Statistical Demography

Bayesian inference, population dynamics, ageing research

EvoDemoS Workshop
Bayesian Demographic Trajectory analysis: BaSTA and BaFTA

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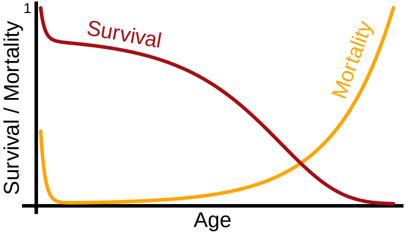
Max Planck Institute for Evolutionary Anthropology

Why do we care?

Demographic rates

Survival: $S(x) = \Pr[X > x]$, where X is a random variable for ages at death and $x \geq 0$.

Mortality: $\mu(x)$ hazard rate
$\left(S(x) = \exp\left[-\int_0^x \mu(t)dt\right]\right)$



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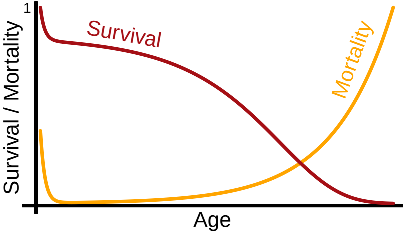
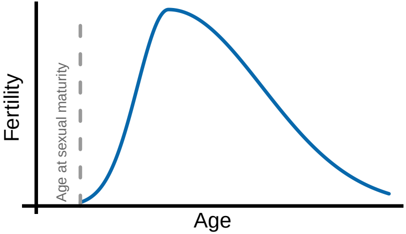
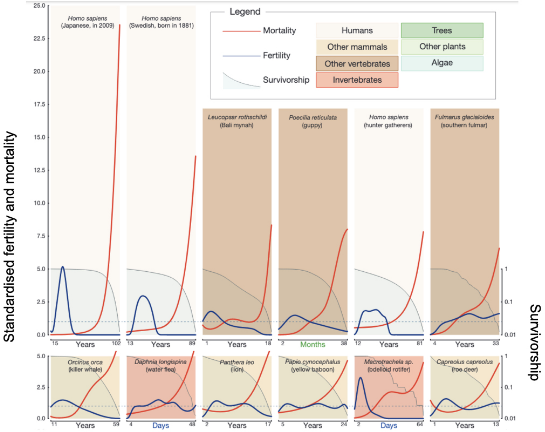
Fertility: $\mathrm{E}[Y_x] = m(x)$ where Y_x is a random variable for the number of offspring produced by adults of age x.





Demography in nature

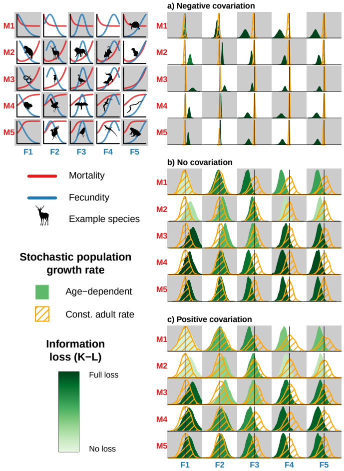
One size **doesn't** fit all...



Jones *et al.* (2014) ***Nature***

Population Dynamics

Implications for population dynamics



Colchero *et al.* (2019) ***Ecol Lett***

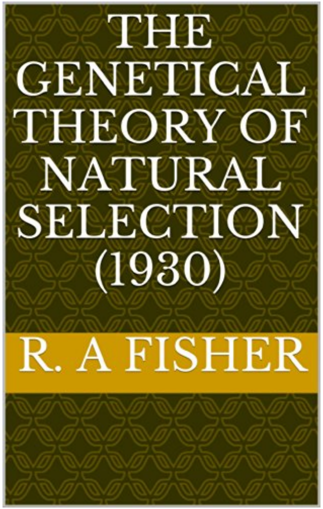
Fitness and demography (stable populations)

- Intrinsic rate of population increase, r (or $\lambda = e^r$), real root of Lotka's (1913) renewal equation

$$\int_0^{\infty} e^{-rx} S(x) m(x) dx = 1$$

- Population reproductive value (Fisher 1930, Leslie 1948), $V = \int_0^{\infty} v(x)/v_0 dx$ where

$$v(x) = \frac{e^{rx}}{S(x)} \int_x^{\infty} e^{-rt} S(t) m(t) dt.$$

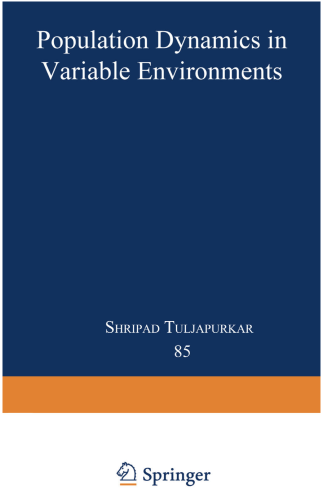


Fitness and demography (stochastic environments)

Long-run population growth rate (Tuljapurkar 1982, 1990)

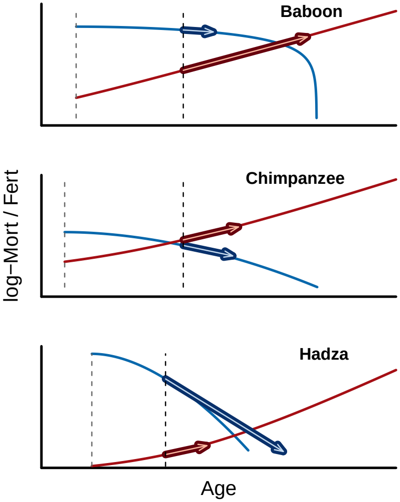
$$a = \tilde{r} - V_s - V_c + S$$

where $\tilde{\lambda} = e^{\tilde{r}}$ is the growth rate from average demographic rates, and V_s, V_c account for the variances and covariances between demographic rates, and S for serial autocorrelation.



Demographic aging rates

Rate of reproductive vs actuarial aging

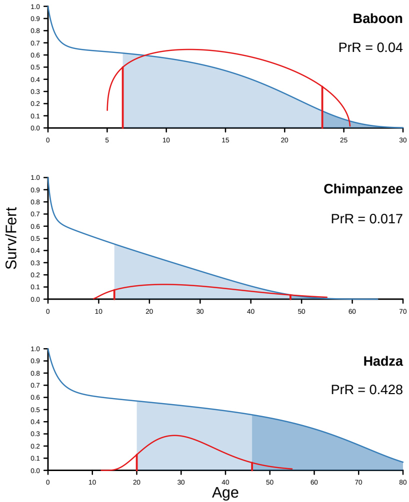


Olive baboons (*P. anubis*) and Chimpanzees (*P. troglodytes*), Gombe Ntl. Park, Tanzania
Hadza, Tanzania

Packer *et al.* (1998), Muller *et al.* (2020), Colchero *et al.* (2021)

Post-reproductive representation (PrR)

Studies on menopause



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Problems

Unknown times of birth and death

Missing births to temporal disappearances

Limited functional forms for age-specific mortality and fertility

Bayesian Survival Trajectory Analysis

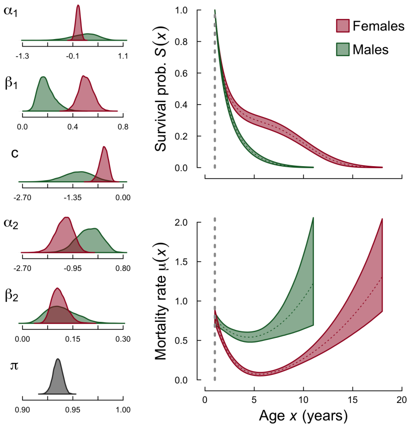
BaSTA - Inference for datasets with incomplete ages

Hazard rate or mortality function

$$\mu(x) = \lim_{\Delta x \to 0} \frac{\Pr[x < X < x + \Delta x | X > x]}{\Delta x},$$

where $S(x) = \exp[-\int_0^x \mu(t)dt]$.

Colchero & Clark (2012) ***J Anim Ecol***, Colchero *et al.* (2012) ***MEE***, Colchero *et al.* (2021) ***Nat Comms***



Soay sheep mortality

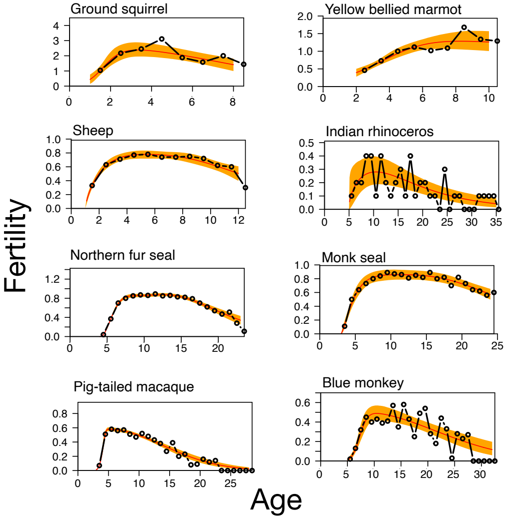
BaFTA - Inference on aggregated and individual level data

W_x random variable for the number of offspring born from n_x adults alive at age x, where

$$W_x \sim \mathrm{NB}(n_x \alpha, \rho),$$

where $\alpha > 0$ and $\rho = \alpha / [b(x) + \alpha]$.

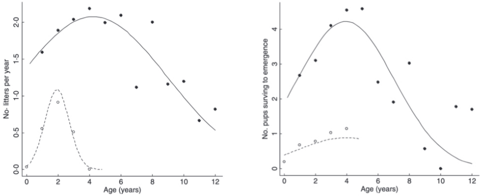
Colchero (2025) ***Biometrics***



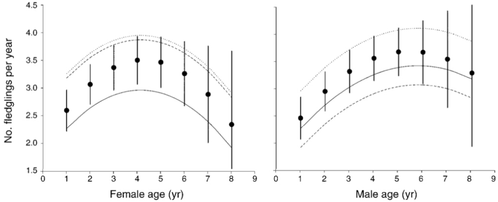
BaFTA

Studies on age-specific fertility

limited to a single model (i.e., quadratic)



Sharp & Clutton-Brock (2010) *J Anim Ecol* 79:176-183



Brown & Roth (2009) *Ecology* 90:218-229

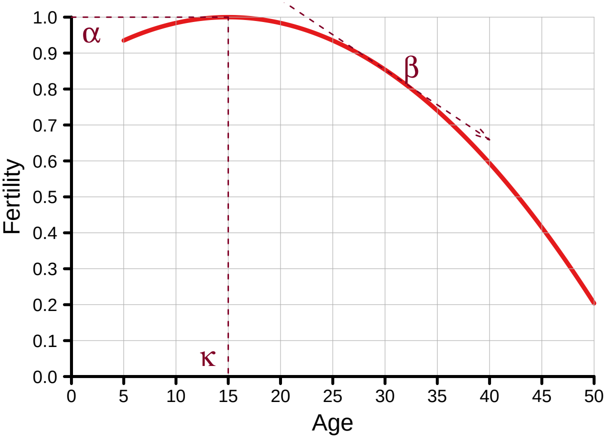
Polynomials

Quadratic:

The most common given by

$$g(x) = \alpha - \beta(x - \kappa)^2,$$

where
- $\alpha \geq 0$ is the maximum fertility,
- $\beta \geq 0$ is the rate of increase - decrease,
- $\kappa > 0$ is the age at maximum fertility.



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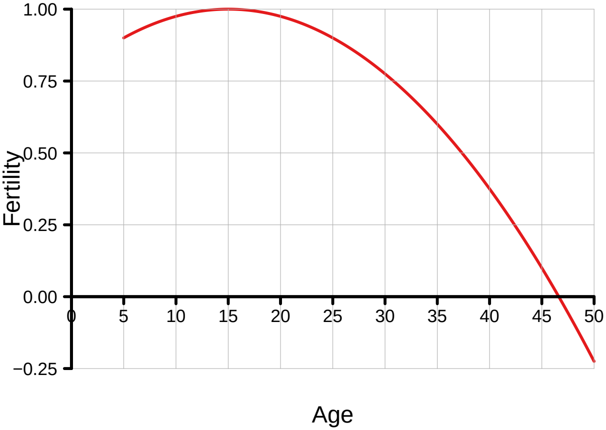


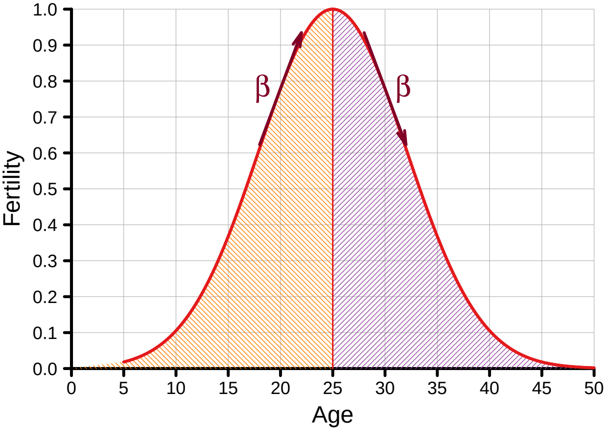
Polynomials

Exponential - quadratic:

Given by

$$g(x) = \alpha e^{-\beta(x-\kappa)^2},$$

It is symmetric around κ.



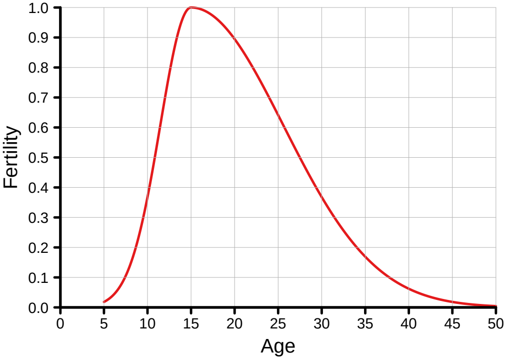
Polynomials

Peristera and Kostaki (2007)

$$g(x) = \alpha \exp\left[-\left(\frac{x-\kappa}{\beta(x)}\right)^2\right],$$

where $\alpha > 0$, $\kappa \geq 0$ and $\beta(x) > 0$, with $\beta(x) = \beta_{11}$ for $x \leq \kappa$ and $\beta(x) = \beta_{12}$ for $x > \kappa$.

(Note: Exponential quadratic is a special case.)



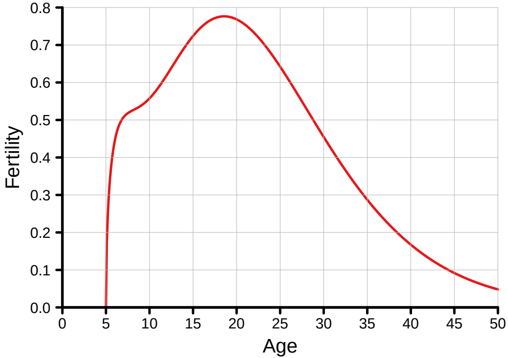
Mixture-density based models

Mixture models

((Chandola *et al.* 1999)

$$g(x) = R\left[m\ f_1(x|\boldsymbol{\theta}_1) + (1-m)f_2(x|\boldsymbol{\theta}_2)\right],$$

where $m \in [0,1]$ is the mixture parameter and $f_1(x|\boldsymbol{\theta}_1)$ and $f_2(x|\boldsymbol{\theta}_2)$ are two distributions with parameter vectors $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.



Inference

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Solution... make an R package