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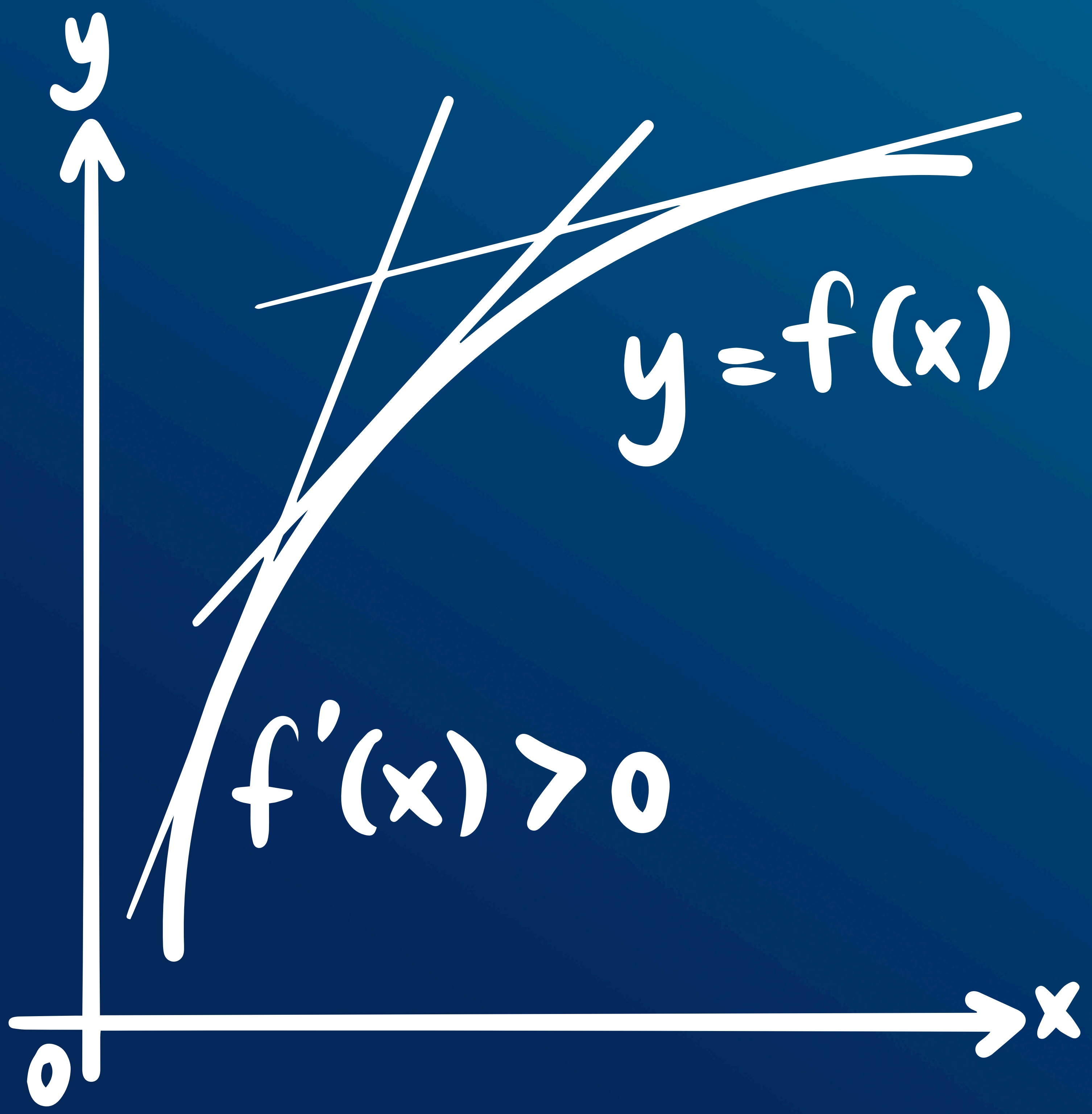
Jujur Adil Wibawa Amanah Religius Akuntabel



Calculus 1

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#1 Meeting

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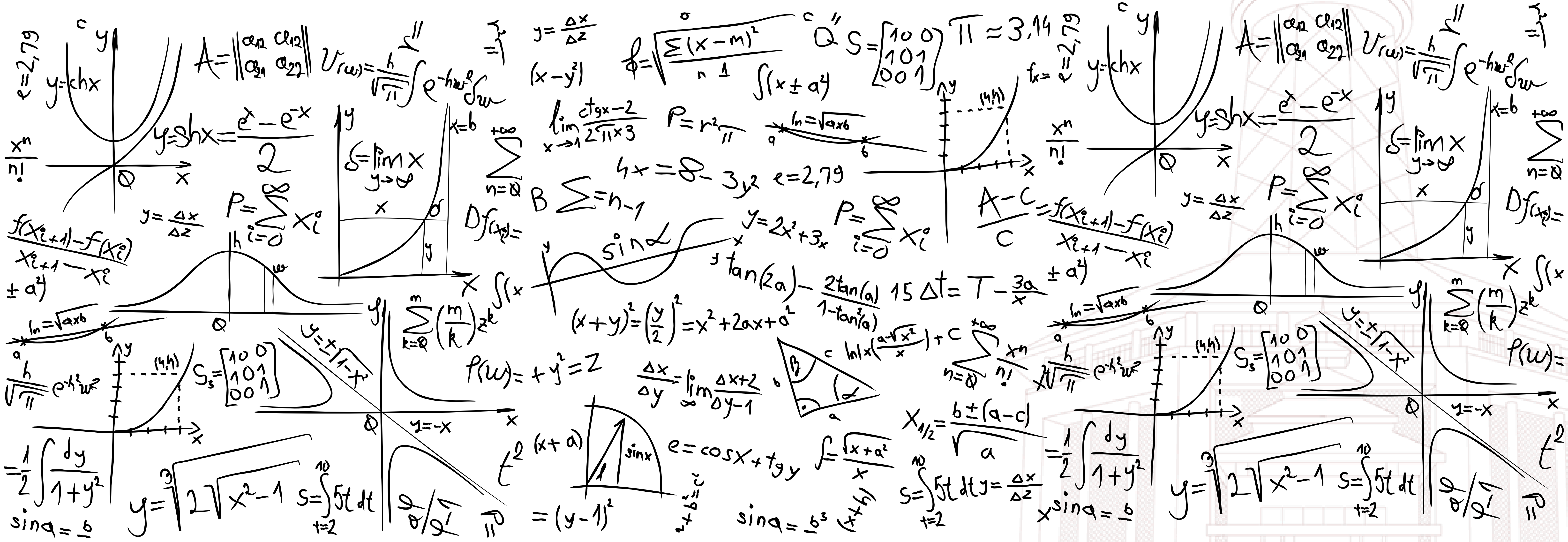
Real Numbers, Inequalities, and Absolute Values



Preliminaries

Real Numbers

Natural Numbers (the simplest numbers)

$1, 2, 3, 4, 5, 6, \ldots$

Integers (Natural numbers + negative & zero)

$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

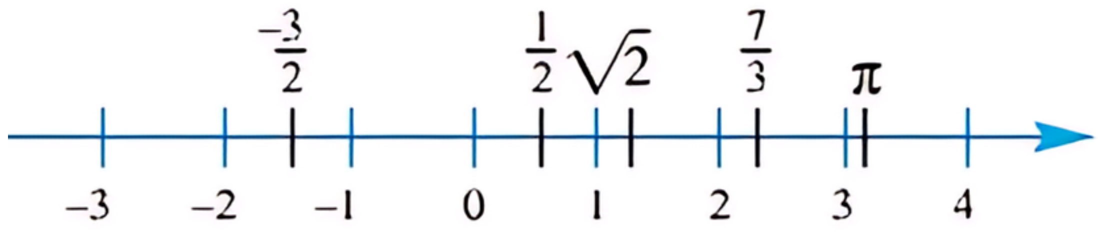
Rational numbers (fractional numbers/ decimals)

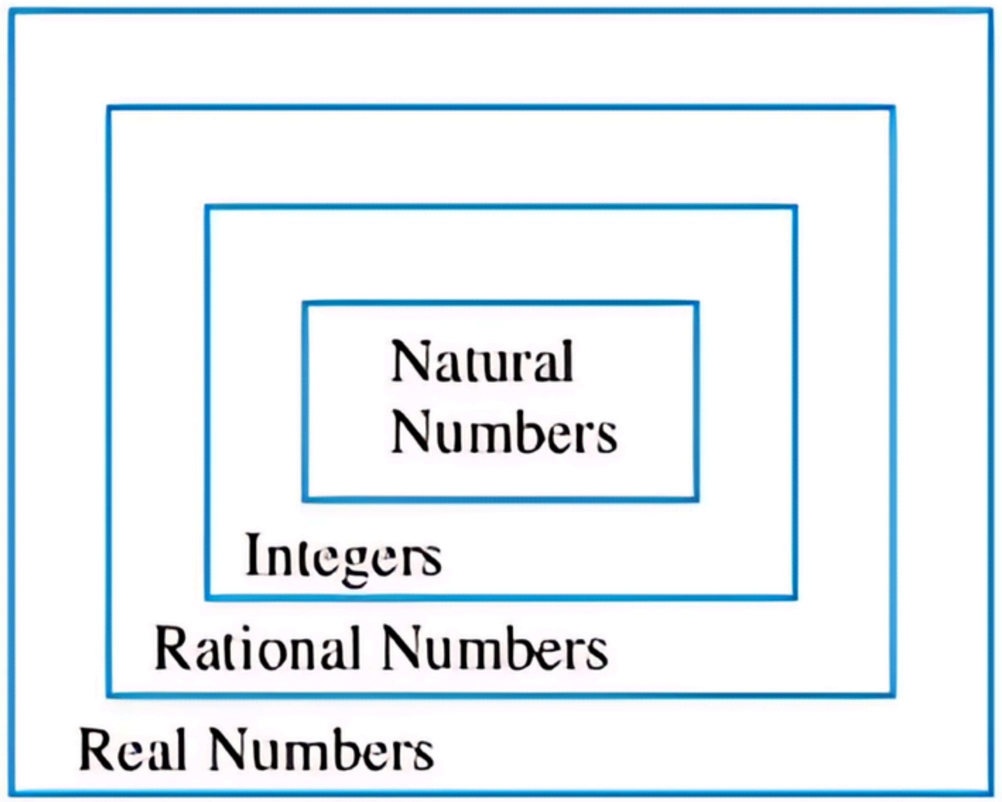
$\frac{3}{4}, \frac{-7}{8}, \frac{21}{5}, \frac{19}{-2}, \frac{16}{2}$, and $\frac{-17}{1}$ form m/n, with $n \neq 0$

Irrational numbers (no-fractional/ random decimals)

$\sqrt{3}, \sqrt{5}, \sqrt[3]{7}, \pi,$

The Real Numbers Consider all numbers (rational and irrational) that can measure lengths, together with their negatives and zero. We call these numbers the **real numbers.**





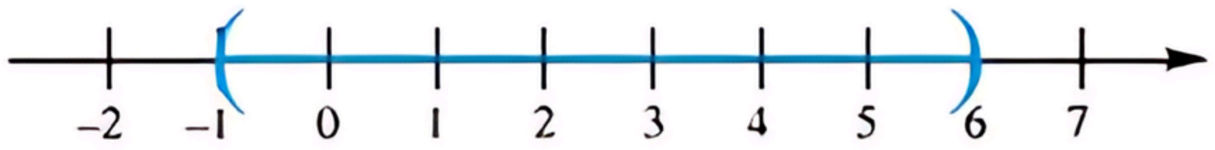
Preliminaries

Inequalities

inequality is to find the set of all real numbers that make the inequality true.

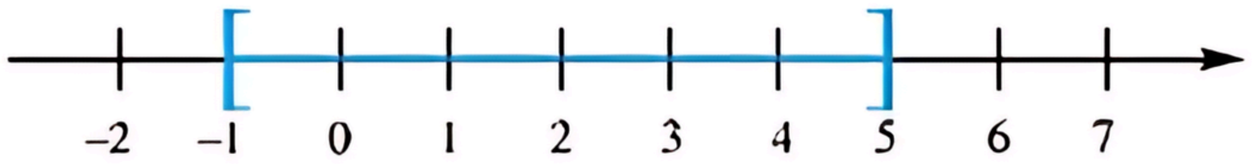
the solution set of an inequality is usually an entire interval of numbers or the union of such intervals.

The inequality $a < x < b$



open interval

the inequality $a \leq x \leq b$



closed interval

Preliminaries

Inequalities

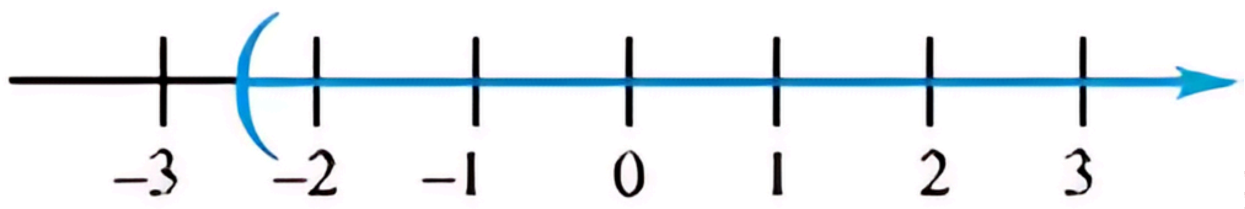
Set Notation	Interval Notation	Graph
$\{x : a < x < b\}$	(a, b)	
$\{x : a \le x \le b\}$	$[a, b]$	
$\{x : a \le x < b\}$	$[a, b)$	
$\{x : a < x \le b\}$	$(a, b]$	
$\{x : x \le b\}$	$(-\infty\ , b]$	
$\{x : x < b\}$	$(-\infty\ , b)$	
$\{x : x \ge a\}$	$[a,\ \infty)$	
$\{x : x > a\}$	$(a,\ \infty)$	
$\mathbb{R}$	$(-\infty,\ \infty)$	

EXAMPLE 1 Solve the inequality $2x - 7 < 4x - 2$ show the graph

SOLUTION

$$2x - 7 < 4x - 2$$
$$2x < 4x + 5$$
$$-2x < 5$$
$$x > -\tfrac{5}{2}$$

Graph



$$\left(-\frac{5}{2}, \infty\right) = \left\{x : x > -\frac{5}{2}\right\}$$

Interval Notation   Set Notation

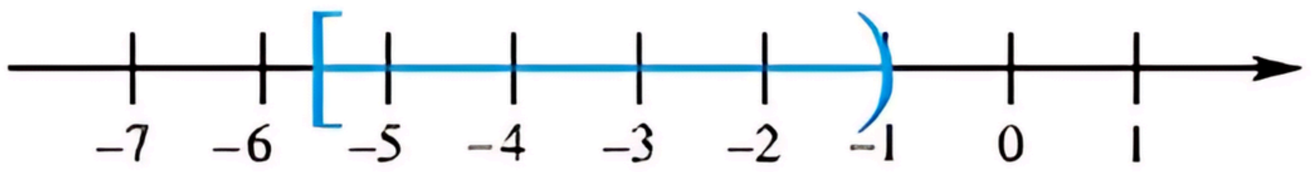
Preliminaries

Inequalities

EXAMPLE 2 Solve $-5 \le 2x + 6 < 4$.
show the graph

SOLUTION

$$-5 \le 2x + 6 < 4$$
$$-11 \le 2x < -2$$
$$-\frac{11}{2} \le x < -1$$

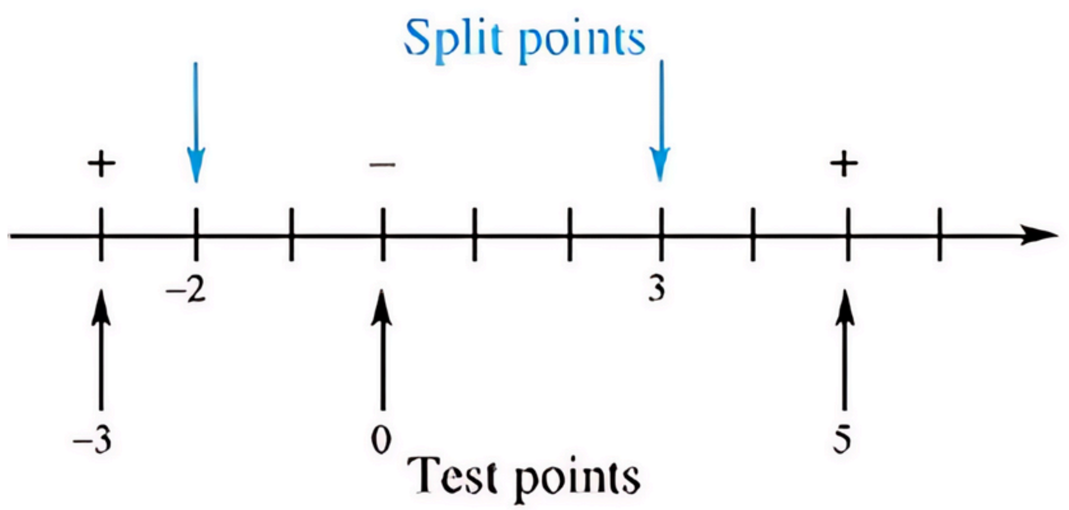


$$\left[-\frac{11}{2}, -1\right) = \left\{ x : -\frac{11}{2} \le x < -1 \right\}$$

EXAMPLE 3 Solve the quadratic inequality $x^2 - x < 6$.
show the graph

SOLUTION

$$x^2 - x < 6$$
$$x^2 - x - 6 < 0$$
$$(x-3)(x+2) < 0$$



(–2, 3)

Preliminaries

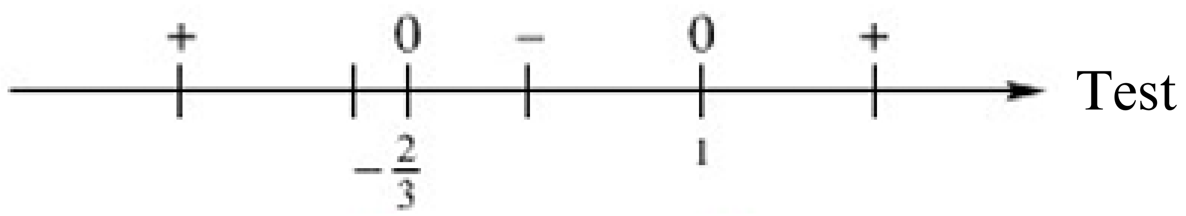
Inequalities

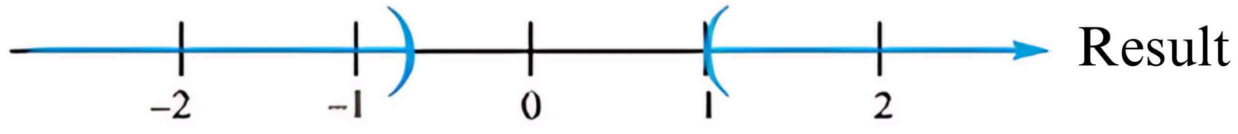
EXAMPLE 4 Solve $3x^2 - x - 2 > 0$.

show the graph

SOLUTION

$$3x^2 - x - 2 > 0.$$
$$(3x + 2)(x - 1) > 0.$$
$$3(x - 1)\left(x + \tfrac{2}{3}\right) > 0.$$

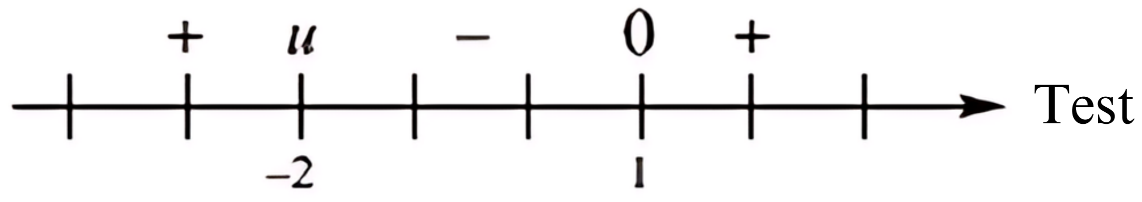


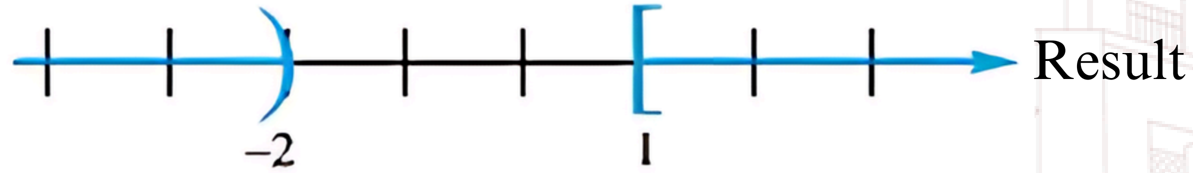


$$\left(-\infty, -\tfrac{2}{3}\right) \cup (1, \infty)$$

EXAMPLE 5 Solve $\dfrac{x - 1}{x + 2} \geq 0$. show the graph

SOLUTION





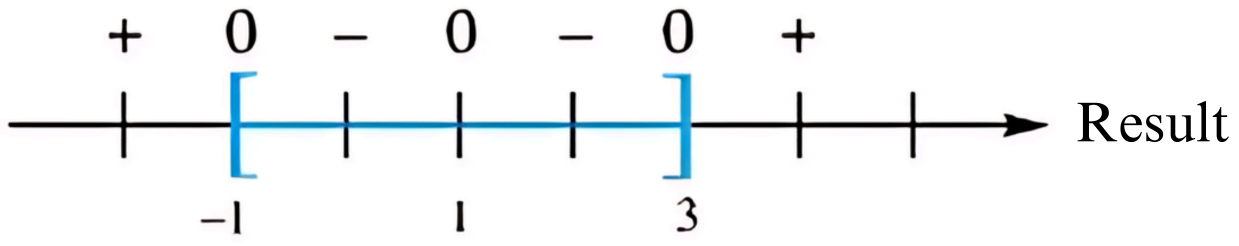
$$(-\infty, -2) \cup [1, \infty)$$

Preliminaries

Inequalities

EXAMPLE 6 Solve $(x+1)(x-1)^2(x-3) \le 0$ show the graph

SOLUTION



$[-1, 3]$

EXAMPLE 7 Solve $2.9 < \frac{1}{x} < 3.1$. show the graph

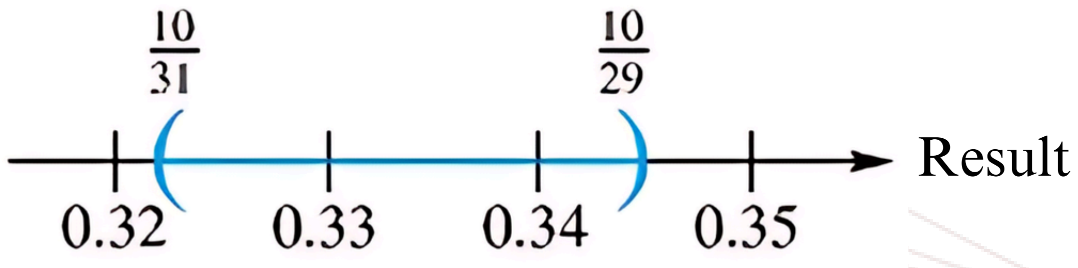
SOLUTION

$$2.9x < 1 < 3.1x$$

$$x < \frac{1}{2.9} \quad \text{and} \quad \frac{1}{3.1} < x$$

$$\frac{1}{3.1} < x < \frac{1}{2.9}$$

$$\frac{10}{31} < x < \frac{10}{29}$$



$\left(\frac{10}{31}, \frac{10}{29}\right)$

Preliminaries

Inequalities (Time to participation)

express the solution set of the given inequality in interval notation and sketch its graph.

1. $$\frac{1}{3x-2} \leq 4$$

2. $$\frac{3}{x+5} > 2$$

3. $$4x^2 - 5x - 6 < 0$$

4. $$2x^2 + 5x - 3 > 0$$

5. $$5x - 3 > 6x - 4$$

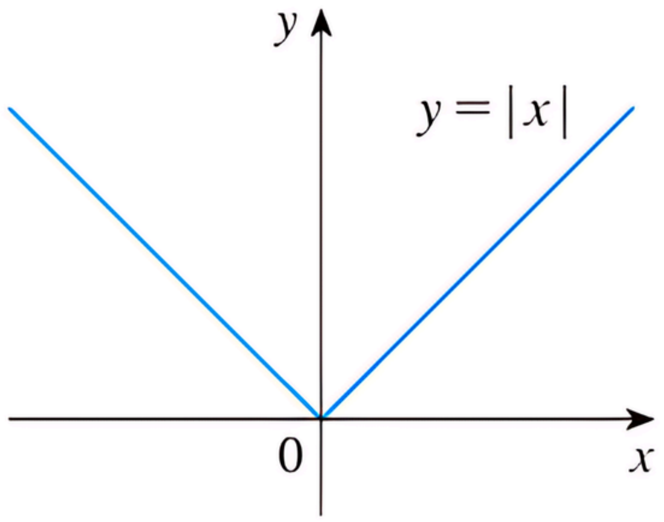
Preliminaries

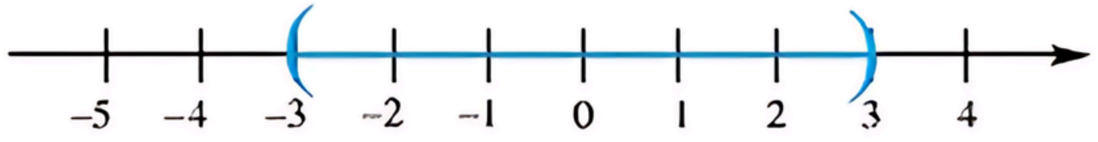
Absolute Values

The **absolute value** of a real number x, denoted by $|x|$, is defined by

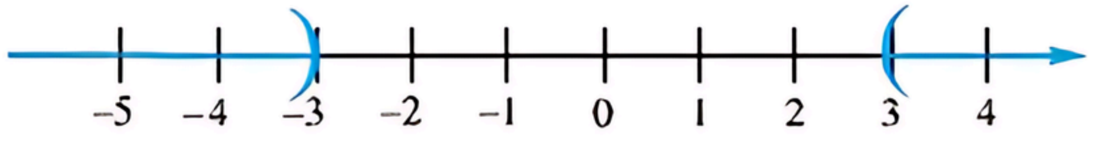
$$|x| = x \quad \text{if } x \geq 0$$
$$|x| = -x \quad \text{if } x < 0$$

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$$





$|x| < 3$



$|x| > 3$

Properties of Absolute Values

1. $|ab| = |a||b|$
2. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$
3. $|a + b| \leq |a| + |b|$
4. $|a - b| \geq ||a| - |b||$

$$|x| < |y| \Leftrightarrow x^2 < y^2$$

$$|x|^2 = x^2 \quad \text{and} \quad |x| = \sqrt{x^2}$$

$$|x| < a \Leftrightarrow -a < x < a$$
$$|x| > a \Leftrightarrow x < -a \quad \text{or} \quad x > a$$

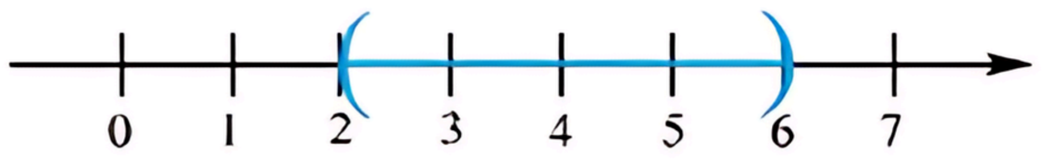
Preliminaries

Absolute Values

EXAMPLE 8 Solve the inequality $|x - 4| < 2$

SOLUTION

$$|x - 4| < 2 \Leftrightarrow -2 < x - 4 < 2$$

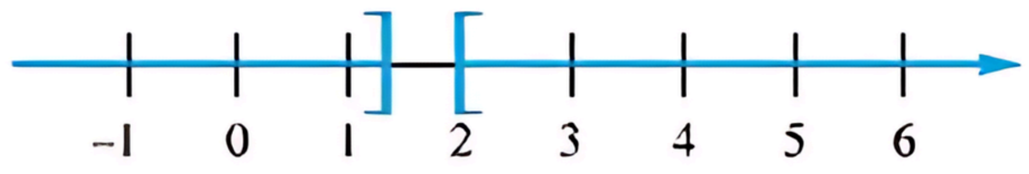


$|x-4| < 2$

EXAMPLE 9 Solve the inequality $|3x - 5| \geq 1$

SOLUTION

$$\begin{array}{rcl} 3x - 5 \leq -1 & \text{or} & 3x - 5 \geq 1 \\ 3x \leq 4 & \text{or} & 3x \geq 6 \\ x \leq \frac{4}{3} & \text{or} & x \geq 2 \end{array}$$



$\left(-\infty, \frac{4}{3}\right] \cup [2, \infty)$

EXAMPLE 10 Solve $x^2 - 2x - 4 \leq 0$.

the quadratic equation $ax^2 + bx + c = 0$ are given by

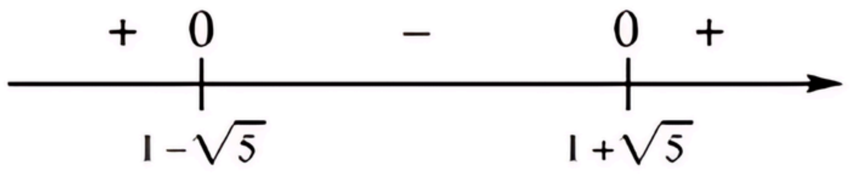
$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

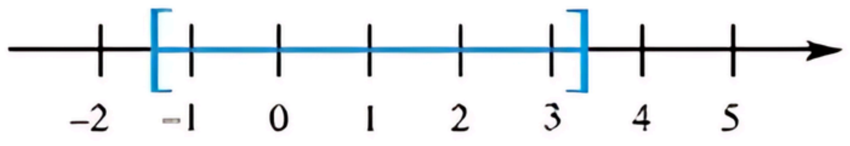
SOLUTION

The two solutions of $x^2 - 2x - 4 = 0$ are

$$x_1 = \frac{-(-2) - \sqrt{4 + 16}}{2} = 1 - \sqrt{5} \approx -1.24$$

$$x_2 = \frac{-(-2) + \sqrt{4 + 16}}{2} = 1 + \sqrt{5} \approx 3.24$$





EXAMPLE 11 Solve the inequality $|3x + 1| < 2|x - 6|$.

SOLUTION

$$\boxed{|x| < |y| \Leftrightarrow x^2 < y^2}$$

$$\begin{aligned}
|3x + 1| < 2|x - 6| &\Leftrightarrow |3x + 1| < |2x - 12| \\
&\Leftrightarrow (3x + 1)^2 < (2x - 12)^2 \\
&\Leftrightarrow 9x^2 + 6x + 1 < 4x^2 - 48x + 144 \\
&\Leftrightarrow 5x^2 + 54x - 143 < 0 \\
&\Leftrightarrow (x + 13)(5x - 11) < 0
\end{aligned}$$

The split points for this quadratic inequality are -13 and $\frac{11}{5}$; they divide the real line into the three intervals: $(-\infty, -13)$, $\left(-13, \frac{11}{5}\right)$, and $\left(\frac{11}{5}, \infty\right)$. When we use the test points -14, 0, and 3, we discover that only the points in $\left(-13, \frac{11}{5}\right)$ satisfy the inequality. ■

Preliminaries

Absolute Values (Time to participation)

express the solution set of the given inequality in interval notation and sketch its graph.

1. $\left|\dfrac{2x}{7} - 5\right| \geq 7$

2. $\left|\dfrac{x}{4} + 1\right| < 1$

3. $3x^2 + 17x - 6 > 0$

4. $|2x - 1| \geq |x + 1|$

5. $2|2x - 3| < |x + 10|$

SEE YOU NEXT WEEK !