Scattering Challenge: Gaussian Well + Inverse-Polynomial Tail

Advanced Undergraduate / Beginning Graduate Project

Physics Goal: Characterize elastic scattering (differential and total cross sections) for a spinless particle of mass m from the potential

$$V(r) = -V_0 e^{-r^2/a^2} + \frac{\lambda}{(r^2+b^2)^{p/2}}. \tag{1}$$

Choose dimensionless units (e.g., set $a = 1$ and $\hbar^2/2m = 1$). Compare analytic approximations with numerical partial-wave solutions.

Tasks:

1. **Non-dimensionalize.** Show units and write radial Schrödinger equation.
2. **Analytic limits.** Derive:
   - First Born approximation for $f(\theta)$.
   - Low-energy s-wave limit and scattering length a_s.
3. **Numerics.** Solve without approximations and find the phase shift $\delta_\ell(E)$, scattering amplitude $f(\theta)$, and then σ_{tot} without approximations. Identify resonances (rapid δ_ℓ through $\pi/2$), also compare to the harmonic potential.

Suggested Parameter Sets (dimensionless, $a = 1$):

- A (Born-friendly): $V_0 = 0.8,\ \lambda = 0.2,\ b = 0.1,\ p = 3$.
- B (resonant s-wave): $V_0 = 6.0,\ \lambda = 0.0,\ b = 0.1,\ p = 2$.
- C (mixed tail): $V_0 = 3.0,\ \lambda = -1.5,\ b = 0.5,\ p = 1.5$.

Energy range: E from near 0 up to $\sim 10V_0$.

Deliverables:

- Short PDF report (2–4 pages) with derivations, method, convergence tests, and plots.
- Code: a link to the code used must be included in the report. With the code, you must plot figures to visualize your solution and compare to what was stated above.

Have fun — the aim is insight and exploration, not a single closed-form solution.