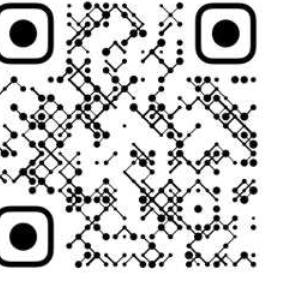


# EquiPy: A Python Package for Sequential Fairness using Optimal Transport with Applications in Insurance

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## MOTIVATION: SEQUENTIAL FAIRNESS

### Insurance regulation

- In certain regions of Canada and the United States (US), discrimination based on **multiple sensitive attributes** (MSA), such as **ethnic origin** ( $A_1$ ), **gender** ( $A_2$ ) and **age** ( $A_3$ ), is prohibited for **insurance pricing**.
- Due to **proxy variables**, eliminating MSA from predictive models does not guarantee fair premiums.

→ There is a need for an approach to **evaluate and mitigate unfairness** in model predictions among groups with shared MSA, thereby ensuring **group fairness**.

**Strong Demographic Parity** Given a model  $f$ , let  $F_f$  denote the distribution of  $f(\mathbf{X}, \mathbf{A})$  and  $F_{f|a_i}$  denote the conditional distribution of  $f(\mathbf{X}, \mathbf{A})|A_i = a_i$ , with  $\mathbf{X} \in \mathcal{X}$  corresponds to ‘non-sensitive’ features and  $\mathbf{A} = (A_1, A_2, A_3) \in \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$ .

- $f$  is strongly fair regarding a **single sensitive attribute** (SSA)  $A_i$ , if and only if: 
$$\mathcal{U}_i(f) = \max_{a_i \in \mathcal{A}_i} \text{distance}(F_f, F_{f|a_i}) = 0$$

- Here,  $f$  is **strongly fair regarding MSA**, if and only if:

$$\mathcal{U}(f) = \mathcal{U}_1(f) + \mathcal{U}_2(f) + \mathcal{U}_3(f) = 0$$

→ **Wasserstein distance** is employed to compute the distance between distributions.

**Mitigating biases for MSA** When  $\mathcal{U}(f) \neq 0$ , the goal is to **correct unfairness** while preserving good model performance. One effective approach is employing **Optimal Transport**, specifically using **Wasserstein barycenter**, as it maintains the order of individuals within groups in terms of risk.

**Sequential fairness** Hu et al. (AAAI-2024) introduce **sequential correction** of unfairness related to **MSA**, using this approach. Unlike **intersectional fairness** this sequential approach allows for:

- interpretability across sensitive features,
- easily adding sensitive attributes to meet changing regulatory demands.

## 2. EQUIPY: METHODOLOGY

**Objective** **EquiPy** is a Python package applying sequential fairness across a set of MSA by transforming predictions from a **regression** or scores from a **binary classifier**, denoted  $f(\mathbf{X}, \mathbf{A}) \in \mathbb{R}$ , into fair ones  $f_B(\mathbf{X}, \mathbf{A})$ .

### Notation

- $\mathbf{A} = (A_1, \dots, A_r) = A_{1:r} \in (\mathcal{A}_1 \times \dots \times \mathcal{A}_r)$ , the sequence of  $r$  **sensitive attributes**,
- $Y$  is the response variable.

**Theoretical results** Fair predictions,  $f_B(\mathbf{X}, \mathbf{A})$ , are defined by:

$$f_B = \arg \inf_f \{\mathcal{R}(f) : \mathcal{U}(f) = 0\}$$

$$\mathcal{R}(f) := \mathbb{E}[Y - f(\mathbf{X}, \mathbf{A})]^2 \quad (\text{risk metric})$$

$$\mathcal{U}_i(f) := \max_{a_i} \mathcal{W}_1(\nu_f, \nu_{f|a_i}) \quad (\text{unfairness regarding } A_i)$$

$$\mathcal{U}(f) := \mathcal{U}_1(f) + \dots + \mathcal{U}_r(f) \quad (\text{unfairness regarding } \mathbf{A})$$

**SSA** ( $r = 1$ ) Given the results in Chzhen et al. (2020),  $f_B$  is computed as:

$$\forall (\mathbf{x}, a_i) \in \mathcal{X} \times \mathcal{A}_i, f_B(\mathbf{x}, a_i) := \left( \sum_{a'_i \in \mathcal{A}_i} \mathbb{P}(A_i = a'_i) Q_{f|a'_i} \right) \circ F_{f|a_i}(f(\mathbf{x}, a_i))$$

calculated with  $\mathcal{W}_2$ -Barycenter between all  $f|a_i$  for  $a_i \in \mathcal{A}_i$ .

**MSA** ( $r > 1$ ) Hu et al. establish that  $f_B$  can be expressed as:

$$\forall (\mathbf{x}, \mathbf{a}) \in \mathcal{X} \times \mathcal{A}_{1:r}, f_B(\mathbf{x}, \mathbf{a}) := f_{B_1} \circ f_{B_2} \circ \dots \circ f_{B_r}(\mathbf{x}, \mathbf{a})$$

$$f_{B_i} \circ f_{B_j}(\mathbf{x}, \mathbf{a}) = \left( \sum_{a'_i \in \mathcal{A}_i} \mathbb{P}(A_i = a'_i) Q_{f_{B_j}|a'_i} \right) \circ F_{f_{B_j}|a_i}(f_{B_j}(\mathbf{x}, \mathbf{a}))$$

with the  $i$ -th component of  $\mathbf{a}$  denoted  $a_i$ .

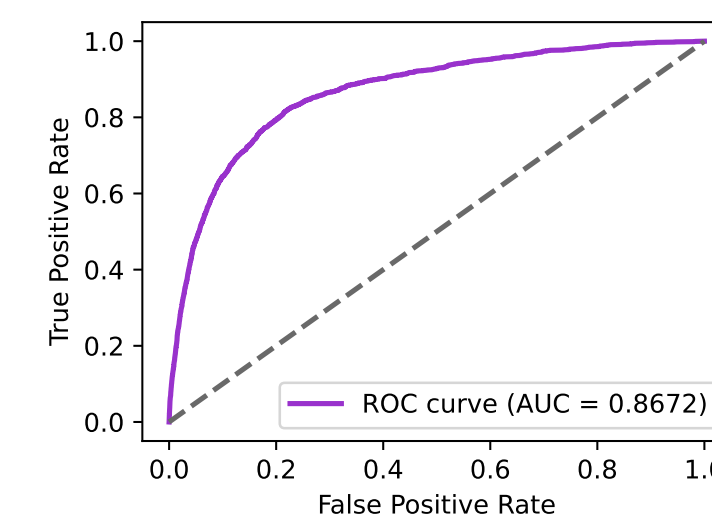
→ Fairness mitigation remains **unaffected by the order of  $A_{1:r}$** , given the property of **associativity** of Wasserstein barycenters.

## 3. LIFE INSURANCE DATA

**Description** The dataset, from the public SEER database, studies the mortality of US individuals with **melanoma skin cancer**. There are  $n = 547,878$  observations from 2004 to 2018, and 16 features describing patient characteristics (age, **gender**, **origin**) and cancer attributes (tumor size, extent).

**Predictive modeling** Utilizing the methodology presented in Sauce et al. (2023), we convert the dataset into **survival data**. Subsequently, we predict the **one-year mortality** while accounting for exposure over the given time interval:

- Split the data into train and test sets,
- Fit **Logistic Regression**  $f$  using exposure as sample weights,
- Apply  $f$  on the test set.



→ **Model-agnosticity**: EquiPy employs a **post-processing methodology** to attain fair predictions, originating from any Machine Learning model.

## 4. APPLICATION IN INSURANCE

### EquiPy import

from equipy import fairness, metrics, graphs → **3 modules**

### Fairness module

- Split the test set into **calibration** and **test** sets,
- SSA**: `x_ssa_calib` and `x_ssa_test` contain values for a unique sensitive attribute ( $A_1$ : **gender** or **origin**)  

```
wst = FairWasserstein()
wst.fit(scores_calib, x_ssa_calib)
y_ssa_fair = wst.transform(scores_test, x_ssa_test)
```
- MSA**: `x_msa_calib` and `x_msa_test` contain values for both sensitive attributes ( $A_1$ : **origin**,  $A_2$ : **gender**)  

```
wst = MultiWasserstein()
wst.fit(scores_calib, x_msa_calib)
y_msa_fair = wst.transform(scores_test, x_msa_test)
```

### Metrics module

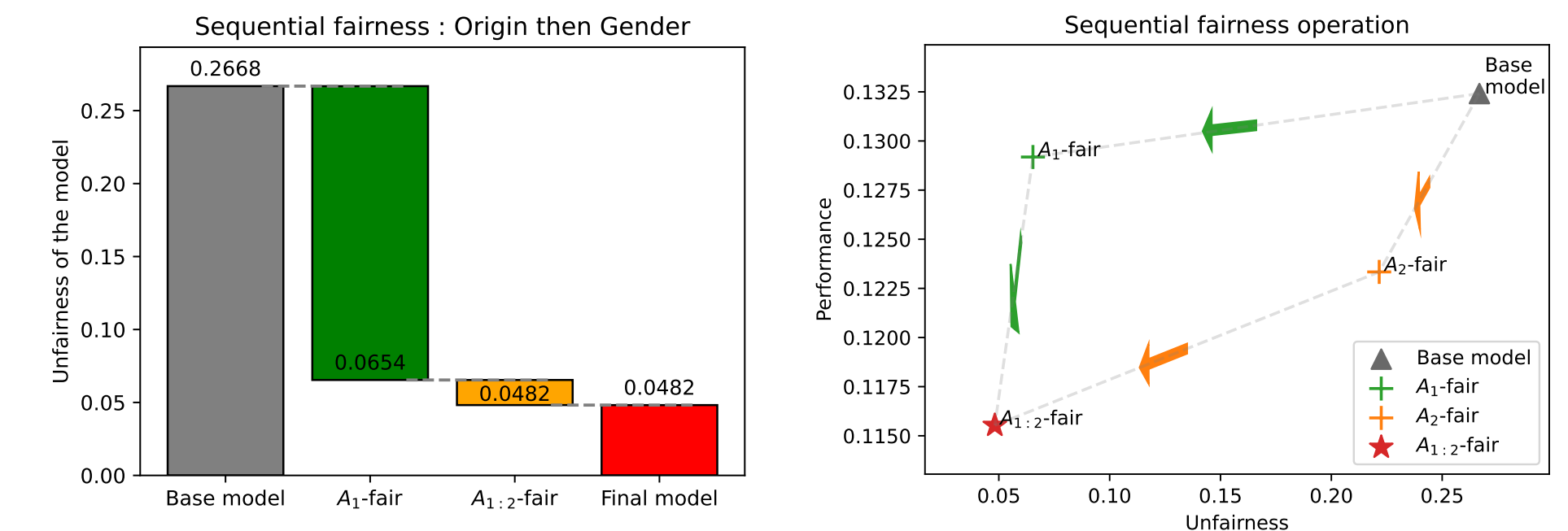
`unfairness(y_msa_fair, x_msa_test) → 0.0482`

`y_pred_fair = (y_msa_fair > 0.102).astype(int)`

`performance(y_pred_fair, y_true_test, f1_score) → 0.1155`

**Graphs module** Visualization of unfairness and metrics calculations:

- `fair_density_plot`: representation of  $f|a_i, a_i \in \mathcal{A}_1 \cup \mathcal{A}_2$ ,
- `fair_waterfall_plot`: sequential gain in fairness,
- `fair_multiple_arrow_plot`: fairness-performance relationship.



Fairness step	Unfairness in <b>origin</b>	Unfairness in <b>gender</b>
Base model	<b>0.2371</b>	0.0297
<b>Origin</b>	<b>0.0345</b>	0.0309
<b>Origin &amp; Gender</b>	<b>0.0469</b>	0.0013

→ **Prioritization across attributes** using **approximate fairness**:  
`wst.transform(scores_calib, x_msa_calib, epsilon = [0, 0.5])`,  
corresponding to exact fairness in  $A_1$  and 0.5-approximate fairness in  $A_2$ :  

$$f_B = 0.5 \cdot (f_{B_2} \circ f_{B_1}) + 0.5 \cdot f_{B_1}$$

## REFERENCES

- Chzhen, E., Denis, C., Hebiri, M., Oneto, L., and Pontil, M. (2020). Fair Regression with Wasserstein Barycenters. In *Advances in Neural Information Processing Systems*.
- Hu, F., Ratz, P., and Charpentier, A. (2024). A Sequentially Fair Mechanism for Multiple Sensitive Attributes. *AAAI-2024*.
- Sauce, M., Chancel, A., and Ly, A. (2023). AI and Ethics in Insurance: a new solution to mitigate proxy discrimination in risk modeling. *arXiv*, 2307.13616.

## 1. BACKGROUND: OPTIMAL TRANSPORT

For univariate distributions  $\nu_1$  and  $\nu_2$ ,  $p$ -**Wasserstein distance** ( $p \geq 1$ ) computes the minimum ‘cost’ required to transform  $\nu_1$  into  $\nu_2$ :

$$\mathcal{W}_p(\nu_1, \nu_2) = \left( \int_{u \in [0,1]} |Q_1(u) - Q_2(u)|^p du \right)^{1/p}$$

In this case, the optimal transport map  $T^*$ , with  $T^*_{\#} \nu_1 = \nu_2$ , for some strictly convex ‘cost’, such as quadratic cost, is defined as  $T^* := Q_2 \circ F_1$ .

The **Wasserstein Barycenter** finds a representative distribution that lies between  $K$  given distributions  $(\nu_1, \dots, \nu_K)$ , and weights  $(w_1, \dots, w_K) \in \mathbb{R}_+^K$ . The  $\mathcal{W}_2$ -Barycenter is the minimizer:

$$\text{Bar}\{(\nu_k, w_k)_{k=1}^K\} = \arg \min_{\nu} \sum_{k=1}^K w_k \cdot \mathcal{W}_2^2(\nu_k, \nu)$$

Hu et al. prove the **associativity** of the  $\mathcal{W}_2$ -Barycenter with univariate measures.

