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DYNAMICS ANALYSIS OF A SIX DEGREE OF FREEDOM ARTICULATED ROBOTIC ARM USING ROBOTIC TOOLBOX

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Abstract - This paper describes dynamics analysis of a six degree of freedom (DOF) articulated robotic arm. The kinematics is derived based on Denavit-Hartenberg (DH) representation of robotic arm. In this paper, Newton Euler formulation is used to solve direct and inverse dynamics problems of manipulator. MATLAB Robotic Toolbox software is used for dynamics analysis on robotic arm. Total workspace of a six DOF robotic arm 500×500×650mm to carry a maximum payload of 460g is used for painting operation. Finally, the results from theoretical and simulation approaches are compared and discussed. The maximum value of driving torque is occurred at joint two and the minimum value is at the last joint. The maximum driving toque is about 25 N-m and the minimum driving torque is about 4.8 N-m of the articulated robotic arm.

Keywords - Denavit-Hartenberg (DH) parameters, Dynamics, Kinematics, Newton Euler Method, Robotic Arm.

I. INTRODUCTION

The Robot Institute of America designates a robot as "a reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks" [8]. The basic components of a robot system are the mechanical linkage which consists of links, joints and end effector. For a six DOF robotic arm, the first three links and joints form the arm and the last three mutually in intersecting joints made a wrist. The end effector is a part attached at the end of the manipulator. Specialized tools like the painting tool attached to the end of the manipulator arm for performing tasks are also considered as end effector [5].

Robot kinematics and dynamics is the basis of robotics research, and robot kinematics is the relative relationship between the end pose of robot and robot kinematics parameters [7]. The robot dynamics is mainly the study of the correlative relationship between the movement of robot arm movement and its torque (or force) [4]. Dynamics is the study of systems that undergo changes of state as time evolves motion. Derivation of the equations of motion for the system is the main step in dynamic analysis of the system, since these equations are essential in the design, analysis, and control of the system. The dynamic equations of motion describe dynamic behavior. They can be used for computer simulation of the robot's motion, design of suitable control equations, and evaluation of the dynamic performance of the design.

In this paper Newton Euler formulation is used to analyze manipulator dynamics of six DOF articulated robotic arm using MATLAB code. The dynamics analysis of six DOF articulated robotic arm is also conducted numerically by using MATLAB Robotics Toolbox software. From this simulation analysis, the values of driving torques are obtained with respect to time. Finally, these two theoretical and numerical analyses are compared and verified for the confirmation of dynamics analysis.

II. ROBOT KINEMATICS MODEL

Robotic arm kinematics deals with the analytical study of the geometry of motion of a robot arm with respect to a fixed reference coordinate system as a function of time without regard to the forces that cause the motion [8]. There are two types of kinematic analysis problems: forward kinematics and inverse kinematics. In this paper, the homogeneous transformations and Denavit-Hartenberg (DH) representation of robotic arm are used to solve direct and inverse kinematics problems of manipulator. DH parameters for the robotic arm are listed in Table 1.

Link	Link Length (a_i), (m)	Joint Distance (d_i), (m)	Twist Angle (α_i) (degree)	Joint Angle (θ_i)
Link 1	0	d_1=0.15	90	θ_1
Link 2	a_2=0.2	0	0	θ_2
Link 3	a_3=0.15	0	0	θ_3
Yaw 4	a_4=0.15	0	90	θ_4
Pitch 5	0	0	90	θ_5
Roll 6	0	d_6=0.055	0	θ_6

Table1: DH Parameters of Six DOF Robotic Arm

2.1. Forward Kinematics

Forward kinematics refers to determine the values of position and orientation of the end effector, given the joint angles and link lengths of the robotic arm. DH parameters are used in this paper to describe the manipulator configuration. Simplified link coordinate diagram of a six DOF robotic arm is shown in Fig.1.

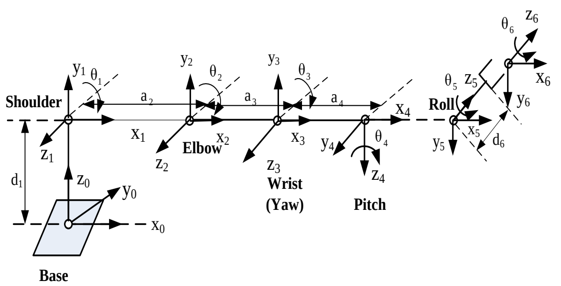

Fig.1. DH Coordinate Frame Assignment of Six DOF Robot

To calculate the values of position and orientation of each joint, the homogeneous transformation matrix is used. Since the robot has six revolute joints, the joint coordinate is [θ_1 θ_2 θ_3 θ_4 θ_5 θ_6]. Forward kinematics analysis for six DOF articulated robot is determined as follows [8];

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$^0T_6 = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$n_x = C_1(S_{234}S_6 + C_{234}C_5C_6) + S_1S_5C_6 \quad (3)$$

$$n_y = S_1(S_{234}S_6 + C_{234}C_5C_6) - C_1S_5C_6 \quad (4)$$

$$n_z = S_{234}C_5C_6 - C_{234}S_6 \quad (5)$$

$$o_x = C_1(S_{234}C_6 - C_{234}C_5S_6) - S_1S_5S_6 \quad (6)$$

$$o_y = S_1(S_{234}C_6 - C_{234}C_5S_6) + C_1S_5S_6 \quad (7)$$

$$o_z = -S_{234}C_5S_6 - C_{234}C_6 \quad (8)$$

$$a_x = C_{234}C_1S_5 - C_5S_1 \quad (9)$$

$$a_y = C_{234}S_1S_5 + C_1C_5 \quad (10)$$

$$a_z = S_{234}S_5 \quad (11)$$

$$P_x = C_1(a_2C_2 + a_4C_{234} + d_6S_5C_{234} + a_3C_{23}) - S_1C_5d_6 \quad (12)$$

$$P_y = S_1(a_2C_2 + a_4C_{234} + d_6S_5C_{234} + a_3C_{23}) + C_1C_5d_6 \quad (13)$$

$$P_z = d_1 + a_2S_2 + a_4S_{234} + d_6S_5S_{234} + a_3S_{23} \quad (14)$$

2.2. Inverse Kinematics

The purpose of inverse kinematics (IK) problem is the determination of joint angles for a given end effector position and orientation with respect to a base coordinate system [12]. To calculate the IK, algebraic method is used, which deals with each robot separately and is suitable for most of industrial robot. The values of joint angles for the given position of robot end effector are evaluated. The inverse kinematics solution of six DOF articulated robot, as follows;

The first to the last joint angle are;

$$\theta_1 = \tan^{-1}\left(\frac{P_y - d_6a_y}{P_x - d_6a_x}\right) \quad (15)$$

$$\theta_5 = \tan^{-1}\left(\frac{S_5}{C_5}\right) \quad (16)$$

$$\theta_{234} = \tan^{-1}\left(\frac{a_z}{a_xC_1 + a_yS_1}\right) \quad (17)$$

$$\theta_3 = \cos^{-1}\left(\frac{\gamma^2 + \beta^2 - a_2^2 - a_3^2}{2a_2a_3}\right) \quad (18)$$

$$\theta_2 = \tan^{-1}\left(\frac{\gamma(a_2 + a_3C_3) - \beta a_3S_3}{\beta(a_2 + a_3C_3) + \gamma a_3S_3}\right) \quad (19)$$

$$\theta_{23} = \tan^{-1}\left(\frac{\gamma - a_2S_2}{\beta - a_2S_2}\right) \quad (20)$$

$$\theta_4 = \theta_{234} - \theta_{23}$$

$$\theta_6 = \tan^{-1}\left(\frac{o_xS_1 - o_yC_1}{n_yC_1 - n_xS_1}\right) \quad (21)$$

$C_5 = a_yC_1 - a_xS_1$, $S_5 = \pm\sqrt{1 - C_5^2}$

$\theta_{23} = \theta_2 + \theta_3$

$\beta = C_1P_x + S_1P_y - d_6S_5C_{234} - a_4C_{234}$,

$\theta_{234} = \theta_2 + \theta_3 + \theta_4$

$\gamma = P_z - d_1 - d_6S_5S_{234} - a_4S_{234}$,

$C_1 = \cos\theta_1$, $S_1 = \sin\theta_1$

III. ROBOT DYNAMICS MODEL

Dynamic modeling means deriving equations that explicitly describes the relationship between force and motion. These equations are important to consider in simulation of robot motion, and in design of control algorithms. There are two problems related to manipulator dynamics analysis. They are inverse dynamics in which manipulator's equations of motion are solved for given motion to determine the generalized force and torque of each joint and direct dynamics in which the equations of motion are integrated to determine the generalized coordinate response (displacement, velocity, and acceleration) to applied generalized forces and torques. There are many methods to analyze manipulator dynamics in robotic system, but the most common methods in robotic manipulator dynamics analysis are Euler Lagrange formulation and Newton Euler formulation. In this paper, Newton Euler formulation is used to solve direct and inverse dynamics problems of manipulator.

The equations of motion for an n-axis manipulator are given by:

$$\tau_i = M(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + F(\dot{q}_i) + G(q) \qquad (22)$$

The rate of change of the angular momentum with respect to the link attached frame is

$$\tau = R^T \tau_0 \qquad (23)$$

The final torque is expressed as

$$\tau = R^T \dot{R} I\omega + I\dot{\omega} \qquad (24)$$

The force balance equation is

$$f_i = R^i_{i+1} f_{i+1} + m_i a_{c,i} - m_i g_i \qquad (25)$$

Angular velocity and acceleration of each link

$$\omega_i = (R^{i-1}_i)^T \omega_{i-1} + b_i \dot{q}_i \qquad (26)$$

$$\alpha_i = (R^{i-1}_i)^T \alpha_{i-1} + b_i \ddot{q}_i + \omega_i \times b_i \dot{q}_i \qquad (27)$$

The Newton Euler formulation of an n-link manipulator can be calculated by using these equations. The material for the robotic arm links used Aluminium alloy and its density is 2770 kg/m^3. The required dynamic parameters are given in Table 2.

Link	Width (w) (mm)	Mass (m_i) (kg)	Moment of Inertia (I_i) (kg-m^2)	Depth (d) (mm)
1	0.05	0.58	0.0054	0.02
2	0.05	0.77	0.0029	0.02
3	0.05	0.42	0.0009	0.02
4	0.05	0.42	0.0015	0.02
5	0.05	0.25	0.0003	0.02
6	0.05	0.15	0.0002	0.02

Table 2: Dynamics Parameters for Six DOF Articulated Robot

In this dynamics section, firstly, forward dynamics problem is computed by Recursive Newton Euler method using MATLAB code. In the forward recursion, the values of angular velocities and accelerations of the links by using Equation 26 and Equation 27. The required results of forward dynamics problem can be obtained by using MATLAB code. These angular velocities and accelerations of the links can be computed from the joint position, velocities and accelerations with respect to time.

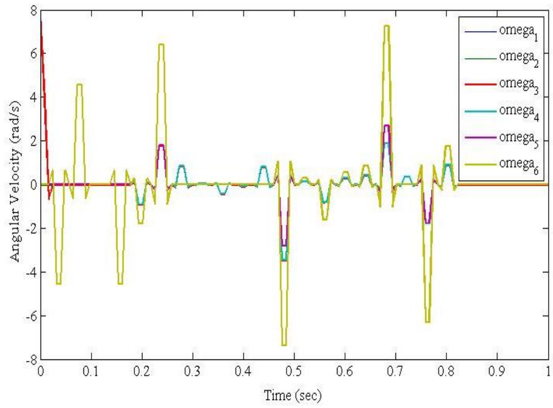

Fig.2. Angular Velocities of Link 1 to 6 with respect to Time

Fig.2 shows the values of angular velocities of the link 1 to 6 with respect to time. The maximum angular velocity occurred at joint-6. The link-6 angular velocity is started at zero position and ended also at zero position during one second. The maximum value is 7 rad/s.

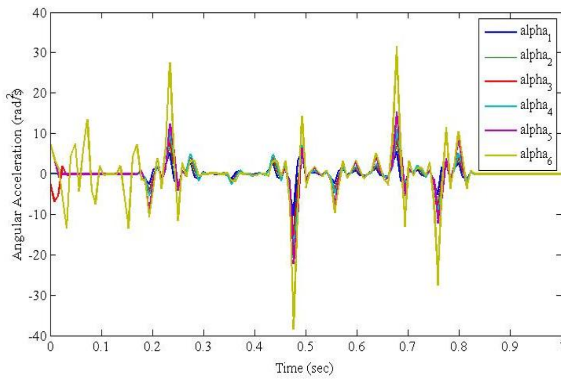

Fig.3. Angular Acceleration of Link 1 to 6 with respect to Time

The angular acceleration of each link is shown in Fig.3. In Recursive Newton Euler Formulation, the angular accelerations of the robot are obtained by taking the derivations of the robot's links angular velocities with respect to time. Therefore, the angular acceleration is fully dependent on the angular velocity of that robot. The maximum angular acceleration is also occurred at link-6 and at the middle of the simulation time. The second maximum value of angular acceleration is at link-5 and the third is at link-4 and so on.

In the inverse dynamics problem, the main target is to compute the values of driving torque of each joint. These values can be obtained from the inverse Recursion Newton Euler Method. Fig.8 shows the values of driving torque of each joint for six DOF articulated robot.

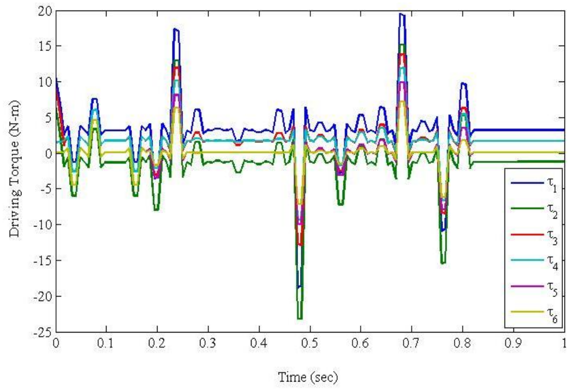

Fig.4. Driving Torque of Joint 1 to 6 with respect to Time

The values of driving torque of each joint can be seen in Fig.4 with respect to time. In this Fig. 4 the maximum value is occurred at joint-2 and its value is 23 N-m and the second maximum value is 19.5 N-m at joint-1. The value of joint-3 is 13 N-m. The value of joint-4 and joint-5 are 10.5 N-m and 7 N-m. The minimum value is 5 N-m at the last joint-6.

IV. ROBOT DYNAMICS SIMULATION

Robot dynamic research is to solve the relationship between the robot's movement and the robot torque, and robot dynamics analysis. And the torque of each link in the process that the robot run from the initial pose to the end pose of the robot.

The MATLAB Robotics Toolbox uses the recursive Newton Euler (RNE) formulation to compute the inverse dynamics problems. Inverse dynamics computes the joint torques required to achieve the specified state of joint position, velocity and acceleration. The recursive Newton Euler formulation is an efficient matrix oriented algorithm for computing the inverse dynamics, and in Robotics Toolbox is implemented in the function "rne". Inverse dynamics requires inertial and mass parameters of each link, as well as the kinematic parameters. This is achieved by augmenting the kinematic description matrix with additional columns for the inertial and mass parameters for each link. The required joint torques can be computed for each joint of the trajectory.

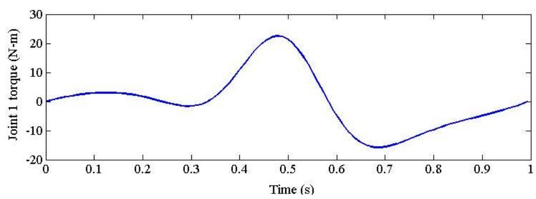

Fig.5. Torque of Joint-1 respect to Time

Fig.5 shows the values of torque of the joint-1 with respect to time. From this Fig.5 the maximum joint torque is occurred at 0.56 sec, and the value is 22 N-m. The minimum value of joint torque is about 0.001 N-m.

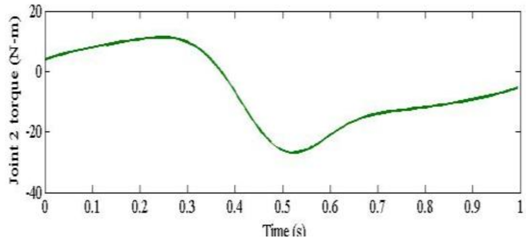

Fig.6. Torque of Joint-2 respect to Time

Fig.6 shows the torque of the joint-2 with respect to time. From this Fig.6, the maximum joint torque is occurred at 0.47 second, and the value is about 25 N-m. The value of joint torque is about 0.005 N-m.

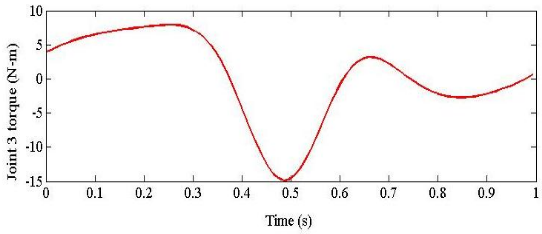

Fig.7. Torque of Joint-3 respect to Time

Fig.7 shows the values of torque of the joint-3 with respect to time. From this Fig.7 the maximum joint torque is occurred at 0.5 sec, and the value is 15 N-m. The minimum value of joint torque is about 0.001 N-m.

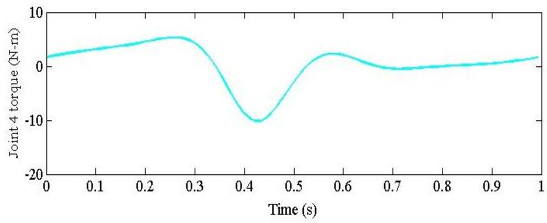

Fig.8. Torque of Joint-4 respect to Time

Fig.8 shows the values of torque of the joint-4 with respect to time. From this Fig.20 the maximum joint torque is occurred at 0.45 sec, and the value is 11 N-m. The minimum value of joint torque is about 0.002 N-m.

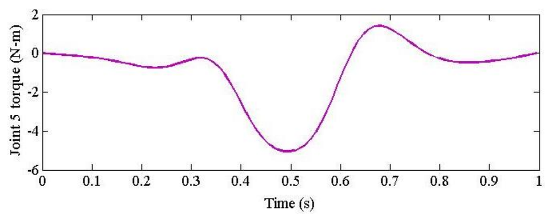

Fig.9. Torque of Joint-5 respect to Time

Fig.9 shows the torque of the joint-5 with respect to time. From this Fig.9 the maximum required joint torque is occurred at 0.5 sec, and the value is 5.7 N-m.

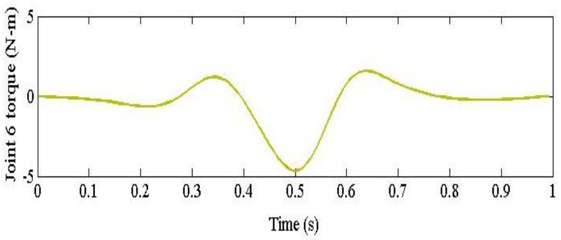

Fig.22. Torque of Joint-6 respect to Time

Fig.22 shows the torque of the joint-6 with respect to time one second. From this Fig.19 the maximum joint torque is occurred at 0.5 second, and the value is 4.8 N-m.

V. COMPARISON OF DYNAMICS ANALYSIS

In this section, Recursive Newton Euler Method is used to analyze dynamics behaviors of six DOF articulated robot for the theoretical analysis. The results from the forward and inverse dynamics are presented and shown in Section 5.4. The main target of dynamics analysis is to compute the values of torques of the robot to rotate the joints to the required positions. Therefore, in this section, the values of torques of the robot are mainly compared with theoretical and numerical approaches.
Table 3 shows the values of driving torque of each joint for six DOF articulated robot by using theoretical and numerical analysis.

Approach	Theoretical (MATLAB Code)	Numerical (MATLAB Robotic Toolbox)	(%) Deviation
Driving Torque, τ_1(N-m)	19.5	22	11.36
Driving Torque, τ_2(N-m)	23	25	8
Driving Torque, τ_3(N-m)	13	15	13.33
Driving Torque, τ_4(N-m)	10.5	11	4.54
Driving Torque, τ_5(N-m)	7	5.7	18.5
Driving Torque, τ_6(N-m)	5	4.8	4

Table 3: Comparison of Driving Torques for Six DOF Articulated Robot

According to the Table 3, the values of torques for six DOF articulated robot by simulating Dynamics using Robotic Toolbox in MATLAB are larger than the values by computing theoretical analysis exact at joint five and six. All the deviations are lower than 20 percent. The highest deviation is only 18.5 percent at joint-5 and the lowest deviation at joint-6 is 4 percent. Therefore, the comparison of the final target of dynamics analysis for six DOF articulated robotic arm by using theoretical and numerical analysis is satisfied.

VI. CONCLUSIONS

In this paper, numerical and theoretical investigations are carried out based on a six degree of freedom robotic arm. Newton Euler formulation is used to solve direct and inverse dynamics problems of manipulator. MATLAB toolbox is used to compute the motion equations of serial-link robots. This study is the multibody dynamics analysis of a six degree of freedom robotic arm, intended for computing of velocities, accelerations and driving torque. All the components of the robotic arm are assumed to be rigid. The dynamics analysis was conducted numerically by using Robotic Toolbox in MATLAB. This simulation method helps to reduce the cost of production, completion time for the same and minimize errors in the physical development of the robotic arms. Maximum driving torque is occurred at joint two and minimum driving torque is at the last joint. Therefore, the dynamics results are satisfied and confirmed with theoretical and numerical simulation analyses.

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