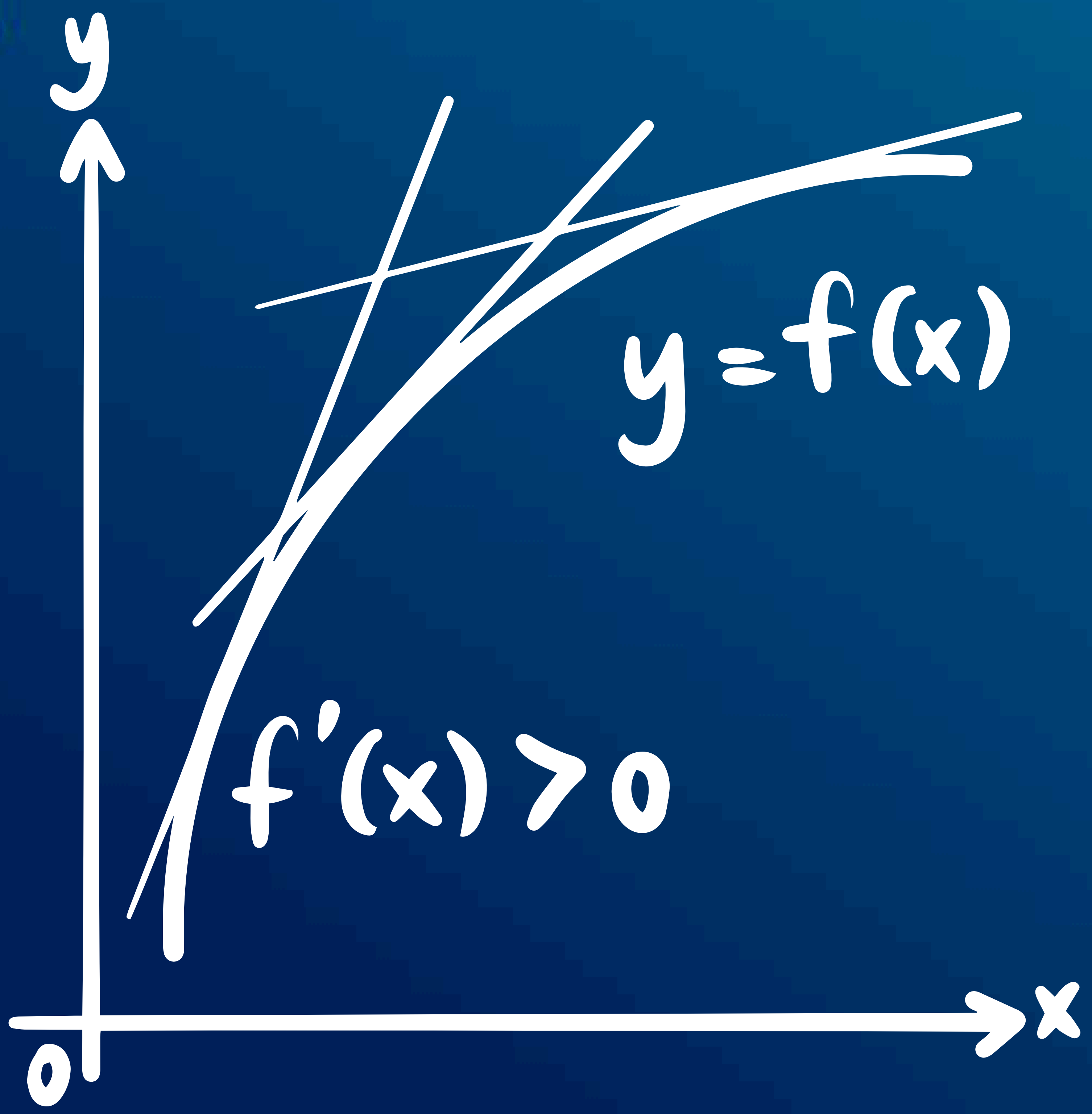

Calculus 1

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UNIVERSITAS SULTAN AGENG TIRTAYASA

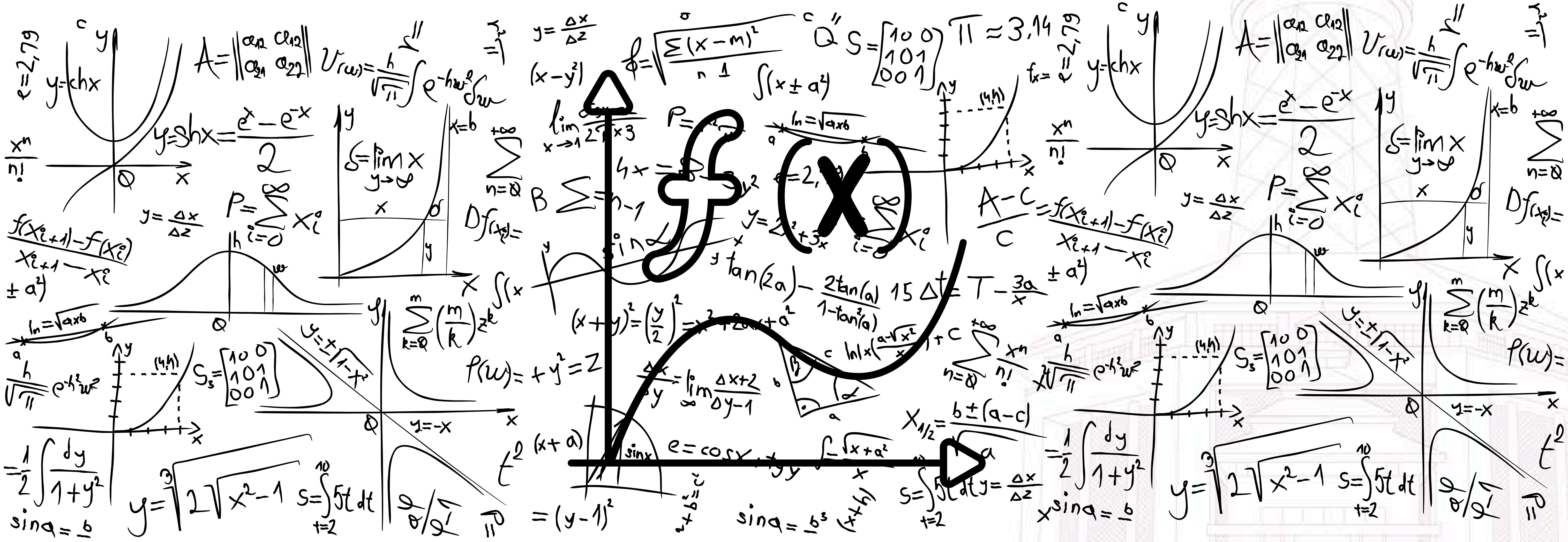
#2 Meeting

UNTIRTA Jawara

Jujur Adil Wibawa Amanah Religius Akuntabel

Function :

Domain & Range, Graph, Trigonometric, Operations

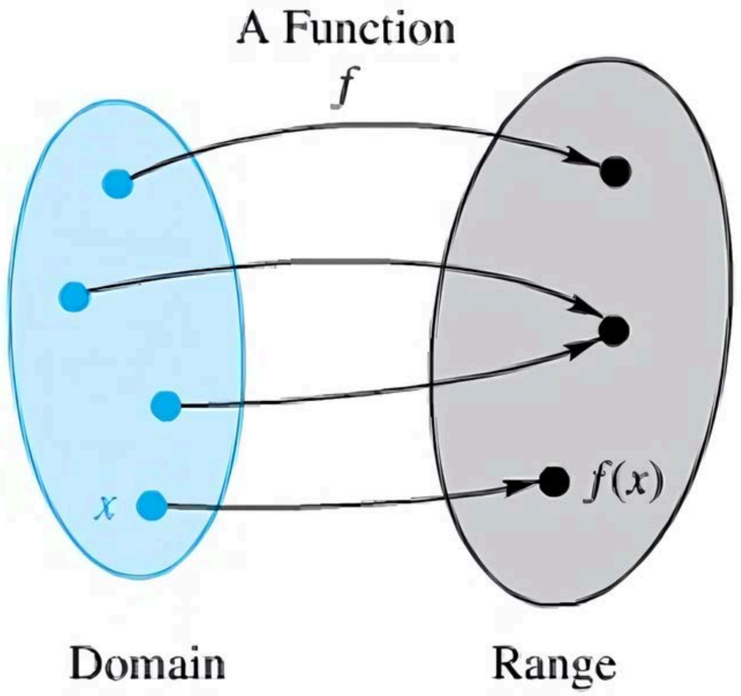


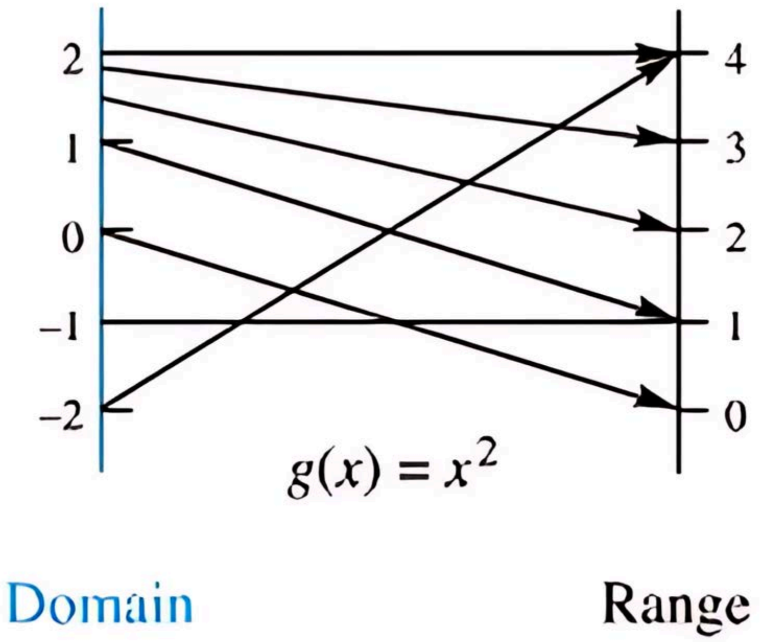
Function

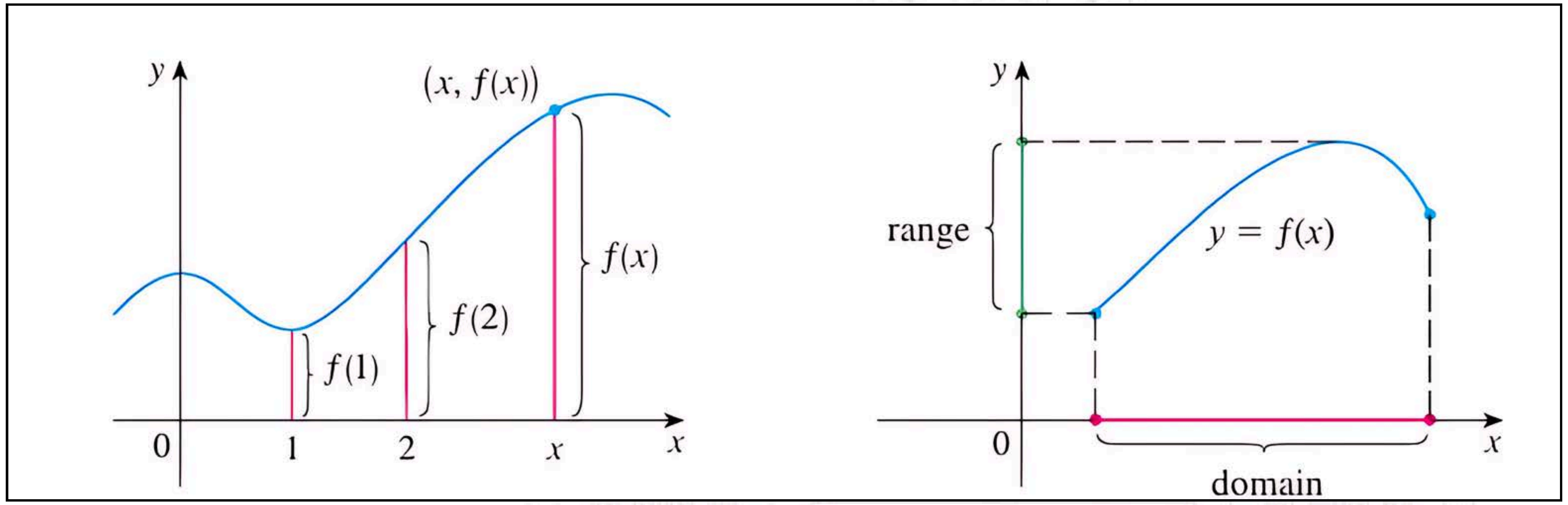
Domain & Range

Definition

A **function** f is a rule of correspondence that associates with each object x in one set, called the **domain,** a single value $f(x)$ from a second set. The set of all values so obtained is called the **range** of the function.







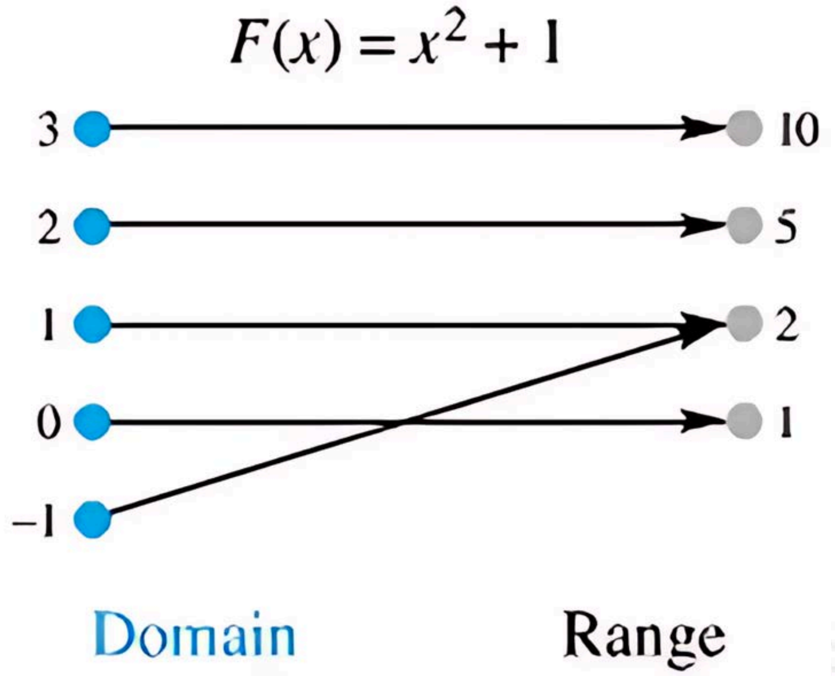
Function

Domain & Range

Function Notation A single letter like f (or g or F) is used to name a function. Then $f(x)$, read "f of x" or "f at x," denotes the value that f assigns to x.

Domain and Range To specify a function completely, we must state, in addition to the rule of correspondence, the domain of the function. For example, if F is the function defined by $F(x) = x^2 + 1$ with domain $\{-1, 0, 1, 2, 3\}$ (Figure), then the range is $\{1, 2, 5, 10\}$. The rule of correspondence, together with the domain, determines the range.

When no domain is specified for a function, we assume that it is the largest set of real numbers for which the rule for the function makes sense. This is called the **natural domain.** Numbers that you should remember to exclude from the natural domain are those values that would cause division by zero or the square root of a negative number.



Function

Domain & Range

EXAMPLE 1 For $f(x) = x^2 - 2x$, find and simplify

(a) $f(4)$ (b) $f(4+h)$

(c) $f(4+h) - f(4)$ (d) $[f(4+h) - f(4)]/h$

SOLUTION

(a) $f(4) = 4^2 - 2 \cdot 4 = 8$

(b) $f(4+h) = (4+h)^2 - 2(4+h) = 16 + 8h + h^2 - 8 - 2h$
$= 8 + 6h + h^2$

(c) $f(4+h) - f(4) = 8 + 6h + h^2 - 8 = 6h + h^2$

(d) $\dfrac{f(4+h) - f(4)}{h} = \dfrac{6h + h^2}{h} = \dfrac{h(6+h)}{h} = 6 + h$

EXAMPLE 2 Find the natural domains for

(a) $f(x) = 1/(x - 3)$ (b) $g(t) = \sqrt{9 - t^2}$

(c) $h(w) = 1/\sqrt{9 - w^2}$

SOLUTION

(a) We must exclude 3 from the domain because it would require division by zero. Thus, the natural domain is $\{x: x \neq 3\}$. This may be read "the set of x's such that x is not equal to 3."

(b) To avoid the square root of a negative number, we must choose t so that $9 - t^2 \geq 0$. Thus, t must satisfy $|t| \leq 3$. The natural domain is therefore $\{t: |t| \leq 3\}$, which we can write using interval notation as $[-3, 3]$.

(c) Now we must avoid division by zero *and* square roots of negative numbers, so we must exclude -3 and 3 from the natural domain. The natural domain is therefore the interval $(-3, 3)$. ■

Function

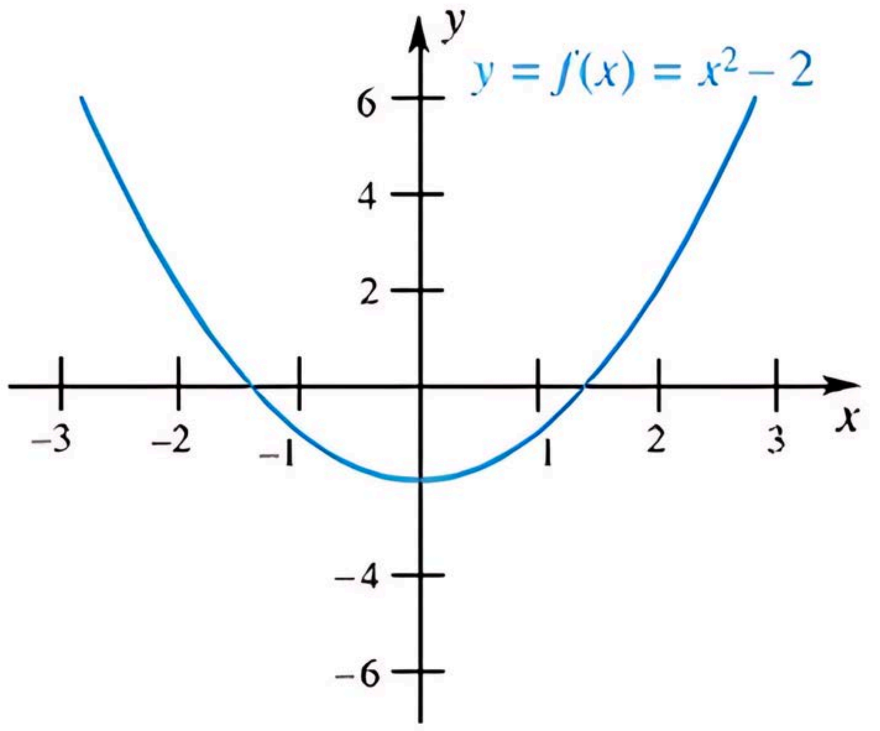
Graph of function

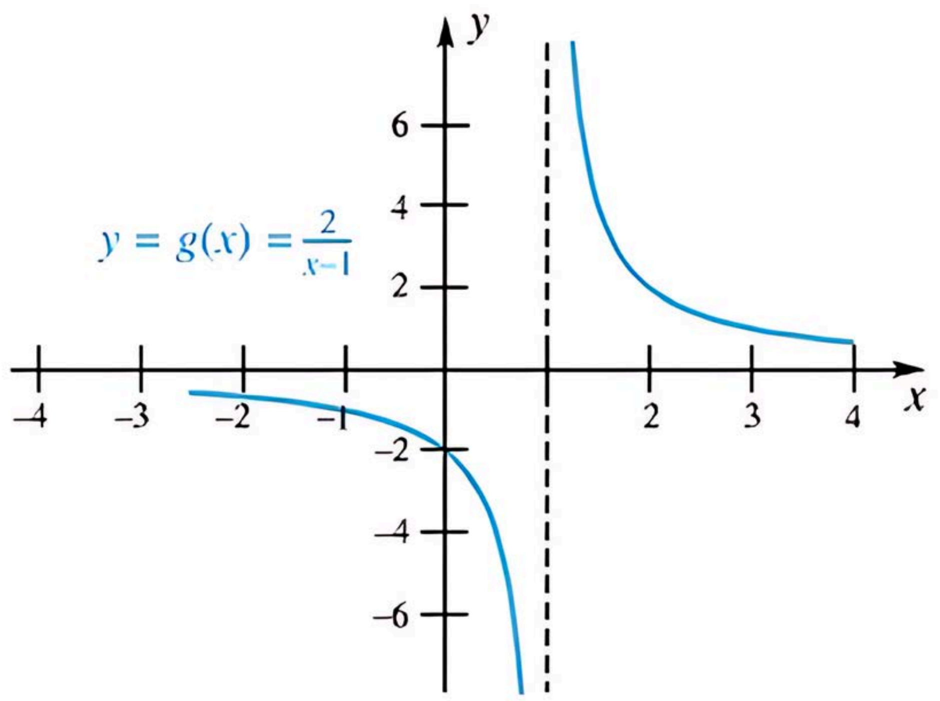
Graphs of Functions When both the domain and range of a function are sets of real numbers, we can picture the function by drawing its graph on a coordinate plane. The **graph of a function** f is simply the graph of the equation $y = f(x)$.

EXAMPLE 3 Sketch the graphs of

(a) $f(x) = x^2 - 2$ (b) $g(x) = 2/(x - 1)$

SOLUTION



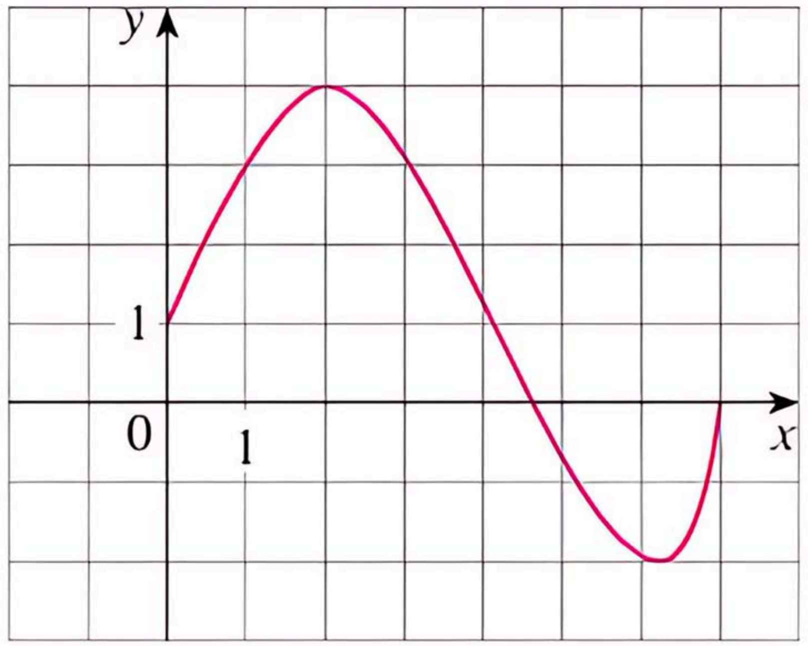


Function	Domain	Range
$f(x) = x^2 - 2$	all real numbers	$\{y: y \geq -2\}$
$g(x) = \dfrac{2}{x - 1}$	$\{x: x \neq 1\}$	$\{y: y \neq 0\}$

Function

Graph of function

EXAMPLE 4



(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of f?

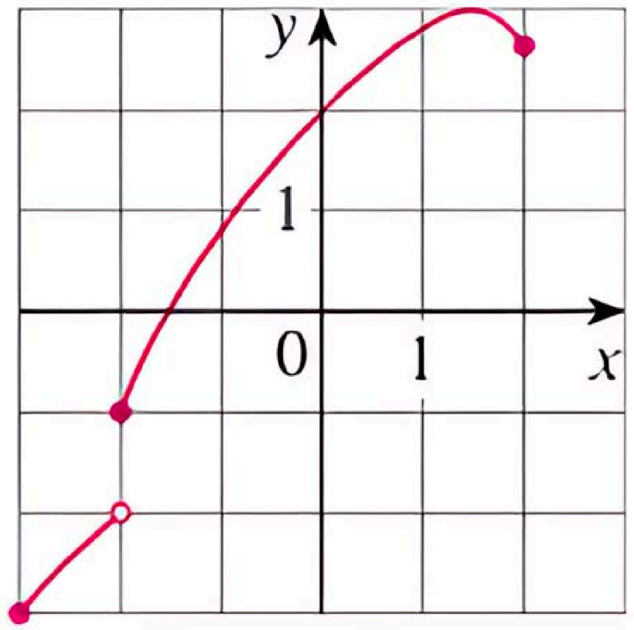
SOLUTION

(a) $f(1) = 3$

$f(5) \approx -0.7$

(b) $\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$

EXAMPLE 5



state the domain and range of the function

SOLUTION

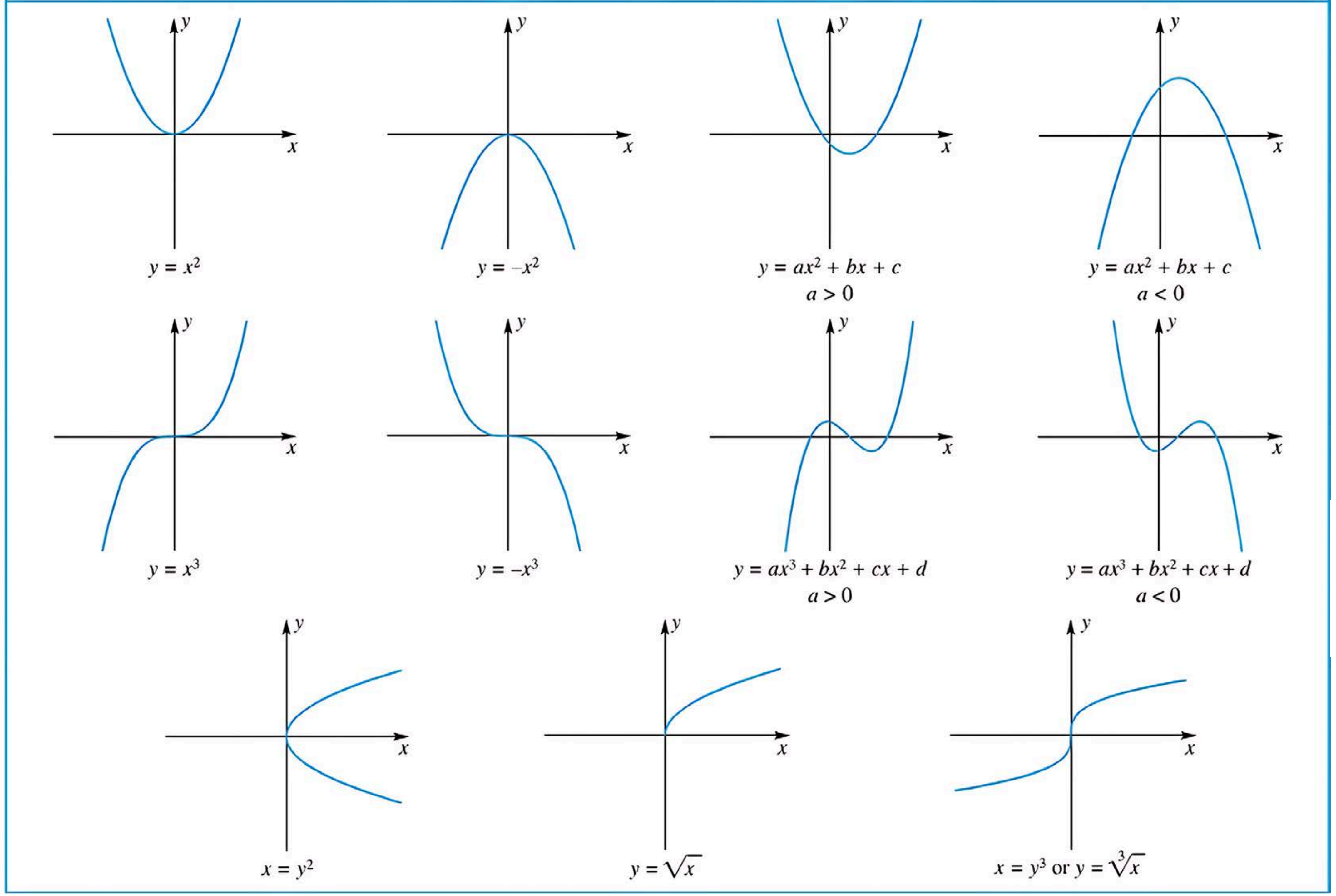
The domain is $[-3, 2]$

the range is $[-3, -2) \cup [-1, 3]$

Function

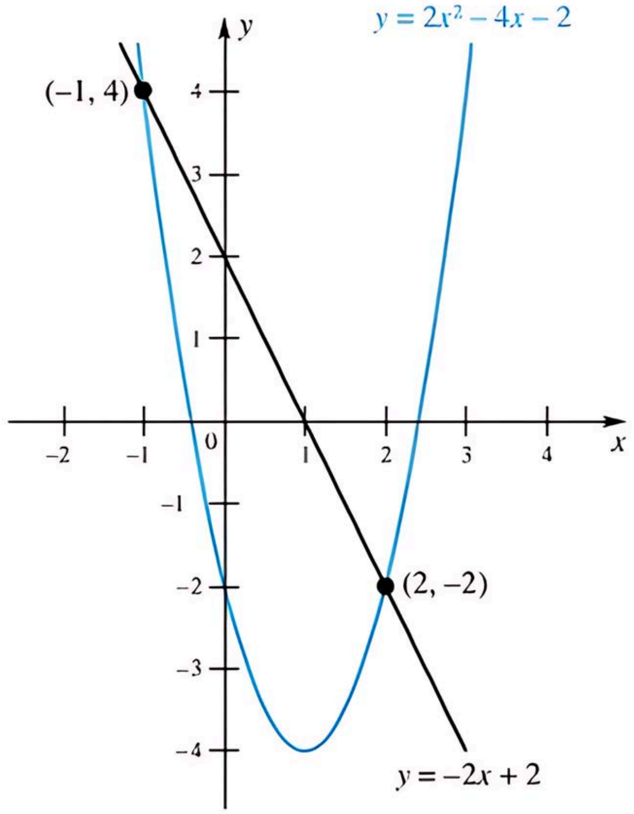
Graph of function

BASIC QUADRATIC AND CUBIC GRAPHS



EXAMPLE 6 Find the points of intersection of the line $y = -2x + 2$ and the parabola $y = 2x^2 - 4x - 2$, and sketch both graphs on the same coordinate plane.

$$-2x + 2 = 2x^2 - 4x - 2$$
$$0 = 2x^2 - 2x - 4$$
$$0 = 2(x + 1)(x - 2)$$
$$x = -1, \qquad x = 2$$

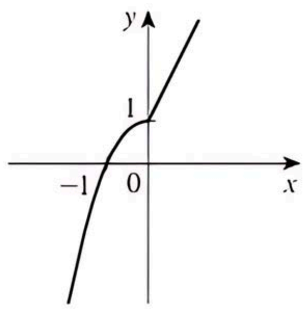


EXAMPLE 7

Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

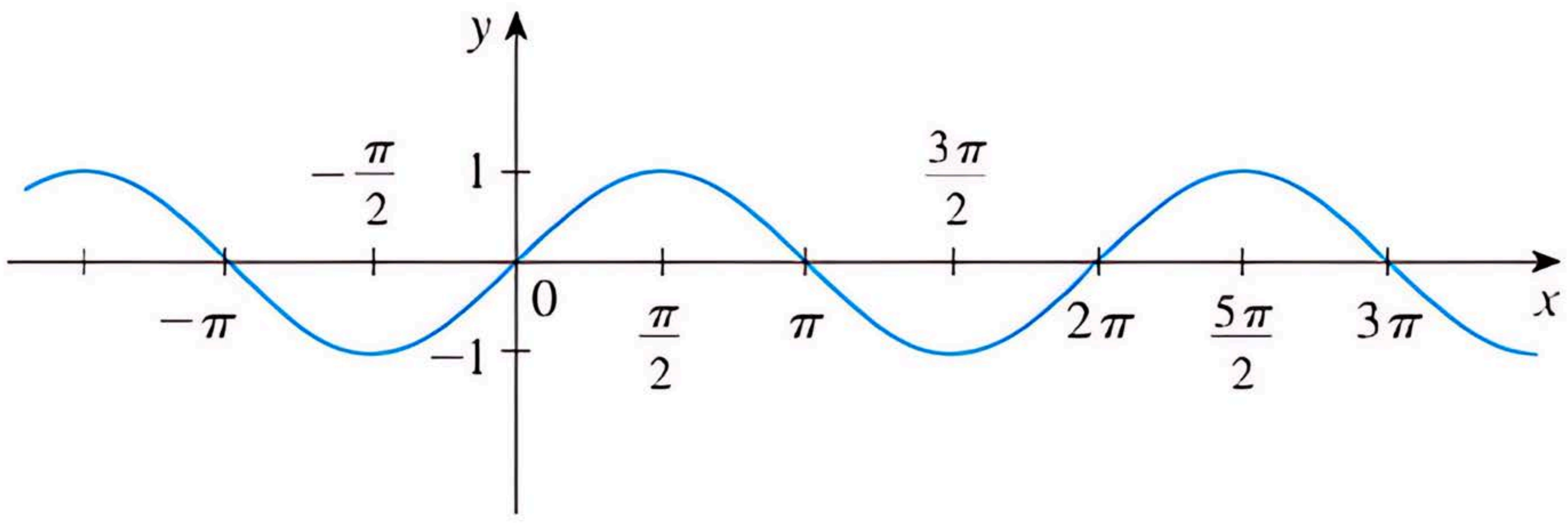
(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

(a) $-3, 3$ (b)

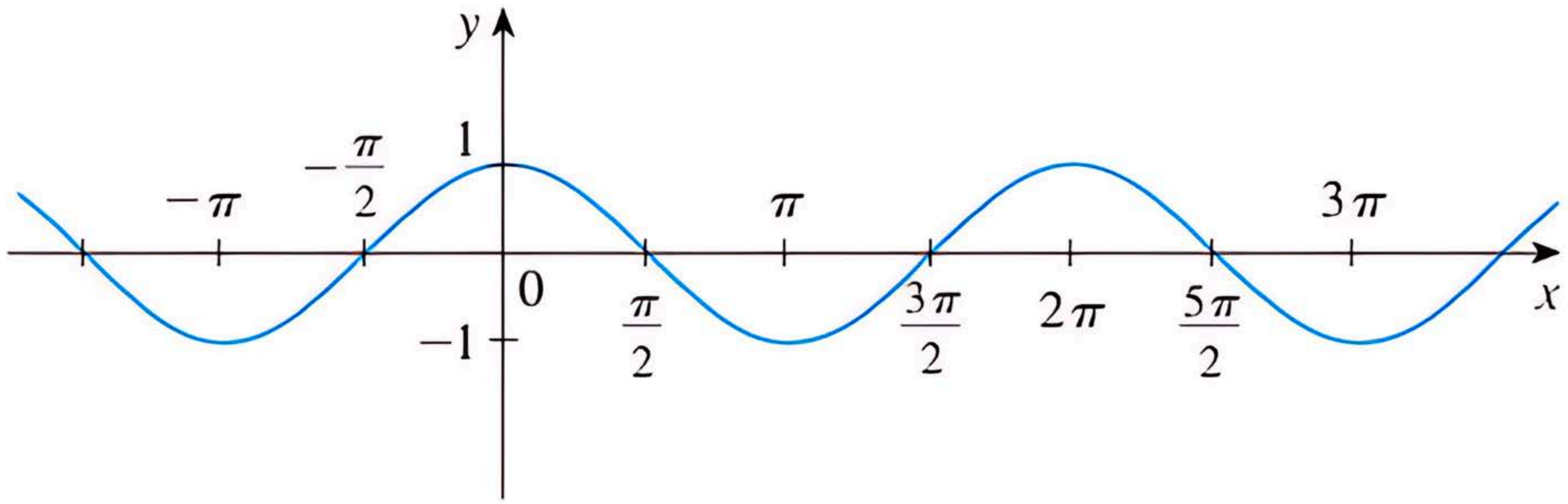


Function

Trigonometric Functions



(a) $f(x) = \sin x$



(b) $g(x) = \cos x$

Degrees	Radians
0	0
30	π/6
45	π/4
60	π/3
90	π/2
120	2π/3
135	3π/4
150	5π/6
180	π
360	2π

$180° = \pi \text{ radians} \approx 3.1415927 \text{ radians}$

$1 \text{ radian} \approx 57.29578° \qquad 1° \approx 0.0174533 \text{ radian}$

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t}$$

$$1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t$$

Sum identities

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Product identities

$$\sin x \sin y = -\tfrac{1}{2}[\cos(x+y) - \cos(x-y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$

Odd–even identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Cofunction identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

Pythagorean identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Double-angle identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

Addition identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Half-angle identities

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

Function

Trigonometric Functions

Trigonometry Ratio Table

Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angles (In Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	1
$\cot$	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined
$\csc$	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
$\sec$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1

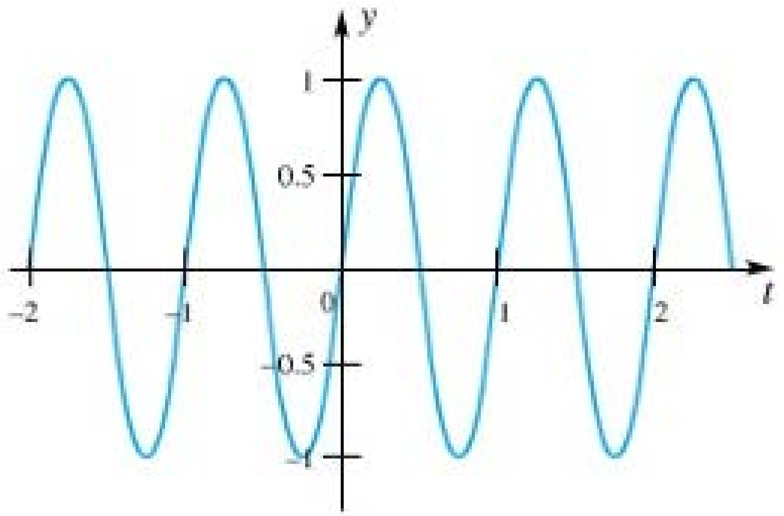
Function

Trigonometric Functions

EXAMPLE 1 Sketch the graphs of

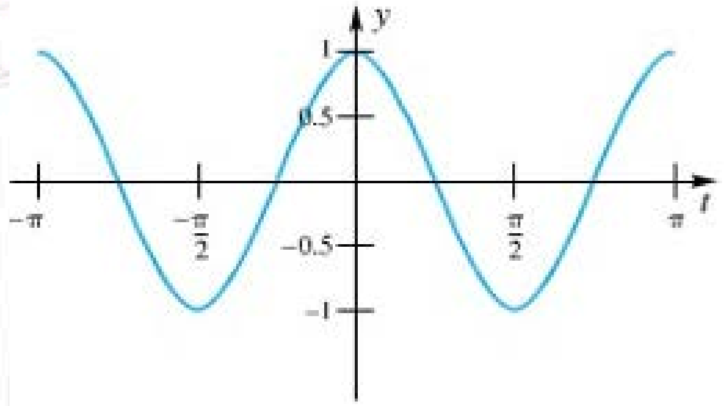
(a) $y = \sin(2\pi t)$

t	$\sin(2\pi t)$	t	$\sin(2\pi t)$
0	$\sin(2\pi \cdot 0) = 0$	$\frac{5}{8}$	$\sin\left(2\pi \cdot \frac{5}{8}\right) = -\frac{\sqrt{2}}{2}$
$\frac{1}{8}$	$\sin\left(2\pi \cdot \frac{1}{8}\right) = \frac{\sqrt{2}}{2}$	$\frac{3}{4}$	$\sin\left(2\pi \cdot \frac{3}{4}\right) = -1$
$\frac{1}{4}$	$\sin\left(2\pi \cdot \frac{1}{4}\right) = 1$	$\frac{7}{8}$	$\sin\left(2\pi \cdot \frac{7}{8}\right) = -\frac{\sqrt{2}}{2}$
$\frac{3}{8}$	$\sin\left(2\pi \cdot \frac{3}{8}\right) = \frac{\sqrt{2}}{2}$	1	$\sin(2\pi \cdot 1) = 0$
$\frac{1}{2}$	$\sin\left(2\pi \cdot \frac{1}{2}\right) = 0$	$\frac{9}{8}$	$\sin\left(2\pi \cdot \frac{9}{8}\right) = \frac{\sqrt{2}}{2}$



(b) $y = \cos(2t)$

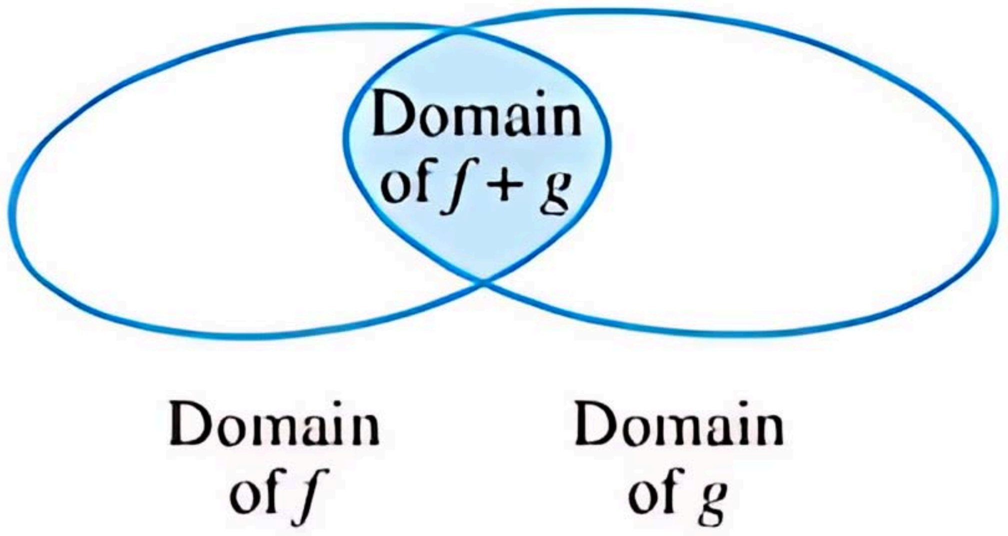
t	$\cos(2t)$	t	$\cos(2t)$
0	$\cos(2 \cdot 0) = 1$	$\frac{5\pi}{8}$	$\cos\left(2 \cdot \frac{5\pi}{8}\right) = -\frac{\sqrt{2}}{2}$
$\frac{\pi}{8}$	$\cos\left(2 \cdot \frac{\pi}{8}\right) = \frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$	$\cos\left(2 \cdot \frac{3\pi}{4}\right) = 0$
$\frac{\pi}{4}$	$\cos\left(2 \cdot \frac{\pi}{4}\right) = 0$	$\frac{7\pi}{8}$	$\cos\left(2 \cdot \frac{7\pi}{8}\right) = \frac{\sqrt{2}}{2}$
$\frac{3\pi}{8}$	$\cos\left(2 \cdot \frac{3\pi}{8}\right) = -\frac{\sqrt{2}}{2}$	π	$\cos(2 \cdot \pi) = 1$
$\frac{\pi}{2}$	$\cos\left(2 \cdot \frac{\pi}{2}\right) = -1$	$\frac{9\pi}{8}$	$\cos\left(2 \cdot \frac{9\pi}{8}\right) = \frac{\sqrt{2}}{2}$



Function

Operations on Functions

Just as two numbers a and b can be added to produce a new number $a + b$, so two functions f and g can be added to produce a new function $f + g$.



Sums, Differences, Products, Quotients, and Powers

$$f(x) = \frac{x-3}{2}, \qquad g(x) = \sqrt{x}$$

We can make a new function $f + g$ $f(x) + g(x) = (x - 3)/2 + \sqrt{x}$; that is,

$$(f+g)(x) = f(x) + g(x) = \frac{x-3}{2} + \sqrt{x}$$

Formula	Domain
$(f+g)(x) = f(x) + g(x) = \frac{x-3}{2} + \sqrt{x}$	$[0, \infty)$
$(f-g)(x) = f(x) - g(x) = \frac{x-3}{2} - \sqrt{x}$	$[0, \infty)$
$(f \cdot g)(x) = f(x) \cdot g(x) = \frac{x-3}{2}\sqrt{x}$	$[0, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x-3}{2\sqrt{x}}$	$(0, \infty)$

raise a function to a power.

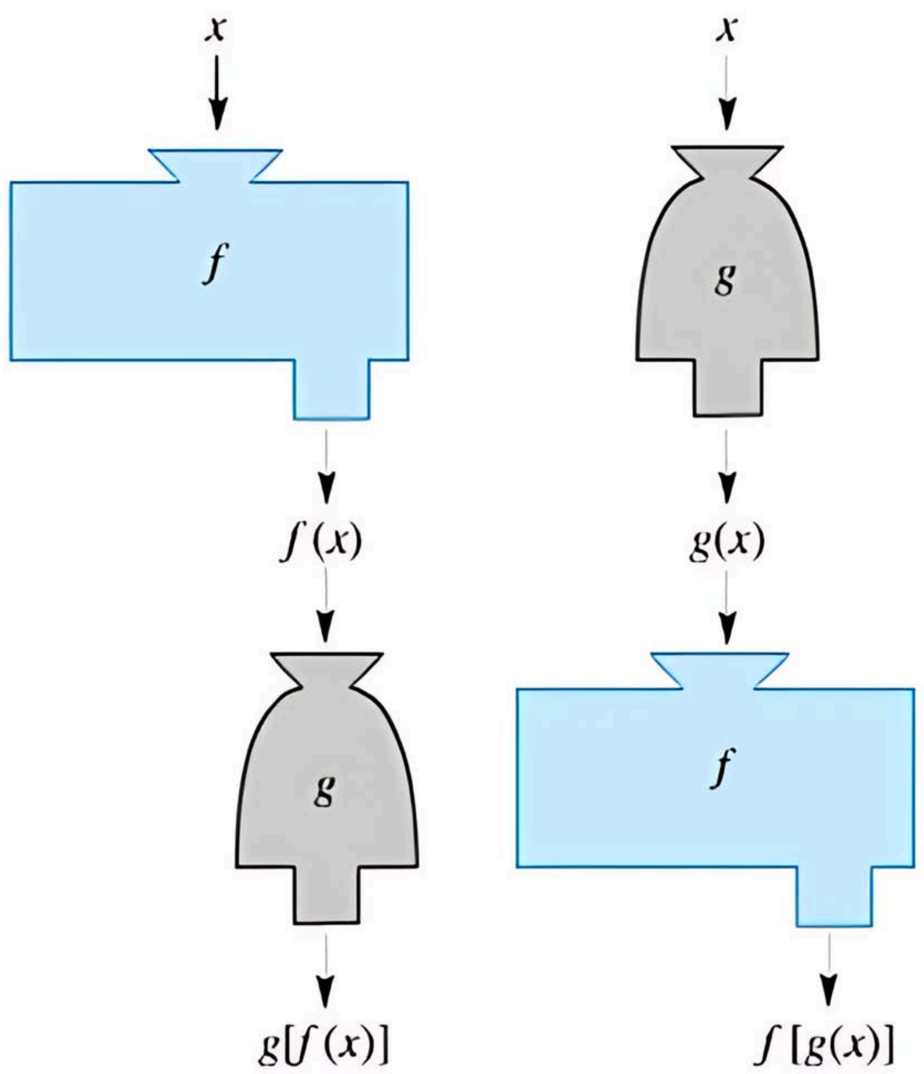
$$g^3(x) = [g(x)]^3 = \left(\sqrt{x}\right)^3 = x^{3/2}$$

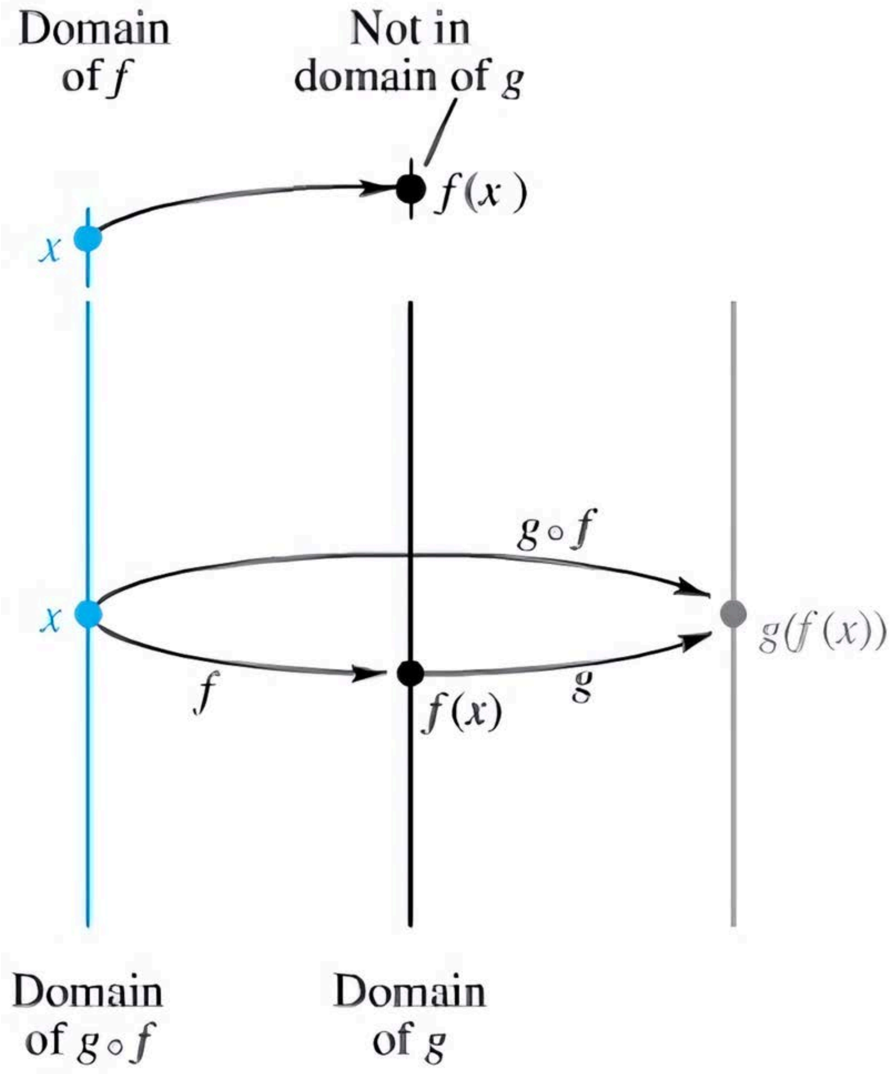
f^{-1} does not mean $1/f$.

Function

Operations on Functions

Composition of Functions





$$\boxed{(g \circ f)(x) = g(f(x))}$$

$$f(x) = \frac{x-3}{2}, \qquad g(x) = \sqrt{x}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x-3}{2}\right) = \sqrt{\frac{x-3}{2}}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right) = \frac{\sqrt{x}-3}{2}$$

The domain for $g \circ f$ is the interval $[3, \infty)$
The domain for $f \circ g$ is the interval $[0, \infty)$

the domains of $g \circ f$ and $f \circ g$ can be different.

Function

Operations on Functions

EXAMPLE 1 Let $F(x) = \sqrt[4]{x+1}$ and $G(x) = \sqrt{9-x^2}$, with respective natural domains $[-1, \infty)$ and $[-3, 3]$. Find formulas for $F+G$, $F-G$, $F \cdot G$, F/G, and F^5 and give their natural domains.

SOLUTION

Formula	Domain
$(F+G)(x) = F(x) + G(x) = \sqrt[4]{x+1} + \sqrt{9-x^2}$	$[-1, 3]$
$(F-G)(x) = F(x) - G(x) = \sqrt[4]{x+1} - \sqrt{9-x^2}$	$[-1, 3]$
$(F \cdot G)(x) = F(x) \cdot G(x) = \sqrt[4]{x+1}\sqrt{9-x^2}$	$[-1, 3]$
$\left(\frac{F}{G}\right)(x) = \frac{F(x)}{G(x)} = \frac{\sqrt[4]{x+1}}{\sqrt{9-x^2}}$	$[-1, 3)$
$F^5(x) = [F(x)]^5 = \left(\sqrt[4]{x+1}\right)^5 = (x+1)^{5/4}$	$[-1, \infty)$

EXAMPLE 2 Let $f(x) = 6x/(x^2 - 9)$ and $g(x) = \sqrt{3x}$, with their natural domains. First, find $(f \circ g)(12)$; then find $(f \circ g)(x)$ and give its domain.

SOLUTION

$$(f \circ g)(12) = f(g(12)) = f\left(\sqrt{36}\right) = f(6) = \frac{6 \cdot 6}{6^2 - 9} = \frac{4}{3}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3x}\right) = \frac{6\sqrt{3x}}{\left(\sqrt{3x}\right)^2 - 9}$$

$$(f \circ g)(x) = \frac{6\sqrt{3x}}{3x - 9} = \frac{2\sqrt{3x}}{x - 3}$$

We must also exclude $x = 3$ from the domain of $f \circ g$ because $g(3)$ is not in the domain of f. (It would cause division by 0.) Thus, the domain of $f \circ g$ is $[0, 3) \cup (3, \infty)$.

Function

Domain and Range, Graph, and Trigonometric (Time to participation)

1 Find the natural domain

$$f(x) = \frac{4 - x^2}{x^2 - x - 6}$$

2 Find the natural domain

$$G(y) = \sqrt{(y + 1)^{-1}}$$

3 *sketch its graph*

$$g(t) = \begin{cases} 1 & \text{if } t \leq 0 \\ t + 1 & \text{if } 0 < t < 2 \\ t^2 - 1 & \text{if } t \geq 2 \end{cases}$$

4 *sketch its graph*

$$h(x) = \begin{cases} -x^2 + 4 & \text{if } x \leq 1 \\ 3x & \text{if } x > 1 \end{cases}$$

5 *sketch its graph*

$$y = 2 \sin t$$

following on $[-\pi, 2\pi]$.

Function

Operations on Functions (Time to participation)

If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = 2/x$, find formulas for the following and state their domains.

1. $(f \cdot g)(x)$
2. $f^4(x) + g^4(x)$
3. $(f \circ g)(x)$
4. $(g \circ f)(x)$

Find f and g so that $p = f \circ g$.

$$p(x) = \frac{2}{(x^2 + x + 1)^3}$$

5.

SEE YOU NEXT WEEK !