Topics in Macro 2

Week 6 - Second Part - TD 1 and TD 2

Oscar Fentanes
www.oscarfentanes.com

TSE

Tuesday (17:00-18:30)

TD Second Part: Fiscal Multipliers (4 Weeks)

Part I

- Exercise I: Habit Persistence and The Keynesian Multiplier (Week 1)
- Exercise II: A Benchmark Model (Week 1)
- Exercise III: Consumption, Labor Supply and the Multiplier

Part II

- Exercise I: Taxes on the Labor Input and the Multiplier
- Exercise II: Public Spending in Utility Function and the Multiplier
- Exercise III: Labor Supply, Public Spending in Utility and the Multiplier

Part III

- Exercise I: Endogenous Public Spending
- Exercise II: Externality in Production and the Multiplier
- Exercise III: Externality in Labor Supply and the Multiplier

Objectives

1. What is a Fiscal multiplier?
2. What is the lag operator
3. What is log-linearization of a function

Exercise I: Habit Persistence and The Keynesian Multiplier

Model with habit persistence in consumption

Consumption:

$$C_t = \lambda C_{t-1} + c(1-\lambda)(Y_t - T_t)$$

Aggregate resources:

$$Y_t = C_t + G_t$$

Lag operator:

$$LC_t = C_{t-1}$$

Question 1. Assume first that $T_t = 0$. Using the lag operator L, determine the equilibrium output as a function of G_t. More precisely, determine the function $B(L)$, such that equilibrium output is expressed as:

$$Y_t = B(L)G_t$$

Answer: $Y_t = \frac{1-\lambda L}{1-c(1-\lambda)-\lambda L} G_t$

Question 2. Compute the short run multiplier. This multiplier is obtained by imposing $L = 0$ in the representation $Y_t = B(L)G_t$.

Answer: $B(0) = \frac{1}{1-c(1-\lambda)}$. Maximum for $\lambda = 0$.

Question 3. Compute the long run multiplier. This multiplier is obtained by imposing $L = 1$ in the representation $Y_t = B(L)G_t$.

Answer: $B(1) = \frac{1-\lambda}{1-c(1-\lambda)-\lambda} = \frac{1}{1-c}$

Question 4. Discuss the two obtained government spending multipliers.

$$B(0) = B(1) \text{ if } \lambda = 0 \text{ and}$$
$$B(0) < B(1) \text{ if } \lambda > 0$$

Question 5. Assume now that $G_t = T_t$. Using the lag operator L, determine the equilibrium output as a function of G_t. More precisely, determine the function $B(L)$, such that equilibrium output is expressed as $Y_t = B(L)G_t$. Compute the short run and long-run multipliers. Discuss.

Answer: $Y_t = G_t$. Fiscal multiplier $B(L) = 1 = B(0) = B(1)$.

Exercise II: A Benchmark Model

Log-linearization in 3 steps

1. Take logs

$$y_t = F(x_t)$$

$$log(y_t) = log(F(x_t))$$

2. 1st order Taylor expansion around the steady state

$$log(y_t) \approx log(\bar{y}) + \frac{1}{\bar{y}} * (y_t - \bar{y})$$

$$log(F(x_t)) \approx log(F(\bar{x})) + \frac{F'(\bar{x})}{F(\bar{x})} * (x_t - \bar{x})$$

3. Express everything in deviations

$$\hat{x}_t = \frac{x_t - \bar{x}}{\bar{x}}$$

Example 1

Production function $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$.

Example 2

Resource constraint: $y_t = c_t + g_t$.

Benchmark model

Representative household:

$$U_t = \ln c_t - \frac{\eta}{1+\nu} n_t^{1+\nu}$$

Budget constraint:

$$c_t \leq w_t n_t - T_t + \pi_t$$

Technology:

$$y_t = a n_t^{\alpha}$$

Profits:

$$\pi_t = y_t - w_t n_t$$

Taxes:

$$T_t = g_t$$

Market clearing condition:

$$y_t = c_t + g_t$$

Question 1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS).

Answer: $\frac{\eta n_t^{\nu}}{1/c_t} = w_t$.

Question 2. Determine the optimality condition of the firm.

Answer: $a\alpha n_t^{\alpha-1} = w_t$.

Question 3. Determine the equilibrium output.

Answer: Solution to $\eta \left(\frac{y_t}{a}\right)^{\frac{\nu+1}{\alpha}} = \frac{\alpha y_t}{y_t - g_t}$.

Question 4. Compute the log-linearization of equilibrium output around the steady-state.

Answer: $\left(\nu + 1 + \frac{\alpha s_g}{1-s_g}\right)\widehat{y}_t = \frac{\alpha s_g}{1-s_g}\widehat{g}_t.$

Question 5. Compute the output multiplier and discuss the value of this multiplier with respect to ν and α.

Answer: $\frac{dy_t}{dg_t} = \frac{\alpha s_g}{(1-s_g)(\nu+1)+\alpha s_g}$.

Question 6. Compute the consumption multiplier and discuss the value of this multiplier with respect to ν and α.

Answer: $\frac{d\hat{c}_t}{d\hat{g}_t}$.

Taylor Approximation (Wikipedia)

Definition

The Taylor series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots,$$

where $n!$ denotes the factorial of n. In the more compact sigma notation, this can be written as

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n,$$

where $f^{(n)}(a)$ denotes the nth derivative of f evaluated at the point a. (The derivative of order zero of f is defined to be f itself and $(x-a)^0$ and $0!$ are both defined to be 1.)

For example, for a function $f(x,y)$ that depends on two variables, x and y, the Taylor series to second order about the point (a, b) is

$$f(a,b)+(x-a)f_x(a,b)+(y-b)f_y(a,b)+\frac{1}{2!}\Big((x-a)^2f_{xx}(a,b)+2(x-a)(y-b)f_{xy}(a,b)+(y-b)^2f_{yy}(a,b)\Big)$$

Lag polynomial

$y_t = \rho y_{t-1} + x_t$ (AR(1) Process) with $\rho \in (0,1)$

$\Rightarrow y_t - \rho y_{t-1} = x_t \Rightarrow (1-\rho L) y_t = x_t \Rightarrow \boxed{y_t = (1-\rho L)^{-1} x_t}$

But what is $(1-\rho L)^{-1}$?

$$
\begin{aligned}
y_t &= \rho y_{t-1} + x_t \\
&= \rho[\rho y_{t-2} + x_{t-1}] + x_t \\
&= \rho^2 y_{t-2} + \rho x_{t-1} + x_t \\
&= \rho^2[\rho y_{t-3} + x_{t-2}] + \rho x_{t-1} + x_t \\
&= \rho^3 y_{t-3} + \rho^2 x_{t-2} + \rho x_{t+1} + x_t \\
&\vdots \\
&= \rho^k y_{t-k} + \rho^{k-1} x_{t-k-1} + \cdots + x_t \\
&= \rho^k y_{t-k} + \sum_{\tau=0}^{k-1} \rho^\tau x_{t-\tau}
\end{aligned}
$$

If $k \to \infty$ $\rho^k \to 0$

$\Rightarrow \boxed{y_t = \sum_{\tau=0}^{\infty} \rho^\tau x_{t-\tau}}$

And $\sum_{\tau=0}^{\infty} \rho^\tau x_{t-\tau} = \sum_{\tau=0}^{\infty} \rho^\tau L^\tau x_t$

So we have:

$\boxed{(1-\rho L)^{-1} = \sum_{\tau=0}^{\infty} \rho^\tau L^\tau}$