac{dy_n}{dt}, \quad \frac{\partial W}{\partial y_i} = U'(y_i)$$

$$\frac{\Delta W}{\cancel{\Delta t}} = U'(y_1) \frac{\Delta y_1}{\cancel{\Delta t}} + \dots + U'(y_n) \frac{\Delta y_n}{\cancel{\Delta t}}$$

$$\Delta W = U'(y_1) \underbrace{\Delta y_1}_{1} + \dots + U'(y_n) \Delta y_n$$

$$\Delta y_i = 1€, \quad \Delta y_k = 0 \ \forall k \neq i$$

Hint: Total derivative. $\frac{dW}{dt} = \sum_{i=1}^{n} \frac{\partial W}{\partial y_i} \frac{dy_i}{dt}$

We consider the following situation: there are n individuals in the economy. We know that individual k has **5 times** the income of individual j. We consider a **transfer of 1 Euro** from individual k to j. We assume that during the transfer from k to j, there are no other income variations in the economy.

Question 4. Explain why under the conditions given for our *SWF*, individual k would be willing to give up on her money in favor of j.

$$\Delta W = u'(y_1)\Delta y_1 + \cdots + u'(y_n)\Delta y_n \,/\, \text{suppose } \Delta y_i = 0 \;\; \forall i \neq j,k$$

$$= \underbrace{u'(y_j)\Delta y_j}_{} + \underbrace{u'(y_k)\Delta y_k}_{}$$

Suppose $-\Delta y_k = \Delta y_j$

$$\Delta W = u'(y_j)\Delta y_j + u'(y_k)(-\Delta y_j)$$

$$= \underbrace{(u'(y_j) - u'(y_k))}_{}\underbrace{\Delta y_j}_{+}$$

Transfer if $\Delta W \geq 0$

$\Rightarrow u'(y_j) > u'(y_k)$

Hint: Different welfare weights $U'(y_j) > U'(y_k)$.

Question 5. How much is individual k willing to give up so that individual j gets exactly 1 Euro when $\varepsilon = 0$?

$$\Delta W = u'(y_j)\Delta y_j + u'(y_k)\Delta y_k$$

$$u'(y) = y^{-\varepsilon}, \quad \text{If } \varepsilon = 0 \Rightarrow u'(y) = 1$$

$$\Delta W = \Delta y_j + \Delta y_k, \quad \Delta y_j = 1$$

$$\Delta W = 0 \Leftrightarrow 1 + \Delta y_k = 0$$

$$\Delta y_k = -1$$

Hint: 1 Euro. Same social marginal utility. No cost of tax collection.

Question 6. How much is individual k willing to give up so that individual j gets exactly 1 Euro when $\varepsilon = 1/2$. What about $\varepsilon = 1$ or $\varepsilon = 5$?

Case where $\varepsilon = 1/2$

$$\Delta W = u'(y_j)\Delta y_j + u'(y_k)\Delta y_k$$

$$u'(y) = y^{-\frac{1}{2}}, \quad y_k = 5y_j, \quad \boxed{\Delta y_j = 1}$$

$$\Delta y_k \text{ s.t. } \boxed{\Delta W = 0} \qquad = 2.23$$

$$0 = y_j^{-1/2}(1) + (5y_j)^{-1/2}\Delta y_k$$

$$\Longrightarrow \Delta y_k = \frac{-y_j^{-1/2}}{5y_j^{-1/2}} = -\left(\frac{\cancel{y_j}}{5\cancel{y_j}}\right)^{-1/2} = -\left(\frac{1}{5}\right)^{-1/2} = -\sqrt{5}$$

Case where $\varepsilon = 1$

$$\Delta W = 0 = u'(y_j)\Delta y_j + u'(y_k)\Delta y_k$$

$$u'(y) = y^{-\varepsilon} = y^{-1}$$

Case where $\varepsilon = 5$

$$u'(y) = y^{-5}$$

Question 7. What about $\varepsilon \to \infty$?

$$\Delta W = 0 = u'(y_j)\Delta y_j + u'(y_k)\Delta y_k$$

Suppose $\Delta y_j = 1$, $y_k > y_j$

$$u'(y_j) + u'(y_k)\Delta y_k = 0$$

$$\Delta y_k = \frac{-u'(y_j)}{u'(y_k)} = \frac{-y_j^{-\varepsilon}}{y_k^{-\varepsilon}} = -\left(\frac{y_j}{y_k}\right)^{-\varepsilon}$$

$$= -\left(\frac{y_k}{y_j}\right)^{\varepsilon} \xrightarrow[\varepsilon \to \infty]{} -\infty$$

Hint: $dy_k = -(\frac{y_k}{y_j})^{\varepsilon}$.

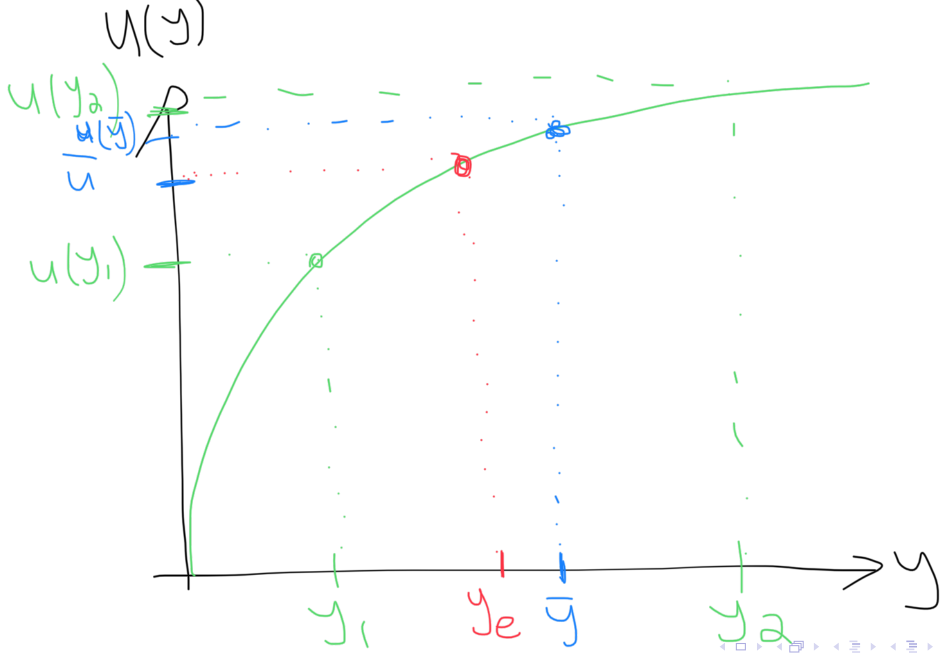
Exercise 9: The Atkinson inequality index

Recall

- The Dalton inequality index
  - It measures how far the current average social utility is from potential average social utility.
  - $D_\varepsilon = 1 - \frac{\bar{U}}{U(\bar{y})}$.

- The Atkinson inequality index
  - How far the equally distributed equivalent income y_e is from the average income $\bar{y}$.
  - $y_e = U^{-1}(\bar{U})$
  - $A_\varepsilon = 1 - \frac{y_e}{\bar{y}}$.

Plot:



We consider a fictitious economy with 10 individuals that each has the income yi that we report in the following table:

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
Gross income	1	5	11	35	40	45	50	90	95	100

We assume that the social utility function is:

$$U(y) = \frac{y^{1-\varepsilon} - 1}{1 - \varepsilon}$$

with ε the social aversion parameter.

Question 1. Compute the Atkinson inequality index in this economy when $\varepsilon = 1.5$.

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	1	5	10	35	40	45	50	90	95	100
Utility	0.000	1.106	1.368	1.662	1.684	1.702	1.717	1.789	1.795	1.800

Average y	47.1000
Average U	1.4622
U(Avg y)	1.7086
Equivalent y	13.8290
Dalton Index	**0.144212**
Atkinson Index	**0.706391**

$$A_\varepsilon = 1 - \frac{y_e}{\bar{y}}$$

$$y_e \text{ s.t. } u(y_e) = \bar{U}$$

$$y_e = u^{-1}(\bar{u})$$

$$u(y) = \frac{y^{1-\varepsilon} - 1}{1-\varepsilon}$$

$$\Rightarrow u\cdot(1-\varepsilon) = y^{1-\varepsilon} - 1$$

$$\Rightarrow u\cdot(1-\varepsilon) + 1 = y^{1-\varepsilon}$$

$$\Rightarrow$$

$$y_e = [\bar{u}\cdot(1-\varepsilon)+1]^{\frac{1}{1-\varepsilon}}$$

Question 2. The government consider 3 alternative transfer schemes.

- Take 1 from the agent earning y_{10} to give it to the agent earning y_8.
- The same between y_7 and y_5.
- Do the same between y_3 and y_1.

According to the Atkinson index, which policy reduces inequality the most? Comment.

Individual	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**	**y9**	**y10**
Income	1	5	10	35	40	45	50	91	95	99
Utility	0.000	1.106	1.368	1.662	1.684	1.702	1.717	1.790	1.795	1.799

$$\frac{\sum_{i=1}^{n} y_i}{n}$$

Average y	47.1000
Average U	1.4622
U(Avg y)	1.7086
Equivalent y	13.8298
Dalton Index	**0.144203**
Atkinson Index	**0.706374**

$$\frac{\sum_{i=1}^{n} U(y_i)}{n}$$

$$\frac{(\bar{y})^{1-\varepsilon} - 1}{1-\varepsilon}$$

$$y_e = \left[\bar{U} \cdot (1-\varepsilon) + 1\right]^{\frac{1}{1-\varepsilon}}$$

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	1	5	10	35	41	45	49	90	95	100
Utility	0.000	1.106	1.368	1.662	1.688	1.702	1.714	1.789	1.795	1.800

Average y	47.1000
Average U	1.4623
U(Avg y)	1.7086
Equivalent y	13.8342
Dalton Index	**0.144153**
Atkinson Index	**0.706281**

Individual	y1	y2	y3	y4	y5	y6	y7	y8	y9	y10
Income	2	5	9	35	40	45	50	90	95	100
Utility	0.586	1.106	1.333	1.662	1.684	1.702	1.717	1.789	1.795	1.800

Average y	47.1000
Average U	1.5173
U(Avg y)	1.7086
Equivalent y	17.1703
Dalton Index	**0.111929**
Atkinson Index	**0.635450**

Question 3. We consider the exact same 3 transfer policies but for an inequality aversion parameter of $\varepsilon = 3$. How does that change inequality rankings.

Atkinson Index

Epsilon	**1.5**	**3**
Initial Distribuion	0.706391	0.934562
Scheme 1	0.706374	0.934562
Scheme 2	0.706281	0.934561
Scheme 3	0.635450	0.878432

Higher Inequality because society is more inequality averse for higher ε