Diffusion processes in the extracellular space

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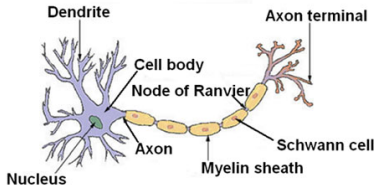# Contents

Brief introduction to the brain
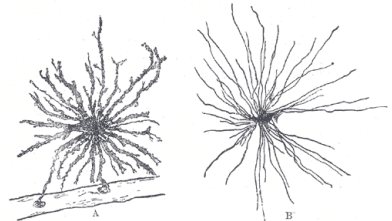
Cells in the brain

Neurons:

- Signal processing

Structure of a Typical Neuron

Neuroglia:

- Janitorial tasks

The extracellular space

- Space surrounding neurons and neuroglia
- Accounting for $\sim 20\%$ of total brain volume
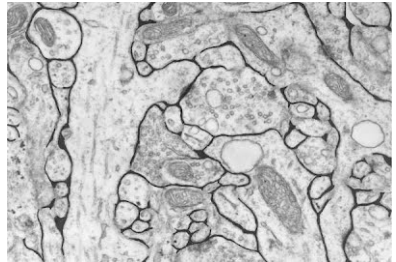- Important for transport of nutrients, medicines etc.

Figure: Extracellular space marked as dark grey.

Basic diffusion

The basic diffusion equation reads

$$\frac{\partial C}{\partial t} = D\nabla^2 C \tag{1}$$

Einstein relations

$$D = \frac{k_B T}{6\pi\eta r} \tag{2}$$

$$\langle r^2 \rangle = 2dDt \tag{3}$$

Diffusion in ECS

Network simulations:

- Verification against experimental results

- Local field potential

- Extracellular conductance

$$\sigma = \frac{cq}{k_B T} D \qquad (4)$$

$$\nabla \cdot (\sigma(\mathbf{r}) \nabla \phi(\mathbf{r}, t)) = -C(\mathbf{r}, t)$$

Modified diffusion equation

A modified version of the basic diffusion equation is needed to account for

- Sources
- Uptake of diffusing molecules
- evt. bulk flow (absent below)

This new equation reads

$$\frac{\partial C}{\partial t} = D^* \nabla^2 C + \frac{s}{\alpha} - k' C \quad (5) \qquad\qquad \lambda = \sqrt{\frac{D}{D^*}} \qquad (6)$$

Numerical simulations

This will be addressed in the Molecular Dynamics part.

Optical measurements

- Developed in 1993 by Nicholson and Tao
- Based on thin slices and macromolecules with fluorescent label
- Problems with slice thickness and photobleaching
- Quantum dot IOI

TMA$^+$ measurements

- Micro-pipette
- Ion sensitive micro-electrodes a known distance away ($\sim 100\mu$m)
- Measures concentration curve at the electrode position
- Time resolution as well as spatial resolution
- Some refinements possible

Radiotracer measurements

- "Oldest trick in the book" - most intuitive
- Good method, but slightly out dated due to radioactivity
- Results still used as verification
- Offers independent reference and cross-species data

Diffusion Tensor Imaging

- Non invasive method based on MRI
- Offers the entire diffusion tensor, but not necessarily of the ECS
- Calculates the gradient of the diffusion tensor in a voxel and reconstructs the tensor using six different gradients
- Method has clear advantages over the other ones, but disadvantages as well
  - Very complicated machinery required
  - Expensive

Molecular dynamics

- Study of systems of atoms and their time-development
- Most research towards fracture mechanics and flow in tight rocks
- Much of the geometry is similar, but the length scale is a bit to small
- Dissipative fluid dynamics

My experiment - motivation

- Results from 2003 article by Hrabětová and Nicholson
- Max value of tortuosity $\lambda \leq 1.225$
- Diffusion modeling on regular geometries
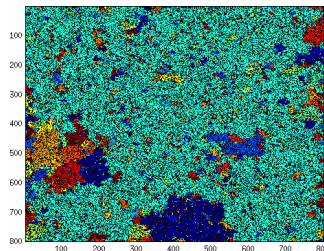

trunc. octahedra
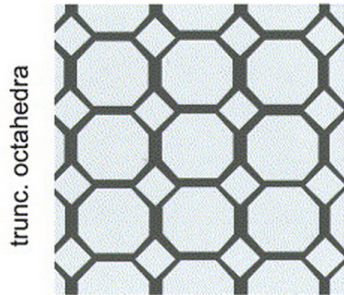
My experiment - results

- Making spheres of stationary atoms
- Measuring self diffusion constant of liquid using Einstein relation
- Comparing to self diffusion constant of bulk fluid
- Found $\lambda \approx 1.41$
- Limitations
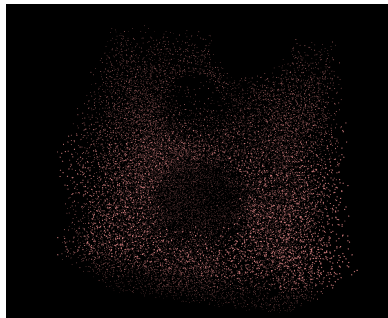
Random walks

- Percolation theory
- Random walks and diffusion
- Spanning cluster
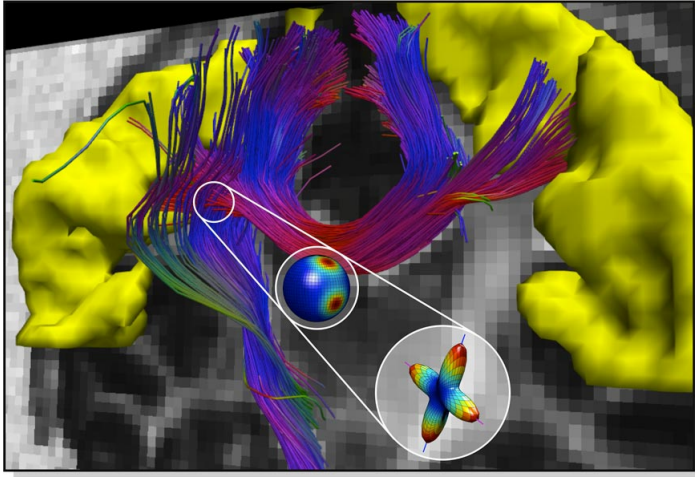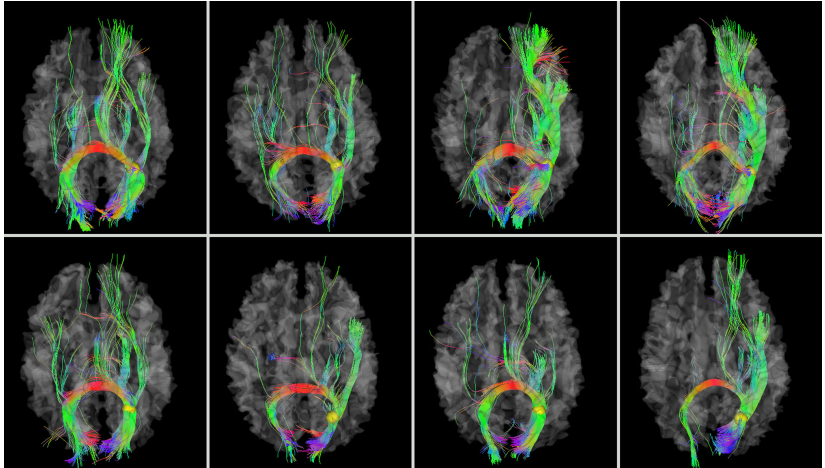- Results from by Hrabětová and Nicholson
- Limitations

Outlook

- Unanswered questions
  - limitation of tortuosity
  - tortuosity constant for increasing α

- Other modeling methods

- Multi scale models; the best from both worlds?

Thank you!
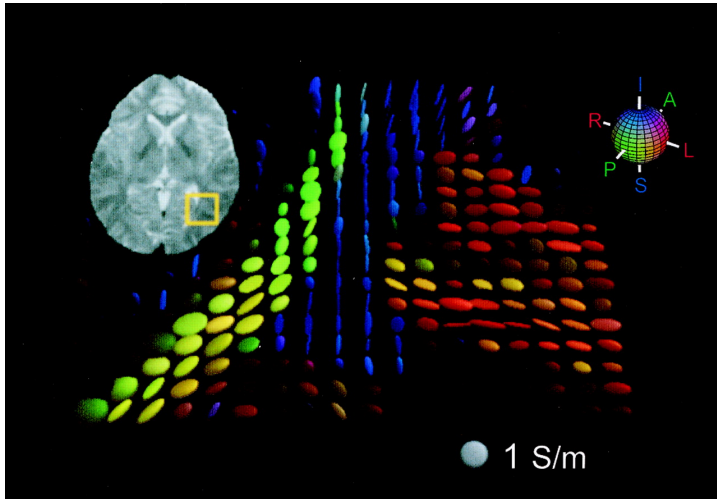
Output from DTI

Output from DTI

Output from DTI

Illustration of voxels