Método de Newton

- Para funciones $\mathbb{R}^2 : \longrightarrow \mathbb{R}$

$$f(x,y) = z$$

Ejemplos:

a) $f(x,y) = 3x^2 - sin(xy)$

b) $f(x,y) = 3e^{3xy} + y^3$

Gradiente $\mathbb{R}^2 : \longrightarrow \mathbb{R}^2$

$$\nabla f(x,y) = \left(\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y} \right)$$

Ejemplos:

a) $\nabla f(x,y) = \big(6x - y\ cos(xy), -x\ cos(xy)\big)$

b) $\nabla f(x,y) = (9ye^{3xy}, 9xe^{3xy} + 3y^2)$

Aproximación lineal

$$f(x,y) = f(x_0,y_0) + \frac{\partial f(x_0,y_0)}{\partial x}(x - x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y - y_0)$$
$$f(x,y) = f(x_0,y_0) + \nabla f(x_0,y_0) \cdot (x - x_0, y - y_0)$$
$$f(x_1,x_2) = f(x_{10},x_{20}) + \nabla f(x_{10},x_{20}) \cdot (x_1 - x_{10}, y_1 - y_{10})$$
$$f(\boldsymbol{x}) = f(\boldsymbol{x_0}) + \nabla f(\boldsymbol{x_0}) \cdot (\boldsymbol{x} - \boldsymbol{x_0})$$

- Para funciones $\mathbb{R}^2 : \longrightarrow \mathbb{R}^2$

$$f(x,y) = \big(f_1(x,y), f_2(x,y)\big)$$

Ejemplos:

a) $f(x,y) = (3x + 2y^2, 4x^2y)$

b) $f(x,y) = (xy, 4 - y + x)$

Matriz Jacobiana $\mathbb{R}^2 : \longrightarrow \mathbb{R}^2 \times \mathbb{R}^2$

$$\boldsymbol{D}f(x,y) = \begin{pmatrix} \frac{\partial f_1(x,y)}{\partial x} & \frac{\partial f_1(x,y)}{\partial y} \\ \frac{\partial f_2(x,y)}{\partial x} & \frac{\partial f_2(x,y)}{\partial y} \end{pmatrix}$$

Ejemplos:

a) $\boldsymbol{D}f(x,y) = \begin{pmatrix} 3 & 4y \\ 8xy & 4x^2 \end{pmatrix}$

b) $\boldsymbol{D}f(x,y) = \begin{pmatrix} y & x \\ 1 & -1 \end{pmatrix}$

Aproximación lineal

$$f(\boldsymbol{x}) = f(\boldsymbol{x_0}) + \boldsymbol{D}f(\boldsymbol{x_0}) \cdot (\boldsymbol{x} - \boldsymbol{x_0})$$
$$f(\boldsymbol{x}) - f(\boldsymbol{x_0}) = \boldsymbol{D}f(\boldsymbol{x_0}) \cdot (\boldsymbol{x} - \boldsymbol{x_0})$$
$$\big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}}\big(f(\boldsymbol{x}) - f(\boldsymbol{x_0})\big) = \big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}}\boldsymbol{D}f(\boldsymbol{x_0}) \cdot (\boldsymbol{x} - \boldsymbol{x_0})$$
$$\big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}}\big(f(\boldsymbol{x}) - f(\boldsymbol{x_0})\big) = \boldsymbol{I} \cdot (\boldsymbol{x} - \boldsymbol{x_0})$$
$$\big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}}\big(f(\boldsymbol{x}) - f(\boldsymbol{x_0})\big) = \boldsymbol{x} - \boldsymbol{x_0}$$
$$\boldsymbol{x} = \boldsymbol{x_0} + \big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}}\big(f(\boldsymbol{x}) - f(\boldsymbol{x_0})\big)$$

Como buscamos los valores de $\boldsymbol{x}$ cuando $f(\boldsymbol{x}) = 0$

$$\boldsymbol{x} = \boldsymbol{x_0} - \big(\boldsymbol{D}f(\boldsymbol{x_0})\big)^{\boldsymbol{-1}} f(\boldsymbol{x_0})$$