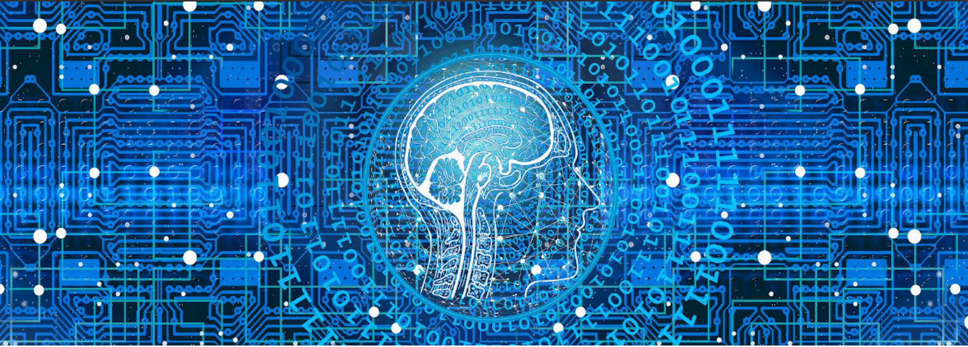


Chapter 11: Fuzzy Logic

Fuzzy-Logik - An Introduction by Mathias Bank and Kimia Lab - Machine Intelligence - Lecture 17

Content

- What is Fuzziness
- Fuzzy Sets
- Fuzzy Logic
- Reasoning with Fuzzy

The origins of fuzziness

- In contrast to precise, limited and constrained language that we use to describe notions, entities, and concepts while building logical models (so far), the real-life concepts and entities are described in much less rigid way.
- Let's consider the following example. In real life sentence: **John is a tall guy**
  - may mean many things, depending of our perspective, the place we live (meaning of tall is different in e.g. Japan) and so on.
  - But, if we want to feed John's data into computer, we have to determine his height precisely – say 190 cm.
  - But what if do not know John's height exactly?
- In real life we are doing perfectly all right with the sentences like:
- It takes about 40 minutes to reach an airport if the traffic is not too heavy.
- But what if we want a computer to understand such a sentence? How do we represent about and too heavy in a machine?

Fuzzy concepts and fuzzy sets

- In 1965 Lotfi Zadeh proposed a different way of looking at notions such as: set, containment, subset. His target was to make it possible to deal with concepts (sets) and dependencies that by nature are imprecise and vague - so called fuzzy concepts (sets).
- Again, the example of such concept is the natural language sentence:
- **John is a tall guy.**
- If we know, that John is 175 cm tall, we may start to wonder about the validity of the above sentence.
- In classical set theory we are forced to make definite decision whether 175 cm qualifies John as tall or not.
- In the fuzzy set theory we may be more subtle and express to what degree 175 cm of height makes John a tall guy.

Fuzzy sets

In classical set theory and with classical binary logic, that we usually employ when doing things with use of computer, the (contents of a) set A in some universe X can be expressed in the form of its characteristic (containment) function:
Such classical, rigidly defined set we will further call crisp or definite. The key step in defining fuzzy set theory is the replacement of characteristic function χ_A by function $\mu_A : X \Rightarrow [0, 1]$
μ_A is called membership function or fuzzy membership.
If

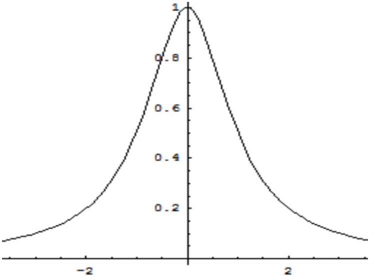
$$\chi_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \\ 0 & \text{if} \quad x \notin A \end{cases}$$

then A is a classical set i.e., crisp (definite) set. If there exists $x \in X$ such that $0 < \mu_A(x) < 1$ - the set A is fuzzy.

Fuzzy sets - examples

A classical example of a fuzzy set near - zero provided by Zadeh for the concept of real number near 0. This set may be defined, for example, by the following membership function:

$$\mu_{near-zero} = \frac{1}{1+x^2}$$



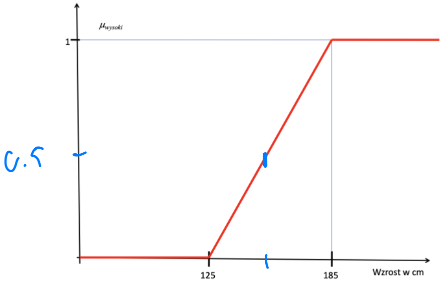
10° 20° 30°

Fuzzy sets - examples

The previously considered notion of tall guy could be given – for height x in centimetres – by membership:

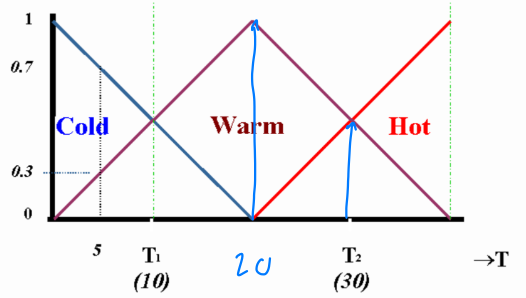
$$\mu_{tall} = \begin{cases} 0 & \text{if} \quad x \leq 125 \\ 1 & \text{if} \quad x \geq 185 \\ \frac{x-185}{2} + 1 & \text{if} \quad 125 < x < 185 \end{cases}$$

:



Fuzzy sets - examples

- Another example of three fuzzy sets for the notion of cold, warm and hot, where x is a temperature.



Height of a fuzzy set

The height of a fuzzy set is the largest degree of membership possessed by all elements of the fuzzy set. Thus, for a fuzzy set μ over a basic set A, its height is defined as:

$$H(F) = \max\{\mu(x) : x \in X\}$$

Accordingly, two-valued logic works with sets whose height is 1.

Comming from Temperature to Words (revisited)

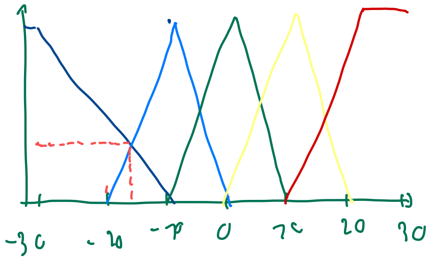
Fuzzy Set

$\boxed{\text{Temperatur}}$

Linguistik Syntax

{ very cold, cold, cool, warm, hot }   Set of term

Semantic



very cold   cold

Interpretation von -15°   [0.3, 0.7, 0, 0, 0]

Fuzzy Sets (revisited)

$$A = \{ x \mid x \in X \text{ und } x \text{ ist } [illegible] \}$$

$$A_{\text{Fuzzy}} = \{ (x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0, 1] \}$$

Membership function μ_A quantifies the degree of belonging ness of x to A

μ_A is known (bright, old, cold)

Rules in 'common' logic and approximation as decision trees

Modus Ponens

Rule: If A is true, then B is true

Observation: A is true

Conclusion: B is true

Modus Tollens:

Rule: If A is true, then B is true

Observation: B is false

Conclusion: A is false

Interpretation of Rules in Fuzzy Logic

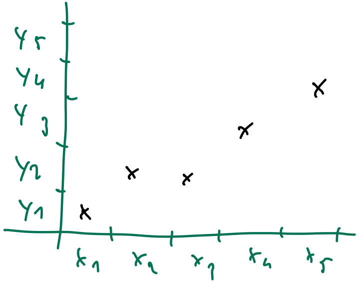
If x_1, then y_1

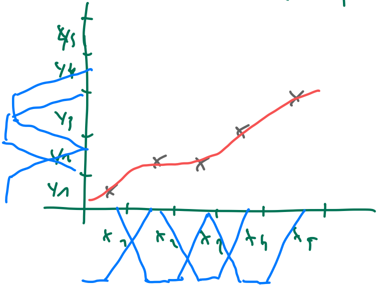
If x_2, then y_2

If x_3, then y_2

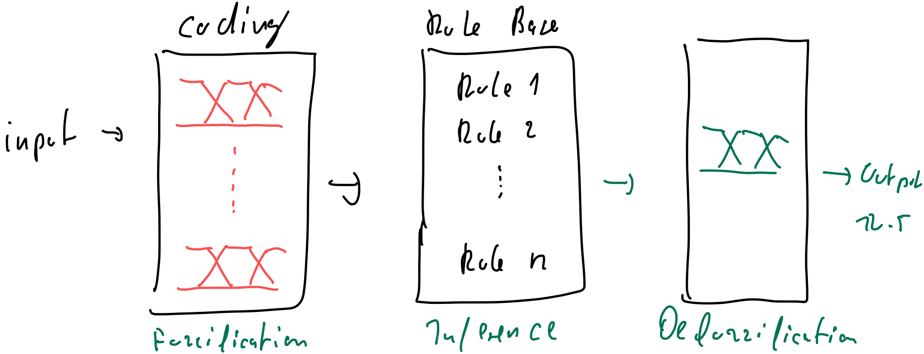
⋮

Set rules



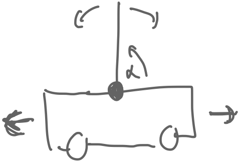


Fuzzification, Inference, Defuzzification



22° = [0,75, 0,25, 0,0,0]

Example: The inverted Pendulum



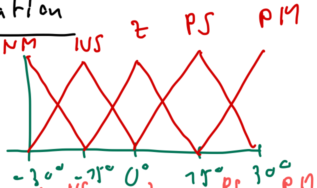
inputs: 1) error (difference) in rad

2) omega (angular velocity)

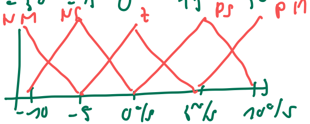
output: current (+/- for direction)

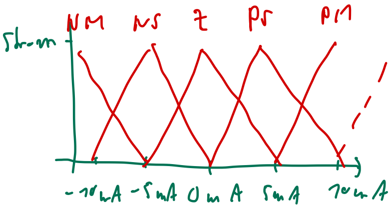
Fuzzification

Error



Omega





Example: Rules 1

$\Delta\Theta$ \ Θ	NL	NM	NS	ZE	PS	PM	PL
NL				PL			
NM				PM			
NS			PM	PS	Z		
ZE	PL	PM	PS	Z	NS	NM	NL
PS			Z	NS	NM		
PM				NM			
PL				NL			

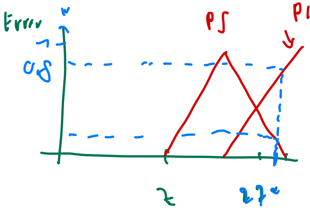
Example: Rules 2

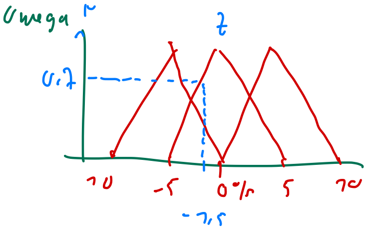
ΔΘ \ Θ	NL	NM	NS	ZE	PS	PM	PL
NL				PL			
NM				PM			
NS				PS	NS		
ZE	PL	PM	PS	ZE	NS	NM	NL
PS			PS	NS			
PM				NM			
PL				NL			

Example: Calculate output

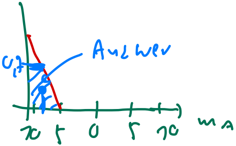
Input: Error $= 27^o$, Omega$=-1.5^o/s$, Current?

Lets look at the rule: If error is PM and Omega is Z, then the current is NM .





Defuzzification:
Center of Gravity



-6.5mA

true, true ⇒ true	1, 1	⇒ 1	min(x_1, x_2)
true, false ⇒ false	1, 0	⇒ 0	
false, true ⇒ false	0, 1	⇒ 0	
false, false ⇒ false	0, 0	⇒ 0	

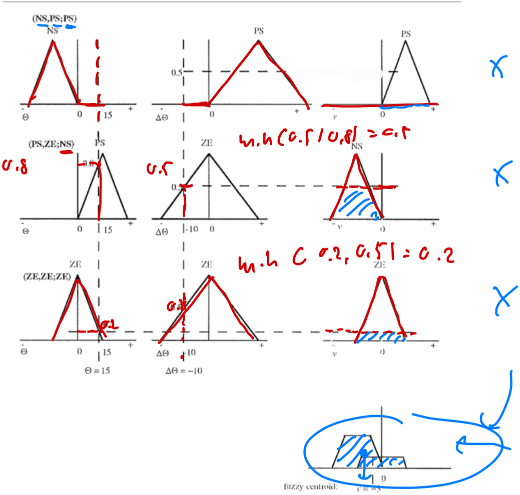
$$AND = \min(0.8,\ 0.7) = 0.7$$

Example: Calculate output when taking all rules into account

$\Theta = 15°$

$w = -10°/s$

$\Delta\Theta$



center of Gravity

Example: Calculate output when taking all rules into account

Correlation-minimum fuzzy inference procedure.

- All rules are activated in parallel, but to varying degrees.
- The degree depends on how well the input value matches the membership function on the left- hand side (input side, antecedent side).
- The first rule is not applied since the input values $\theta = 15$ and $\Delta theta = -10$ do not intersect any of their membership functions.
- Next rule: the membership function for PS is intersected by $\theta = 15$ at 0.8, the one for ZE is intersected by $\Delta theta = -10$ at 0.5.
- These values are combined by using the minimum (logically speaking the AND) function: $min(0.8, 0.5) = 0.5$. Thus, this rule is applied to degree 0.5.
- This degree of application of the rule is propagated to the output side as shown geometrically. Similarly for the third rule.
- Now take maximum over all consequent sides of the rules (shaded areas): get fuzzy centroid at bottom. This is the geometrical output function that we are looking for. This is a kind of min- max procedure.

Summary

- Fuzzy Logic successful in many applications (in particular controlling)
- takes intrinsic uncertainty of real world into account
- fundamental difference between probabilistic and fuzzy logic thinking
- fuzzy sets differ from classical crisp sets by their membership function
- five steps in development of fuzzy rule system
- fuzzy inference procedure apply all rules in parallel to varying degrees