Joint Probabilistic Data Association Filter

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JPDAF defines a joint association event with following:

$$\boldsymbol{\theta} = \bigcap_{j=1}^{m_k} \theta j t_j \tag{1}$$

where

$$\begin{aligned} \theta jt &\triangleq \{\text{measurement } j \text{ originated from target } t\}, \\ j &= 1, ..., m_k; \qquad t = 0, 1, ..., T \end{aligned} \tag{2}$$

T is the number of targets being tracked and m_k is the number of measurements in time step k (latest time step). Instead of considering all joint association events, JPDAF uses a validation matrix to eliminate association events with negligible probability in order to reduce computational complexity. If a measurement j is outside of the validation gate of target t, association probability of the event θ_{jt} is considered negligible. JPDAF defines a validation matrix as follows:

$$\boldsymbol{\Omega} \triangleq [\omega_{jt}], \qquad j = 1, ..., m_k; \qquad t = 0, 1, ..., T \tag{3}$$

where ω_{jt} is a binary value that indicates if measurement j is in the validation gate of target t.

A joint association event is called *feasable* if a measurement can only be originated from one source, i.e.,

$$\sum_{t=0}^{T} \hat{\omega}_{jt}(\boldsymbol{\theta}) = 1, \qquad j = 1, ..., m_k \tag{4}$$

and no more than one measurement can originate from a target, i.e.,

$$\delta_t(\boldsymbol{\theta}) \triangleq \sum_{j=1}^{m_k} \hat{\omega}_{jt}(\boldsymbol{\theta}) \leq 1, \qquad t = 1, ..., T \tag{5}$$

$\delta_t(\boldsymbol{\theta})$ is called *target detection indicator*. It indicates that target t is associated with a measurement in the joint event $\boldsymbol{\theta}$. For the convenience of the main equation of JPDAF, a binary *measurement association indicator* $\tau_j(\boldsymbol{\theta})$ is also defined. It indicates that measurement j is associated with a target.

$$\tau_j(\boldsymbol{\theta}) \triangleq \sum_{t=1}^{T} \hat{\omega}_{jt}(\boldsymbol{\theta}), \qquad j = 1, ..., m_k \tag{6}$$

$$P\{\boldsymbol{\theta}(k)|Z^k\} = \frac{1}{c}\frac{\phi!}{V^{\phi}} \prod_{j=1}^{m_k} \Big[N_{t_j}[\mathbf{z}_j(k)]\Big]^{\tau_j} \prod_{t=1}^{T} (P_D^t)^{\delta_t}(1 - P_D^t)^{1-\delta_t} \tag{7}$$