Topics in Macro 2

Week 8 - Second Part - Part I - Exercise III

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TSE

Tuesday (17:00-18:30)

TD Second Part: Fiscal Multipliers (Weeks 6 to 10)

Part I

- Exercise I: Habit Persistence and The Keynesian Multiplier (Week 6)
- Exercise II: A Benchmark Model (Week 7)
- **Exercise III: Consumption, Labor Supply and the Multiplier (Week 7)**

Part II

- Exercise I: Taxes on the Labor Input and the Multiplier (Week 8)
- Exercise II: Public Spending in Utility Function and the Multiplier (Week 8)
- Exercise III: Labor Supply, Public Spending in Utility and the Multiplier (Week 9)

Part III

- Exercise I: Endogenous Public Spending (Week 9)
- Exercise II: Externality in Production and the Multiplier (Week 10)
- Exercise III: Externality in Labor Supply and the Multiplier (Week 10)

Exercice III: Consumption, Labor Supply and the Multiplier

The model

Utility:

$$U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\nu} n_t^{1+\nu}, \text{ where } \eta > 0, \ \sigma > 0$$

Budget constraint:

$$c_t \leq w_t n_t - T_t + \Pi_t$$

Technology:

$$y_t = a n_t$$

Profits:

$$\Pi_t = y_t - w_t n_t$$

Government:

$$T_t = g_t$$

Market clearing:

$$y_t = c_t + g_t$$

Question 1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS).

$$L = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\nu} n_t^{1+\nu} - \lambda_t [c_t - w_t n_t + T_t - \Pi_t]$$

F.O.C.

$$c_t : \ c_t^{-\sigma} = \lambda_t$$
$$n_t : \eta n_t^{\nu} = \lambda_t w_t$$

Then:

$$w_t = \frac{\eta n_t^{\nu}}{c_t^{-\sigma}}$$

Question 2. Determine the optimality condition of the firm.

$$\max_{n_t} an_t - w_t n_t$$

Notice that:

$$\Pi_t = y_t - w_t n_t = (a - w_t) n_t$$

So, if:

$$\begin{aligned} w_t < a \text{ then } n_t &\to \infty \\ w_t = a \text{ then } n_t &\in (0, \infty) \\ w_t > a \text{ then } n_t &= 0 \end{aligned}$$

Suppose from now on that $w_t = a$.

Question 3. Determine the equilibrium output.

Definition of a **competitive equilibrium**: An equilibrium are quantities $c_t, n_t^s, n_t^d, g_t, T_t$ and prices w_t such that:

1. c_t, n_t^s solve the consumer's problem.
2. n_t^d solve the firm's problem.
3. Government budget balance: $g_t = T_t$.
4. Goods and labor market clearing: $n_t^s = n_t^d$ and $y_t = c_t + g_t$.

Equilibrium output:

We Know $w_t = \frac{\eta n_t^{\nu}}{c^{-\sigma}}$

and $w_t = a$

$\Rightarrow \eta n_t^{\nu} c_t^{\sigma} = a$

By Mkt clearing:

$\Rightarrow \eta n_t^{\nu} [y_t - g_t]^{\sigma} = a$

By Production function:

$\eta [y_t/a]^{\nu} [y_t - g_t]^{\sigma} = a$

$\Downarrow$

Answer: $\frac{y_t^{\nu}}{(y_t - g_t)^{-\sigma}} = \frac{a^{\nu+1}}{\eta}$.

Question 4. Compute the log-linearization of equilibrium output around the determinist steady-state.

We know:

$$\frac{a^{\nu+1}}{\eta} = \frac{y_t^{\nu}}{(y_t - g_t)^{-\sigma}}$$

Step 1. Take logs.

$$log\left(\frac{a^{\nu+1}}{\eta}\right) = \nu log(y_t) + \sigma log(y_t - g_t)$$

Step 2. Taylor approximation.

$$log\left(\frac{a^{\nu+1}}{\eta}\right) = \nu\left(log(\bar{y}) + \frac{1}{\bar{y}}(y_t - \bar{y})\right) + \sigma\left(log(\bar{y} - \bar{g}) + \frac{1}{\bar{y} - \bar{g}}(y_t - \bar{y}) - \frac{1}{\bar{y} - \bar{g}}(g_t - \bar{g})\right)$$

Step 3. Deviations from the steady state.

Notice that in the steady state:

$$\frac{a^{\nu+1}}{\eta} = \frac{\bar{y}^{\nu}}{(\bar{y}-\bar{g})^{-\sigma}}$$

Then:

$$log\left(\frac{a^{\nu+1}}{\eta}\right) = \nu log(\bar{y}) + \sigma log(\bar{y}-\bar{g})$$

So we have:

$$0 = \nu\frac{1}{\bar{y}}(y_t-\bar{y}) + \sigma\left(\frac{1}{\bar{y}-\bar{g}}(y_t-\bar{y}) - \frac{1}{\bar{y}-\bar{g}}(g_t-\bar{g})\right)$$

Which is equivalent to:

$$0 = \nu \frac{1}{\bar{y}}(y_t - \bar{y}) + \sigma \left(\frac{1}{\bar{y} - \bar{g}}(y_t - \bar{y}) * \frac{\bar{y}}{\bar{y}} - \frac{1}{\bar{y} - \bar{g}}(g_t - \bar{g}) * \frac{\bar{g}}{\bar{g}} \right)$$

Using the fact that $\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}}$:

$$0 = \nu \hat{y}_t + \sigma \left(\frac{\bar{y}}{\bar{y} - \bar{g}} \hat{y}_t - \frac{\bar{g}}{\bar{y} - \bar{g}} \hat{g}_t \right)$$

Define $s_g = \frac{\bar{g}}{\bar{y}}$. Then:

$$0 = \nu \hat{y}_t + \sigma \left(\frac{1}{1 - s_g} \hat{y}_t - \frac{s_g}{1 - s_g} \hat{g}_t \right)$$

Then:

$$
\begin{aligned}
0 &= \nu(1 - s_g)\hat{y}_t + \sigma\left(\hat{y}_t - s_g\hat{g}_t\right) \\
0 &= \nu(1 - s_g)\hat{y}_t + \sigma\hat{y}_t - \sigma s_g\hat{g}_t \\
(\sigma + \nu(1 - s_g))\hat{y}_t &= \sigma s_g\hat{g}_t \\
\frac{\hat{y}_t}{\hat{g}_t} &= \frac{\sigma s_g}{\sigma + \nu(1 - s_g)}
\end{aligned}
$$

Question 5. Compute the output multiplier and discuss the value of this multiplier with respect to ν and σ.

We know:

$$\frac{\hat{y}_t}{\hat{g}_t} = \frac{\sigma s_g}{\sigma + \nu(1 - s_g)}$$

Notice that:

$$\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}} = \frac{\Delta x_t}{\bar{x}} = \frac{dx_t}{\bar{x}}$$

Then:

$$dx_t = \hat{x}_t \bar{x}$$

Then:

$$\frac{dy_t}{dg_t} = \frac{\hat{y}_t \bar{y}}{\hat{g}_t \bar{g}} = \frac{\sigma s_g}{\sigma + \nu(1 - s_g)} * \frac{1}{s_g} = \frac{\sigma}{\sigma + \nu(1 - s_g)}$$

Interpretation. We know:

$$\frac{dy_t}{dg_t} = \frac{\sigma}{\sigma + \nu(1 - s_g)}$$

- σ The intertemporal elasticity of consumption is $1/\sigma$

  If $\sigma \to \infty$, $1/\sigma \to 0$, so consumers smooth consumption beause there is no intertemporal substitution.

  As $\sigma \to \infty$, $\frac{dy_t}{dg_t} \to 1$, so fiscal policy is very effective. Individuals increase n_t to smooth c_t.

- ν

  The Frisch elasticity of labor supply is $\frac{1}{\nu}$.

  If $\nu \to \infty$, $\frac{1}{\nu} \to 0$. As $\nu \to \infty$, individuals dislike more to supply labor. Fiscal policy becomes less effective because n_t does not increase.

Question 6.. Compute the consumption multiplier and discuss the value of this multiplier with respect to ν and σ.

We know:

$$y_t = c_t + g_t$$

Then

$$\begin{aligned} dy_t &= dc_t + dg_t \\ \frac{dy_t}{dg_t} &= \frac{dc_t}{dg_t} + 1 \\ \frac{dc_t}{dg_t} &= \frac{dy_t}{dg_t} - 1 \end{aligned}$$

Then:

$$\frac{dc_t}{dg_t} = \frac{\sigma}{\sigma + \nu(1 - s_g)} - 1$$

So same interpretation.