Active Rear Wheel Steering and Direct Yaw Moment Robust Control using Automated Driving Toolbox

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***Abstract*— Considering the demand for vehicle stability control, an integrated control system of active rear wheel steering (4WS) and direct yaw moment control (DYC) is presented in this paper. The proposed control system is integrated in a LPV model, which makes the vehicle follow the desired trajectory, using the state feedback and feedforward gains compute by H-inf and L1 of yaw rate, side slip angle and tracking error states. Finally, numerical simulations based on MATLAB/Simulink and Automated Driving Toolbox[1] experiments were performed with the proposed control strategy to identify its performance. The simulation and experimental results indicate that the handling stability of the 4WS vehicle is improved by the optimal controllers.**

Keywords*— direct yaw moment, model matching controller, side slip angle, yaw rate, MATLAB/Simulink, optimal controller)

I. INTRODUCTION

With the development of electronic and control technologies, the four-wheel steering (4WS) system for automobiles has been actively studied as an effective vehicle maneuvering technology that can improve the maneuvering of vehicles at low speeds and enhance their stability at high speeds. In order to enhance control performance, many studies have focused on the control strategies of the 4WS system since the first 4WS system became popular, especially its steering stability controller design, which has recently been a hot research topic.

Various control strategies have been performed on vehicle stability control, such as PID control, fuzzy control, and neural network control. Although these control approaches enhance vehicle stability to a certain extent, neural network control needs a wide variety of experimental data, fuzzy control requires human experience to construct the fuzzy rules, and PID control shows weak robustness against external disturbances. Modern robust control theories have demonstrated some effective methods to deal with the above drawbacks. Moreover, it is well known that the handling stability control containing various uncertainties is a highly nonlinear and complex process. The 4WS vehicle stability control system is vulnerable to uncertainties, such as modeling uncertainties, external disturbances, measurement noises, and parameter variations. Hence, robust control urgently needs to be introduced into the 4WS system so as to reduce the influences of uncertainties on handling stability control.

Especially various active control systems for rear wheel steering have been developed by feed forward and/or feedback compensations for side slip motion in vehicle body. Although the improvement of handling and stability results in quite a good effect on driver steering manoeuvres, so-called, a closed loop performance of driver-vehicle system, it is limited within a linear dynamical system because of the decrease of cornering stiffness of tires during a high lateral acceleration.

Direct yaw moment control systems (DYC) using driving/braking forces have been researched and developed also to improve handling and stability. One of them is active traction control system of each wheel through the feedback of state variables, such as yaw rate. The other is active braking control system through the feedback of state variables, such as yaw rate and/or side slip angle in vehicle body. These active control systems can generate yaw moment directly to compensate for vehicle yaw motion not only in linear ranges but also in nonlinear ranges of tire performance. Therefore, the direct yaw moment control systems are expected to suppress the deterioration of the steering control effects in nonlinear or large lateral acceleration ranges. Fig. 1 shows the effective areas of 4WS and DYC.

II. MODEL

In this article a robust control H_inf and L1 with integral effect is implemented in a LPV system. To this end, we have proceeded to:

- Obtain the LPV Model and implement in the Simulink environment.
- Integrate an integral effect to the LPV model.
- Develop the Feedback and Feedforward state controller using optimal control techniques.
- Generate a velocity reference to observe the robustness of the system and implement a gain scheduling control.
- Finally, generate a simulation of a real environment in the Automated Driving Toolbox to prove the Robust control.

A. *modeling*

Four-wheel steering systems have been researched based on the two-degree-of-freedom (2DOF) steering model shown in fig. 1. Systems equations of this model governing the side slip angle and the yaw rate are developed as follows

Nomenclature

$\boldsymbol{\beta}$ Side slip angle of the vehicle body

$\boldsymbol{\gamma}$	Yaw rate of the vehicle body
$\boldsymbol{\delta sw}$	Steering wheel angle
$\boldsymbol{\delta f}$	Front wheel steering angle input
$\boldsymbol{\delta r}$	Rear wheel steering angle input
$\boldsymbol{N}$	Direct yaw moment input
$\boldsymbol{V}$	Vehicle velocity
$\boldsymbol{m}$	Vehicle mass
$\boldsymbol{Iz}$	Yaw moment of inertia
$\boldsymbol{af}$,$\boldsymbol{ar}$	Distance from the center of gravity to front/rear axle.
$\boldsymbol{cf}$, $\boldsymbol{cr}$	Cornering stiffness at front/rear tire
αf ,α $\boldsymbol{r}$	Slip angle at front/real wheels

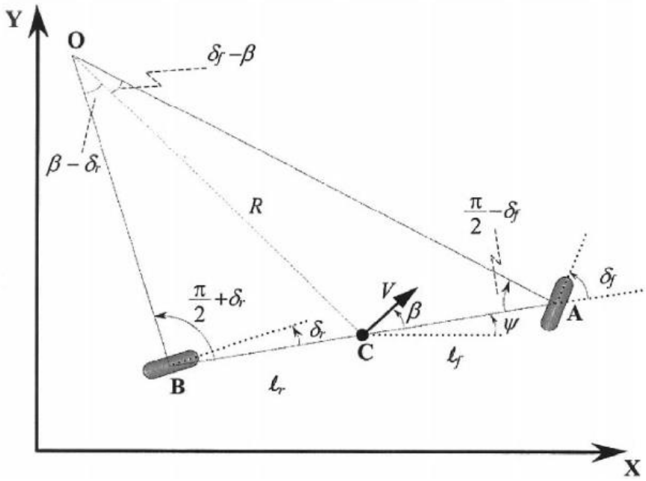

Fig.1 Kinematics lateral motion

B. *Matematical model*

System equations of this model governing the side slip angle and the yaw rate are written as follows.

$$m\dot{V}\sin(\beta) + mVcos(\gamma + \dot{\beta}) = C_f(\delta_f - \beta_f) + C_r(\delta_r - \beta_r) \quad (1)$$

$$I_z\dot{\gamma} = C_f a_f(\delta_f - \beta_f) + C_r a_r(\delta_r - \beta_r) \quad (2)$$

For small variations of angles β_f , β_f , β and longitudinal acceleration null, the lineal model is given for the equations 3 and 4:

$$mV(\gamma + \dot{\beta}) = C_f(\delta_f - \beta - \frac{a_f\gamma}{V}) + C_r(\delta_r - \beta + \frac{a_r\gamma}{V}) \quad (3)$$

$$I_z\dot{\gamma} = C_f a_f\left(\delta_f - \beta - \frac{a_f\gamma}{V}\right) + C_r a_r\left(\delta_r - \beta + \frac{a_r\gamma}{V}\right) + \mathrm{N} \quad (4)$$

The equations of the 2DOF motions can be written in the following state space from.

$$\dot{x} = Ax + Bu + E\delta_f \quad (5)$$

where the state vector including the side slip angle and the yaw rate, and the input vector including the rear steer angle and the direct yaw moment are written as follows.

$$x = \begin{bmatrix}\beta\\ \gamma\end{bmatrix}, \qquad u = \begin{bmatrix}\delta_r\\ N\end{bmatrix} \quad (6)$$

And the coefficient matrices are:

$$A = \begin{bmatrix} -\frac{C_f+C_r}{mV} & -\frac{a_fC_f-a_rC_r}{mV^2} - 1 \\ -\frac{a_fC_f-a_rC_r}{I_z} & -\frac{{a_f}^2C_f-{a_r}^2C_r}{I_zV}\end{bmatrix}$$
$$B = \begin{bmatrix} \frac{C_r}{mV} & 0 \\ -\frac{a_rC_r}{I_z} & \frac{1}{I_z}\end{bmatrix}, \; E = \begin{bmatrix}\frac{C_f}{mV}\\ \frac{a_fC_f}{I_z}\end{bmatrix}, \quad (7)$$

The parameters of the model are shown in the table below.

Table.1. Vehicle model parameters

Original vehicle
$\boldsymbol{m}$ = 1562 kg
$\boldsymbol{I_z}$ = 2630 kg m^2
$\boldsymbol{a_f}$ = 1.104 m
$\boldsymbol{a_r}$ = 1.421 m
$\boldsymbol{c_f}$ = 42,000 N/rad
$\boldsymbol{c_r}$ = 64,000 N/rad
V = 80 k/h

The eigenvalues of the model are:

$$\lambda_1 = -3.0704 + 3.9962i$$
$$\lambda_2 = -3.0704 - 3.9962i$$

These have real negative parts, then the system is stable. Also, if we check the rank of the reachability and observability matrices, it is found that the system is fully reachable and observable then is possible to design any control law.

C. *LPV model system*

The Linear Parameter Varying version of the system is stated to give an uncertain modelling equal to the robust case, with the difference of hypotheses that we can measure the uncertain parameter online, which varies over time. Such information will be exploited in the synthesis by building a time-varying gain.

Therefore $p = p(t)$ consequently we will have $A(p(t))$ where:

$$\dot{x}(t) = A(p(t))x(t) + B(p(t))u(t) + E(p(t))\delta_f(t)$$
$$y(t) = C(p(t))x(t) + D(p(t))u(t) + D_w(p(t))\,\delta_f(t \quad (8)$$

Is the LPV system equations. It must be remembered that the value of the speed is determined through sensors in real time, which can measure the speed at any instant of time.

The matrices of the system depend on the ρn parameter through the following relationship.

$$A(\rho n) = (1-\rho n)\underline{A} + \rho n\overline{A} \qquad B(\rho n) = (1-\rho n)\underline{B} + \rho n\overline{B}$$
$$E(\rho n) = (1-\rho n)\underline{E} + \rho n\overline{E} \qquad D(\rho n) = (1-\rho n)\underline{D} + \rho n\overline{D} \qquad (9)$$
$$D_w(\rho n) = (1-\rho n)\underline{D_w} + \rho n\overline{D_w} \quad C(\rho n) = (1-\rho n)\underline{C} + \rho n\overline{C}$$
$$\rho = \frac{v - v_{min}}{v_{max} - v_{min}}$$

Then, the control gain will be obtained with the following equation.

$$k(\rho n) = (1-\rho n)\underline{k} + \rho n\overline{k} \qquad (10)$$

III. CONTROL DESING

A. *Implementation of Integral Effect*

In this section it is introduced an offset free effect in the control problem. Many tracking problems can be converted to classic regulation problems and solved as shown below:

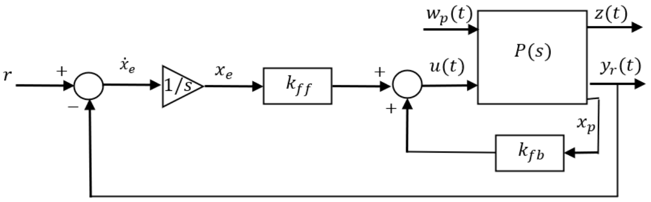

Fig.2 Integral effect with 1 pole Schematic

Where r is the reference signal, yr is the reference output and wp exogenous disturbance. The model in the state space is.

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_u u(t) + B_w w_p(t) \\ z(t) = C_z x_p(t) + D_z u(t) + F_z w_p(t) \\ y_r(t) = C_r x_p(t) + D_r u(t) + F_r w_p(t) \end{cases} \qquad (11)$$

In the scalar case, given a reference signal r(t)=r, the idea is regulating the error when the time tends to infinite.

$$\lim_{t\to\infty} e(t) = 0 \qquad (12)$$

The error is given by $e(t) = r(t) - y_r(t)$ This problem can be converted in a regulation problem by introducing a new state component and finding an extended plant.

$$\dot{x}_e(t) = r(t) - C_r x_p(t) + D_r u(t) + F_r w_p(t) \qquad (13)$$

The extended plant is represented as in the Eq,14

$$\begin{bmatrix} \dot{x}_e \\ \dot{x}_p \end{bmatrix} = \begin{bmatrix} 0 & -C_r \\ 0 & A_p \end{bmatrix} \begin{bmatrix} x_e \\ x_p \end{bmatrix} + \begin{bmatrix} -D_r \\ B_p \end{bmatrix} u + \begin{bmatrix} -F_r & 1 \\ B_w & 0 \end{bmatrix} \begin{bmatrix} w_p \\ r \end{bmatrix} \qquad (14)$$
$$z = \begin{bmatrix} 0 & C_r \end{bmatrix} \begin{bmatrix} x_e \\ x_p \end{bmatrix} + D_z u + \begin{bmatrix} F_z & 0 \end{bmatrix} \begin{bmatrix} w_p \\ r \end{bmatrix}$$

And the control llaw become as follows, with feedback and feedforward gain

$$u(t) = kff x_e(t) + kfb x_p(t) \qquad (15)$$

B. *Optimal control*

Integral control gains are obtained by using LMI (Linear Matrix Inequalities) with optimal control approach H8, H2 and L1.

For design paper reasons, the controls will be designed for a static velocity of 80km, and it will be shown the behavior of each one at this velocity and for a Moose test shown in the Fig.3. Then, in the simulation section, these will be implemented at different variations of speed to make robust control.

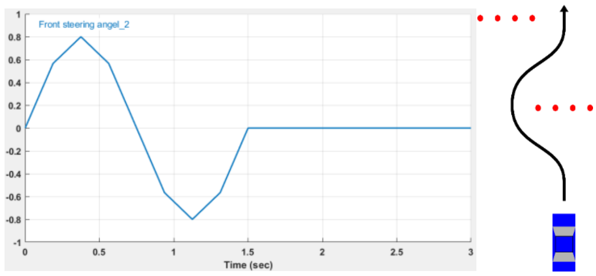

Fig.3 Front steering angle - Moose test

H_∞ *– controller*

The objective of this control is to minimize the norm H_∞ of the transfer function $W(s)$. From the induced norms is possible found the relationship that link the input and output with H_∞ norm.

$$\|y\|_2 \le \|W\|_{H_\infty} \|u\|_2 \qquad (16)$$

Given a finite dimensional LTI system, it is necessary to find static feedback of the state such that in closed loop the transfer function W has the norm H_∞ smaller than an upper limit fixed a priori.

$$\|W\|_{H_\infty} < \gamma \qquad (17)$$

It is wanted to minimize the output of the error with respect to the disturbance according to the Eq.5. Then, it is possible set the following LMI problem:

$$[x^*, y^*] = \min_{X,Y,\gamma} \gamma$$
$$s.t$$
$$\begin{bmatrix} AX + B_1Y + (AX + B_1Y)^T & B_2 & (C_1X + D_{12}Y)^T \\ B_2^T & -\gamma I & D_{11}^T \\ (C_1X + D_{12}Y) & D_{11} & -\gamma I \end{bmatrix} < 0 \qquad (18)$$
$$\gamma > 0$$
$$X = X^T > 0$$

The results using MATLAB are:

$$k_\infty^* = 1.0e + 12 \begin{bmatrix} 0.000063 & 0.000005 & -0.000002 & -0.0000001 \\ 5.525402 & 0.45955 & -0.218756 & -0.0115747 \end{bmatrix}$$

Where the tow first columns correspond to feedforward gain and the others to feedback gain.

To show the behavior of the control it is plot a system response considering the yaw rate as a function of the front wheel steering angle.

Fig.3 Yaw rate versus steering wheel angle, H_∞ controller

$H_2 - controller$

The objective of this control is to minimize the norm H_2 , i.e., the amplitude of the output of interest, considering the input signals as limited in energy, through the following relationship.

$$\|y\|_\infty \le \|W\|_{H_2} \|u\|_2 \tag{19}$$

The zero regulation of the states (errors signals) is the main objective of the control design, following this idea the controller H_2 allows to have outputs with limited amplitudes. Therefore, the controller that make the closed loop (A+Bk) asymptotically stable and minimizes the H_2-norm is obtained by stablishing the following LMI condition:

$$\begin{cases} [x^*, y^*] = \min_{X,Q,Y,\gamma} \gamma \\ s.t \\ \begin{bmatrix} AX + B_1Y + (AX + B_1Y)^T & B_2 \\ B_2^T & -I \end{bmatrix} < 0 \\ \begin{bmatrix} Q & (C_2X + D_{22}Y) \\ (C_2X + D_{22}Y)^T & X \end{bmatrix} > 0 \\ tr\{Q\} < \gamma \\ X > 0 \\ Q > 0 \end{cases} \tag{20}$$

The results using MATLAB are:

$$k_2^* = 1.0\mathrm{e}+07 \begin{bmatrix} 0.33200 & -3.1439 & -0.0043 & -0.00009 \\ 3.14479 & 0.3320 & -0.1657 & -0.00884 \end{bmatrix}$$

Where the tow first columns correspond to feedforward gain and the others to feedback gain.

To show the behavior of the control as follows:

Fig.4 Yaw rate versus steering wheel angle, H_2 controller

$L1 - controller$

The objective of this control is to minimize the norm $L1$ of the transfer function W(s). From the induced norms is possible found the relationship that link the input and output with $L1$ norm.

$$\|y\|_\infty \le \|W\|_{L_1} \|u\|_\infty \tag{21}$$

This induced norm relates to the amplitude of the output of interest, also considering a small amplitude of the control signal. The zero regulation of the states (errors signals) is the main objective of the control design, following this idea the $L1$ allows to have outputs with limited amplitudes. Therefore, the controller that make the closed loop (A+Bk) asymptotically stable and minimizes the $L1$-norm is obtained by stablishing the following LMI condition:

$$\begin{cases} [x^*, y^*] = \min_{X,Y,\gamma,\mu} \gamma \\ s.t \\ \begin{bmatrix} \lambda X & 0 & (C_3X + D_{32}Y)^T \\ 0 & (\gamma - \mu)I & D_{31}^T \\ (C_3X + D_{32}Y) & D_{31} & -\gamma I \end{bmatrix} < 0 \\ \begin{bmatrix} AX + B_1Y + (AX + B_1Y)^T + \lambda X & B_2 \\ B_2^T & -\mu I \end{bmatrix} > 0 \\ \gamma > 0 \\ X = X^T > 0 \end{cases} \tag{22}$$

The results using MATLAB are:

$$k_1^* = 1.0\mathrm{e}+11 \begin{bmatrix} 0.00002 & 0.00000009 & -0.00000002 & -0.000000 \\ 2.13526 & 0.11360699 & -0.00205312 & -0.000108 \end{bmatrix}$$

Where the tow first columns correspond to feedforward gain and the others to feedback gain.

To show the behavior of the control as follows:

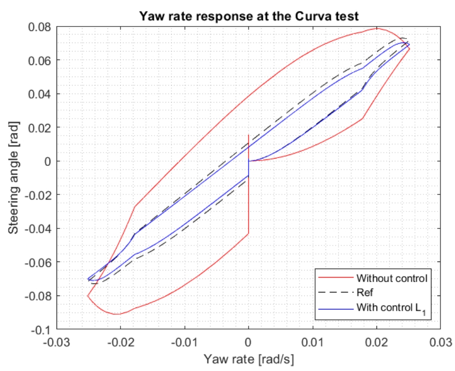

Fig.5 Yaw rate versus steering wheel angle, $L1$ controller

IV. Simulation Phase

The simulation phase is carried out in different stages as shown below.

A. Implementation of the robust lpv controller

With MATLAB StateFlow the finite state machine with five states has been implemented which allows to select the feedback and feedforward control gains k based on the speed. This gain is computed according the previous optimal controller.

Fig.6 shows the StateFlow machine configuration implemented in the control. The control is the so-called Gain scheduling control.

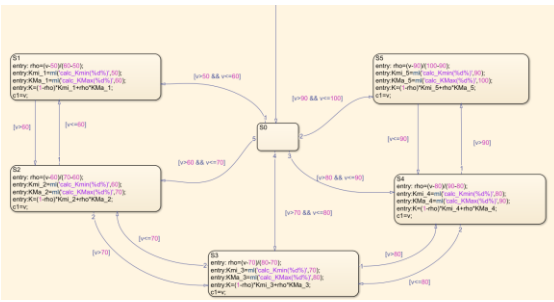

Fig.6 StateFlow machine configuration

Unfortunly, the sumulation take long time using this proccess, then it was made a function for computing the dinamic gain acording to the input velocity and computing all control gains off-line.

due to the plant being an LPV model the matrices A, B, and E must be computed for each change in the velocity, this was achieved by implemented functions in Simulink that have as input the velocity that is shown in the Fig7.

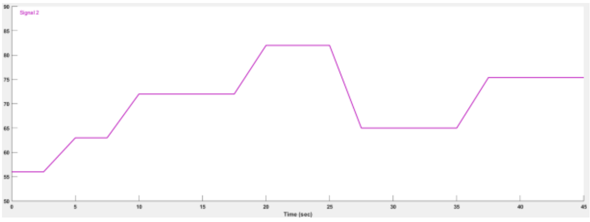

Fig.7 Velocity variation

B. Simulation enviroment

For the implementation of the system with the Simulink- Automated Driving Toolbox, it is necessary to create several sub-blocks that include the simulation of the longitudinal and lateral speed of the vehicle, the block of sensors that allow knowing the outputs and subsequently comparing them with the reference signals, the generator of the reference signal according to the trajectory that the camera captures with respect to the limits of the lines on the road, the reader of the implemented scenario on which the vehicle moves, the package of the Ego Car to which it is transfer the data for the previously calculated simulation, controller and vehicle model.

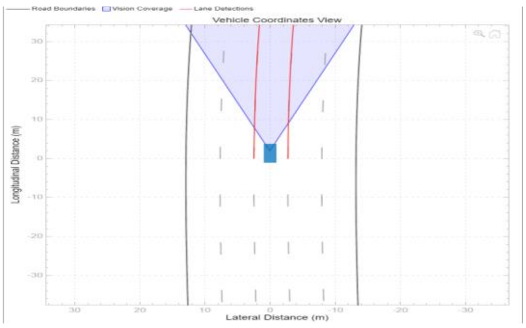

Fig.8 Trajectory tracking

Fig.9 shows the proposed path to be followed by the vehicle, while Fig.8 shows the behavior of the vehicle following the path. The camera is responsible for detecting the lines as a reference to maintain the trajectory. In this project the control of the position is carried out based on the Side slip angle and the Yaw rate.

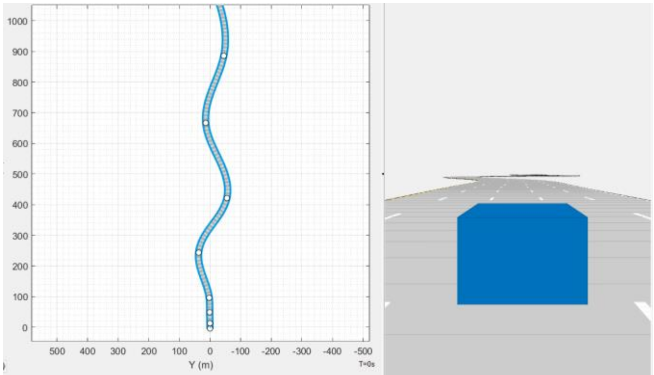

Fig.9 Proposed Trajectory

In this case, the generation of the reference signal was carried out through the images captured by the camera in front of the vehicle, which detects the lines on the tracks to generate the trajectory. The disturbance signal, that is, the front wheel steering angle, was generated by calculating the curvature of the track and the equation.

$$\delta_f = arctan\left(\frac{\sin(\beta) + a_f curvatura}{\cos(\beta)}\right) \quad (23)$$

C. Simulation Results

Once the velocity reference, front steering angle, LPV control, time variant matrices computation, and simulation environment are created and configurated. The project is simulated, however, only the results for the H_∞ control is shown due to the high computational cost required to simulate the entire process.

First, Fig.10 shows the front steering angle generated by the trajectory due to the camera. This is a system that monitors the vehicle position with respect to the line on the street and provides the curvature of the street.

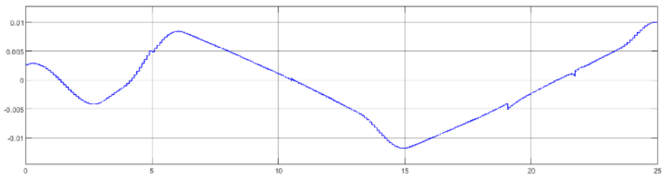

Fig.10 front steering angle

Fig.11 shows the response of the system (side slip angle and Yaw rate) due to the Gain Scheduling LPV H_∞-controller with integral effect and without controller for a disturbance related to simulated environment in the front steer angle for different velocities.

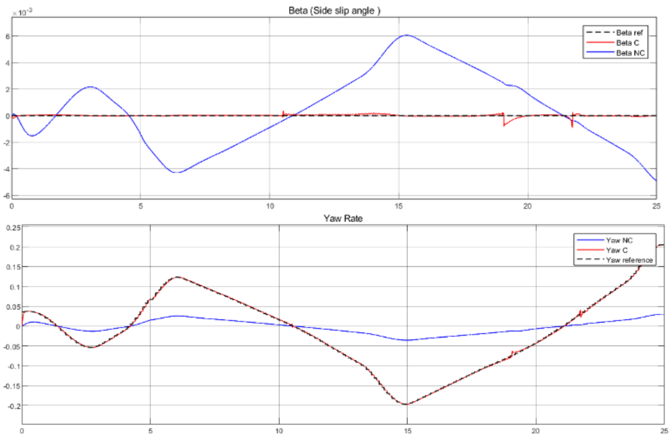

Fig.11 Side slip angle [rad] and Yaw rate [rad/s]

The tracking error of the Yaw rate and side slip angle signal reference signal can be seen in Fig.12

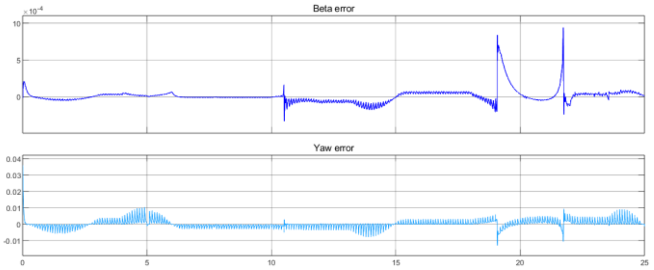

Fig.12 tracking errors of the states

The moment generated as a control signal for the stability of the vehicle in curves can be seen in Fig.13. while the rear wheel steering angle generated by the controller is presented in Fig14.

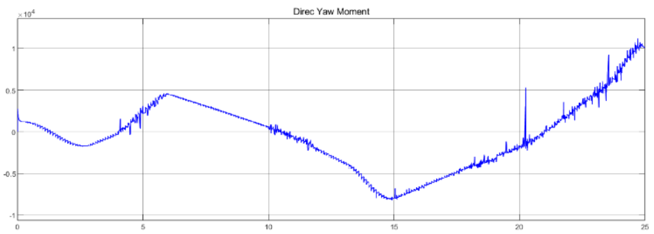

Fig.13 direct Yaw Moment

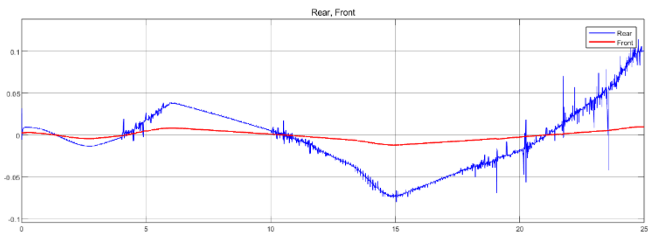

Fig.14 front and rear steering angle

Finally, Fig.15 shows the lateral and longitudinal displacement of the vehicle.

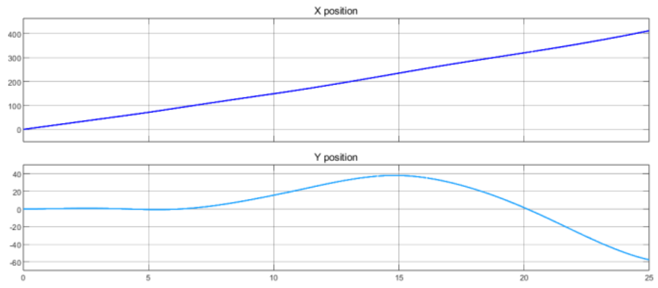

Fig.15 X and Y position of the Ego car

V. CONCLUTIONS

a) The model used has been a single track in order to observe the angles of rotation in the front and rear tires. The rear tire compensates the rotation of the front tire in order to maintain the vehicle's stability in curves, in this case analyzing the performance of the controller at a diferent velocities. The direct moment of Yaw is the second control signal of the system in order to keep the vehicle within the parameters established to follow the reference signal. As we can observe fron the Fig. 12 the error produced after the control is very small for all variation of velocities so it is concluded that the control performance is very high.

b) The reference tracking problem by increasing an Offset Free effect with a pole and a Feedforward gain, result in a vastly improved of the performance of the optimal controls H_inf, H_2, and L1 However, the L1 controller has the best behavior in the regularization of the Beta state reducing the error to almost 0. But Unfortunately, the computational cost produced by this control is very high, so the h_inf control was implemented, which also has excellent results.

c) Finally, it can be observed that the robust control carried out with H_inf was greatly improved by using an LPV plant design and considering that velocity is a parameter that we can measure online. Then, by applying a Gain Scheduling Controller was possible to generate a time-variant control gain that regulates the plant for each variation of velocity. In this way, the performance of the robust control was highly improved.

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