CSE208: Data Structures and Algorithms II Sessional

Online week 5: Generalized shortest path (B1/B2)

Time: 35 minutes

Generalized shortest-paths problem. In Internet routing, there are delays on lines but also, more significantly, delays at routers. This motivates a generalized shortest-paths problem.

Suppose that in addition to having edge lengths $\{l_e : e \in E\}$, a graph also has ***vertex costs*** $\{c_v : v \in V\}$. Now define the cost of a path to be the sum of its edge lengths, ***plus*** the costs of all vertices on the path (including the endpoints). Give an efficient algorithm for the following problem.

> *Input:* A directed graph $G = (V, E)$; positive edge lengths l_e and positive vertex costs c_v; a starting vertex $s \in V$.
>
> *Output:* An array $\texttt{cost}[\cdot]$ such that for every vertex u, $\texttt{cost}[u]$ is the least cost of any path from s to u (i.e., the cost of the cheapest path), under the definition above.

Notice that $\texttt{cost}[s] = c_s$.

Input: The first line of the input file will contain the number of vertices *n* (≤ 1000) and the number of edges *m* (≤ 10000) followed by *n* lines containing vertex *u* and cost *c*, and then *m* lines each containing origin *u*, end *v* and weight *w* (≤ 100000) of an edge of the directed graph. The last line will contain a source vertex *s* and a destination vertex *d.*

Sample input and output:

5 10 0 5 1 3 2 4 3 10 4 6 0 1 4 0 2 2 1 2 3 1 3 2 1 4 3 2 1 1 2 4 5 2 3 4 3 4 5 4 3 1 0 4	Shortest path cost: 21 0 -> 1 -> 4