UNIVERSITAS SULTAN AGENG TIRTAYASA

UNTIRTA Jawara
Jujur Adil Wibawa Amanah Religius Akuntabel

Analisis Deret Waktu

#11 Meeting

Model AR, MA, ARMA

Ferdian Bangkit Wijaya, S.Stat., M.Si
NIP. 199005202024061001

Model ARMA (p,q)

Ide Dasar

Model ARMA merupakan kombinasi dari $AR(p)$ dan $MA(q)$, sehingga biasa dituliskan $ARMA(p,q)$

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \ldots + \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}$$

Proses ARMA(1,1)

Model ARMA(1,1)

- Bila $p = 1$ dan $q = 1$ maka

$$Z_t = \phi Z_{t-1} + a_t - \theta a_{t-1}$$

- Struktur koragam

$$\gamma_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-\phi^2}\phi^{k-1}\sigma_a^2 \text{ untuk } k \geq 1$$

- Struktur korelasi

$$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1} \quad \text{ untuk } k \geq 1$$

- Dengan demikian, autokorelasi Model ARMA juga menurun eksponensial

Proses AR(1)

Diketahui sebuah proses deret waktu stasioner Z(t) mengikuti model AR(1) sebagai berikut:

$$Z(t) = 0.8\, Z(t-1) + a(t)$$

Dimana a(t) adalah white noise (WN) dengan rata-rata 0 dan varians 3.6.

Tugas:

Hitunglah:

1. Ragam (Varian) proses.
2. Koragam (Autokovarians).
3. Korelasi (ACF).
4. Korelasi Parsial (PACF).

Proses AR(1)

Kita memiliki parameter:

- $\phi_1 = 0.8$
- $\sigma_a^2 = 3.6$

1. Perhitungan Ragam (γ_0)

$$\gamma_0 = \frac{\sigma_a^2}{1-\phi_1^2}$$

$$\gamma_0 = \frac{3.6}{1-(0.8)^2}$$

$$\gamma_0 = \frac{3.6}{1-0.64}$$

$$\gamma_0 = \frac{3.6}{0.36} \qquad \gamma_0 = \mathbf{10}$$

2. Perhitungan Koragam (γ_k)

gunakan rumus rekursif Yule-Walker: $\gamma_k = \phi_1 \gamma_{k-1}$

Untuk γ_1 (Lag 1)

$$\gamma_1 = \phi_1 \gamma_0$$
$$\gamma_1 = 0.8 \times 10$$
$$\gamma_1 = \mathbf{8}$$

Untuk γ_2 (Lag 2)

$$\gamma_2 = \phi_1 \gamma_1$$
$$\gamma_2 = 0.8 \times 8$$
$$\gamma_2 = \mathbf{6.4}$$

3. Perhitungan Korelasi / ACF (ρ_k)

gunakan rumus $\rho_k = \phi_1^k$

atau bisa juga $\rho_k = \gamma_k / \gamma_0$

Untuk ρ_1 (Lag 1)

$\rho_1 = \phi_1^1 = 0.8$ atau $\gamma_1/\gamma_0 = 8/10 = 0.8$

Untuk ρ_2 (Lag 2)

$$\rho_2 = \phi_1^2 = (0.8)^2 = 0.64$$

atau

$$\gamma_2/\gamma_0 = 6.4/10 = 0.64$$

Untuk ρ_3 (Lag 3)

$$\rho_3 = \phi_1^3 = (0.8)^3 = 0.512$$

$$\rho_k = \{0.8, 0.64, 0.512, \ldots\}$$

Nilainya **meluruh (tails off)**

Proses AR(1)

UNIVERSITAS SULTAN AGENG TIRTAYASA

UNTIRTA Jawara

Jujur Adil Wibawa Amanah Religius Akuntabel

4. Perhitungan Korelasi Parsial / PACF (ϕ_{kk})

$$\phi_{kk} = \begin{cases} \phi_1 & \text{jika } k = 1 \\ 0 & \text{jika } k \geq 2 \end{cases}$$

Untuk ϕ_{11} (Lag 1)

$$\phi_{11} = \phi_1 = 0.8$$

Untuk ϕ_{22} (Lag 2)

$$\phi_{22} = 0$$

Untuk ϕ_{33} (Lag 3)

$$\phi_{33} = 0$$

$$\phi_{kk} = \{0.8, 0, 0, \ldots\}$$

Nilainya **terputus (cuts off)** setelah lag 1

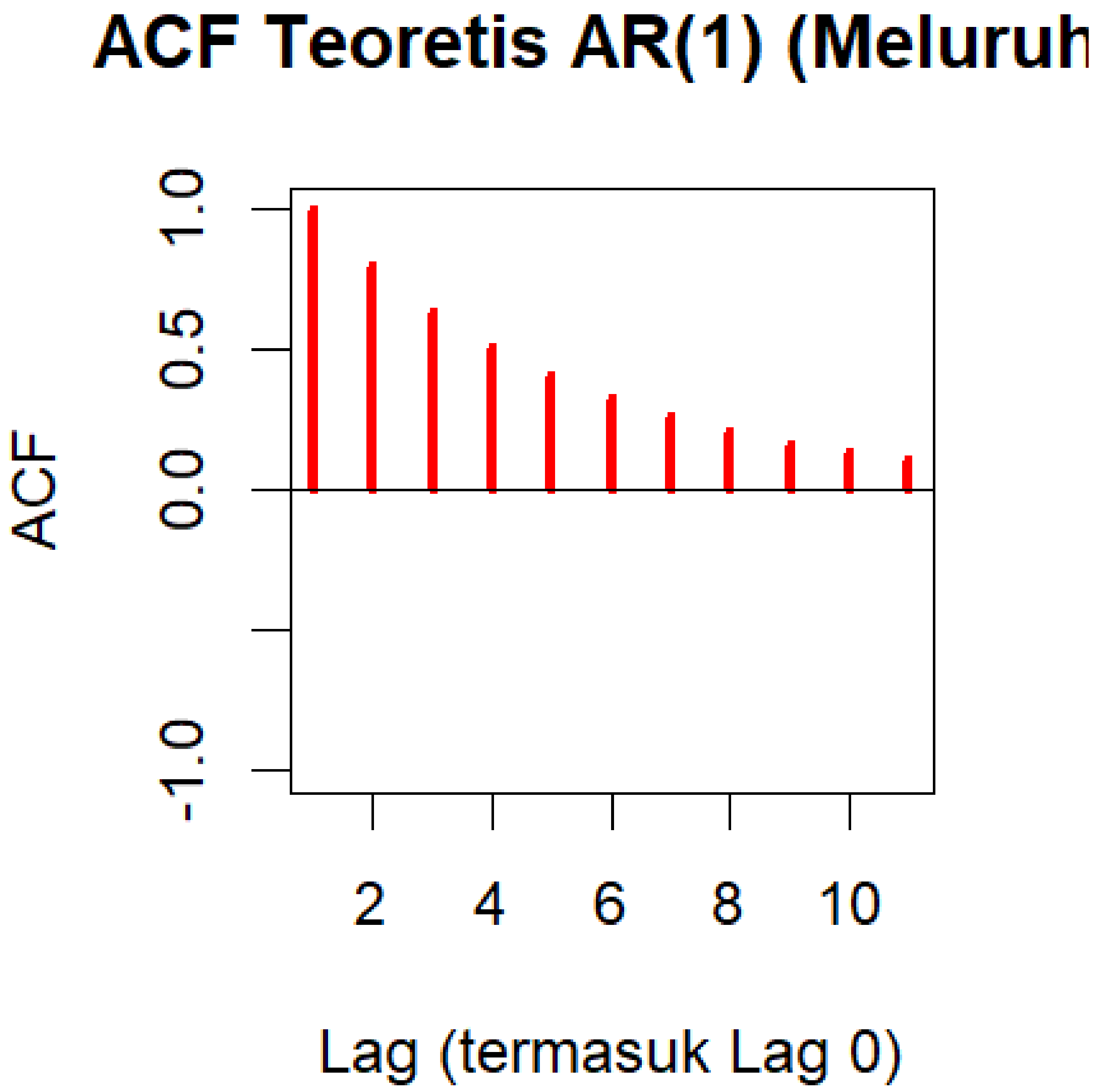
Proses AR(1) dalam R

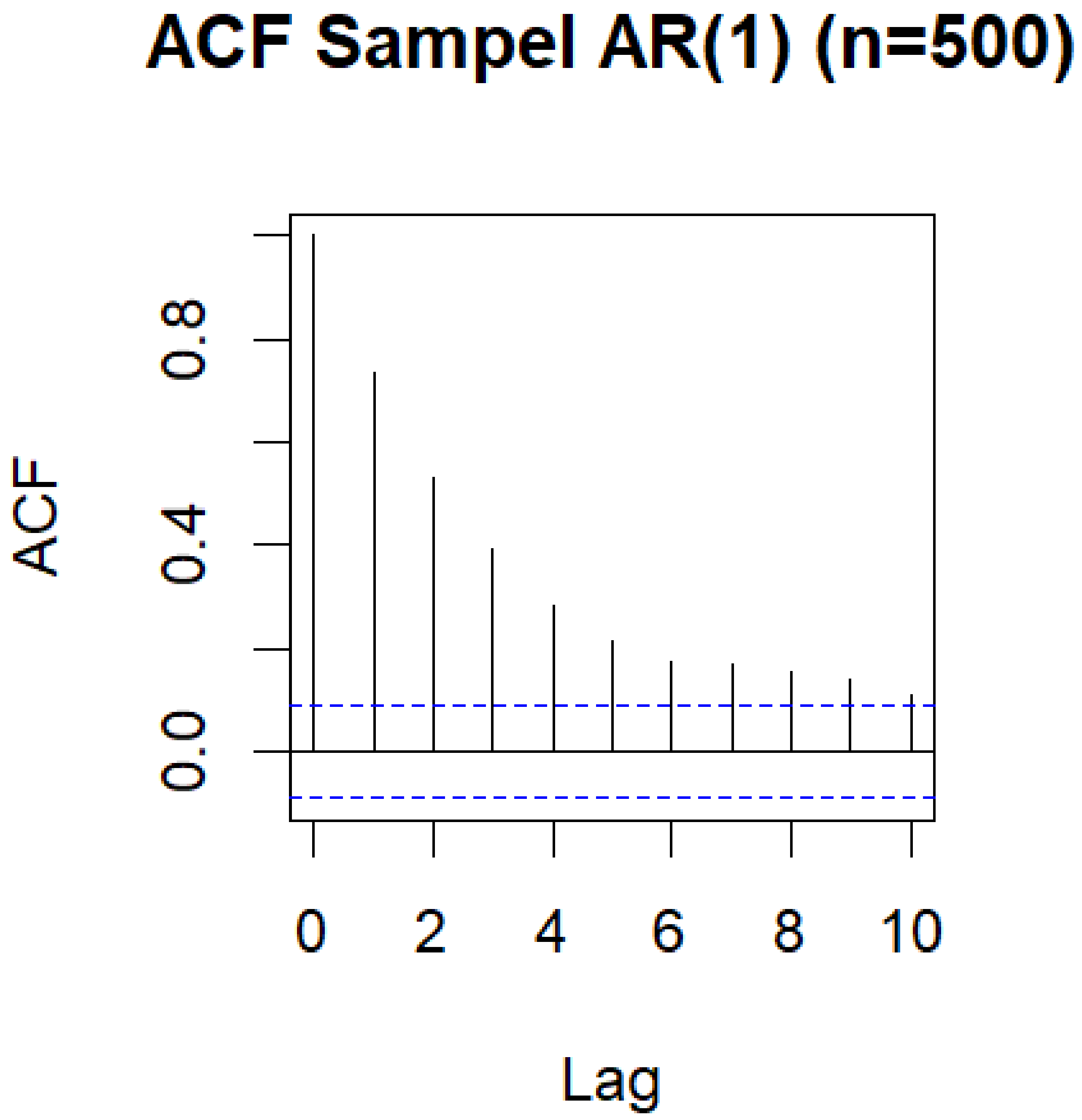
```
# ---------------------------------------------------------------
# Verifikasi Perhitungan Teoretis AR(1)
# Model: Z_t = 0.8 * Z_{t-1} + a_t
# Varians error: sigma_a^2 = 3.6
# ---------------------------------------------------------------

# --- 1. Definisikan Parameter ---
phi <- 0.8
sigma_a_sq <- 3.6
lag_maks <- 3 # Kita ingin hitung sampai lag 3

# --- 2. Perhitungan Ragam (gamma_0) ---
# Rumus: gamma_0 = sigma_a^2 / (1 - phi^2)

gamma_0 <- sigma_a_sq / (1 - phi^2)
print(paste("Ragam (gamma_0):", gamma_0))

# --- 3. Perhitungan ACF (rho_k) ---
rho_k <- ARMAacf(ar = phi, lag.max = lag_maks)
print("--- Nilai ACF (rho_k) ---")
print(rho_k)

# --- 4. Perhitungan Koragam (gamma_k) ---
gamma_k <- rho_k * gamma_0
print(gamma_k)

# --- 5. Perhitungan PACF (phi_kk) ---
phi_kk <- ARMAacf(ar = phi, lag.max = lag_maks, pacf =
TRUE)
print(phi_kk)
```

Plot ACF AR(1) dalam R

```
dev.new()
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# ACF akan meluruh secara eksponensial (0.8^k)
acf_teori <- ARMAacf(ar = 0.8, lag.max = 10, pacf =
FALSE)
plot(acf_teori, type = "h",
     main = "ACF Teoretis AR(1) (Meluruh)",
     xlab = "Lag (termasuk Lag 0)", ylab = "ACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)

# --- PLOT KANAN (SAMPEL) ---
set.seed(123) # Menggunakan seed yang sama
# Simulasi data dari model AR(1) dengan ar = 0.8
data_sim <- arima.sim(model = list(ar = 0.8), n = 500)
acf(data_sim, lag.max = 10,
 main = "ACF Sampel AR(1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot ACF AR(1) dalam R



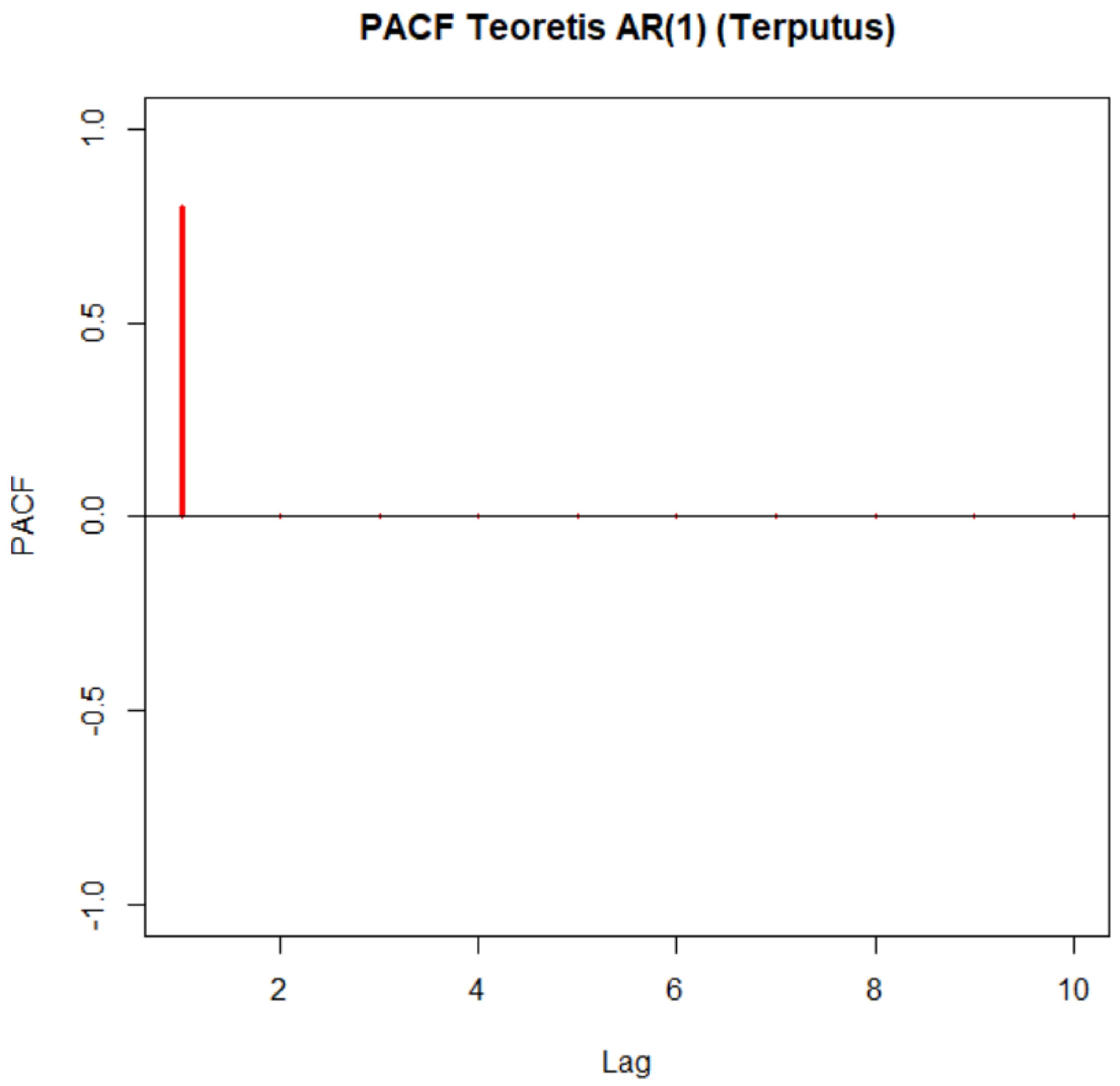


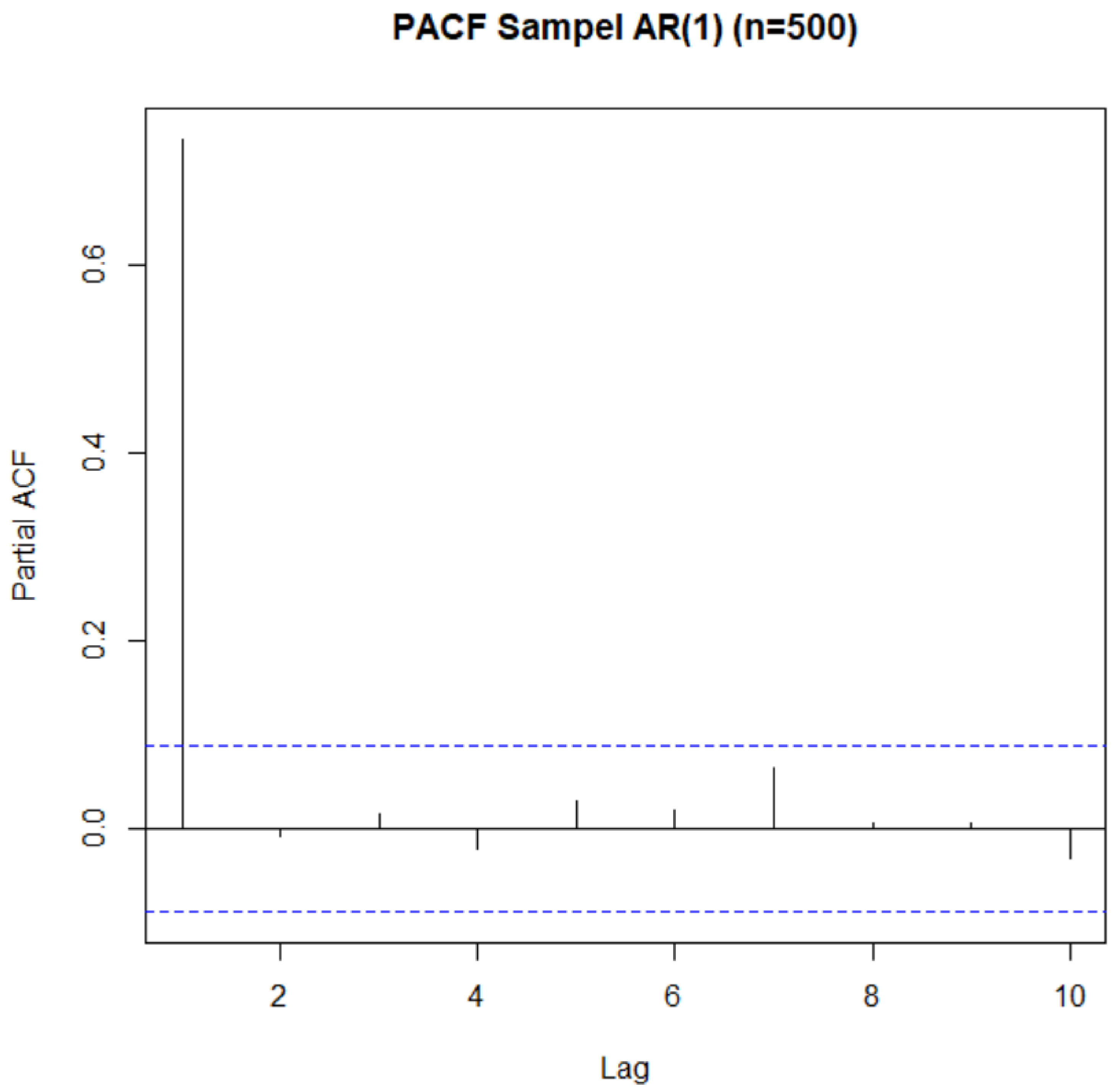
Plot PACF AR(1) dalam R

```
dev.new()
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# PACF akan terputus setelah Lag 1
pacf_teori <- ARMAacf(ar = 0.8, lag.max = 10, pacf =
TRUE)
plot(pacf_teori, type = "h",
     main = "PACF Teoretis AR(1) (Terputus)",
     xlab = "Lag", ylab = "PACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)
```

```
# --- PLOT KANAN (SAMPEL) ---
# Kita gunakan data simulasi yang sama dari atas
(data_sim)
pacf(data_sim, lag.max = 10,
 main = "PACF Sampel AR(1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot PACF AR(1) dalam R





Proses MA(1)

Diketahui sebuah proses deret waktu stasioner Z(t) mengikuti model MA(1) sebagai berikut:

Z(t) = a(t) - 0.5 a(t-1)

Dimana a(t) adalah white noise (WN) dengan rata-rata 0 dan varians 8.

Tugas:

Hitunglah:

1. Ragam (Varian) proses.
2. Koragam (Autokovarians).
3. Korelasi (ACF).
4. Korelasi Parsial (PACF).

Proses MA(1)

Kita memiliki parameter:

- $\theta_1 = 0.5$
- $\sigma_a^2 = 8$

1. Perhitungan Ragam (γ_0)

$$\gamma_0 = \sigma_a^2(1 + \theta_1^2)$$

$$\gamma_0 = 8 \times (1 + (0.5)^2)$$

$$\gamma_0 = 8 \times (1 + 0.25)$$

$$\gamma_0 = 8 \times (1.25)$$

$$\boldsymbol{\gamma_0 = 10}$$

2. Perhitungan Koragam (γ_k)

Untuk γ_1 (Lag 1)

$$\gamma_1 = -\theta_1 \sigma_a^2$$

$$\gamma_1 = -0.5 \times 8$$

$$\boldsymbol{\gamma_1 = -4}$$

Untuk γ_2 (Lag 2)

koragamnya nol untuk lag $k \geq 2$.

$$\boldsymbol{\gamma_2 = 0}$$

3. Perhitungan Korelasi / ACF (ρ_k)

Untuk ρ_1 (Lag 1)

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{-4}{10} \qquad \boldsymbol{\rho_1 = -0.4}$$

atau

$$\frac{-\theta_1}{1+\theta_1^2} = \frac{-0.5}{1.25} = -0.4$$

Untuk ρ_2 (Lag 2)

$$\rho_2 = 0$$

Untuk ρ_3 (Lag 3)

$$\rho_3 = 0$$

$$\rho_k = \{-0.4, 0, 0, \ldots\}$$

Nilainya **terputus (cuts off)** setelah lag 1

Proses MA(1)

UNIVERSITAS SULTAN AGENG TIRTAYASA

UNTIRTA Jawara

Jujur Adil Wibawa Amanah Religius Akuntabel

4. Perhitungan Korelasi Parsial / PACF (ϕ_{kk})

$$\phi_{kk} = \frac{-\theta^k(1-\theta^2)}{1-\theta^{2(k+1)}}$$

Untuk ϕ_{11} (Lag 1)

$$\phi_{11} = \rho_1 = -0.4$$

Untuk ϕ_{22} (Lag 2)

$$\phi_{22} = \frac{-(0.5)^2(1-0.5^2)}{1-0.5^{2(2+1)}} = \frac{-(0.25)(1-0.25)}{1-0.5^6}$$

$$\phi_{22} = \frac{-0.25 \times 0.75}{1-0.015625} = \frac{-0.1875}{0.984375}$$

$$\phi_{22} \approx \mathbf{-0.1905}$$

Untuk ϕ_{33} (Lag 3)

$$\phi_{33} = \frac{-(0.5)^3(1-0.5^2)}{1-0.5^{2(3+1)}} = \frac{-(0.125)(0.75)}{1-0.5^8}$$

$$\phi_{33} = \frac{-0.09375}{1-0.00390625}$$

$$\phi_{33} \approx \mathbf{-0.0941}$$

$$\phi_{kk} = \{-0.4, -0.19, -0.09, \ldots\}$$

Nilainya **meluruh (tails off)**

Proses MA(1) dalam R

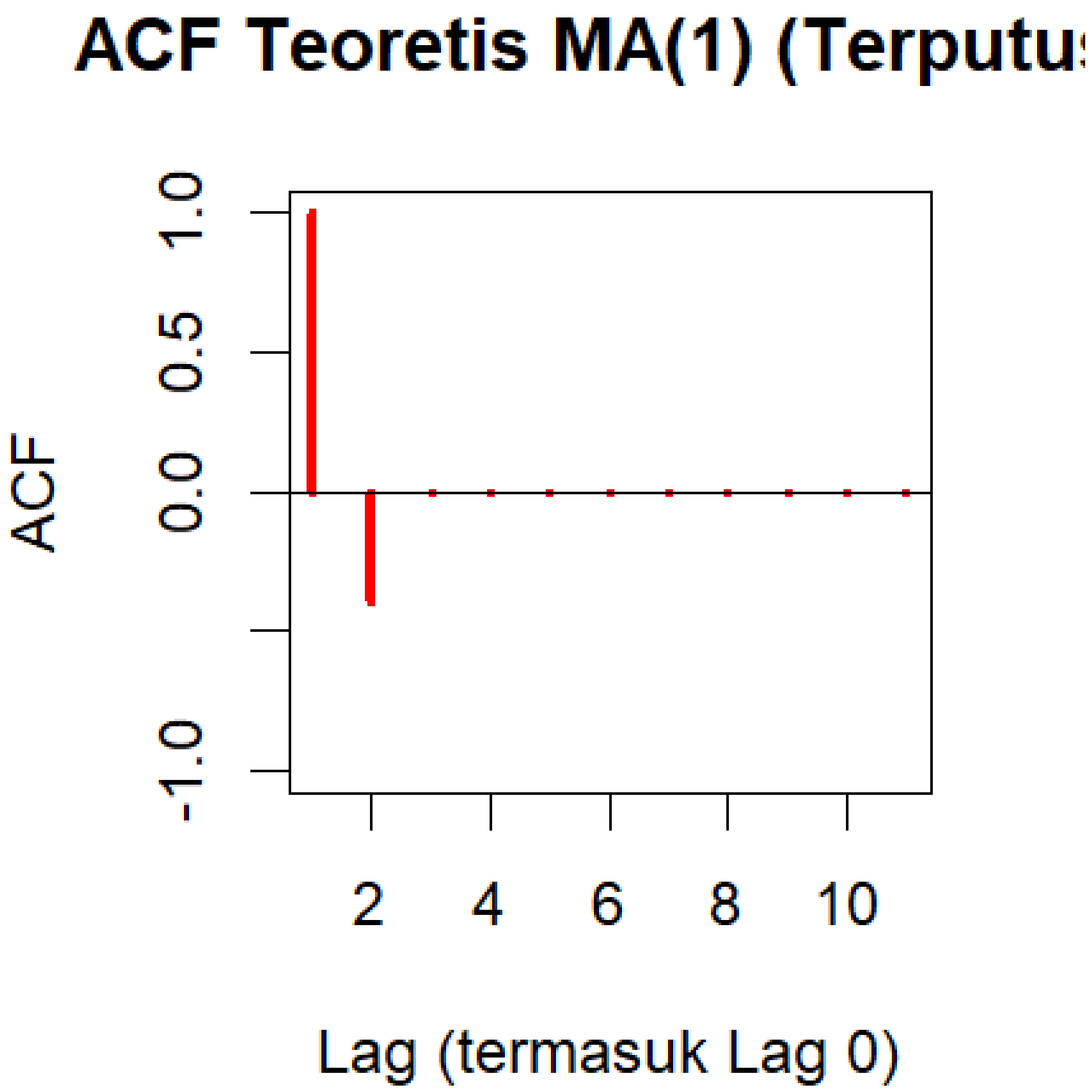
```r
# ----------------------------------------------------------------
# Verifikasi Perhitungan Teoretis MA(1)
# Model: Z_t = a_t - 0.5 * a_{t-1}
# Konvensi R: ma = -0.5
# ----------------------------------------------------------------

# --- 1. Definisikan Parameter ---
theta_bj <- 0.5
koef_r <- -theta_bj  # Koefisien untuk R
lag_maks <- 3

# --- 2. Perhitungan ACF (rho_k) ---
# Kita gunakan fungsi ARMAacf()
rho_k <- ARMAacf(ma = koef_r, lag.max = lag_maks)

print(rho_k)

# --- 3. Perhitungan PACF (phi_kk) ---
# Kita gunakan ARMAacf() dengan 'pacf = TRUE'
phi_kk <- ARMAacf(ma = koef_r, lag.max = lag_maks, pacf
= TRUE)

print("--- Nilai PACF (phi_kk) ---")
print(phi_kk)
```

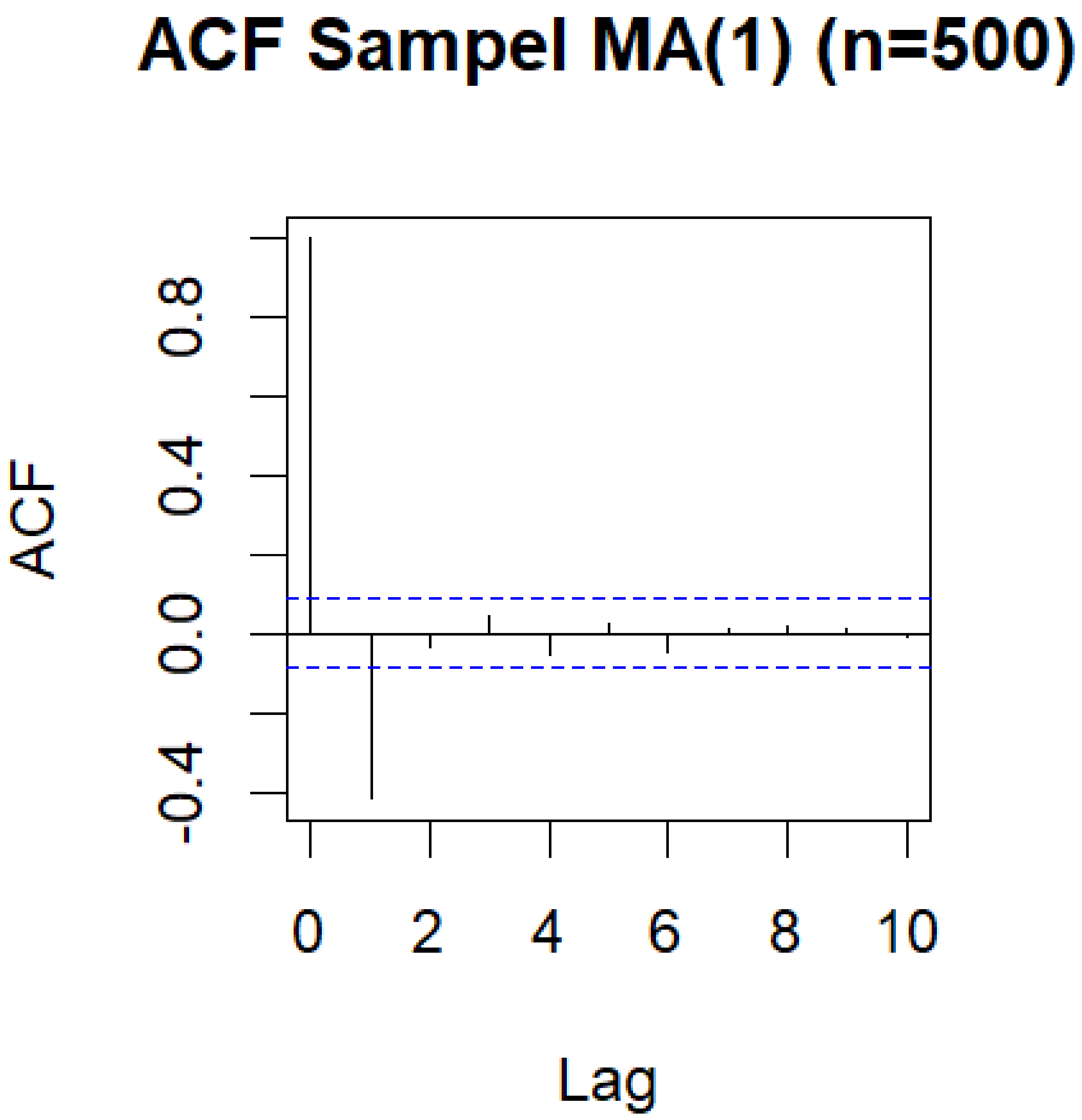
Plot ACF MA(1) dalam R

```
dev.new()
# Atur layout jadi 1 baris, 2 kolom
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# ACF akan terputus setelah Lag 1
acf_teori <- ARMAacf(ma = -0.5, lag.max = 10, pacf =
FALSE)
plot(acf_teori, type = "h",
     main = "ACF Teoretis MA(1) (Terputus)",
     xlab = "Lag (termasuk Lag 0)", ylab = "ACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)
```

```
# --- PLOT KANAN (SAMPEL) ---
set.seed(123)
# Simulasi data dari model MA(1) dengan ma = -0.5
data_sim <- arima.sim(model = list(ma = -0.5), n = 500)
acf(data_sim, lag.max = 10,
 main = "ACF Sampel MA(1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot ACF MA(1) dalam R

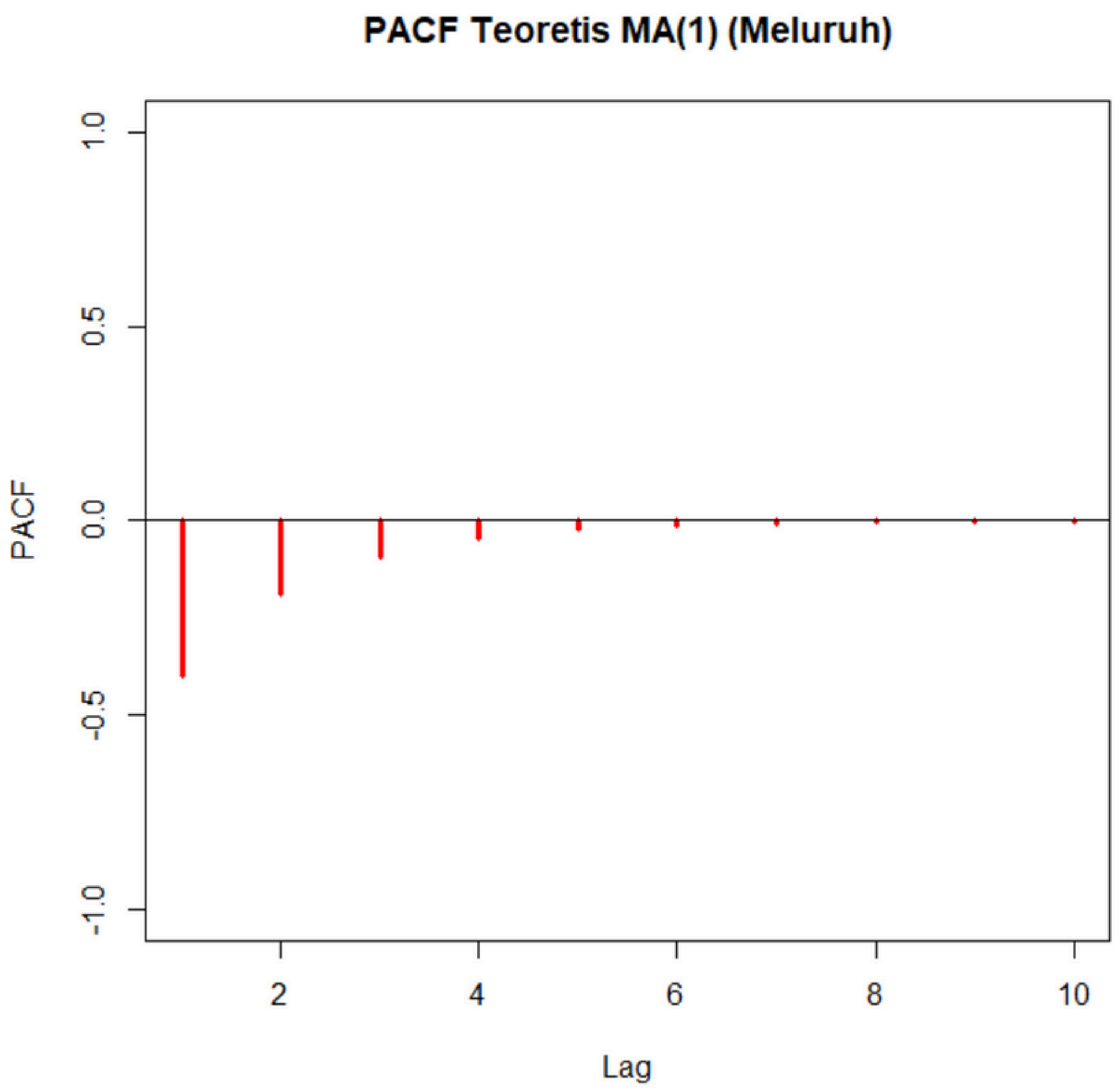


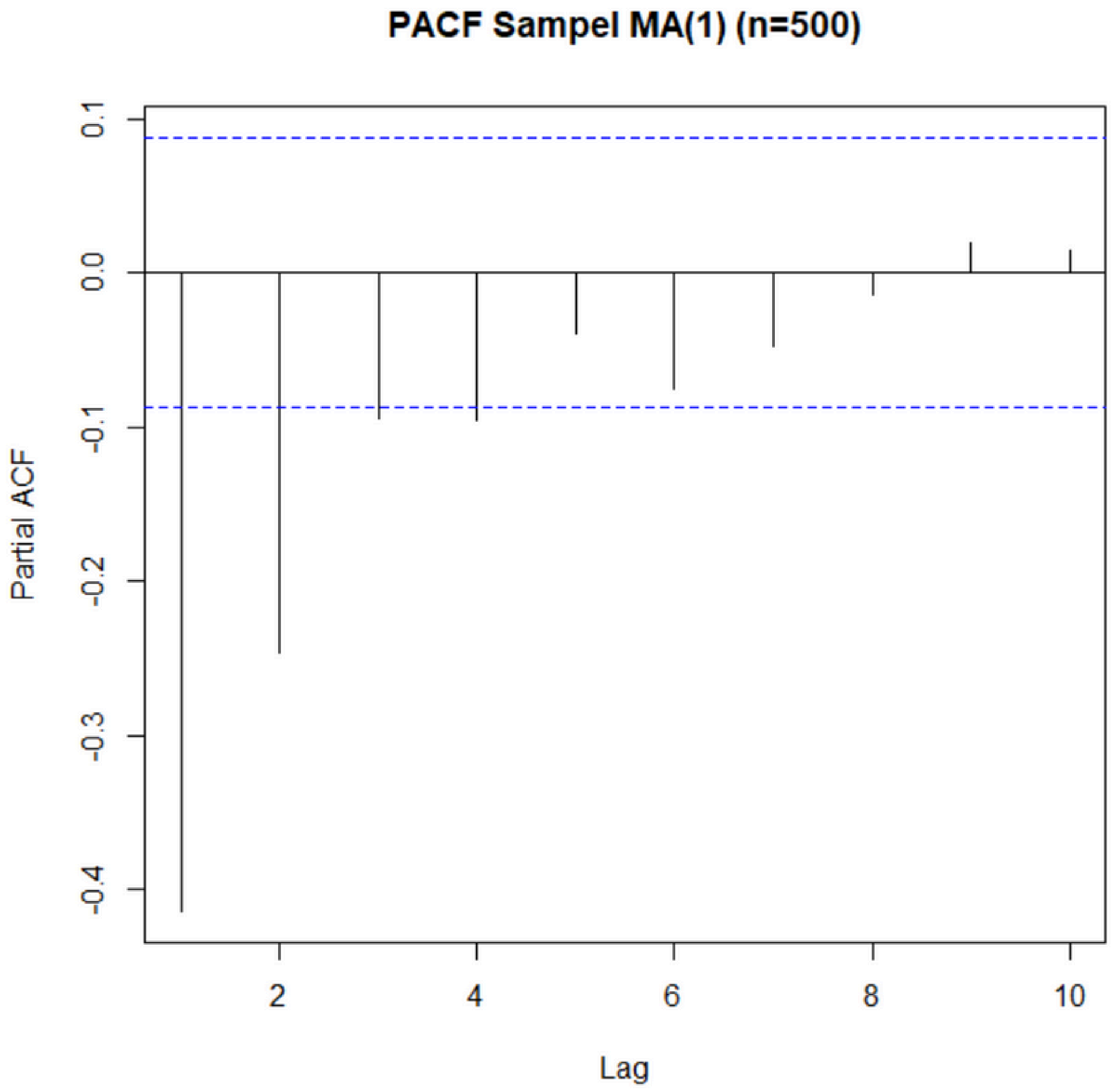


Plot PACF MA(1) dalam R

```
dev.new()
# Atur layout jadi 1 baris, 2 kolom
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# PACF akan meluruh
pacf_teori <- ARMAacf(ma = -0.5, lag.max = 10, pacf =
TRUE)
plot(pacf_teori, type = "h",
     main = "PACF Teoretis MA(1) (Meluruh)",
     xlab = "Lag", ylab = "PACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)

# --- PLOT KANAN (SAMPEL) ---
# Kita gunakan data simulasi yang sama dari atas
(data_sim)
pacf(data_sim, lag.max = 10,
 main = "PACF Sampel MA(1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot PACF MA(1) dalam R





Proses ARMA(1,1)

Diketahui sebuah proses deret waktu stasioner Z(t) mengikuti model MA(1) sebagai berikut:

$$Z(t) = 0.5\, Z(t-1) + a(t) - 0.4\, a(t-1)$$

Dimana a(t) adalah white noise (WN) dengan rata-rata 0 dan varians 1.

Tugas:

Hitunglah:

1. Ragam (Varian) proses.
2. Koragam (Autokovarians).
3. Korelasi (ACF).
4. Korelasi Parsial (PACF).

Proses ARMA(1,1)

Kita memiliki parameter:

- $\phi_1 = 0.5$
- $\theta_1 = 0.4$
- $\sigma_a^2 = 1$

1. Perhitungan Ragam (γ_0)

Rumus γ_0:

$$\begin{aligned}\gamma_0 &= Var(Z_t) \\ &= Var(\phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1}) \\ &= Var(\phi_1 Z_{t-1}) + Var(a_t) + Var(\theta_1 a_{t-1}) - 2Cov(\phi_1 Z_{t-1}, \theta_1 a_{t-1})\end{aligned}$$

(Catatan: $Cov(Z_{t-1}, a_{t-1}) = \sigma_a^2$)

$$\begin{aligned}\gamma_0 &= \phi_1^2\gamma_0 + \sigma_a^2 + \theta_1^2\sigma_a^2 - 2\phi_1\theta_1\sigma_a^2 \\ \gamma_0 &= \phi_1^2\gamma_0 + \sigma_a^2(1 + \theta_1^2 - 2\phi_1\theta_1) \\ \gamma_0(1 - \phi_1^2) &= \sigma_a^2(1 + \theta_1^2 - 2\phi_1\theta_1) \\ \gamma_0(1 - 0.5^2) &= 1(1 + 0.4^2 - 2 \times 0.5 \times 0.4) \\ \gamma_0(1 - 0.25) &= 1(1 + 0.16 - 0.4) \\ \gamma_0(0.75) &= 0.76 \\ \gamma_0 &= \frac{0.76}{0.75} \\ &\approx \mathbf{1.0133}\end{aligned}$$

Proses ARMA(1,1)

2. Perhitungan Koragam (γ_k)

$$\gamma_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-\phi^2}\phi^{k-1}\sigma_a^2$$

Untuk γ_1 (Lag 1, set $k=1$)

$$\begin{aligned}
\gamma_1 &= \frac{(1-(0.4)(0.5))(0.5-0.4)}{1-(0.5)^2}(0.5)^{1-1}(1) \\
&= \frac{(1-0.2)(0.1)}{1-0.25}(0.5)^0(1) \\
&= \frac{(0.8)(0.1)}{0.75}(1) \\
&= \frac{0.08}{0.75} \\
&\approx \mathbf{0.1067}
\end{aligned}$$

Untuk γ_2 (Lag 2, set $k=2$)

$$\begin{aligned}
\gamma_2 &= \frac{(1-(0.4)(0.5))(0.5-0.4)}{1-(0.5)^2}(0.5)^{2-1}(1) \\
&= \left(\frac{0.08}{0.75}\right) \times (0.5)^1 \\
&= \frac{0.04}{0.75} \\
&\approx \mathbf{0.0533}
\end{aligned}$$

Proses ARMA(1,1)

UNIVERSITAS SULTAN AGENG TIRTAYASA

UNTIRTA Jawara

Jujur Adil Wibawa Amanah Religius Akuntabel

3. Perhitungan Korelasi / ACF (ρ_k) $\rho_k = \gamma_k/\gamma_0$

Untuk ρ_1 (Lag 1)

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{0.08/0.75}{0.76/0.75} = \frac{0.08}{0.76} \approx \mathbf{0.1053}$$

Untuk ρ_2 (Lag 2)

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{0.04/0.75}{0.76/0.75} = \frac{0.04}{0.76} \approx \mathbf{0.0526}$$

Perilaku ACF (seperti AR(1))

pola **meluruh (tails off)**

$\rho_k = \phi_1 \rho_{k-1}$ (untuk $k \geq 2$)

4. Perhitungan Korelasi Parsial / PACF (ϕ_{kk})

Untuk ϕ_{11} (Lag 1)

$$\phi_{11} = \rho_1 \approx \mathbf{0.1053}$$

Untuk ϕ_{22} (Lag 2)

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} = \frac{0.0526 - (0.1053)^2}{1 - (0.1053)^2} = \frac{0.0526 - 0.0111}{1 - 0.0111} = \frac{0.0415}{0.9889} \approx \mathbf{0.042}$$

Perilaku PACF (seperti MA(1))

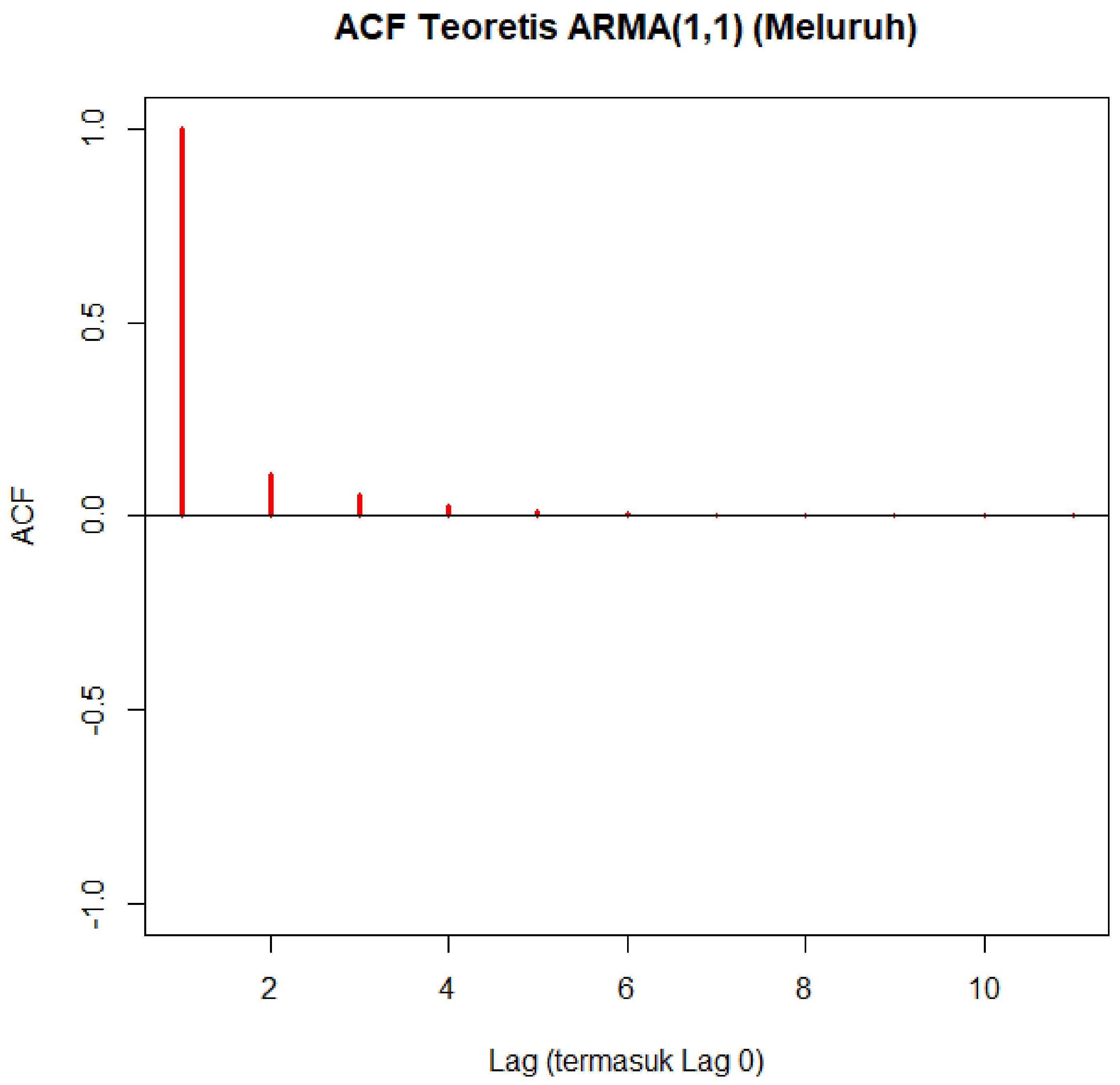
pola **meluruh (tails off)**

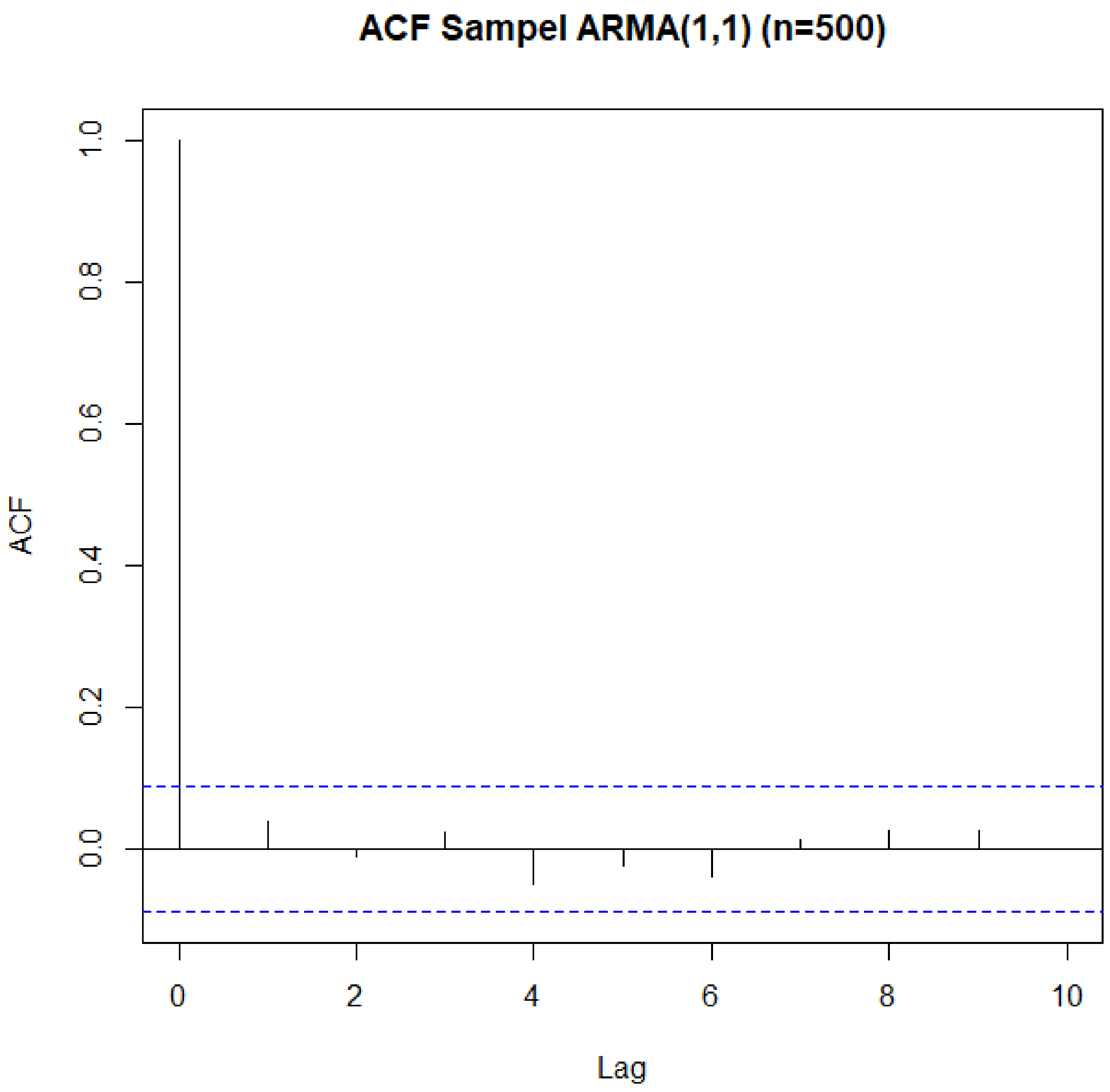
$$\phi_{kk} = \frac{-\theta^k(1-\theta^2)}{1-\theta^{2(k+1)}} \quad \text{untuk } k \geq 1$$

Plot ACF ARMA(1,1) dalam R

```
dev.new()

# Atur layout jadi 1 baris, 2 kolom
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# ACF akan meluruh
acf_teori <- ARMAacf(ar = 0.5, ma = -0.4, lag.max = 10,
pacf = FALSE)

plot(acf_teori, type = "h",
     main = "ACF Teoretis ARMA(1,1) (Meluruh)",
     xlab = "Lag (termasuk Lag 0)", ylab = "ACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)

# --- PLOT KANAN (SAMPEL) ---
set.seed(123)

# Simulasi data dari model
data_sim <- arima.sim(model = list(ar = 0.5, ma = -0.4), n =
500)

# Plot ACF dari sampel
acf(data_sim, lag.max = 10,
 main = "ACF Sampel ARMA(1,1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot ACF ARMA(1,1) dalam R

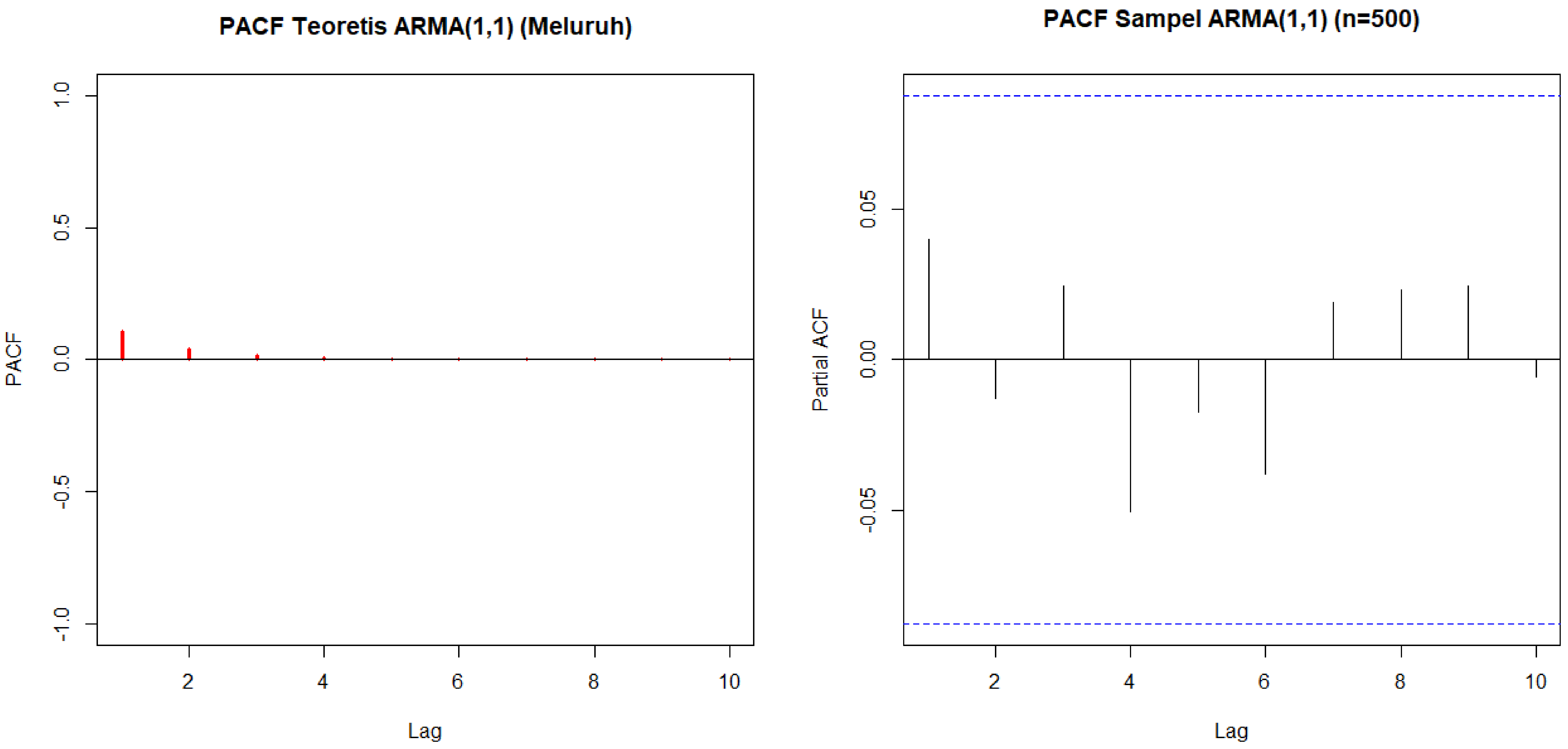




Plot PACF ARMA(1,1) dalam R

```
dev.new()

# Atur layout jadi 1 baris, 2 kolom
par(mfrow = c(1, 2))

# --- PLOT KIRI (TEORETIS) ---
# PACF akan meluruh
pacf_teori <- ARMAacf(ar = 0.5, ma = -0.4, lag.max = 10,
pacf = TRUE)

plot(pacf_teori, type = "h",
     main = "PACF Teoretis ARMA(1,1) (Meluruh)",
     xlab = "Lag", ylab = "PACF",
     ylim = c(-1, 1),
     lwd = 3, col = "red")
abline(h = 0)

# --- PLOT KANAN (SAMPEL) ---
set.seed(123)

# Simulasi data dari model
data_sim <- arima.sim(model = list(ar = 0.5, ma = -0.4), n =
500)

# Plot PACF dari sampel
pacf(data_sim, lag.max = 10,
 main = "PACF Sampel ARMA(1,1) (n=500)")

# Kembalikan layout ke normal
par(mfrow = c(1, 1))
```

Plot PACF ARMA(1,1) dalam R



SEE YOU NEXT WEEK !

Ferdian Bangkit Wijaya, S.Stat., M.Si
NIP. 199005202024061001
ferdian.bangkit@untirta.ac.id