5.8 The saturation concentration of dissolved oxygen in freshwater can be calculated with the equation

$$\ln o_{sf} = -139.34411 + \frac{1.575701 \times 10^5}{T_a} - \frac{6.642308 \times 10^7}{T_a^2} + \frac{1.243800 \times 10^{10}}{T_a^3} - \frac{8.621949 \times 10^{11}}{T_a^4}$$

where o_{sf} = the saturation concentration of dissolved oxygen in freshwater at 1 atm (mg L^{-1}); and T_a = absolute temperature (K). Remember that $T_a = T + 273.15$, where T = temperature (°C). According to this equation, saturation decreases with increasing temperature. For typical natural waters in temperate climates, the equation can be used to determine that oxygen concentration ranges from 14.621 mg/L at 0 °C to 6.949 mg/L at 35 °C. Given a value of oxygen concentration, this formula and the bisection method can be used to solve for temperature in °C.

(a) If the initial guesses are set as 0 and 35 °C, how many bisection iterations would be required to determine temperature to an absolute error of 0.05 °C?

(b) Based on **(a)**, develop and test a bisection M-file function to determine T as a function of a given oxygen concentration. Test your function for o_{sf} = 8, 10 and 14 mg/L. Check your results.

5.10 Water is flowing in a trapezoidal channel at a rate of $Q = 20\ m^3/s$. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - \frac{Q^2}{g A_c^3} B$$

where $g = 9.81\ m/s^2$, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth y by

$$B = 3 + y$$

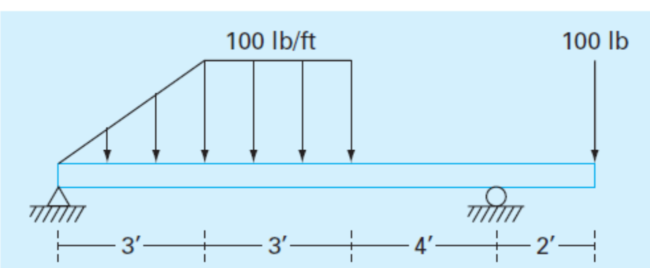


FIGURE P5.9

and

$$A_c = 3y + \frac{y^2}{2}$$

Solve for the critical depth using **(a)** the graphical method, **(b)** bisection, and **(c)** false position. For **(b)** and **(c)** use initial guesses of $x_l = 0.5$ and $x_u = 2.5$, and iterate until the approximate error falls below 1% or the number of iterations exceeds 10. Discuss your results.

5.11 The Michaelis-Menten model describes the kinetics of enzyme mediated reactions:

$$\frac{dS}{dt} = -v_m \frac{S}{k_s + S}$$

where S = substrate concentration (moles/L), v_m = maximum uptake rate (moles/L/d), and k_s = the half-saturation constant, which is the substrate level at which uptake is half of the maximum [moles/L]. If the initial substrate level at $t = 0$ is S_0, this differential equation can be solved for

$$S = S_0 - v_m t + k_s \ln(S_0/S)$$

Develop an M-file to generate a plot of S versus t for the case where $S_0 = 8$ moles/L, $v_m = 0.7$ moles/L/d, and $k_s = 2.5$ moles/L.

5.13 Figure P5.13*a* shows a uniform beam subject to a linearly increasing distributed load. The equation for the resulting elastic curve is (see Fig. P5.13*b*)

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x) \tag{P5.13}$$

Use bisection to determine the point of maximum deflection (i.e., the value of x where $dy/dx = 0$). Then substitute this value into Eq. (P5.13) to determine the value of the maximum deflection. Use the following parameter values in your

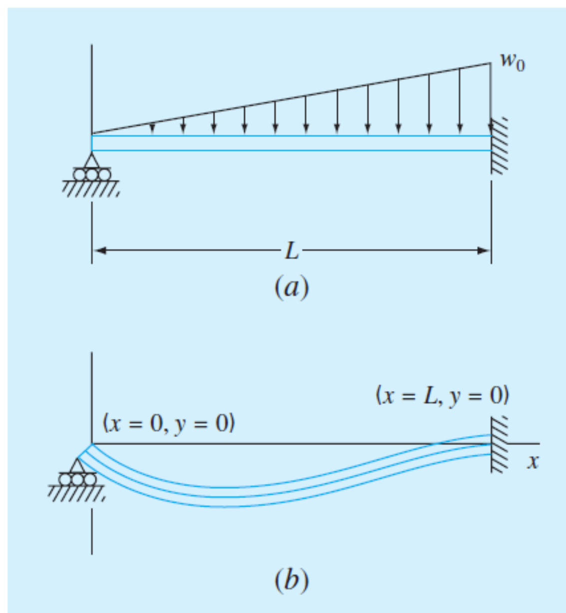


FIGURE P5.13

computation: L = 600 cm, E = 50,000 kN/cm², I = 30,000 cm⁴, and w_0 = 2.5 kN/cm.

5.14 You buy a \$35,000 vehicle for nothing down at \$8,500 per year for 7 years. Use the `bisect` function from Fig. 5.7 to determine the interest rate that you are paying. Employ initial guesses for the interest rate of 0.01 and 0.3 and a stopping criterion of 0.00005. The formula relating present worth P, annual payments A, number of years n, and interest rate i is

$$A = P \frac{i(1+i)^n}{(1+i)^n - 1}$$

5.16 The resistivity ρ of doped silicon is based on the charge q on an electron, the electron density n, and the electron mobility μ. The electron density is given in terms of the doping density N and the intrinsic carrier density n_i. The electron mobility is described by the temperature T, the reference temperature T_0, and the reference mobility μ_0. The equations required to compute the resistivity are

$$\rho = \frac{1}{qn\mu}$$

where

$$n = \frac{1}{2}\left(N + \sqrt{N^2 + 4n_i^2}\right) \quad \text{and} \quad \mu = \mu_0 \left(\frac{T}{T_0}\right)^{-2.42}$$

Determine N, given $T_0 = 300$ K, $T = 1000$ K, $\mu_0 = 1360\ \text{cm}^2\ (\text{V s})^{-1}$, $q = 1.7 \times 10^{-19}$ C, $n_i = 6.21 \times 10^9\ \text{cm}^{-3}$, and a desired $\rho = 6.5 \times 10^6$ V s cm/C. Employ initial guesses of $N = 0$ and 2.5×10^{10}. Use **(a)** bisection and

5.17 A total charge Q is uniformly distributed around a ring-shaped conductor with radius a. A charge q is located at a distance x from the center of the ring (Fig. P5.17). The force exerted on the charge by the ring is given by

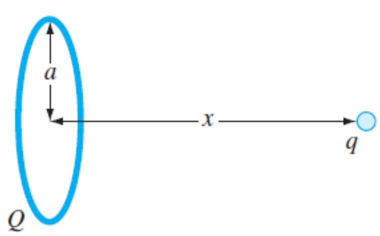
$$F = \frac{1}{4\pi e_0} \frac{qQx}{(x^2 + a^2)^{3/2}}$$

where $e_0 = 8.9 \times 10^{-12}\ \text{C}^2/(\text{N m}^2)$. Find the distance x where the force is 1.25 N if q and Q are 2×10^{-5} C for a ring with a radius of 0.85 m.

5.18 For fluid flow in pipes, friction is described by a dimensionless number, the *Fanning friction factor f*. The Fanning friction factor is dependent on a number of parameters related to the size of the pipe and the fluid, which can all be represented by another dimensionless quantity, the *Reynolds number* Re. A formula that predicts f given Re is the *von Karman equation:*

$$\frac{1}{\sqrt{f}} = 4 \log_{10}\left(\text{Re}\sqrt{f}\right) - 0.4$$

Typical values for the Reynolds number for turbulent flow are 10,000 to 500,000 and for the Fanning friction factor are 0.001 to 0.01. Develop a function that uses bisection to solve for f given a user-supplied value of Re between 2,500 and



1,000,000. Design the function so that it ensures that the absolute error in the result is $E_{a,d} < 0.000005$.

5.19 Mechanical engineers, as well as most other engineers, use thermodynamics extensively in their work. The following polynomial can be used to relate the zero-pressure specific heat of dry air c_p kJ/(kg K) to temperature (K):

$$c_p = 0.99403 + 1.671 \times 10^{-4}T + 9.7215 \times 10^{-8}T^2$$
$$-9.5838 \times 10^{-11}T^3 + 1.9520 \times 10^{-14}T^4$$

Develop a plot of c_p versus a range of $T = 0$ to 1200 K, and then use bisection to determine the temperature that corresponds to a specific heat of 1.1 kJ/(kg K).

5.20 The upward velocity of a rocket can be computed by the following formula:

$$v = u \ln \frac{m_0}{m_0 - qt} - gt$$

where v = upward velocity, u = the velocity at which fuel is expelled relative to the rocket, m_0 = the initial mass of the rocket at time $t = 0$, q = the fuel consumption rate, and g = the downward acceleration of gravity (assumed constant = 9.81 m/s^2). If $u = 1800$ m/s, $m_0 = 160{,}000$ kg, and $q = 2600$ kg/s, compute the time at which $v = 750$ m/s. (Hint: t is somewhere between 10 and 50 s.) Determine your result so that it is within 1% of the true value. Check your answer.