

Project 3 - Percolation

FYS4460

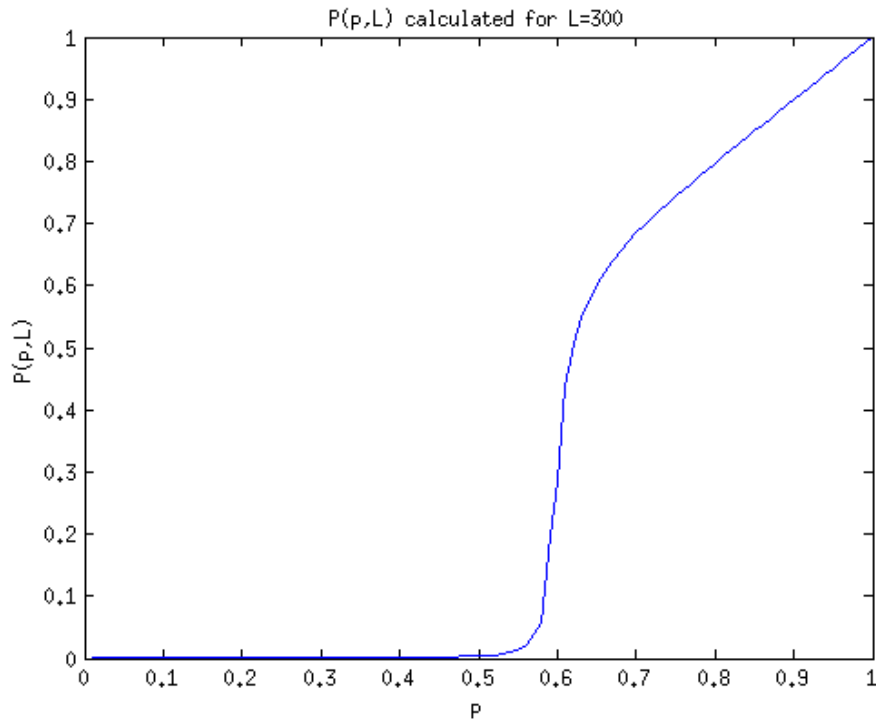
Fredrik E Pettersen

April 24, 2013

a

Starting out with a uniform random matrix which we transform to a porous matrix consisting of either filled (1) or empty (0) places we will now have clusters of connected, filled places. The ammount of filled and empty places depends on our chosen porosity for the random matrix. An example of a porous, labelled matrix is shown in figure ??.

We want to find the probability distribution for a site in the porous matrix to be in a spanning cluster. A spanning cluster is a cluster of connected sites which makes a path from one side of the matrix to the other. This can be done by finding the area of the spanning cluster, and divide it by the area of the entire matrix. We find the spanning cluster by labelling the connected clusters and checking if the same labels can be found on each side of the matrix. The resulting “distribution” is shown in figure . This method lets me calculate at least for any rectangular matrix boundary, though I do not know how periodic boundary conditions will affect the calculations.



b

For $p > p_c$ we have that $P(p, L) \propto (p - p_c)^\beta$ ($p_c = 0.59275$). We can find β by doing a double-logarithmic plot of our measured $P(p, L)$ versus $(p - p_c)$.

1 c

We are now gives a collection of random numbers on the form

$$z = \text{rand}(1\text{e}6, 1) \cdot (-2);$$

and area asked to find the distribution function $f_z(z)$ using that $f_z(z) = \frac{dP(Z > z)}{dz}$ and that we know f is on the form $f_z(z) \propto z^\alpha$. We can plot the cumulative distribution function $P(Z > z)$ (see figure INSERT REFERENCE) and use this to calculate the distribution function. Since

$$z^\alpha \propto \frac{dP(Z > z)}{dz} \implies \int z^\alpha dz \propto \int dP(Z > z)$$

$$\frac{1}{\alpha + 1} z^{\alpha+1} \propto P(Z > z) \implies (\alpha + 1) \log(z) \propto \log(P(Z > z))$$

So we see that we are actually measuring $\alpha + 1$ if we do a double logarithmic plot of z and the cumulative distribution function. From this plot (figure 1) we find $\alpha = -3/2$. In figure ?? we have plotted the measured cumulative distribution function and the modelled $f_z(z) = z^\alpha$. We see that there is a good fit between the two graphs.

