Topics in Macro 2

Week 3 - TD 6 and TD 7

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TSE

Tuesday (17:00-18:30)

TD First Part: Inequality (6 Weeks)

What's going on?

- Exercise 1: The Height of Inequality (Week 1)
- Exercise 2: Skilled biased technological change and wage dispersion (Week 2)
- Exercise 3: Top wages and incomes (Week 3)

How do we measure it?

- Exercise 4: Interdecile ratios of income (Week 3)
- Exercise 5: Lorenz curve and Gini basics (Week 3)

What do we do?

- **Exercise 6: Taxes, consumption and inequality** (Week 4)
- **Exercise 7: Variance based inequality indicators** (Week 4)
- Exercise 8: Social Welfare Function and inequality aversion (Week 5)
- Exercise 9: The Atkinson inequality index (Week 5-6)

Recall Gini Index

5 Steps

1. Put income in increasing order (for Lorenz curve).
2. Compute the mean income:

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} \lambda y_i \quad \Rightarrow \quad \bar{Y} \to \lambda \bar{Y}$$

3. Compute the absolute difference of all pairs of income:

$$M = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} |\lambda y_i - \lambda y_j| \quad \Rightarrow \quad M \to \lambda M$$

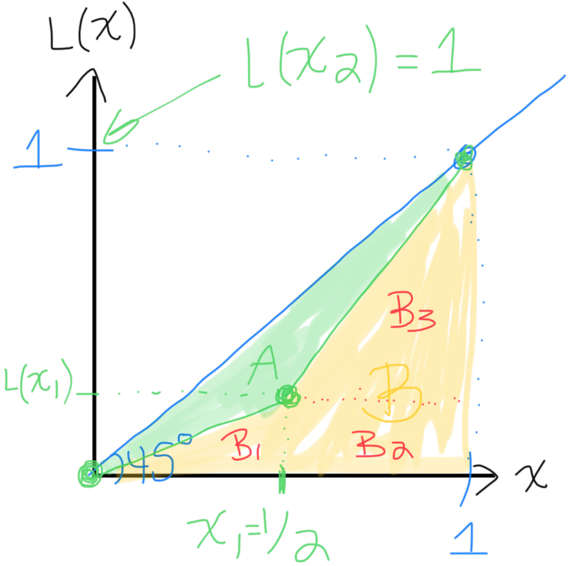
4. Compute the relative mean absolute difference:

$$RMAD = \frac{M}{\bar{Y}} \cancel{\frac{\lambda}{\lambda}}$$

5. Gini is half the $RMAD$:

$$Gini = \frac{1}{2} RMAD$$

Relationship area and Gini formula



Cumulative population: x_i.
Cumulative ~~wealth~~: $L(x_i)$. (Income)
Lorenz curve: $(x_i, L(x_i))$.

With n=2, incomes are $\{y_1, y_2\}$:
$x_1 = \frac{1}{1+1} = 0.5$ and $L(x_1) = \frac{y_1}{y_1+y_2}$.
$Gini = 2A = 1 - 2B$.

$= 1 - 2(B_1 + B_2 + B_3)$

$$\boxed{\frac{y_1}{y_1+y_2}} \quad L(x_1)$$

$$Gini = 2A = 1 - 2(B_1 + B_2 + B_3)$$

$$B_1 = (1/2) * x_1 * L(x_1) = (1/2) * (1/2) * L(x_1)$$

$$B_2 = (x_2 - x_1) * (L(x_2) - L(x_1)) = (1 - 1/2) * L(x_1) = \frac{1}{2} L(x_1)$$

$$B_3 = (1/2) * (x_2 - x_1) * (L(x_2) - L(x_1)) = (1/2) * (1 - 1/2) * (1 - L(x_1))$$

$$B = B_1 + B_2 + B_3 = 1/4 + (1/2) * L(x_1)$$

$$Gini = 1 - 2[1/4 + (1/2) * L(x_1)]$$

$$= 1/2 - L(x_1) = \frac{1}{2} - \frac{y_1}{y_1 + y_2}$$

Relationship area and Gini formula

$$Gini = (1/2) * M/\bar{Y} = (1/2) * (1/\bar{Y}) * \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|$$

For $n = 2$:

$$\begin{aligned}
Gini &= (1/2) * \frac{2}{y_1 + y_2} * (1/4) * [|y_1 - y_2| + |y_2 - y_1|] \\
&= \frac{1}{y_1 + y_2} * \frac{y_2 - y_1}{2} \\
&= \frac{1}{2} * \left(\frac{y_2}{y_1 + y_2} - \frac{y_1}{y_1 + y_2}\right) \\
&= \frac{1}{2} * \left(1 - \frac{y_1}{y_1 + y_2} - \frac{y_1}{y_1 + y_2}\right) \\
&= 1/2 - L(x_1)
\end{aligned}$$

$2(y_2 - y_1)$

$\frac{y_2}{y_1+y_2} = 1 - \frac{y_1}{y_1+y_2}$

4 Principles of an Inequality Index

Suppose the following distribution of income:

$$y = (y_1, y_2, ..., y_i, ..., y_k, ..., y_n)$$

- **1. Weak principle of transfers.** Don't change the ordering

$$y^* = (y_1, y_2, ..., y_i + \delta, ..., y_k - \delta, ..., y_n)$$
$$\boxed{I(y^*)} < \boxed{I(y)}$$

- **2. Decomposability.**

$$y = (g_1(y_1, ..., y_i), g_2(y_{i+1}, ..., y_j), ..., g_m(y_{k+1}, ..., y_n))$$
$$I(y) = I(g_1(.)) + I(g_2(.)) + ... + I(g_m(.))$$

- **3. Scale invariance.**

$$y = (y_1, y_2, ..., y_n)$$
$$y^* = (\lambda y_1, \lambda y_2, ..., \lambda y_n)$$
$$I(y^*) = I(y)$$

- **4. The principle of population.**

$$y = (y_1, y_2, ..., y_n)$$
$$y^* = (y_1, y_1, y_2, , y_2, ..., y_n, y_n)$$
$$I(y^*) = I(y)$$

n

2n

Exercise 6. Taxes,consumption and inequality

Question 1. Define disposable income.

$y_i^d = y_i - \tau_i$

τ_i → Proportional

τ_i → Lump-Sum

TVA, Incom Tax,

Lump-Sum

$y^d = y - \tau y = (1-\tau)y$ ← Prop

$y^d = y - \bar{\tau}$ ← Fixed Amount

Hint: Income net of taxes. $y_i - \tau_i$. Lump-sum or proportional.

Suppose we have the following 10 individuals.

Individual	1	2	3	4	5	6	7	8	9	10
Income	10	15	25	35	60	130	160	200	230	400

For simplicity, we will assume that y_i is the only income of our agents and that there are no other capital income or transfers. We also assume that the government has a constant and exogenous level of **public spending** G. For now, $G = 50$. To pay for its spending, the government levies **taxes** T and always has a **balanced budget** such that $T = G$.

$G = 50 = T$

Question 2. We first assume that the government levies a **lump-sum tax** $\bar{T}$ on all individuals. Compute the **level** of this tax and the resulting disposable income. Is the level of inequality higher, lower or identical before and after the taxes? Construct the disposable income **Lorenz curve** and the disposable income **Gini coefficient**. Comment.

$$G = 50 = T = 10\bar{\tau} \Rightarrow \bar{\tau} = \frac{50}{10} = 5$$

$$y^d = y - \bar{\tau} = 10 - 5 = 5$$

$$y^d = (1-\tau)y = 5, \quad y = 10$$

$$\Rightarrow \tau = 50\%$$

$$y^d = y - \tau = 400 - 5 = 395$$

$$y(1-\tau) = 395$$

$$\tau = -\frac{395}{400} + 1$$

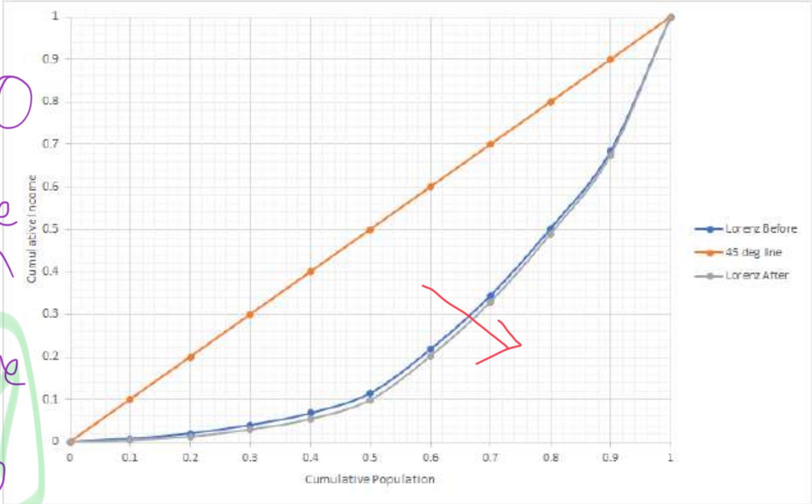
$$\tau = 1.25\%$$

Individual	1	2	3	4	5	6	7	8	9	10
Income	10	15	25	35	60	130	160	200	230	400
After Tax	5	10								395

Hint: $G = 50 = T$. $\bar{T} = 5$. Higher inequality.

$$\frac{\partial \tau(y)}{\partial y} < 0$$

Regressive Taxation



Progressive

$$\frac{\partial \tau(y)}{\partial y} > 0$$

Gini: from 0.5008 to 0.5214

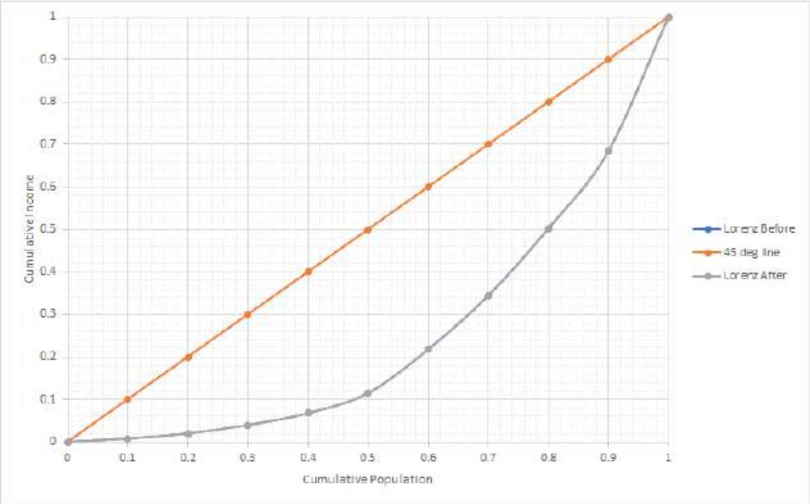
Question 3. We now assume that the government levies a **proportional tax** τ on all individuals. Compute the resulting **tax rate** and disposable income. Is the level of inequality higher, lower or identical before and after the taxes? Construct the disposable income **Lorenz curve** and the disposable income **Gini coefficient**. Comment.

$$G = 50 = T = \sum_{i=1}^{n} \tau y_i = \tau \sum_{i=1}^{n} y_i$$

$$\Rightarrow \tau = \frac{50}{\sum_{i=1}^{n} y_i} = \frac{50}{1{,}265} = 0.0395$$

Individual	1	2	3	4	5	6	7	8	9	10
Income	10	15	25	35	60	130	160	200	230	400
After Tax										

Hint: $\mathcal{T} = 50 = \tau * Y = \tau * 1,265 \rightarrow \tau = 0.0395$. Same inequality.



Gini=0.5008 for both.

Finally, we look at consumption and abstract from taxes. We assume that agents have a constant marginal propensity to consume $\bar{c}$ their income y_i. On top of that, we assume that there can also be a constant non proportional consumption $\bar{C}$ in each period such that individual consumption c_i:

$c_i = \bar{c}y_i + \bar{C}$

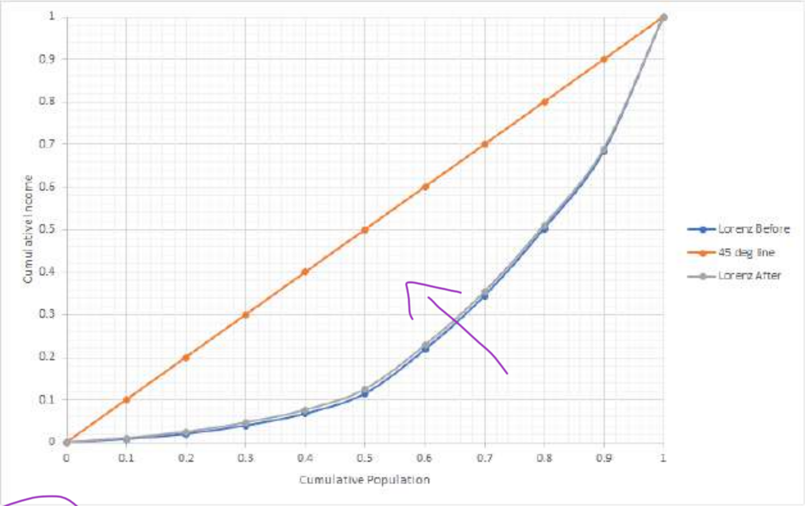
MPC

Question 4. Compute the **consumption distribution** in this economy when $\bar{c} = 0.85$ and $\bar{C} = 3$. Is the level of inequality higher, lower or identical between the income distribution and the consumption distribution? Construct the consumption **Lorenz curve** and the consumption **Gini** coefficient. Comment.

$$C_i = 0.85(10) + 3 = 8.5 + 3 = 11.5 \qquad \bar{C} = 0$$

Individual	1	2	3	4	5	6	7	8	9	10
Income	10	15	25	35	60	130	160	200	230	400
Consumption										

Hint: Lower inequality. Reason: transfer as progressive redistribution.



Gini from 0.5008 to 0.4871975

Exercise 7. Variance based inequality indicators

We consider a population with discrete income distribution $\{y_1, y_2, ..., y_n\}$ ordered such that $y_1 \leq y_2 \leq y_3 ... \leq y_n$.

Question 1. Define the variance V of an income distribution.

$$V = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

Hint: classic definition $V = \frac{1}{n} \sum_{i=1}^{n} [y_i - \bar{y}]^2$.

Question 2. We consider the variance V as an inequality indicator. Derive an **experiment** that show that V is not a good inequality indicator.

$$V_1 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$V_2 = \frac{1}{n}\sum_{i=1}^{n}(\lambda y_i - \lambda \bar{y})^2$$

$$= \lambda^2 \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})$$

$$= \lambda^2 V_1$$

Hint: Multiplicative transformation.

Question 3. We consider a new inequality indicator derived from the variance: $c = \frac{\sqrt{V}}{\bar{y}}$. We call c the **coefficient of variation**. Is it a better inequality indicator than the variance ?

$$C_1 = \frac{\sqrt{V_1}}{\bar{y}_1}$$

$$C_2 = \frac{\sqrt{V_2}}{\bar{y}_2} = \frac{\sqrt{\lambda^2 V_1}}{\lambda \bar{y}_1} = \frac{\cancel{\lambda}\sqrt{V_1}}{\cancel{\lambda}\bar{y}_1}$$

$$= C_1$$

Coeff of Var is Scale Invar.

Hint: Yes. Satisfies scale invariance.

We now consider the following (discrete) income distribution:

Individual	1	2	3	4	5	6	7	8	9	10
Gross Income	1	5	11	35	40	45	50	90	95	100

Question 4. Compute the **coefficient of variation** for this income distribution.

Hint: $\bar{y} = 47.1$, $V = 1241.69$, and $c = \frac{\sqrt{V}}{\bar{y}} = 0.7481448$.

Question 5. The government considers **3 alternative transfer schemes**. The first scheme is to take 1 from the agent earning y_{10} to give it to the agent earning y_8. The second scheme is to do the same between y_7 and y_5. And the last scheme is to do the same between y_3 and y_1. According to the *coefficient of variation*, which policy **reduces inequality** the most? Comment.

Individual	1	2	3	4	5	6	7	8	9	10
Gross Income	1	5	11	35	40	45	50	90	95	100
Scheme 1	1	5	11	35	40	45	50	91	95	99
Scheme 2	1	5	11	35	41	45	49	90	95	100
Scheme 3	2	5	10	35	40	45	50	90	95	100

Weak Principle of transfers

$$C > \underbrace{C_1 = C_2 = C_3}$$

Hint: $c = 0.74\boxed{381}$ for every scheme (principle of transfers).

Question 6. We consider the same transfer policies as in the previous question but now we want to **evaluate the change in inequality** level using the Gini index. Compare the policies using the Gini index.

Hint: Same $Gini = 0.41568$ (principle of transfers).