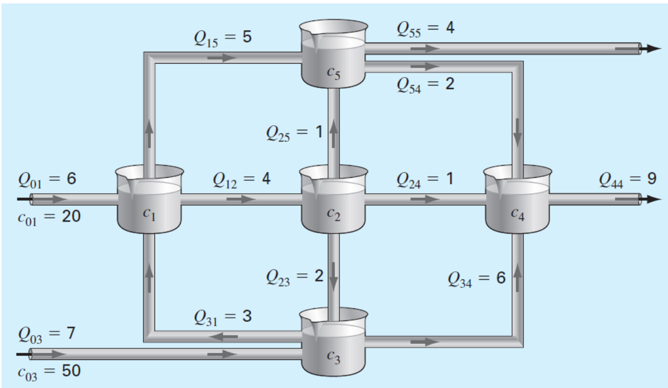
8.9 Five reactors linked by pipes are shown in Fig. P8.9. The rate of mass flow through each pipe is computed as the product of flow (Q) and concentration (c). At steady state, the mass flow into and out of each reactor must be equal. For example, for the first reactor, a *mass balance* can be written as

$$Q_{01}c_{01} + Q_{31}c_3 = Q_{15}c_1 + Q_{12}c_1$$

Write mass balances for the remaining reactors in Fig. P8.9 and express the equations in matrix form. Then use MATLAB to solve for the concentrations in each reactor.



8.10 An important problem in structural engineering is that of finding the forces in a statically determinate truss (Fig. P8.10). This type of structure can be described as a system of coupled linear algebraic equations derived from force balances. The sum of the forces in both horizontal and vertical directions must be zero at each node, because the system is at rest. Therefore, for node 1:

$$\sum F_H = 0 = -F_1 \cos 30^\circ + F_3 \cos 60^\circ + F_{1,h}$$
$$\sum F_V = 0 = -F_1 \sin 30^\circ - F_3 \sin 60^\circ + F_{1,v}$$

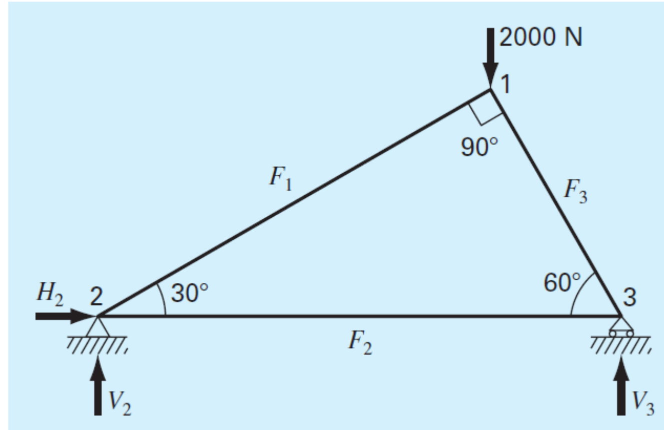
for node 2:

$$\sum F_H = 0 = F_2 + F_1 \cos 30^\circ + F_{2,h} + H_2$$
$$\sum F_V = 0 = F_1 \sin 30^\circ + F_{2,v} + V_2$$

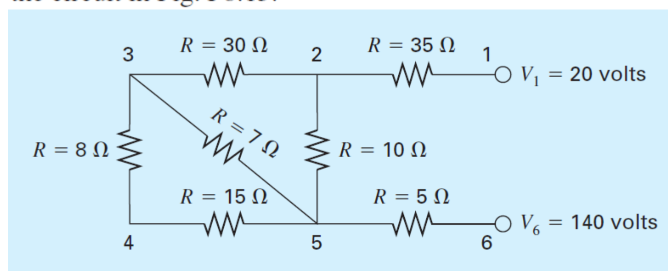
for node 3:

$$\sum F_H = 0 = -F_2 - F_3 \cos 60^\circ + F_{3,h}$$
$$\sum F_V = 0 = F_3 \sin 60^\circ + F_{3,v} + V_3$$

where $F_{i,h}$ is the external horizontal force applied to node i (where a positive force is from left to right) and $F_{i,v}$ is the external vertical force applied to node i (where a positive force is upward). Thus, in this problem, the 2000-N downward force on node 1 corresponds to $F_{i,v} = -2000$. For this case, all other $F_{i,v}$'s and $F_{i,h}$'s are zero. Express this set of linear algebraic equations in matrix form and then use MATLAB to solve for the unknowns.



8.15 Perform the same computation as in Sec. 8.3, but for the circuit in Fig. P8.15.



9.4 Given the system of equations

$$\begin{aligned} 2x_2 + 5x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 1 \\ 3x_1 + x_2 \qquad &= 2 \end{aligned}$$

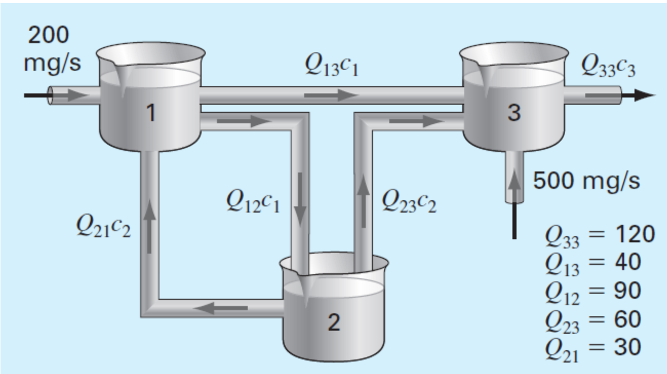
(a) Compute the determinant.
(b) Use Cramer's rule to solve for the x's.
(c) Use Gauss elimination with partial pivoting to solve for the x's. As part of the computation, calculate the determinant in order to verify the value computed in (a)
(d) Substitute your results back into the original equations to check your solution.

9.6 Given the equations

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -3x_1 - 5x_2 + 2x_3 &= -61.5 \\ x_1 + x_2 + 6x_3 &= -21.5 \end{aligned}$$

(a) Solve by naive Gauss elimination. Show all steps of the computation.
(b) Substitute your results into the original equations to check your answers.

9.9 Figure P9.9 shows three reactors linked by pipes. As indicated, the rate of transfer of chemicals through each pipe is equal to a flow rate (Q, with units of cubic meters per second) multiplied by the concentration of the reactor from which the flow originates (c, with units of milligrams per cubic meter). If the system is at a steady state, the transfer into each reactor will balance the transfer out. Develop mass-balance equations for the reactors and solve the three simultaneous linear algebraic equations for their concentrations.



9.10 A civil engineer involved in construction requires 4800, 5800, and 5700 m^3 of sand, fine gravel, and coarse gravel, respectively, for a building project. There are three pits from which these materials can be obtained. The composition of these pits is

	Sand %	Fine Gravel %	Coarse Gravel %
Pit1	52	30	18
Pit2	20	50	30
Pit3	25	20	55

How many cubic meters must be hauled from each pit in order to meet the engineer's needs?

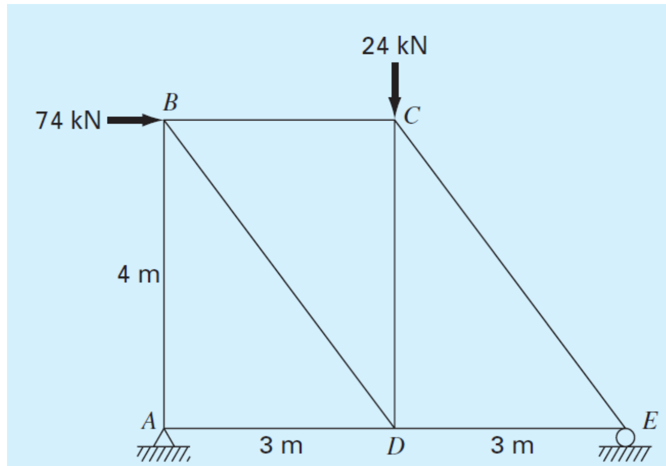
9.11 An electrical engineer supervises the production of three types of electrical components. Three kinds of material—metal, plastic, and rubber—are required for production. The amounts needed to produce each component are

Component	Metal (g/ component)	Plastic (g/ component)	Rubber (g/ component)
1	15	0.25	1.0
2	17	0.33	1.2
3	19	0.42	1.6

If totals of 2.12, 0.0434, and 0.164 kg of metal, plastic, and rubber, respectively, are available each day, how many components can be produced per day?

9.15 A truss is loaded as shown in Fig. P9.15. Using the following set of equations, solve for the 10 unknowns, AB, BC, AD, BD, CD, DE, CE, A_x, A_y, and E_y.

$$A_x + AD = 0 \qquad -24 - CD - (4/5)CE = 0$$
$$A_y + AB = 0 \qquad -AD + DE - (3/5)BD = 0$$
$$74 + BC + (3/5)BD = 0 \qquad CD + (4/5)BD = 0$$
$$-AB - (4/5)BD = 0 \qquad -DE - (3/5)CE = 0$$
$$-BC + (3/5)CE = 0 \qquad E_y + (4/5)CE = 0$$



11.3 The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day):

$$\begin{aligned} 15c_1 - 3c_2 - c_3 &= 4000 \\ -3c_1 + 18c_2 - 6c_3 &= 1500 \\ -4c_1 - c_2 + 12c_3 &= 2400 \end{aligned}$$

(a) Determine the matrix inverse.
(b) Use the inverse to determine the solution.
(c) Determine how much the rate of mass input to reactor 3 must be increased to induce a 10 g/m^3 rise in the concentration of reactor 1.
(d) How much will the concentration in reactor 3 be reduced if the rate of mass input to reactors 1 and 2 is reduced by 500 and 250 g/day, respectively?

11.12 The Lower Colorado River consists of a series of four reservoirs as shown in Fig. P11.12.

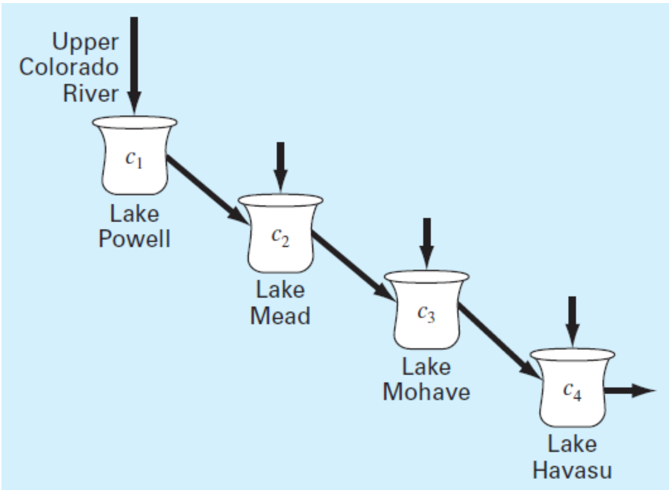
Mass balances can be written for each reservoir, and the following set of simultaneous linear algebraic equations results:

$$\begin{bmatrix} 13.422 & 0 & 0 & 0 \\ -13.422 & 12.252 & 0 & 0 \\ 0 & -12.252 & 12.377 & 0 \\ 0 & 0 & -12.377 & 11.797 \end{bmatrix} \times \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 750.5 \\ 300 \\ 102 \\ 30 \end{Bmatrix}$$

where the right-hand-side vector consists of the loadings of chloride to each of the four lakes and c_1, c_2, c_3, and c_4 = the resulting chloride concentrations for Lakes Powell, Mead, Mohave, and Havasu, respectively.

(a) Use the matrix inverse to solve for the concentrations in each of the four lakes.

(b) How much must the loading to Lake Powell be reduced for the chloride concentration of Lake Havasu to be 75?



12.2 **(a)** Use the Gauss-Seidel method to solve the following system until the percent relative error falls below $\varepsilon_s = 5\%$:

$$\begin{bmatrix} 0.8 & -0.4 & \\ -0.4 & 0.8 & -0.4 \\ & -0.4 & 0.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 25 \\ 105 \end{Bmatrix}$$

12.3 Use the Gauss-Seidel method to solve the following system until the percent relative error falls below $\varepsilon_s = 5\%$:

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -3x_1 - 6x_2 + 2x_3 &= -61.5 \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

12.5 The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day):

$$\begin{aligned} 15c_1 - 3c_2 - c_3 &= 3800 \\ -3c_1 + 18c_2 - 6c_3 &= 1200 \\ -4c_1 - c_2 + 12c_3 &= 2350 \end{aligned}$$

Solve this problem with the Gauss-Seidel method to $\varepsilon_s = 5\%$.

12.6 Use the Gauss-Seidel method **(a)** without relaxation and **(b)** with relaxation ($\lambda = 1.2$) to solve the following system to a tolerance of $\varepsilon_s = 5\%$. If necessary, rearrange the equations to achieve convergence.

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

12.8 Determine the solution of the simultaneous nonlinear equations

$$\begin{aligned} y &= -x^2 + x + 0.5 \\ y + 5xy &= x^2 \end{aligned}$$

Use the Newton-Raphson method and employ initial guesses of $x = y = 1.2$.

12.9 Determine the solution of the simultaneous nonlinear equations:

$$\begin{aligned} x^2 &= 5 - y^2 \\ y + 1 &= x^2 \end{aligned}$$

(a) Graphically.

(b) Successive substitution using initial guesses of $x = y = 1.5$.

(c) Newton-Raphson using initial guesses of $x = y = 1.5$.