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PHYS-300: Computational Methods in Physics
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Investigating Deterministic Chaos in Three-Body Systems

Overview

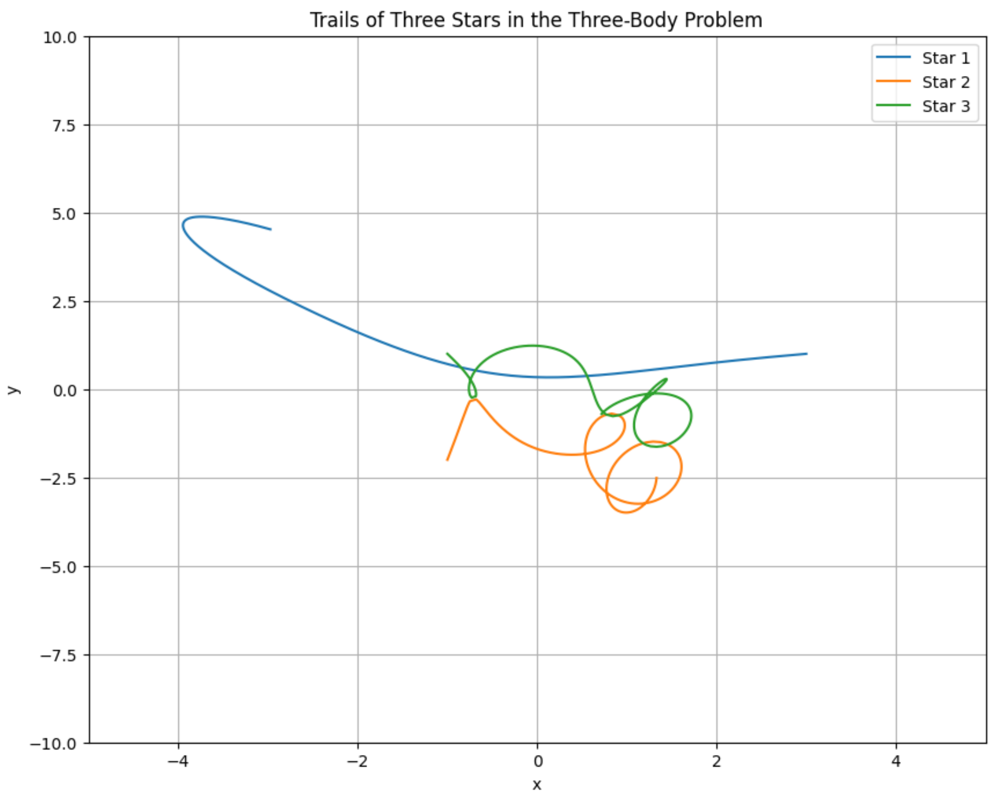
Kepler's laws of planetary motion showed us that orbits are ellipses with the Sun at one focus and that an object in orbit sweeps equal areas in equal time intervals.

Building on Kepler's foundation, we move on to exploring Newton's Law of Universal Gravitation. Newton provided a mathematical framework to understand binary gravitational interactions. However, extending these principles to systems with more than two bodies (where $N > 2$) poses significant challenges due to the absence of a singular, comprehensive formula to accurately describe the interactions in such systems. We can, however, describe a three-body system using a set of differential equations.

Our approach is inspired by a textbook problem, where we utilize a set of differential equations to model a three-body problem. After the physics is derived, we attempt to visualize the motion of the three-body system in the textbook problem. We then move to generalizing the Three-Body System by exploring the sensitivity of these systems to variations in initial conditions, specifically focusing on the parameters of mass and position. Finally, we attempt to draw connections to the concept of Deterministic Chaos, thereby illuminating the intricate and often unpredictable nature of three-body gravitational dynamics.

While our Jupyter Notebook is a detailed submission of our entire approach and code, below is a concise summary of our main approach and findings.

Problem 8.16 in Newman's *Computational Physics* walks us through modeling a specific three-body problem. The system consists of three stars of varying but similar masses (150, 200, 250, respectively), and determined starting positions on a cartesian grid. Part (a) of the question asks to derive the six first-order differential equations that govern the system. The full derivation is in the attached Jupyter Notebook. Part (b) asks for a snapshot of the system's trajectories at $t = 2s$. A snapshot of the trajectories is attached below. Part (c) asks for a 10-second animation of the bodies interacting under gravity. The full mp4 animation can be found on the Jupyter notebook.

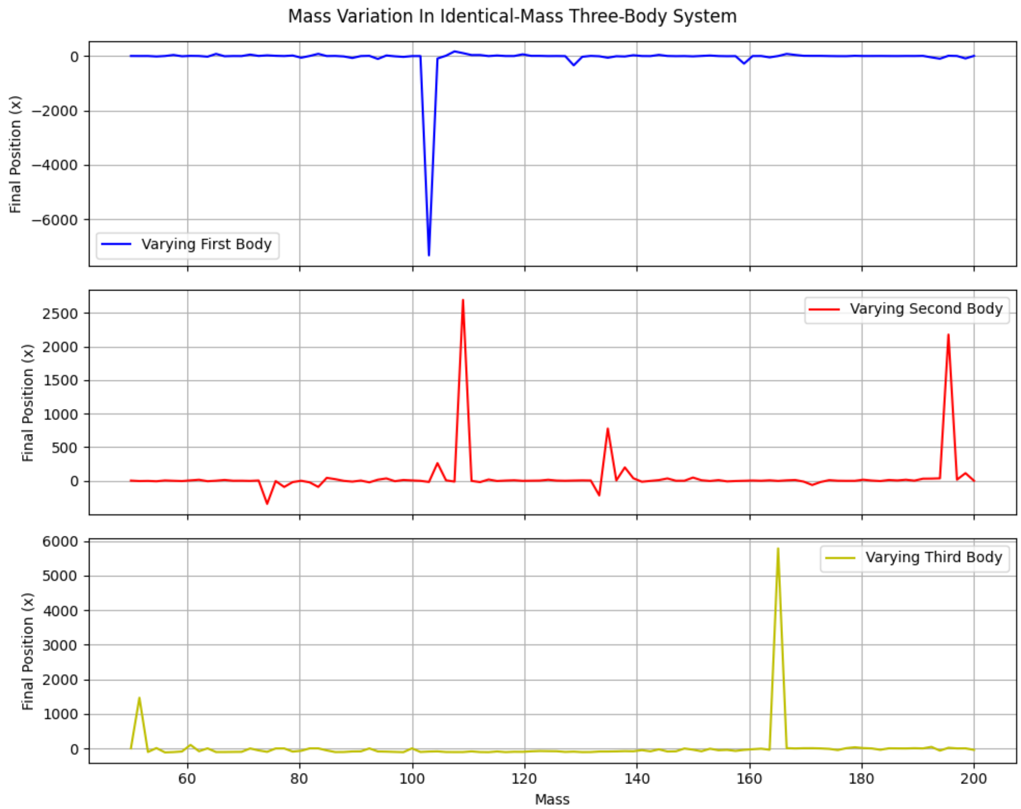


Generalized Three-Body Systems

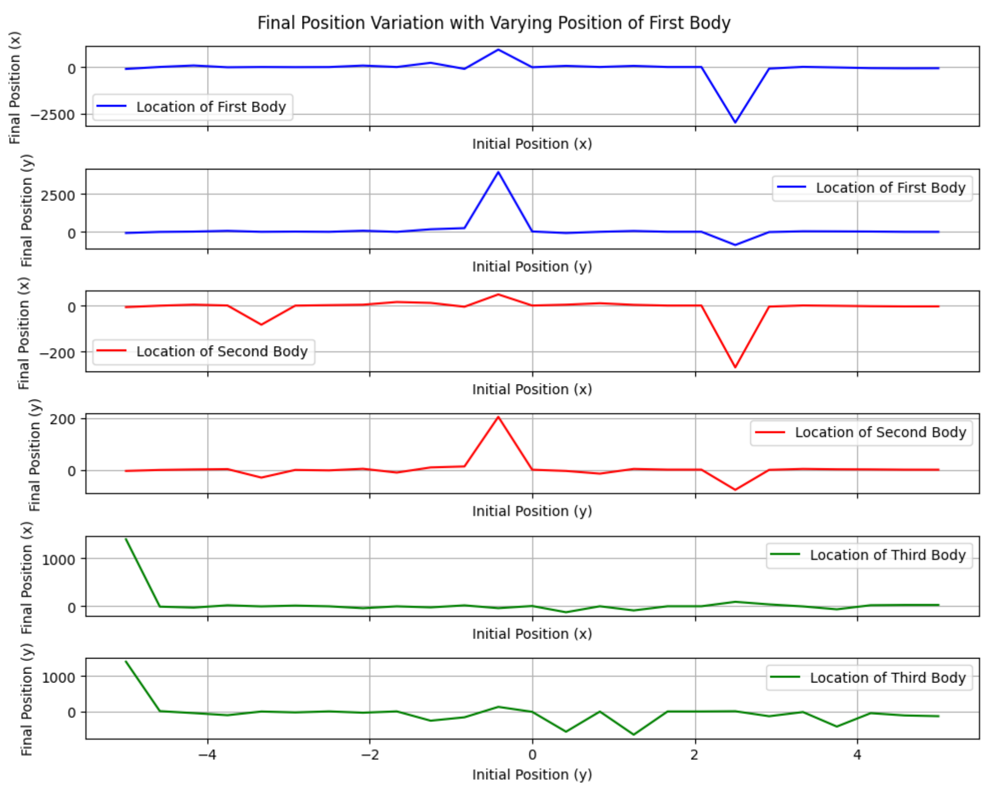
The system exhibits chaotic behavior that could be connected to the deterministic chaos concepts we visited in class. Consider a three-body system, this time the three masses are identical (m1 = m2 = m3 = 100). The starting position for each body is arbitrarily chosen. We fixed two bodies at that mass, while we ran a loop on the third body that varies its mass by assigning 100 equally-spaced masses to it. The final position of that body in the x-direction is plotted against the y-axis, while the x-axis is the corresponding mass for that iteration. The same procedure is repeated two more times, each time varying the mass of a different body. The following subplots show the chaotic behavior the system exhibits due to its extreme sensitivity to initial conditions. A small increment in one of the initial conditions of one of the three bodies could lead to a drastically different behavior. The magnitude in which the final positions could be found was extremely large, up over 10^4 even over a short amount of time and at a position very near the origin.

A similar analogous procedure was conducted, this time concerning the initial positions of the bodies rather than mass. We varied the initial position of the first body in the range (-5,5) for both x and y for 25 evenly spaced points. More points would have been ideal, but proved to be very computationally demanding. What resulted was again more evidence of chaotic behavior in the three body system, with very small differences in the initial position of the first body reflecting massively different positional changes of all three bodies. It is hard to see the finer details of these graphs as they appear relatively flat due to the large magnitude of the final positions. The results are shown below.

Mass Variation:



Position Variation:



We conclude that Three-Body Systems are inherently chaotic. Their deterministic chaos allows us to predict their behavior through computational methods such as numerical integration. Such systems rarely have a closed-form solution, so the numerical integration is to be performed on a set of differential equations that govern the system. Our investigation shows patterns of chaos due to the nature of such systems. Three-Body Systems are extremely sensitive to initial conditions, thus granting them their chaotic nature. Such sensitivity could be explored by slightly altering an initial condition for one of the bodies, and observing the totally different behavior the system exhibits.

Throughout our investigation, we attempted to illustrate the nature and importance of these systems. The fact that most physical three-body systems result in the ejection of one body and the formation of a stable boundary suggests that some binary systems in the universe might have been a three-body system at some point if we run the movie backwards. A lot of systems in the universe, especially on large cosmological scales, resemble three-body systems. Galaxy clusters gravitating towards each other (M33, M31 and us in the Milky Way, for example), resemble a three-body system where the bodies, slowly but surely, are gravitating towards each other. Generalizing three-body systems to systems of more bodies gets us to N-body systems, where comprehending the system becomes overwhelming and demands a profound cosmological understanding of such physical systems in the universe.