

Università della Calabria



**DIPARTIMENTO DI INGEGNERIA INFORMATICA,
MODELLISTICA, ELETTRONICA E SISTEMISTICA**

"2D Cutting Machine" Project Report

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INTRODUCTION

Nowadays the cutting machines are tools used commonly by people as also in the industry due to the ease of changing cutting parameters through programming.

there are different kinds of cutting machines like CNC, delta, 3D, robotic arm, but this report is focused on a 2D axial cutting machine as is shown in fig 1.



Fig 1: Example of laser cutting machine.

A laser cutting machine controlled by a computer is known as (CNC), this can control and monitor the movements of the cut. An amplified beam of light generates cuts depending on the movement of each axis. these axes are controlled by motors which give the position, velocity, and acceleration of the cut. So that, correct control of the motors means a precise cut.

Chapter 1 “Objective”

1.1. Description of the axes

the motor that controls the X-axis will have to carry the charge of the laser generator along its rail. While the motor that controls Y-axis will carry the structure of the X-axis and the laser generator, so that we should consider this in the election of the motors, nevertheless in the project is considered the same motor for both axes.



Fig 2: Movement mechanism.

1.2. Technical specification

The aim of this project is to simulate in MATLAB-Simulink the cut of three pieces a triangle, a semicircle, and a circle from a rectangular slab (3m long and 2m high). The size, placement, and orientation of the figures are arbitrary. The rest position of the machine is in the upper right corner of the plate.

The design to be made is as follows:

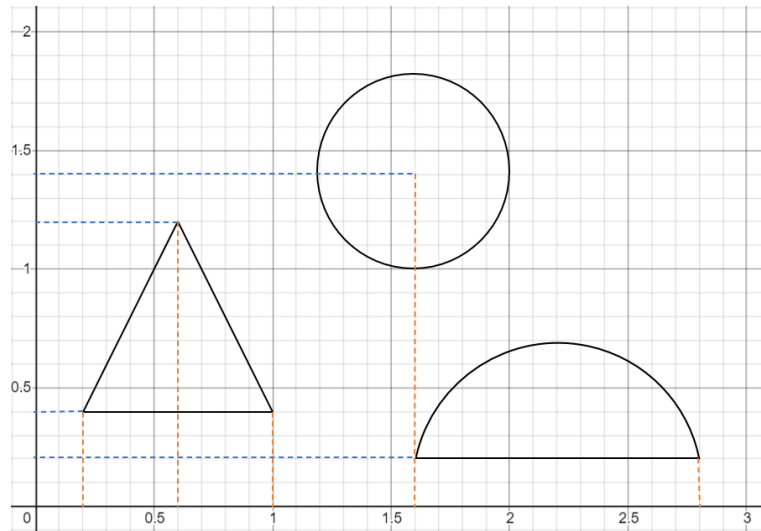


Fig 3: Cut to be made.

Process

1. The machine is considered in position (0,0). Then, the first action is to locate the machine in the rest position (3,2).
2. To start the cut is necessary to put a variable that identifies when the machine is cutting or only moving.
3. The reference generator will send data of position, velocity, and acceleration according to design to cut the figures.
4. Finally, the cut machine comes back to the rest position.

Chapter 2 “Reference Generator”

2.1 Polynomial

To reach the final position from the initial time to the final time in an axis, the trajectory has been described using the polynomial interpolation between the initial point and final point. This is achieved using a polynomial of three, five, or seven degrees for the generation of the reference.

The chosen polynomial is one of fifth degree because this gives us a good continuity in the acceleration signal:

$$\lambda(\sigma) = a\sigma^5 + b\sigma^4 + c\sigma^3 + d\sigma^2 + e\sigma + f$$

First, we assume a generic polynomial where the trajectory starts in 0 in “ $T_i = 0$ ” and end in 1 in “ $T_f = 1$ ” as is shown in fig 3.

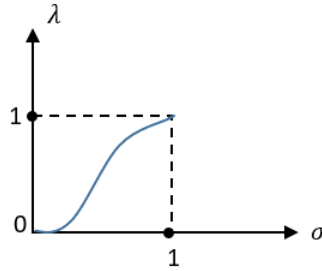


Fig 4: Generic polynomial

Hence, the extremes are $T_i = 0$ y $T_f = 1$ and it must be satisfied that:

Position: $\lambda(0) = 0$ $\lambda(1) = 1$

Velocity: $\dot{\lambda}(0) = 0$ $\dot{\lambda}(1) = 0$

Acceleration: $\ddot{\lambda}(0) = 0$ $\ddot{\lambda}(1) = 0$

We have the following system of linear equations:

$$\lambda(0)=f=0 \qquad \lambda(1)=aT_f^5 + bT_f^4 + cT_f^3=1$$

$$\dot{\lambda}(0)=e=0 \qquad \dot{\lambda}(1)=5aT_f^4 + 4bT_f^3 + 3cT_f^2=0$$

$$\ddot{\lambda}(0)=d=0 \qquad \ddot{\lambda}(1)=20aT_f^3 + 12bT_f^2 + 6cT_f=0$$

The solution of the linear equations system is:

$$a = 6, \quad b = -15, \quad c = 10, \quad d = 0, \quad e = 0; \quad f = 0$$

$$\lambda(\sigma) = 6\sigma^5 - 15\sigma^4 + 10\sigma^3$$

2.2 Polynomial location

Second, the polynomial is located in original time and position using the following equations.

Position

$$\lambda = \frac{X - X_i}{X_f - X_i} \quad \text{where} \quad X = X_i + \lambda(X_f - X_i) \quad \text{and} \quad Y = Y_i + \lambda(Y_f - Y_i)$$

Time

$$\sigma = \frac{t - T_i}{T_f - T_i} \quad \text{where} \quad t = T_i + \sigma(T_f - T_i)$$

Equation for calculating the circular forms:

$$X(\theta) = X_c + R \cos(\theta) \quad \text{where} \quad 0 \leq \theta \leq 2\pi$$

$$Y(\theta) = Y_c + R \sin(\theta) \quad \text{where} \quad 0 \leq \theta \leq 2\pi$$

The programming was made in Simulink as is shown in the fig 5.

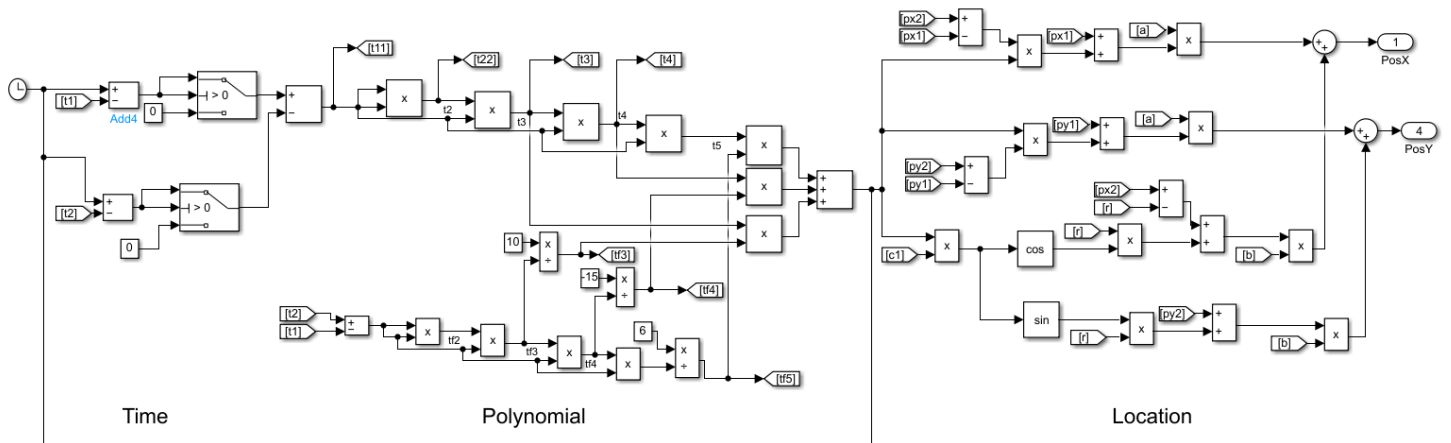


Fig 5: reference generator programming in Simulink

To do each interaction was used "Stateflow". This function is known as a state machine and allows to change the cutting parameters in the required time without the necessity of going out of Simulink. Besides, it was added a variable that shows when the machine is cutting or not.

the state machine consists of eleven states each one has:

- px1= initial position X
- px2= final position X
- py1= initial position Y
- py2= final position y
- t1= time initial

t2= time final
a= variable when is a straight cut
b= variable when is a circular cut
cut= cut or not

The following graphs plotted using Simulink represent the behavior of a fifth-degree polynomial for a signal with parameters: $t_1=0$, $t_2=10$, $px_1=0$, $px_2=3$ in terms of position, velocity, and acceleration profiles.

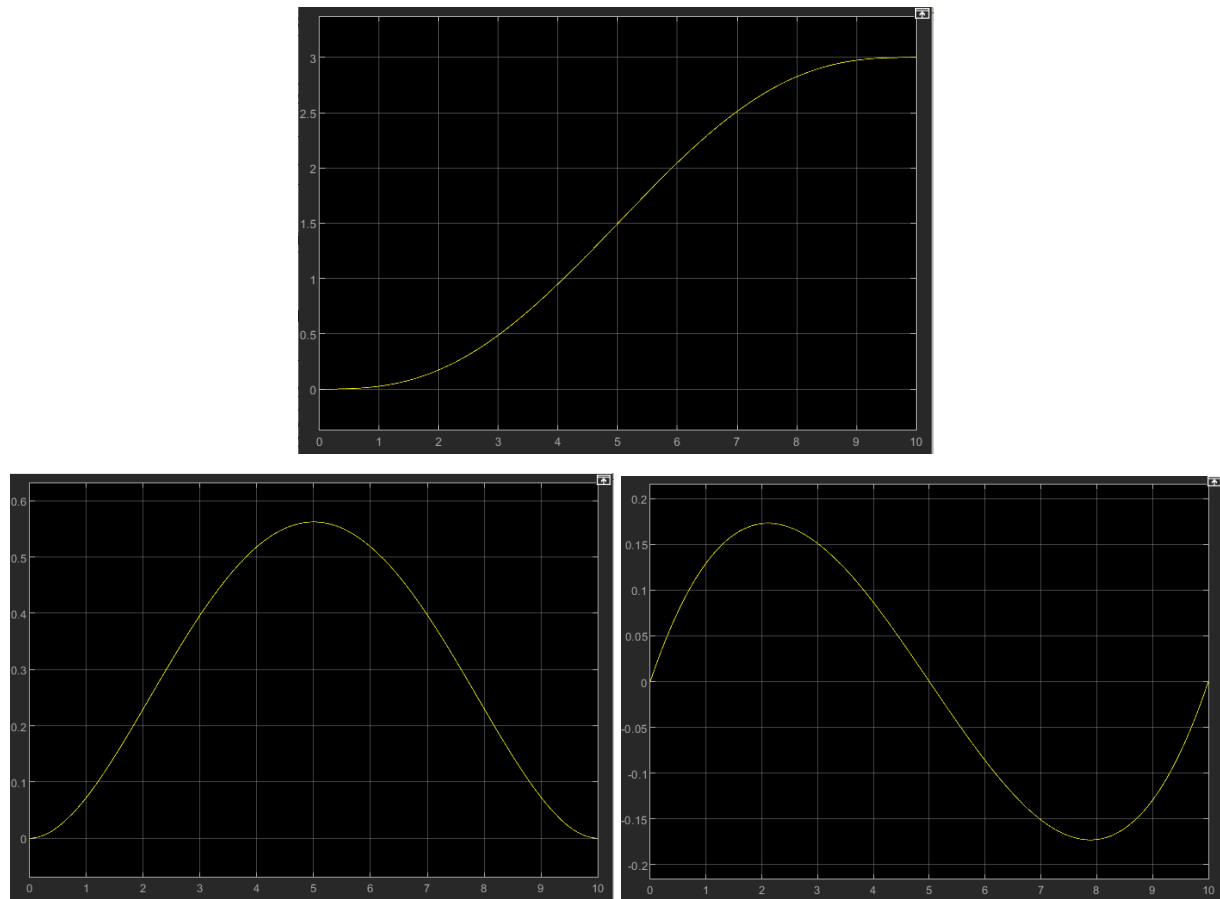


Fig 6. Graphics of position, velocity and acceleration in a 5th order polynomial.

Chapter 3 “Motor”

3.1 Structure of a DC motor.

The direct current electric motor is essentially a machine that converts electrical energy into motion or mechanical work, through an electromagnetic field. This is mainly composed of two parts, a stator that gives mechanical support to the device and has a hole in the center,

generally cylindrical. In the stator, there are also the poles, which can be permanent magnets or windings with copper wire on an iron core.

This DC machine is one of the most versatile in the industry. Its easy control of position, torque, and speed have made it one of the best options in process control and automation application

Mathematical model of the DC engine

the motor can be represented as an electrical and mechanical circuit.

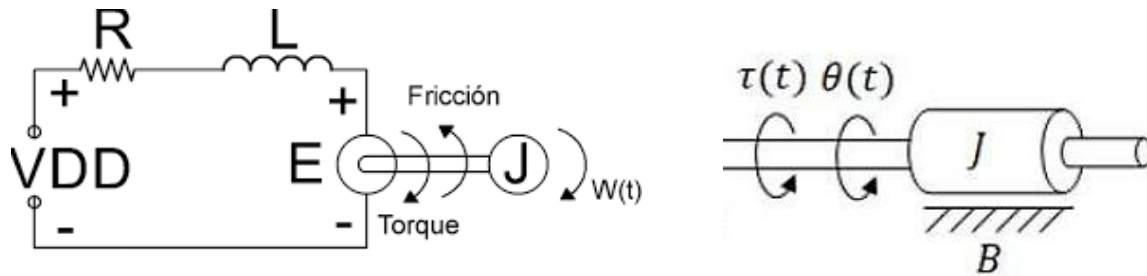


Fig 7: Direct current motor system.

To find the **electrical part**, we applied mesh law.

$$V_a = L_a \frac{di_a}{dt} + R_a i_a + V_e \quad \text{where} \quad V_e = K_m * w$$

Find the transfer function:

$$V_a(s) = sL_a i_a(s) + R_a i_a(s) + V_e(s)$$

$$V_a(s) - V_e(s) = (sL_a + R_a) i_a(s)$$

The **mechanical part** is related with the torque, which depend on load and armature inertia and the friction coefficient. Besides the torque is directly related with the current.

$$\frac{d\theta}{dt} = \omega$$

$$\tau_r = J \frac{d\omega}{dt} + b\omega + K_m i_a$$

Find the transfer function:

$$\tau_r + K_m i_a(s) = (sJ + b)\omega(s)$$

Where:

L_a	Armature inductance
K_m	Torque coefficient and electromotive force
R_a	Armature Resistance
J	Load and armature Inertia

b Friction coefficient
 i_a Armature current
 Ω Rotor speed
 θ Rotor position
 V_a Armature voltage
 τ_r Disturbance

The motor transfer function is expressed as an input-output form, that is:

$$G(s) = \frac{K_m}{(sL_a + R_a)(Js + b) + K_m^2}$$

The direct current motor scheme is the following:

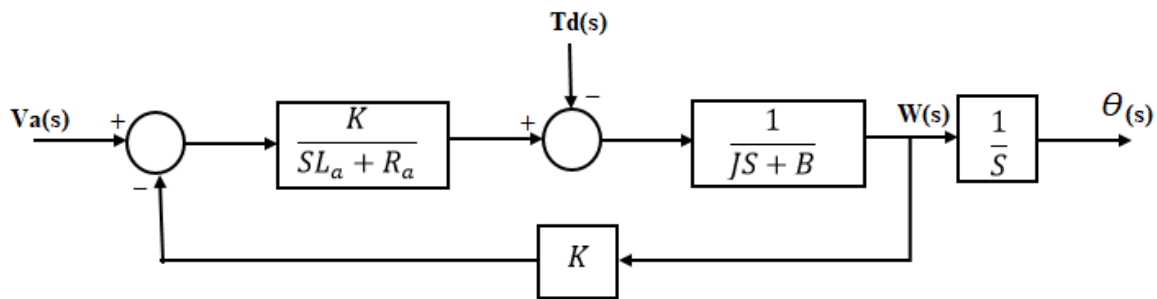


Fig 8: Block diagram of a permanent DC motor.

As we can see an electric motor can be described in terms of two first order dynamics blocks, where the electric pole is $P_e = -\frac{R_a}{L_a}$ and the mechanical pole is $P_m = -\frac{b}{J}$.

Specifications of the motors used

the motors for each axes have the same characteristics

Ra	La	J	B	K
1Ω	1.0mH	0.1Kg m^2	0.029 $\frac{Nms}{rad}$	0.6

- Max voltage 150v
- Max armature current 20A
- Max power 4kw
- Coefficient of transformation for both axes $1m = 2000 \text{ revolutions}$

Implementation in Simulink

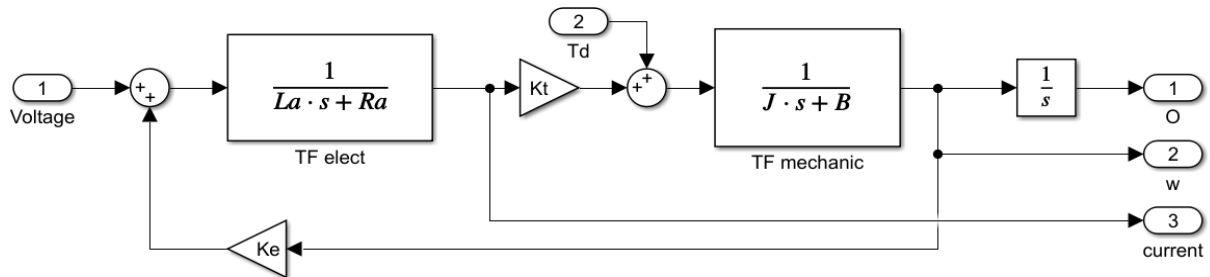


Fig 7: Direct current motor design in Simulink

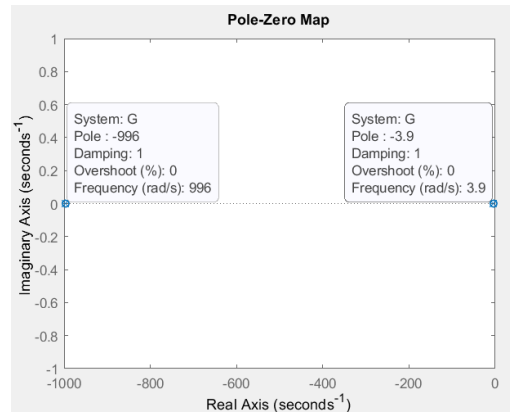


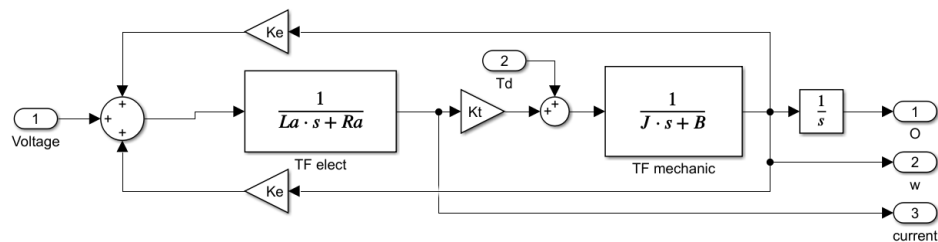
Fig 8: Pole diagram.

The poles of the closed-loop system are: Electric pole $P_e = -996$ and mechanical pole: $P_m = -3.9$, the system is asymptotically stable because its poles are negative.

Besides it's possible to see that electric pole is much bigger than mechanical pole, it works in order of milliseconds, while that mechanical pole work in order of the seconds, because of that the transient is governed by the mechanical pole.

3.2 The motor's control

first step is eliminating the internal feedback of the motor because this will do changes in the poles when the motor is working.



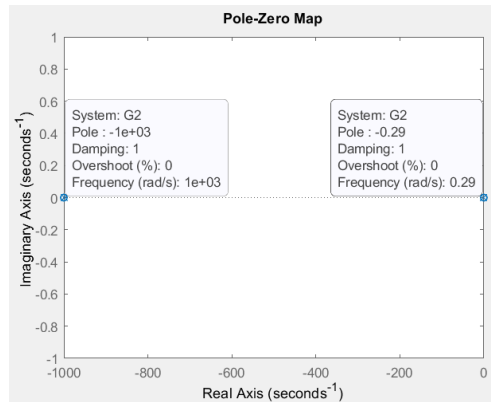


Fig 8: Eliminating of the internal feedback and new poles

The new poles of the open-loop system are: Electric pole $P_e = -1000$ and mechanical pole: $P_m = -0.29$, the system is asymptotically stable because its poles are negative.

3.3 Control of current, velocity and position.

Current control

The current control is necessary because we don't want variations in this to avoid the Joule effect, to get this goal the mechanical part is not taken into count, because the dynamic of this part doesn't have an effect on the electric part, due to the electric pole is at least three order bigger.

The control is a PI because the objective is to keep a fixed electrical pole in the same position which is gotten by a gain K_i .

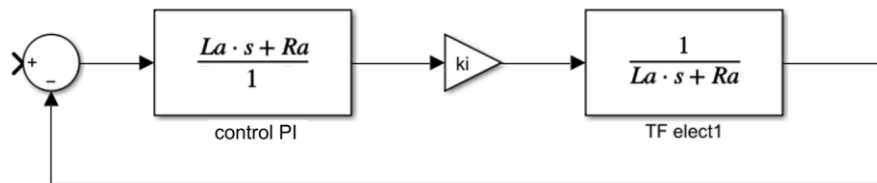


Fig 8: current control

Controller PI

$$Ci = \frac{\tau_e s + 1}{s} * ki = La * ki + \frac{Ra * ki}{s}$$

$$We = \frac{1}{\frac{1}{ki}s + 1}$$

$$\frac{1}{ki} = \frac{Ra}{La}$$

$$ki = 1000$$

Generally, you are looking not to modify the electrical part so much, the integral effect tends to compensate the errors made in the measurement of R_a y L_a .

Velocity control

We consider the control of the electrical part as a state-gain α because the dynamic of the electrical system is much slower than the mechanical system.

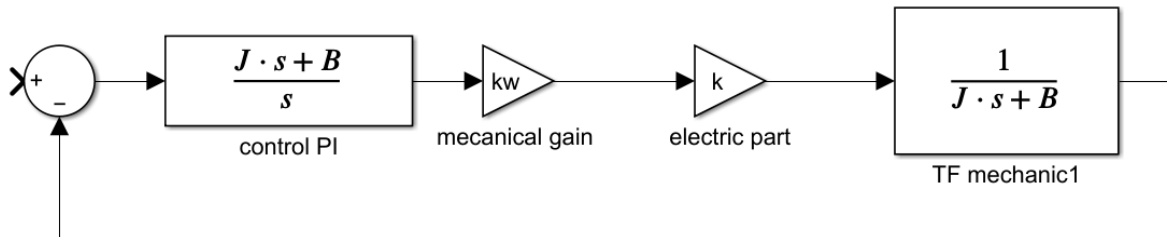


Fig 9: Velocity control

$$C_W = \frac{\tau_w s + 1}{s} * k_w = J * k_w + \frac{B * k_w}{s}$$

$$W_m = \frac{1}{\frac{1}{k_w k} s + 1}$$

K_w is chosen in such a way as to allow the mechanical part to be given a little speed, of at least a decade, In this case $K = 10$. Then our mechanical pole is located $P_m = 6.036$

With this type of feedback are compensated eventual errors or ignorance concerning nominal values of J and B ; it also reduces or eliminates the disturbance effects that are present.

Position control

Finally, we consider the current and velocity control as a constant because there is at least 1 order of difference between the dynamic of velocity and dynamic of position.

Only a proportional action is chosen, setting the value of K_p at least 1 decade to the right of the velocity pole, In This case $K_p = 1.1$

Simulink model of the Motor control scheme:

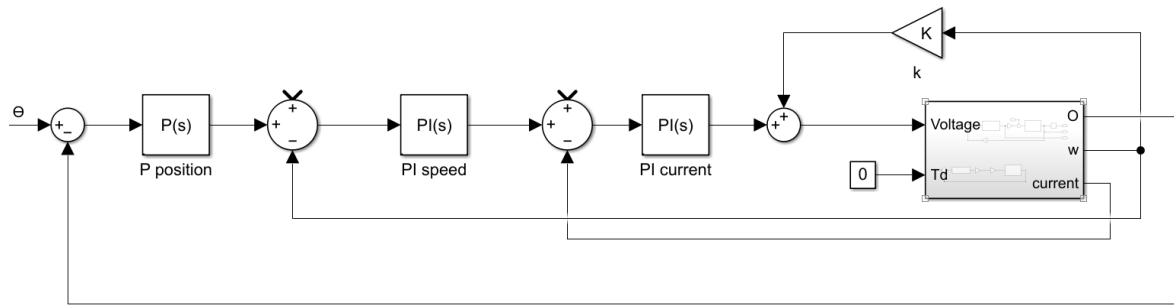


Fig 9: Motor control

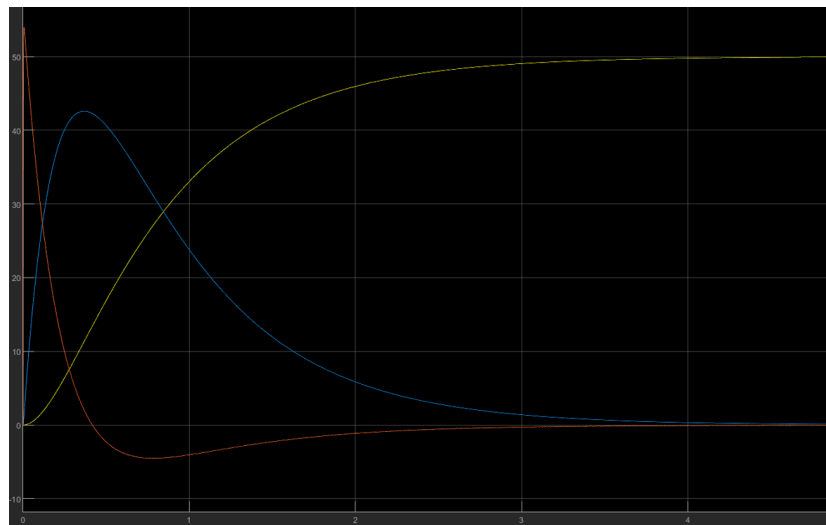


Fig 10: System stability analysis.

As can be seen, the System is stable before a chosen random reference of 50 rad, so it can be concluded that the selected parameters of the controller are correct.

3.4 Inverse model

The Inverse model is to create an auxiliary controller in the closed control loop that allows attenuating or eliminating the entrance of measured or known disturbances to the control loop that is why it is called anticipatory control because it tries to anticipate the disturbances that go to affect the system.

The general control scheme is shown below:

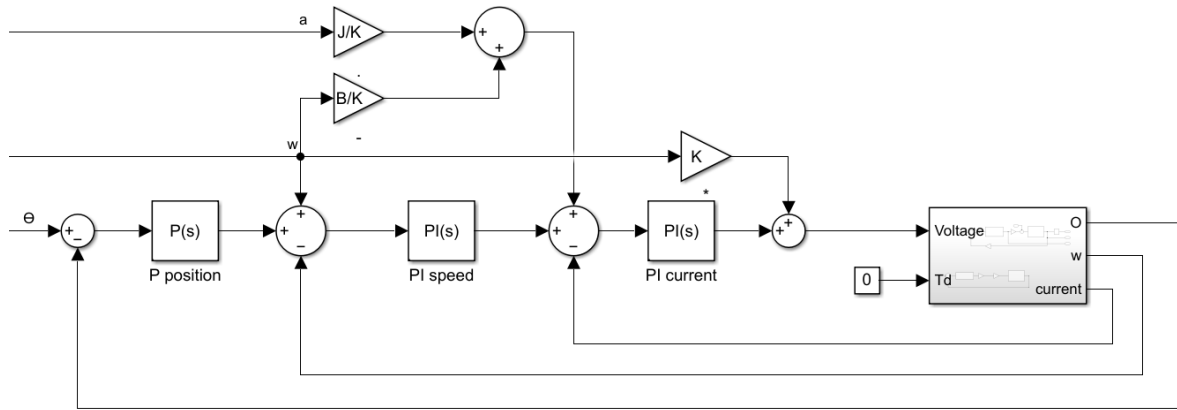


Fig 10: Inverse model.

Thus, if there are more charges in the motor, this will be not affected in the position and velocity.

Finally, the following scheme implemented in Simulink represent the complete cutting machine including the reference generator, a motor for each axis, the control of each motor plus the inverse model, and in the end the coefficient transformation of the motors.

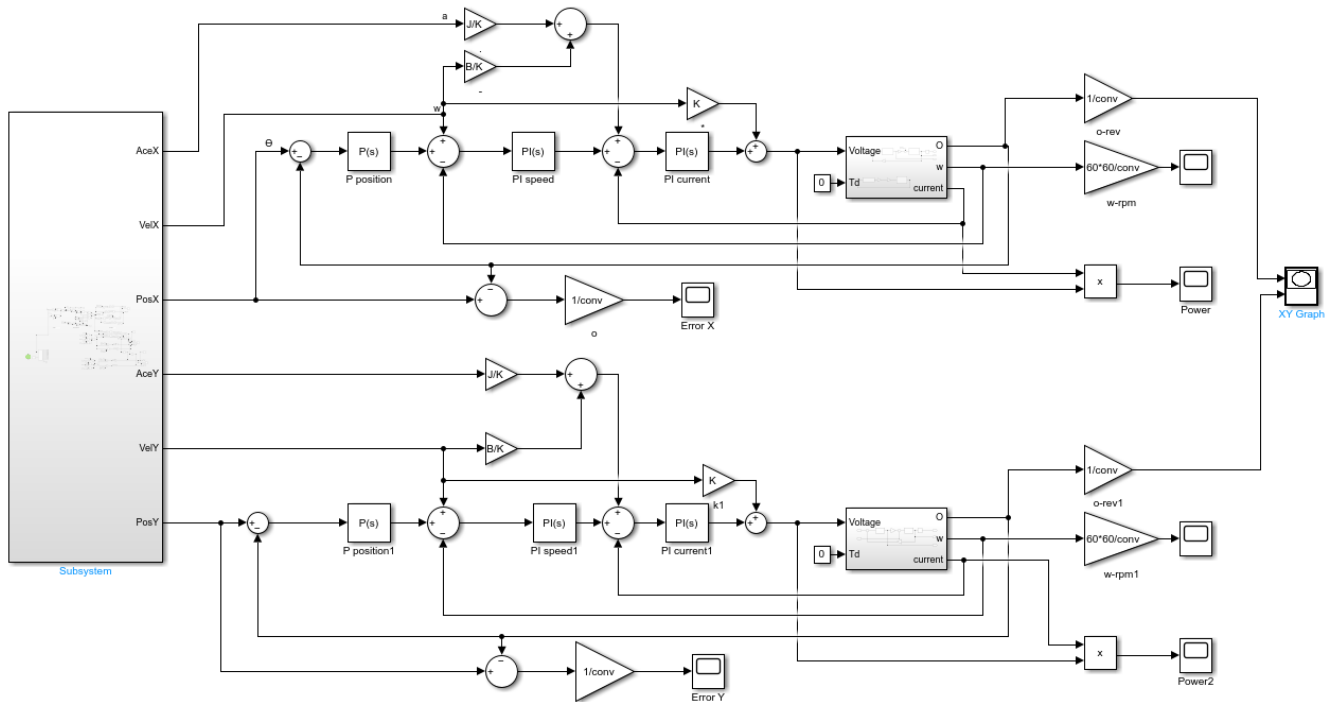


Fig 11: Cutting machine block diagram.

Results

Result of the reference generator

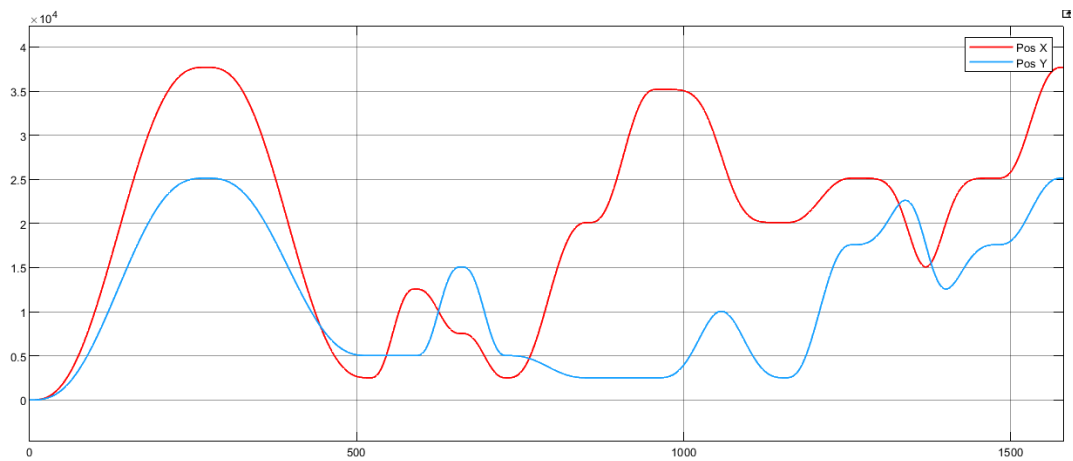


Fig 11: Position of axes X and Y of reference generator

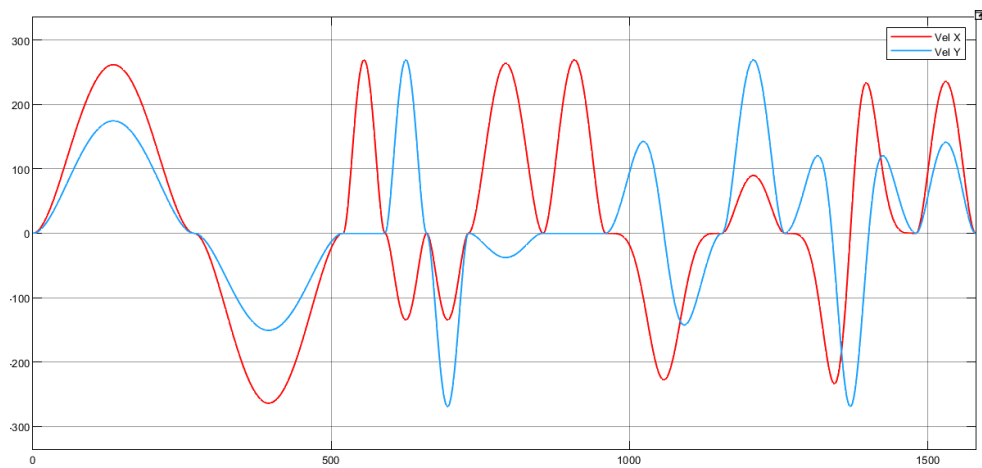


Fig 11: Velocity of axes X and Y of reference generator

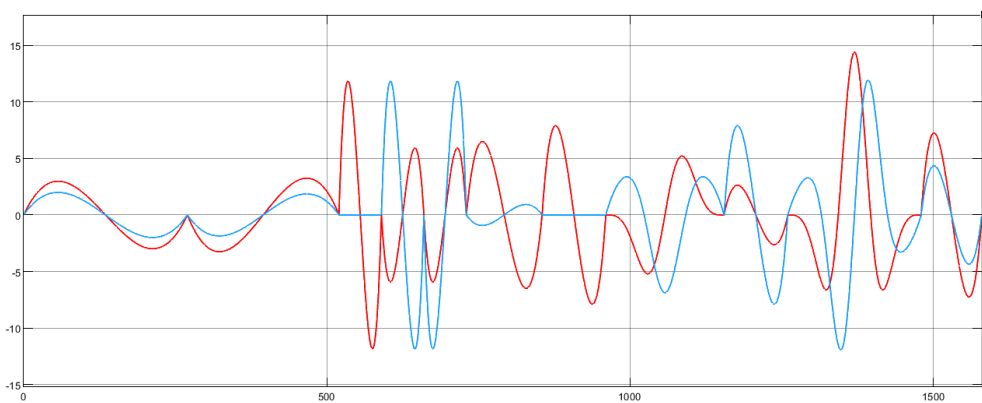


Fig 12: Acceleration of axes X and Y of reference generator

Result of the motor after control

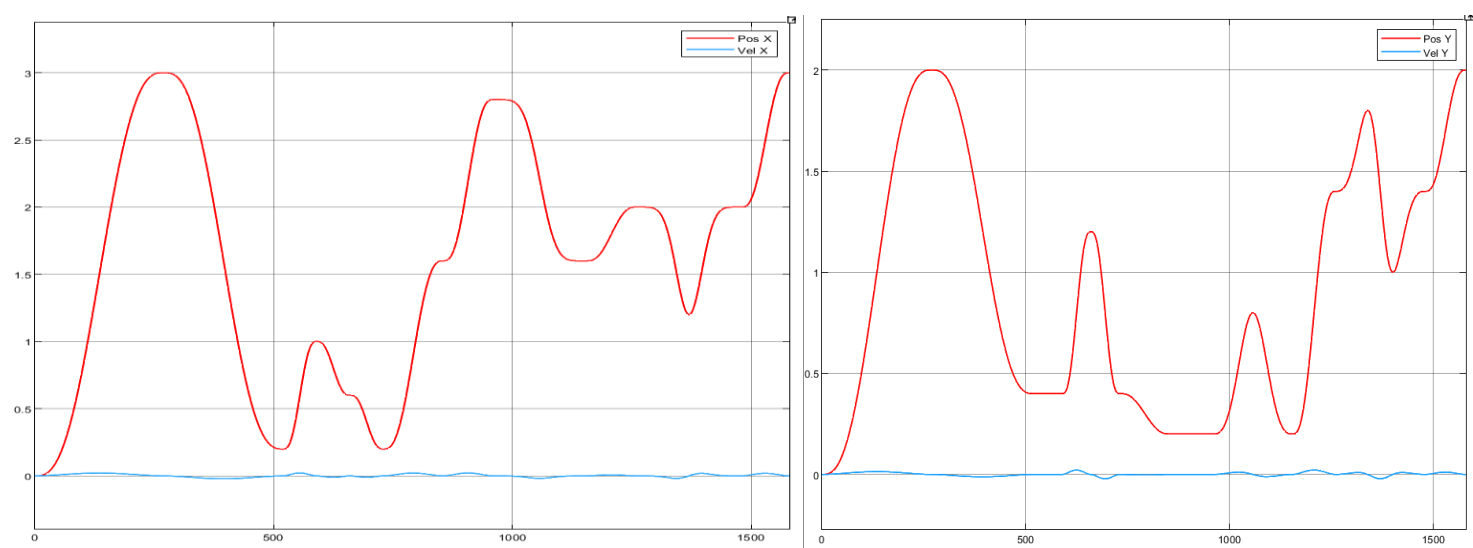


Fig 13: Position and velocity of the motor in X and Y axes

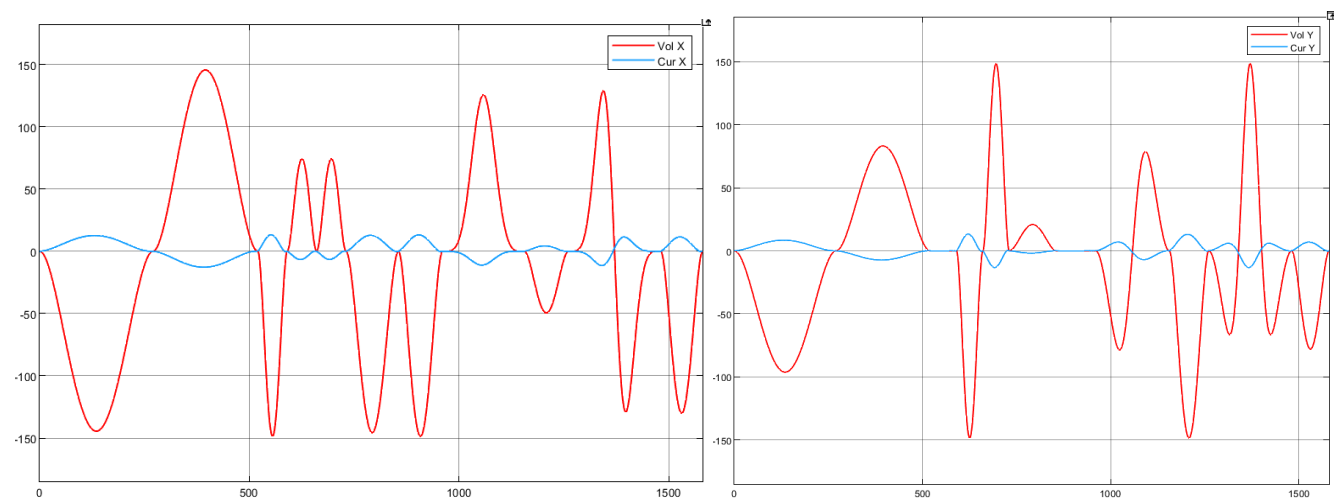


Fig 14: Voltage and current of the motor in X and Y axes

Results of the tracking error without changing J and B parameters

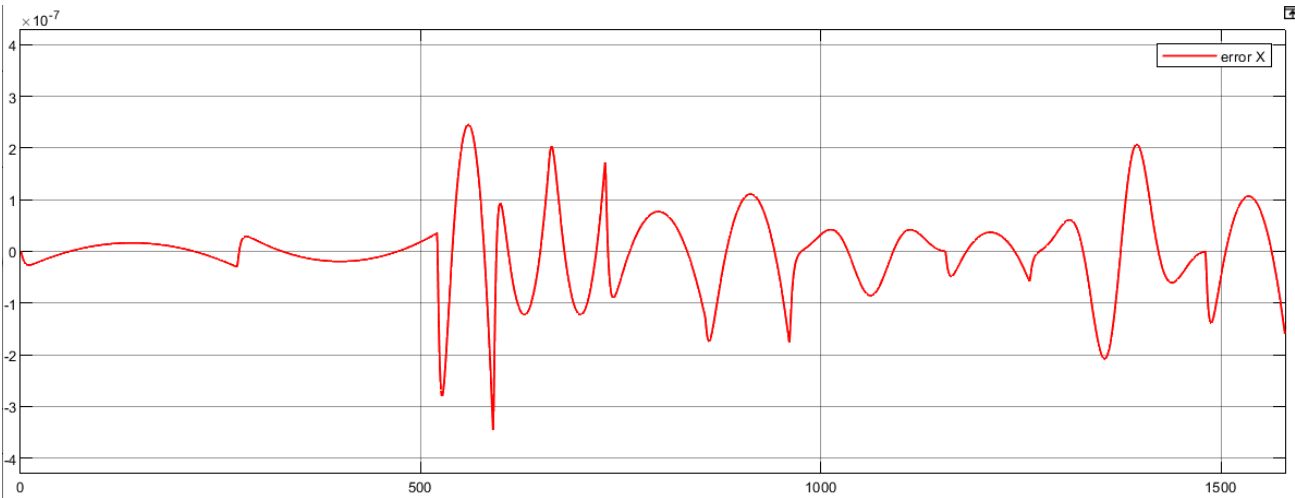


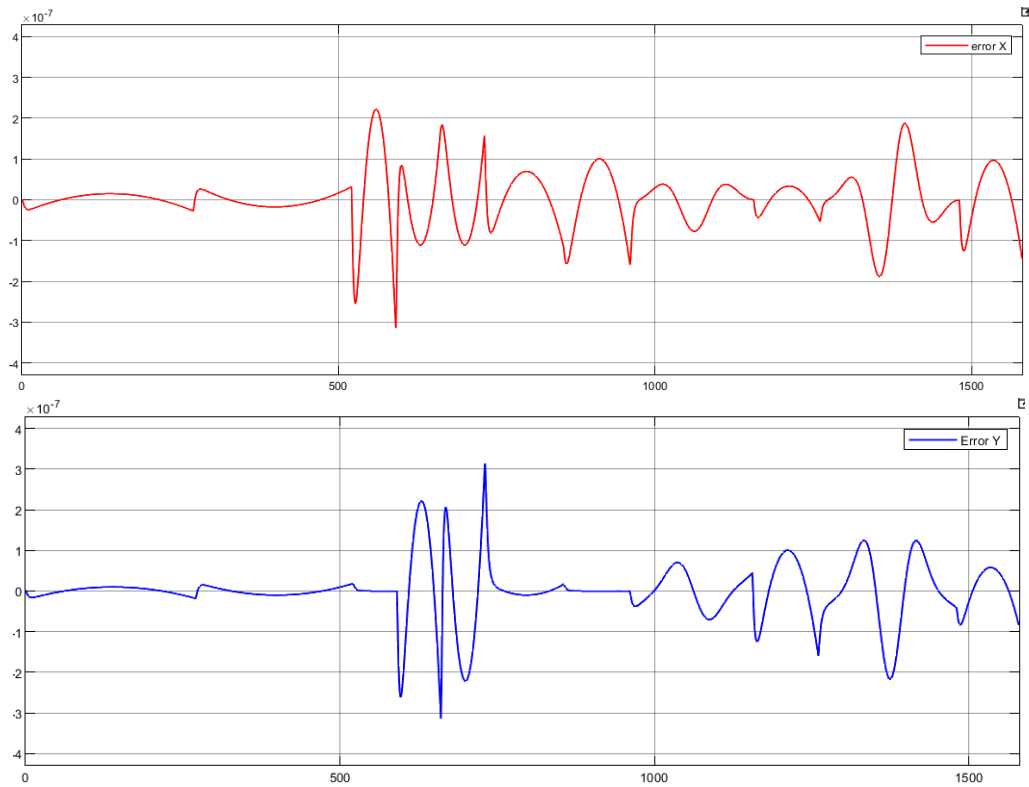
Fig 15: tracking error in X axis



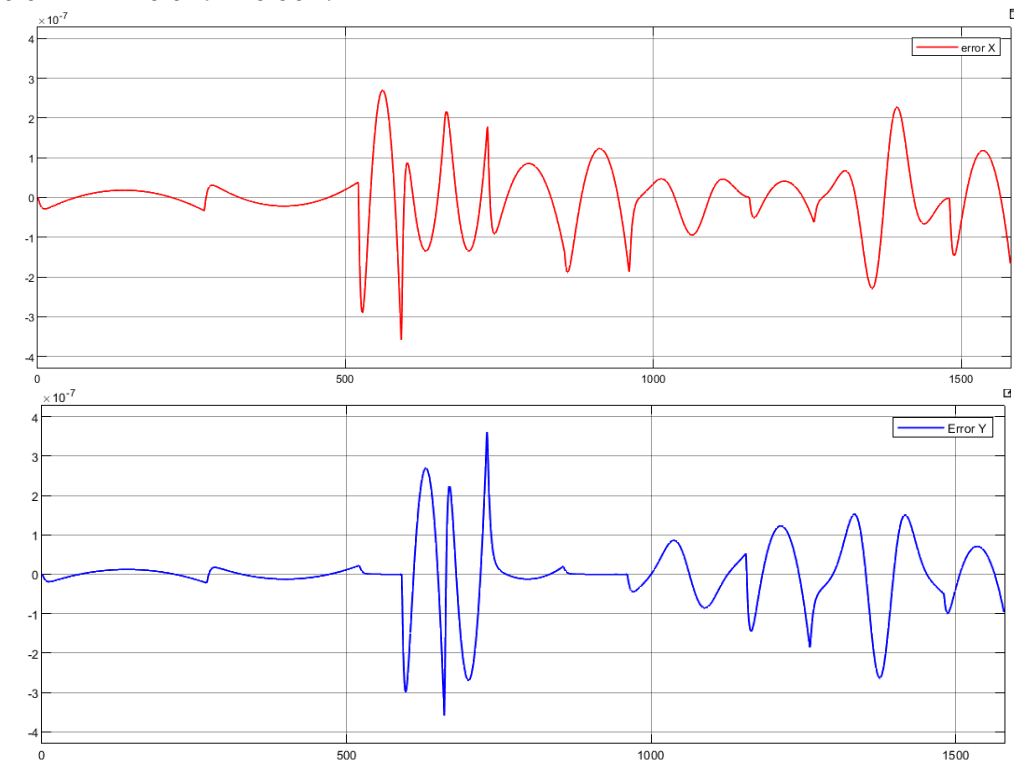
Fig 15: tracking error in Y axis

Results of the tracking error changing J and B parameters in 10% positive and negative

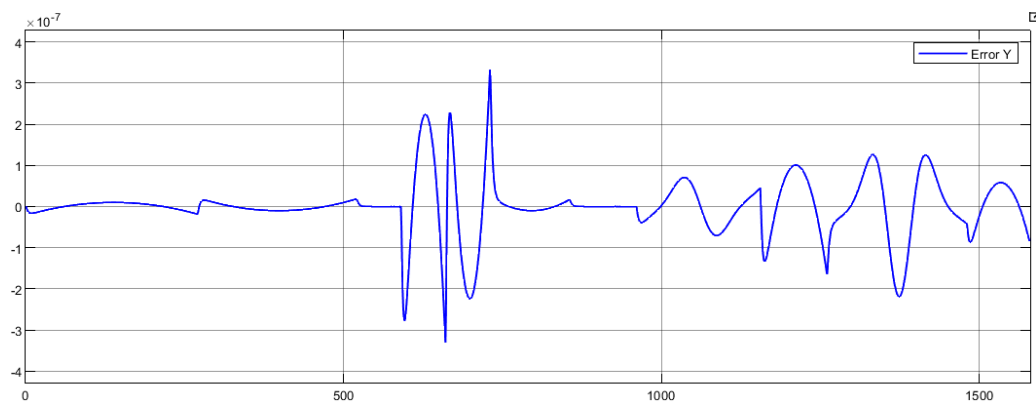
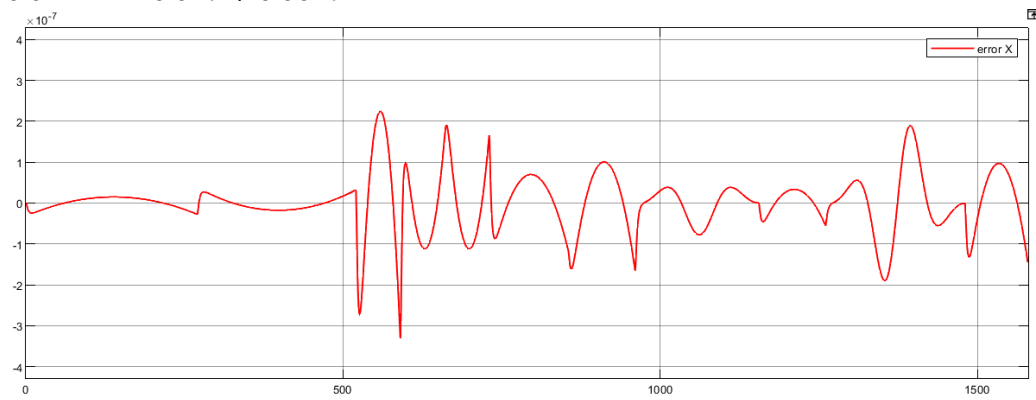
$J = 0.1 + 0.01 \quad B = 0.029 + 0.0029$



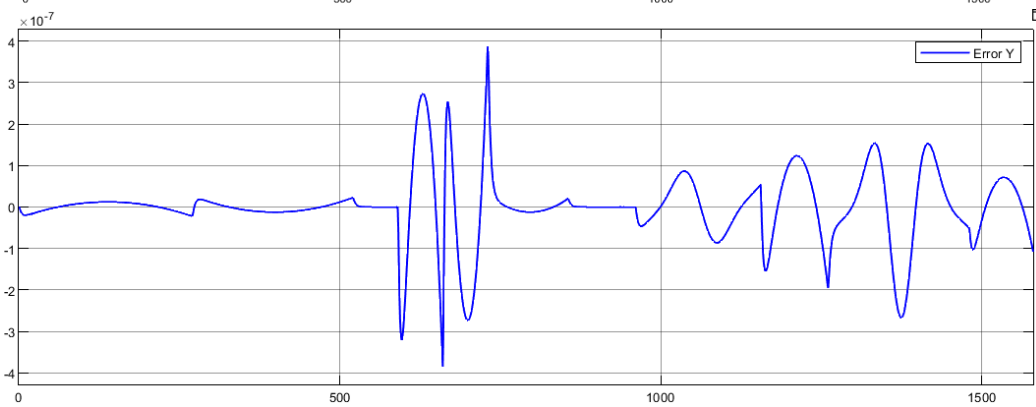
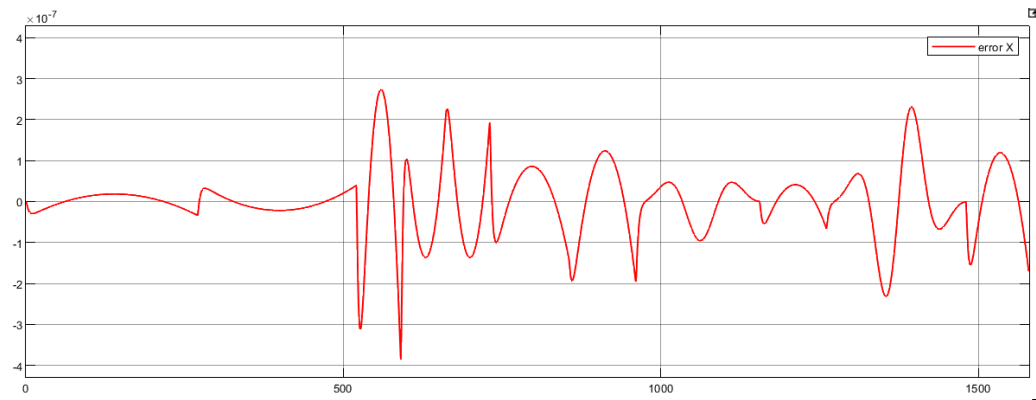
$J = 0.1 + 0.01 \quad B = 0.029 - 0.0029$



$$J = 0.1 - 0.01 \quad B = 0.029 + 0.0029$$

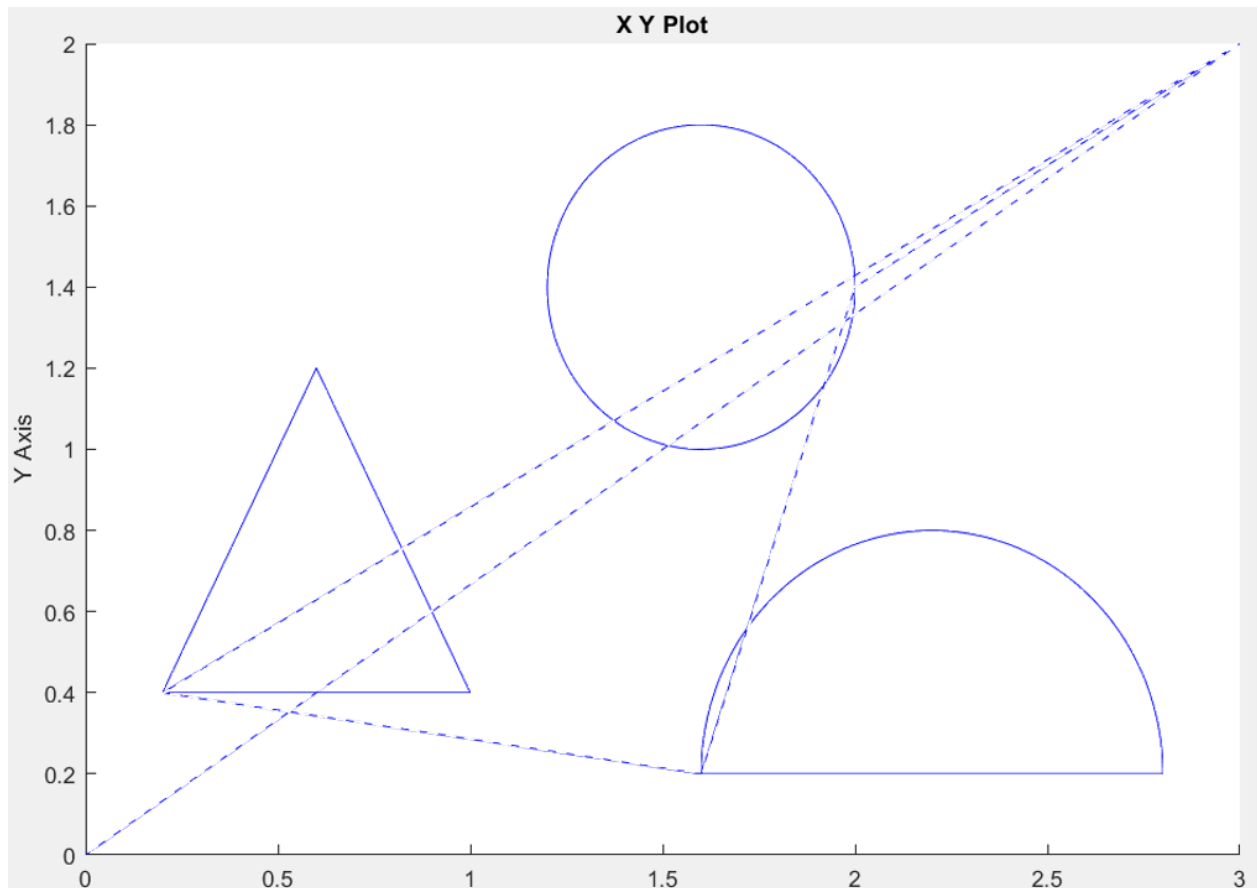


$$J = 0.1 - 0.01 \quad B = 0.029 - 0.0029$$



Final result of the cutting machine.

The result of the cutting machine implemented in Simulink is shown below, where the broken lines are the movements that do the machine when isn't cutting and the solid lines are the figures cut.



Conclusions

- The reference generator was implemented totally in Simulink, we can see that the time for each interaction is modified by displacing it depending on the parameters, obtaining a good result. A fifth-order polynomial was created to plan the trajectory for the position. furthermore, the polynomials for the velocity and acceleration and how it is shown in the results the signals are continuously and without disturbances.
- The control action was implemented in both motors of identical characteristics located on each of the axes of the cutting machine, the adjustment of parameters of each controller were successful giving, as a result, tracking errors less than 0.01 mm. the addition of inverse model in the control assure that the tracking error continues being small even if we change the parameters B, J in 10%.
- The laser cutting machine has been implemented successfully inside of MATLAB-Simulink environment, the extraction of figures was achieved with millimetric precision in a time of 26 minutes due to for the characteristic of the motors it is assumed that these are small, and it is necessary putting a long time to prevent damage.

BIBLIOGRAPHY

1. Prof Pietro M. Muraca, Notes from the Industrial Automation Course.
2. K.Ogata. Modern Control Engineering, 5ta Edition.