Machine Learning

Machine Learning deals with developing automatic algorithms for identifying patterns in complex data and for making prediction based on previous observations. Applications include search engines, image recognition, speech recognition, natural language processing, and much more. Machine Learning algorithms typically fit in either the supervised or unsupervised paradigm.

Supervised Learning

In Supervised Learning, algorithms are trained on data labeled with a desired output. Here the machine (i.e., the algorithm) is expected to derive structure from the data using the presence of labels. A typical application of Supervised Learning is classification, i.e., the task of classifying a new test data point into one of the possible categories that are present in the training data. Image classification is the task of assigning a label to an image from a gives set of categories. For instance, given as input an image of a handwritten digit from 0 to 9, a classification algorithm should identify which category this data belongs to, i.e., which digit the image represents, out of the ten possible choices. Let's see what classification entails!

Classifying points

While Python has many built-in datatypes and functions, typically applications in scientific computing and data science rely on datatypes and functions from external libraries (also called modules in Python). numpy is the core library for scientific computing, and it provides a high-performance array datatype (together with functions on them) that we will use repeadetly. matplotlib is a plotting library to plot numpy datatypes.

The following code imports these Python modules.

```
In [1]: import numpy as np
   import matplotlib.pyplot as plt
   # Configure matplotlib to embed the plots in the output cells of the pre
   sent notebook
%matplotlib notebook
```

Assume we are given a collection of labeled two-dimensional points such as the one that we define below. Each point in X_train is labeled by a corresponding color in Y_train, either red or blue. The collections X_train and Y train represent our training dataset.

```
In [2]: X_train = np.array([[1,1], [2,2.5], [3,1.2], [5.5,6.3], [6,9], [7,6]]) #
    Define numpy array of two-dim points
    Y_train = ['red', 'red', 'red', 'blue', 'blue', 'blue'] # Define a Pytho
    n built-in list (i.e., array) of strings
```

We can think of X_train as a two dimensional array. Notice that Python is a 0-indexed programming language, much like C!

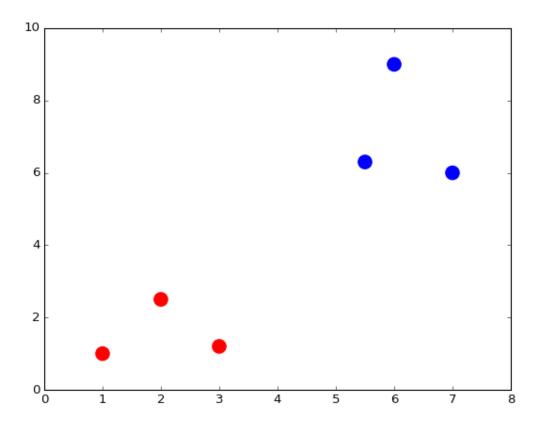
```
In [3]: print(X_train[5,0]) # Extract the 0-th coordinate of the 5-th point in t
he array
print(X_train[5,1]) # Extract the 1-st coordinate of the 5-th point in t
he array

7.0
6.0
```

Python has a slicing syntax that allows us to extract multiple elements in an array at once. See the following code for instance.

The following lines of code use the slicing syntax to conviently plot this colletion of points, and their color label.

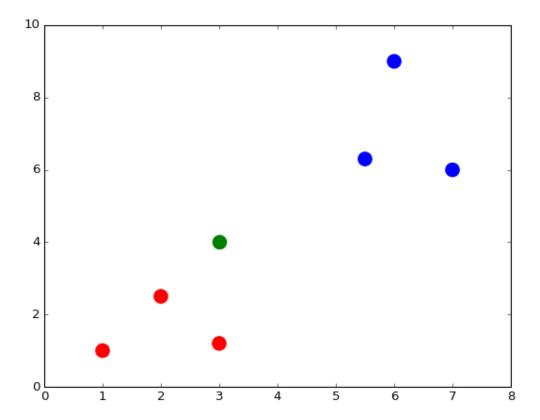
```
In [6]: plt.figure() # Define a new figure
    plt.scatter(X_train[:,0], X_train[:,1], s = 170, color = Y_train[:]) # P
    lot points with Python slicing syntax
    plt.show() # Display plot
```



Now, assume we are given a new test point X_test.

Let us see where this new point lands in the plot.

```
In [8]: plt.figure() # Define a new figure
   plt.scatter(X_train[:,0], X_train[:,1], s = 170, color = Y_train[:])
   plt.scatter(X_test[0], X_test[1], s = 170, color = 'green')
   plt.show() # Display plot
```



Say we want to classify x_{test} as a being red or blue. One reasonable way to do so is to assign to x_{test} the same color of the point in x_{train} that is closest to x_{test} (its nearest neighbor). The following blocks of code precisely do this! First, we define a function that takes two generic points x and y and returns their Euclidean distance.

```
In [9]: def dist(x, y):
    return np.sqrt(np.sum((x - y)**2)) # np.sqrt and np.sum are numpy fu
nctions to work with numpy arrays
```

Then, for each point in x_{train} we compute its distance from x_{test} , and we store this distance in the corresponding component of the array distance.

```
In [10]: num = len(X_train) # Compute the number of points in X_train
    distance = np.zeros(num) # Initialize a numpy arrays of zeros
    for i in range(num):
        distance[i] = dist(X_train[i], X_test) # Compute distance from X_tra
    in[i] to X_test
    print(distance)

[ 3.60555128 1.80277564 2.8 3.39705755 5.83095189 4.4721359
5]
```

As an aside, using the so-called vectorization syntax of Python we can write the output of the previous two blocks of code in only one line! In fact, this would be a preferred way of writing code, as for loops tend to be quite slow in Python (as compared to for loops in C). Just to spark up your interest in learning more about Python...

Second, we choose the point in X train with the minimal distance from X new, and we look at the color of it.

```
In [12]: min_index = np.argmin(distance) # Get the index with smallest distance
    print(Y_train[min_index])
    red
```

Per our specification, we should assign to X test the color red!

Congratulations! What we have just implemented is the so-called Nearest Neighbor Classifier, our first Machine Learning algorithm! The Nearest Neighbor Classifier --- using precisely the same lines of code just described --- can be used to classify other type of datasets, such as the handritten digits dataset, as we see next.

Classifying images

sklearn is a Python modules for machine learning. This module comes with a few standard datasets, such as the digits dataset. The following code loads this dataset from sklearn.

```
In [13]: from sklearn import datasets
digits = datasets.load_digits()
```

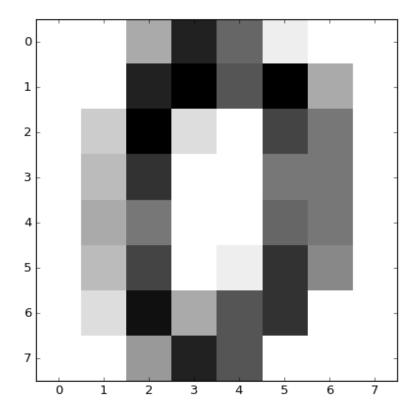
The digits dataset contains 1797 images representing handwritten digits, together with numerical labels representing the true number associated with each image. digits.images is the array of images, while digits.target is the array of labels. Python is a 0-indexed language, such as C, so these arrays are indexed from 0 to 1796.

Each element in the array digits.images is on its own a 8 by 8 array of pixels, where each pixel is an integer between 0 and 16. Let us look what the first image, indexed by 0, looks like:

```
In [14]:
          print(digits.images[0])
           ] ]
                0.
                       0.
                             5.
                                  13.
                                         9.
                                                1.
                                                      0.
                                                            0.1
                                  15.
                                        10.
                                               15.
                                                      5.
                                                            0.1
                0.
                       0.
                            13.
                0.
                       3.
                            15.
                                   2.
                                          0.
                                               11.
                                                      8.
                                                            0.1
                       4.
                            12.
                                   0.
                                         0.
                                                8.
                                                      8.
                                                            0.]
                       5.
                             8.
                                   0.
                                         0.
                                                9.
                                                      8.
                                                            0.1
                       4.
                            11.
                                   0.
                                         1.
                                               12.
                                                      7.
                                                            0.1
                       2.
                            14.
                                   5.
                                               12.
                                                      0.
                                                            0.1
                0.
                                        10.
                0.
                       0.
                             6.
                                  13.
                                        10.
                                                0.
                                                      0.
                                                            0.]]
```

Can you see that the above represents number zero? It is easier if we plot this array by assigning to each number an intensity of black.

```
In [15]: plt.figure()
  plt.imshow(digits.images[0], cmap = plt.cm.gray_r, interpolation = 'near
  est')
  plt.show()
```



Each element in the array digits.target is a numerical label representing the digit associated to the respective image.

```
In [16]: print(digits.target[0])
0
```

To evaluate the performance of an algorithm in Machine Learning, a common practice is to form subsets of the original dataset. One subset is the training set on which the data properties (i.e., the hidden structure) are learnt. Another subset is the testing set on which we test the performance of the algorithm. Here, let us form the training set by selecting all the labeled images from 0 to 9 (note that the slicing syntax [i:j] in Python means that we consider elements from i to j-1; so, in the following code the index 10 is not counted).

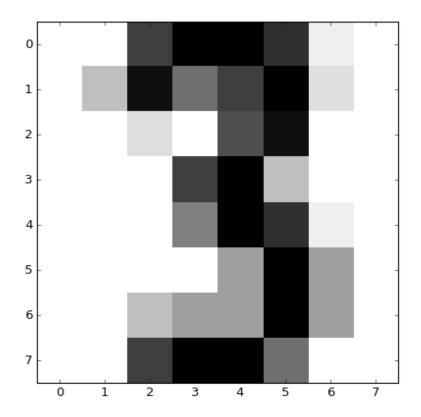
```
In [17]: X_train = digits.data[0:10]
Y_train = digits.target[0:10]
```

Suppose we now want to run the Nearest Neighbor Classifier described above to classify the following test data point (neglecting the fact that for this test data point we also have the true label available!)

```
In [18]: X_test = digits.data[345]
```

We can see what this test point looks like.

```
In [19]: plt.figure()
  plt.imshow(digits.images[345], cmap = plt.cm.gray_r, interpolation = 'ne
  arest')
  plt.show()
```



We now run the Nearest Neighbor Classifier using exactly the same code written above to classify two dimensional points. In particular, note that we use the same function dist that was previously defined!

```
In [20]: num = len(X_train) # Compute the number of points in X_train
    distance = np.zeros(num) # Initialize an arrays of zeros
    for i in range(num):
        distance[i] = dist(X_train[i], X_test) # Compute distance from X_tra
    in[i] to X_test
    min_index = np.argmin(distance) # Get the index with smallest distance
    print(Y_train[min_index])
```

Per our specification, we should assign to X_test the label 3. We can see that this does indeed corresponds to the true solution!

```
In [21]: print(digits.target[min_index])
3
```

So at least for the test point we tried, the algorithm worked well! To have a better idea of how well this algorithm does, we can repeat the above procedure to each element in the test dataset, which we consider being the set containing picture no. 1697 to no. 1796. The following code counts how many errors the algorithm commits over the test set. Note that the function range(i,j) creates a list of integers from i included to j-1, much like the slicing syntax we saw earlier.

So we have incurred in 37 errors out of 100 images in the test dataset! Not bad for a simple algorithm. But we can do much better if we enlarge the training set, as we now see.

Improving the performance

Much like humans get better at learning something the more they are exposed to it, also Machine Learning algorithms improve their performance with the amount of training data they are exposed to. This is the reason why much of the Machine Learning applications we use get better with time, the more we use them! Just think of speech recognition...

We can immediately observe this phenomenon in the case of the Nearest Neighbor Classifier, noticing that if we enlarge the amount of training data available to the algorithm, then the performance of the algorithm increases! For instance, let us see what happens if, instead of training the classifier with a set of 10 images (from 0 to 9), we train the classifier with a set of 1000 images (from 0 to 999).

```
In [23]: X_train = digits.data[0:1000]
Y_train = digits.target[0:1000]
```

We can now run the same code as above to count how many errors the algorithm commits over the same test set previously used.

Much better! As expected, now we have committed only 3 errors out of 100 images tried in the test set.

Unsupervised Learning

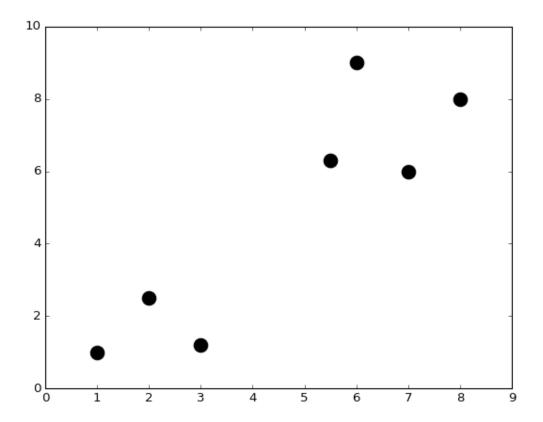
In Unsupervised Learning, algorithms are trained with unlabeled data. Here the machine (i.e., the algorithm) is expected to derive structure from the data on its own, without the presence of labels that can guide the process as in Supervised Learning. A typical application of Unsupervised Learning is clustering, i.e., the task of grouping the data into k categories, where k can be given be the user or inferred directly by the algorithm. Let's see what clustering entails!

Clustering points

First, we import the main Python modules we need for what follows.

```
In [25]: import numpy as np
   import matplotlib.pyplot as plt
   # To configure matplotlib to embed the plots in the output cells of the
        present notebook
%matplotlib notebook
```

Assume we are given a collection of (unlabeled!) two-dimensional points such as the one here defined and plotted.



Let us stress once again that this collection of points is unlabelled, that is, if you wish, each point now does not come with a pre-assigned color! (recall what was the case in Supervised Learning). Our job now is to cluster these points, that is, to assign colors to all of them!

Let's say that we want to cluster this collection of points into 2 groups, based on their vicinity. To the human eye this is an easy tak. What about computers? A popular Machine Lerning algorithm to do clustering is k-means. While the details of the k-means algorithm are not at all that difficult (also its implementation is not difficult), a proper exposition of k-means goes beyond the purpose of this class. As with SVM above, we now show how k-means can be easily imported and run in Python!

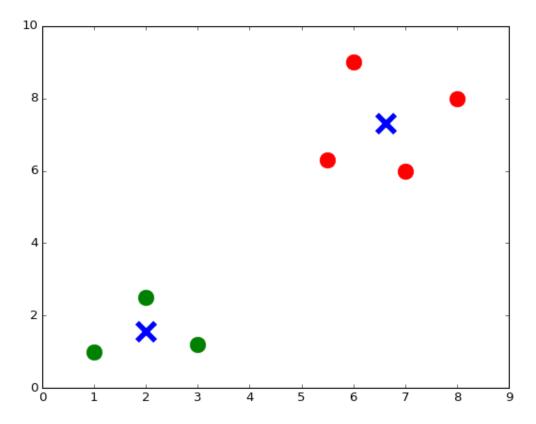
```
In [27]: from sklearn.cluster import KMeans
```

k-means divides the data points into k clusters. We begin with k=2.

```
In [28]: k = 2 # Define the number of clusters in which we want to partion the da
    ta
    kmeans = KMeans(n_clusters = k) # Run the algorithm kmeans
    kmeans.fit(X);
    centroids = kmeans.cluster_centers_ # Get centroid's coordinates for eac
    h cluster
    labels = kmeans.labels_ # Get labels assigned to each data
```

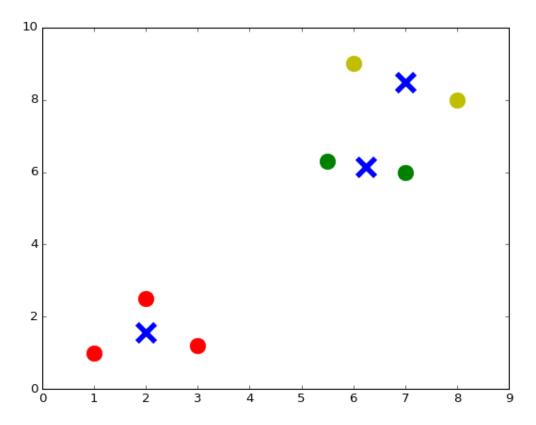
Let us now plot the points with label assignments as given by kmeans.

```
In [29]: colors = ['r.', 'g.'] # Define two colors for the plot below
   plt.figure()
   for i in range(len(X)):
        plt.plot(X[i,0], X[i,1], colors[labels[i]], markersize = 30)
   plt.scatter(centroids[:,0],centroids[:,1], marker = "x", s = 300, linewidths = 5) # Plot centroids
   plt.show()
```



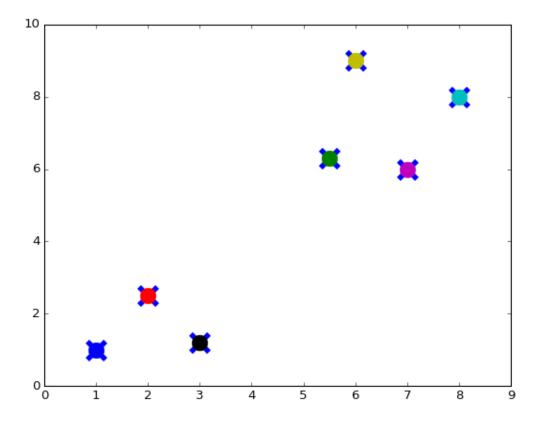
The algorithm k-means divides the data into as many clusters as we want! Let us repeat the procedure above with k=3, group the code together.

```
In [30]: k = 3
    kmeans = KMeans(n_clusters = k)
    kmeans.fit(X);
    centroids = kmeans.cluster_centers_
    labels = kmeans.labels_
    colors = ['r.', 'g.', 'y.']
    plt.figure()
    for i in range(len(X)):
        plt.plot(X[i,0], X[i,1], colors[labels[i]], markersize = 30)
    plt.scatter(centroids[:,0],centroids[:,1], marker = "x", s = 300, linewidths = 5)
    plt.show()
```



Clearly, with k=7 we have the following.

```
In [31]: k = 7
kmeans = KMeans(n_clusters = k)
kmeans.fit(X);
centroids = kmeans.cluster_centers_
labels = kmeans.labels_
colors = ['r.', 'g.', 'y.', 'c.', 'b.', 'k.', 'm.']
plt.figure()
for i in range(len(X)):
    plt.plot(X[i,0], X[i,1], colors[labels[i]], markersize = 30)
plt.scatter(centroids[:,0],centroids[:,1], marker = "x", s = 300, linewidths = 5)
plt.show()
```



The fact that k-means divides the data into as many clusters as we want is prototypical of many applications in Machine Learning. It is typically the case that users have some high-level control on the structure to be inferred. For instance, in this example it was evident that k=2 was a good choice! In other applications we may have other type of domain-specific insights that can guide the output of the algorithm. Of course, there are also algorithms that try to figure out the best k on their owns.. another time!

Clustering documents

Ok, we know how to cluster points! What about documents? Suppose I am given a list of documents and I want to cluster these documents by their topic. Let us consider a quick example. Suppose we have the following corpus of documents, where each document is here represented by a string.

This set represents the data we want to cluster based on their topic. To the human eye it is clear that the documents can be grouped into two clusters: one group is about CS50, and it is made by document 0 and document 2; the other group is about dogs and cats, and it contains document 1 and document 3. Can we use the k-means algorithm to this end?

To run the k-means algorithm we need to transform this collection of documents into data that can be interpreted by the algorithm. That is, we need to transform each document into a numerical vector (i.e., a point in a certain high-dimensional space). The most intuitive way to do so is by using the bags of words representation, where each document is represented by its word count. First, we build a vocabulary from the words used in the corpus, when we do not consider the so-called "stop words" such as "a," "the," or "in," as they do not convey meaningful information. Then, for each document i in the collection, we count the number of occurrences of each word w in the vocabulary and we store it in Z[i,j], where j is the index of the word w in the vocabulary. The matrix z represents the bags-of-words matrix, where each row represents a document in the corpus and each column represents a word in the vocabulary.

```
In [33]: # Create bags-of-words matrix
    from sklearn.feature_extraction.text import CountVectorizer
    count_vect = CountVectorizer(stop_words = 'english')
    Z = count_vect.fit_transform(corpus)
    # The function fit_transform() takes as input a list of strings and does
    two things:
    # first, it "fits the model," i.e., it builds the vocabulary; second, it
    transforms the data into a matrix.
```

We can take a look at the words in the vocabulary.

And we can look at the bags-of-words matrix Z.

The bags-of-words matrix is a good start to associate a numerical value to each document, but also the lenght of each document should be taken into account. In fact, longer documents will tend to have higher absolute count values than shorter ones, even though they might talk about the same topics! To avoid these potential discrepancies we can divide the number of occurrences of each word in a document by the total number of words in the document, so that we get a frequency matrix. Another refinement is to have downscale weights for words that occur in many documents in the corpus, as these words tend to be less informative than those that occur only in a smaller portion of the corpus. The final weighted frequency matrix, which we call x, is tipically called tf-idf for "Term Frequency times Inverse Document Frequency".

```
In [36]: # Create tf-idf matrix
    from sklearn.feature_extraction.text import TfidfVectorizer
    vectorizer = TfidfVectorizer(stop_words = 'english')
    X = vectorizer.fit_transform(corpus)
```

And we can look at the tf-idf matrix.

```
In [37]:
          X.todense()
Out[37]: matrix([[ 0.89469821,
                                   0.
                                                 0.
                                                               0.
                                                                              0.
                     0.23513012,
                                                                              0.2982327
          4,
                     0.
                                   0.235130121,
                   [ 0.
                                                               0.70710678,
                                   0.70710678,
                     0.
                     0.
                                                 0.35415727,
                   0.
                                   0.35415727,
                                                                              0.
                     0.55844332,
                     0.35415727,
                                   0.55844332],
                                                               0.38274272,
                                                                              0.4854606
                   [ 0.
          1,
                     0.
                                   0.38274272, 0.48546061,
                                                               0.48546061,
                     0.
                                   0.
                                              11)
```

Now we have a good numerical representation of our corpus. In order to run k-means on this representation it is convenient to choose a different notion of distance between points. We can now run the k-means algorithm to divide the data points into k=2 clusters.

```
In [38]: k = 2 # Define the number of clusters in which we want to partion THE da
    ta
    # Define the proper notion of distance to deal with documents
    from sklearn.metrics.pairwise import cosine_similarity
    dist = 1 - cosine_similarity(X)
    # Run the algorithm KMeans
    model = KMeans(n_clusters = k)
    model.fit(X);
```

To visualize the outcome of the algorithm, we can print the top terms per cluster.

```
In [39]: print("Top terms per cluster:\n")
    order_centroids = model.cluster_centers_.argsort()[:, ::-1]
    terms = vectorizer.get_feature_names()
    for i in range(k):
        print ("Cluster %i:" % i, end='')
        for ind in order_centroids[i, :3]:
            print (' %s,' % terms[ind], end='')
        print ("")

Top terms per cluster:

Cluster 0: awesome, staff, cs50,
Cluster 1: dog, cat, keeps,
```

Example: clustering movies based on their IDMB synopses

Ok, now that we have an idea of how to cluster collection of documents, why not trying with something more interesting? After all, the example above is so small that the human eye is more than enough! Something more interesting is the following. This is a list of movies I remember watching at some point in my life https://docs.google.com/spreadsheets/d/1udJ4nd9EKIX_awB90JCbKaStuYh6aVjh1X6j8iBUXIU/, with their synopses taken from the IDMB website. We can easily import this table to Python with the followin code.

```
In [40]: # Import the data set into a Panda data frame
import pandas as pd
from io import StringIO
import requests

act = requests.get('https://docs.google.com/spreadsheets/d/ludJ4nd9EKlX_
awB90JCbKaStuYh6aVjh1X6j8iBUXIU/export?format=csv')
dataact = act.content.decode('utf-8') # To convert to string for Stringi
o
frame = pd.read_csv(StringIO(dataact))
```

Take a look at this data frame to realize that it has the same content as the Google spreedsheet.

```
In [41]: print(frame)
```

```
Title
                                                                      Sy
nopsis
0
        Mad Max: Fury Road
                            Max Rockatansky (Tom Hardy) explains in a v
oic...
                The Matrix
                            The screen is filled with green, cascading
cod...
         The King's Speech
                            The film opens with Prince Albert, Duke of
Yor...
    No Country for Old Men
                            The film opens with a shot of desolate, wid
e-o...
4
          A Beautiful Mind
                            John Nash (Russell Crowe) arrives at Prince
ton...
                 Inception
                            A young man, exhausted and delirious, washe
5
s u...
                            The Walt Disney Pictures logo and the movie
                    Frozen
6
ti...
             The Lion King
                            The Lion King takes place in the Pride Land
7
s o...
                   Aladdin
                            The film starts with a street peddler, guid
ing...
                Cinderella
                            The film opens in a tiny kingdom, and shows
us...
10
              Finding Nemo
                            Two clownfish, Marlin (Albert Brooks) and h
is ...
11
                 Toy Story
                            A boy called Andy Davis (voice: John Morri
s) u...
12
                Robin Hood Told with animals for it's cast, the story
 tel...
```

We are going to apply the k-means algorithm to this data, clustering according to the topic inferred by the movie synopsis. To do so, we need to select the Synopsis and convert them into a list of strings, such as the corpus of four documents given in the previous example.

```
In [42]: # Loop over each synopsis and append its content to a list of string nam
    ed 'corpus'
    corpus = []
    for i in range(0, frame["Synopsis"].size):
        corpus.append(frame["Synopsis"][i])
```

Then we create the tf-idf matrix as done previously, with the additional requirement that we consider only the words that appear in at least 20% of the documents. You can try to remove this requirement and see what happens!

```
In [43]: # Create tf-idf matrix
    from sklearn.feature_extraction.text import TfidfVectorizer
    vectorizer = TfidfVectorizer(stop_words = 'english', min_df = 0.2)
    # min_df = 0.2 means that the term must be in at least 20% of the docume
    nts
    X = vectorizer.fit_transform(corpus)
```

Now we are ready to run the k-means algorithm.

```
In [44]: k = 2 # Define the number of clusters in which we want to partion our da
    ta
    # Define the proper notion of distance to deal with documents
    from sklearn.metrics.pairwise import cosine_similarity
    dist = 1 - cosine_similarity(X)
    # Run the algorithm kmeans
    model = KMeans(n_clusters = k)
    model.fit(X);
```

To visualize the outcome of the algorithm, we can print the title of the movie in each cluster, plus the top words per cluster.

```
In [45]: no words = 4 # Number of words to print per cluster
         order_centroids = model.cluster_centers_.argsort()[:, ::-1] # Sort clust
         er centers by proximity to centroid
         terms = vectorizer.get feature names()
         labels = model.labels # Get labels assigned to each data
         print("Top terms per cluster:\n")
         for i in range(k):
             print("Cluster %d movies:" % i, end='')
             for title in frame["Title"][labels == i]:
                 print(' %s,' % title, end='')
             print() #add a whitespace
             print("Cluster %d words:" % i, end='')
             for ind in order_centroids[i, :no words]:
                 print (' %s' % terms[ind], end=','),
             print()
             print()
```

Top terms per cluster:

```
Cluster 0 movies: Mad Max: Fury Road, The Matrix, No Country for Old Me n, A Beautiful Mind, Inception, Frozen, Finding Nemo, Toy Story, Cluster 0 words: room, tank, says, joe,

Cluster 1 movies: The King's Speech, The Lion King, Aladdin, Cinderell a, Robin Hood,
Cluster 1 words: king, prince, john, palace,
```

Ok, the algorithm figured out that my tastes split into the Walt Disney type and action movies;)