Nonlinear diffusion equation with finite elements

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Abstract

In this project we will look at a simple linear second order differential equation.

About the problem

In this project we will solve a nonlinear diffusion equation using the finite element method and the FEniCS software package. More precicely we will be looking at the equation

$$\rho u_t = \nabla \cdot (\alpha(u)\nabla u) + f(\mathbf{x}, t) \tag{1}$$

where ρ is a constant, $\alpha(u)$ is a known function of u, and where we have the initial condition $u(\mathbf{x}, 0) = I(\mathbf{x})$, and Neuman boundary conditions $\frac{\partial u}{\partial n} = 0$.

Discretization

In order to solve equation (1) we will need to discretize it. The standard approach is then to pick a nummerical approximation to the derivatives. We will use a finite difference discertization in time, more specifically the Backward Euler (BE) discretization, and a finite element approximation in space. The BE discertization gives us

$$\rho u^n = \Delta t \nabla \cdot (\alpha(u)\nabla u) + \Delta t f(\mathbf{x}, t) + \rho u^{n-1}$$
$$\rho u^n - \Delta t \nabla \cdot (\alpha(u)\nabla u) - \Delta t f(\mathbf{x}, t) - \rho u^{n-1} = R$$

we now approximate R on a functionspace V and minimize the error by demanding

$$\big(R, v\big) = 0 \quad \forall\, v \in V$$

Source code

Analytic solution

Results

Final comments