

derivation for the potential of a point charge above thin layer of uniform dielectric above an anisotropic substrate bulk material

$$\begin{aligned} & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \end{aligned}$$

using the furrier transform

$$\begin{aligned} & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \end{aligned}$$

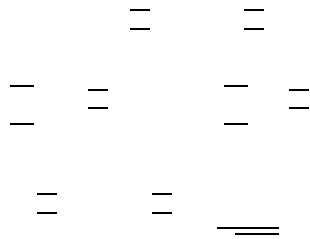
apply boundary conditions as and as

$$\begin{aligned} & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \end{aligned}$$

applying the conditions ,

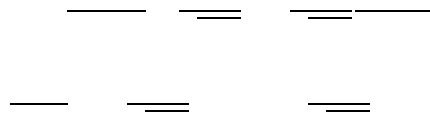
and , , ,

$$\begin{aligned} & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \\ & \frac{1}{4\pi\epsilon_0} \frac{q}{r} \end{aligned}$$

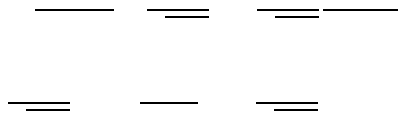


let

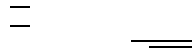
1+2



1-2



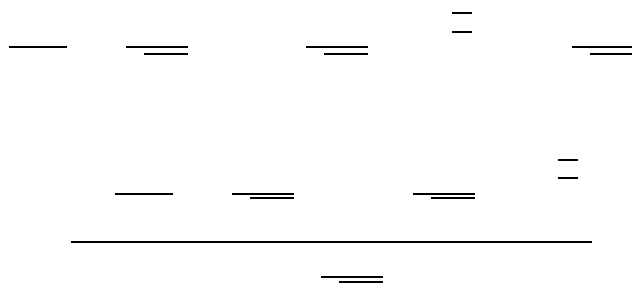
3+4



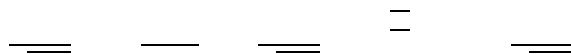
3-4



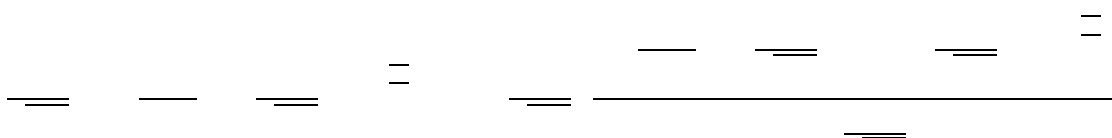
sub 5 in to 7



sub 6 in to 8



sub 9 in to 10



sub 11 in to

The diagram consists of 10 horizontal bars of varying lengths and positions, arranged in a staggered fashion. The bars are black and the background is white. The bars are arranged in a sequence that suggests a signal or data stream. The bars are positioned at different vertical levels, with some appearing above and some below a central horizontal axis. The lengths of the bars vary, with some being short and others being longer. The bars are arranged in a way that suggests a progression or a sequence of events.

Figure 1: Schematic representation of the experimental design. The figure is divided into two main sections: 'Pretest' and 'Main Experiment'. The 'Pretest' section shows a timeline for 'Pretest 1' (10 days) and 'Pretest 2' (10 days), with a 'Control' group and an 'Experimental' group. The 'Main Experiment' section shows a timeline for 'Main Experiment 1' (10 days) and 'Main Experiment 2' (10 days), with a 'Control' group and an 'Experimental' group. The 'Experimental' group is further divided into 'Experimental 1' and 'Experimental 2'.

test cases

let

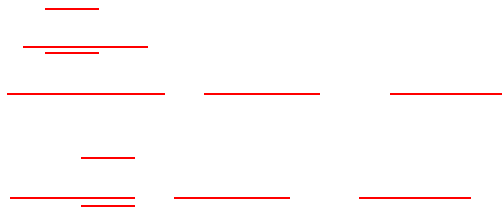
The diagram illustrates a 3D coordinate system with three primary axes: a vertical axis labeled 'Z', a horizontal axis labeled 'X', and a diagonal axis labeled 'Y'. The origin is marked with '0'. Several planes and points are labeled with letters and numbers:

- Planes:**
 - Plane XY is the horizontal base plane.
 - Plane XZ is the vertical plane.
 - Plane YZ is the diagonal plane.
 - Other planes are labeled AB , CD , EF , GH , IK , LM , NO , PQ , RST , UV , WX , YZ , AB , CD , EF , GH , IK , LM , NO , PQ , RST , UV , WX , YZ .
- Points:**
 - Points $A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z$ are marked at various intersections and endpoints.
 - Points $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100$ are marked along the axes and planes.
- Other Labels:**
 - Labels $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100$ are placed near the axes and planes.

returns expected results

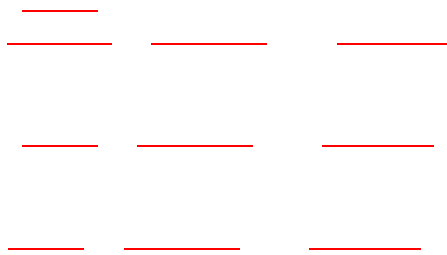
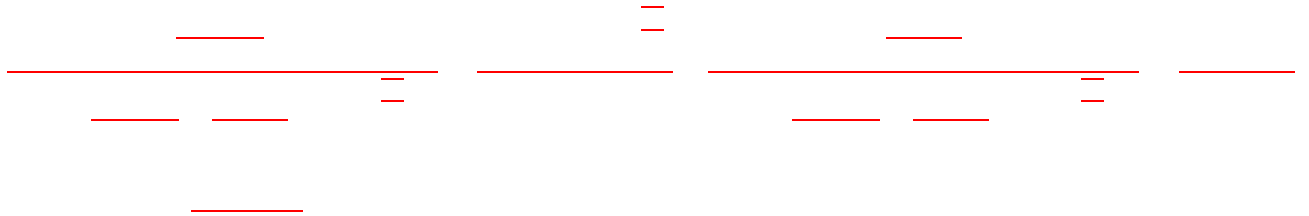
let

The diagram consists of several horizontal line segments of different lengths and positions, arranged in a way that suggests a mathematical sequence or series. The segments are arranged in a way that suggests a pattern or progression, with some segments appearing to be part of a larger structure that is being built up or broken down.



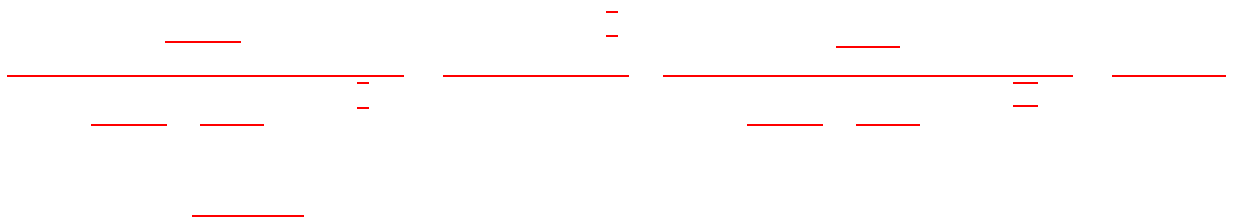
returns expected results

let



returns expected results

let



giving

The diagram consists of two horizontal timelines. The top timeline is represented by a single horizontal line with several short vertical tick marks. The bottom timeline is also represented by a single horizontal line with several short vertical tick marks. Some of the tick marks on the bottom timeline are aligned with the tick marks on the top timeline, while others are not.

as

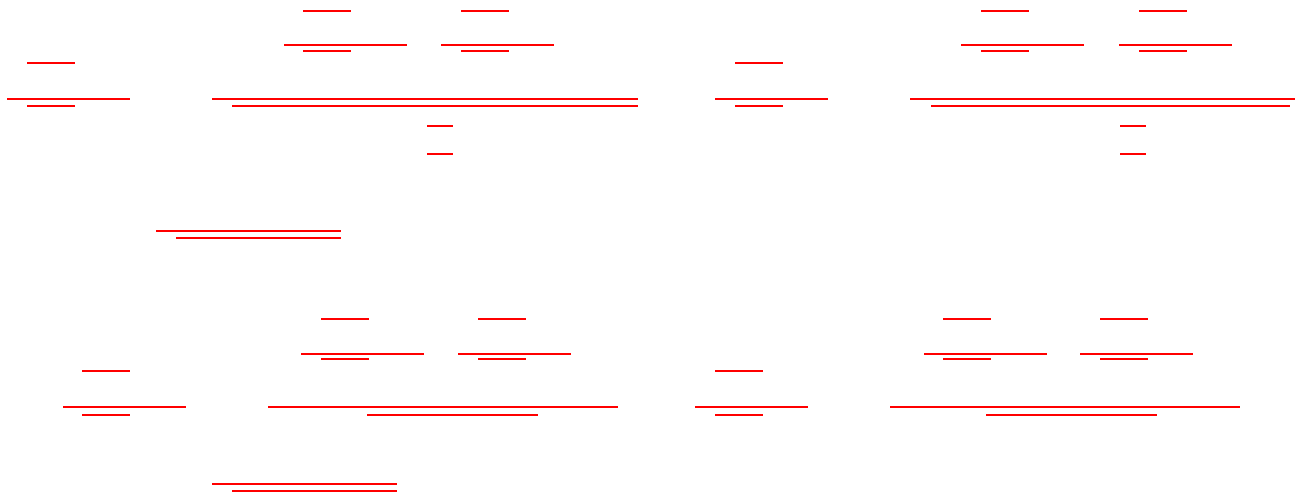
then

A diagram of a 2D hexagonal lattice. A central hexagon is highlighted in red and labeled '1'. It is surrounded by six hexagons, each labeled '2'. These six hexagons are further surrounded by a ring of twelve hexagons, each labeled '3'. The lattice extends to the edges of the frame.

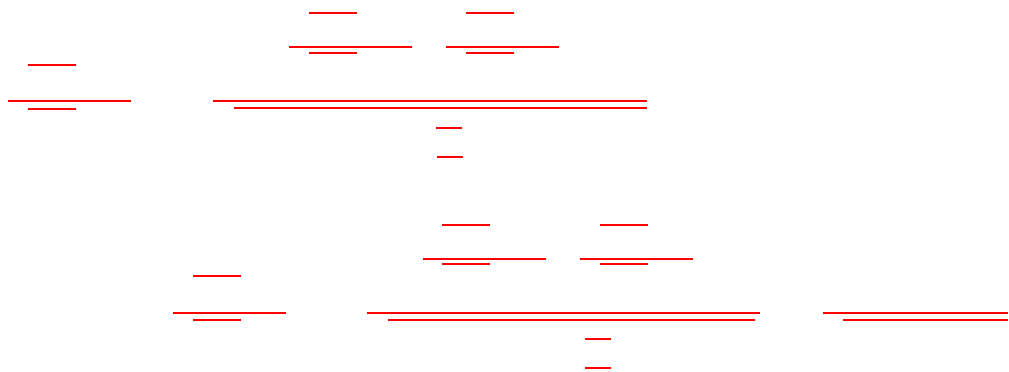
where

trivial cases:

$h=0$



let

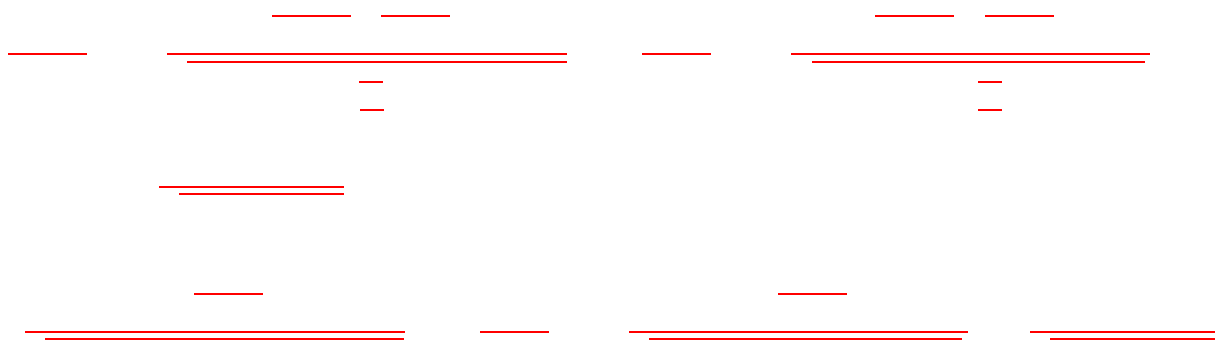


when using the case of $k=0$



returns expected results

let



take case where $k=0$ as otherwise $=0$

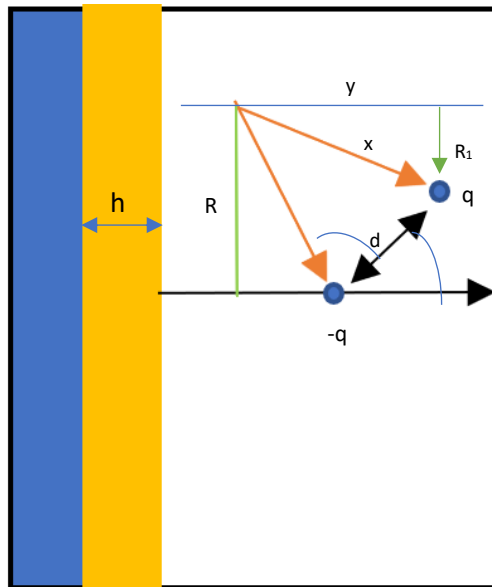
returns expected results

let

A diagram showing a 4x4 grid of squares. The top row has 4 squares, the second row has 3 squares, the third row has 2 squares, and the bottom row has 1 square. The squares are arranged in a staircase pattern from top-left to bottom-right.

returns expected results

Dipole approximation using the principle of superposition

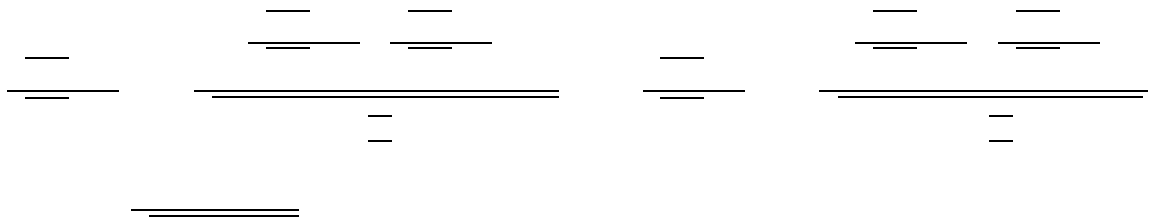


where

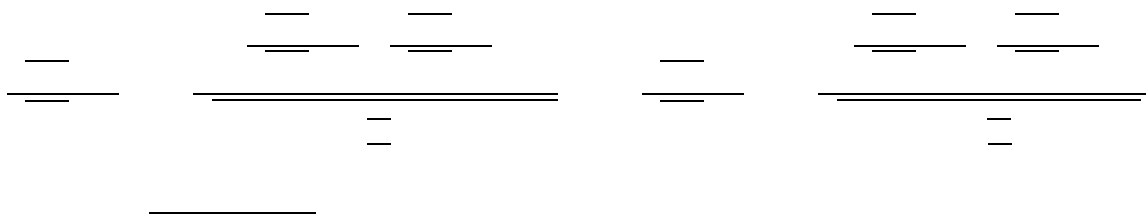
as then

let _____

first particle



second particle



using the principle of superposition

$$\frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \frac{1}{96} + \frac{1}{192} + \frac{1}{384} + \frac{1}{768} + \frac{1}{1536} + \frac{1}{3072} + \frac{1}{6144} + \frac{1}{12288} + \frac{1}{24576} + \frac{1}{49152} + \frac{1}{98304} + \frac{1}{196608} + \frac{1}{393216} + \frac{1}{786432} + \frac{1}{1572864} + \frac{1}{3145728} + \frac{1}{6291456} + \frac{1}{12582912} + \frac{1}{25165824} + \frac{1}{50331648} + \frac{1}{100663296} + \frac{1}{201326592} + \frac{1}{402653184} + \frac{1}{805306368} + \frac{1}{1610612736} + \frac{1}{3221225472} + \frac{1}{6442450944} + \frac{1}{12884901888} + \frac{1}{25769803776} + \frac{1}{51539607552} + \frac{1}{103079215104} + \frac{1}{206158430208} + \frac{1}{412316860416} + \frac{1}{824633720832} + \frac{1}{1649267441664} + \frac{1}{3298534883328} + \frac{1}{6597069766656} + \frac{1}{13194139533312} + \frac{1}{26388279066624} + \frac{1}{52776558133248} + 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+ \frac{1}{2410407066388485416061722207951743903783304490674189252952064} + \frac{1}{4820814132776970832123444415903487807566608981348378505904128} + \frac{1}{9641628265553941664246888831806975615133217962696757011808256} + \frac{1}{19283256531107883328493777663613951230266435925393514023616512} + \frac{1}{38566513062215766656987555327227902460532871850787028047233024} + \frac{1}{77133026124431533313975110654455804921065743701574056094466048} + \frac{1}{154266052248863066627950221308911609842131487403148112188932096} + \frac{1}{308532104497726133255900442617823219684262974806296224377864192} + \frac{1}{617064208995452266511800885235646439368525949612592448755728384} + \frac{1}{1234128417990904533023601770471292878737051899225184897511456768} + \frac{1}{2468256835981809066047203540942585757474103798450369795022913536} + \frac{1}{4936513671963618132094407081885171514948207596900739590045827072} + \frac{1}{9873027343927236264188814163770343029896415193801479180091654144} + \frac{1}{19746054687854472528377628327540686059792830387602958360183308288} + \frac{1}{39492109375708945056755256655081372119585660775205916720366616576} + \frac{1}{78984218751417890113510513310162744239171321550411833440733233152} + \frac{1}{157968437502835780227021026620325488478342643100823666881466466304} + \frac{1}{315936875005671560454042053240650976956685286201647333762932932608} + \frac{1}{631873750011343120908084106481301953913370572403294667525865865216} + \frac{1}{1263747500022686241816168212962603907826741144806589335051731730432} + \frac{1}{2527495000045372483632336425925207815653482289613178670103463460864} + \frac{1}{5054990000090744967264672851850415631306964579226357340206926921728} + \frac{1}{10109980000181489934529345703700831262613929158452714680413853843456} + \frac{1}{20219960000362979869058691407401662525227858316905429360827707686912} + \frac{1}{40439920000725959738117382814803325050455716633810858721655415373824} + \frac{1}{80879840001451919476234765629606650100911433267621717443310830747648} + \frac{1}{161759680002903838952469531259213300201822866535243434886621661495296} + \frac{1}{323519360005807677904939062518426600403645733070486869773243322990592} + \frac{1}{647038720011615355809878125036853200807291466140973739546486645981184} + \frac{1}{1294077440023230711619756250073706401614582932281947479092973291962368} + \frac{1}{2588154880046461423239512500147412803229165864563894958185946583924736} + \frac{1}{5176309760092922846479025000294825606458331729127789916371893167849472} + \frac{1}{10352619520185845692958050000589651212916663458255579832743786335698944} + \frac{1}{20705239040371691385916100001179302425833326916511159665487572671397888} + \frac{1}{41410478080743382771832200002358604851666653833022319330975145342795776} + \frac{1}{82820956161486765543664400004717209703333307666044638661950290685591552} + \frac{1}{165641912322973531087328800009434419406666615332089277323900581371183104} + \frac{1}{331283824645947062174657600018868838813333230664178554647801162742366208} + \frac{1}{662567649291894124349315200037737677626666461328357109295602325484732416} + \frac{1}{1325135298583788248698630400075475355253332922656714218591204650969464832} + \frac{1}{2650270597167576497397260800150950710506665845313428437182409301938929664} + \frac{1}{5300541194335152994794521600301901421013331690626856874364818603877859328} + \frac{1}{10601082388670305989589043200603802842026663381253713748729637207755718656} + \frac{1}{21202164777340611979178086401207605684053326762507427497459274415511437312} + \frac{1}{42404329554681223958356172802415211368106653525014854994918548831022874624} + \frac{1}{84808659109362447916712345604830422736213307050029709989837097662045749248} + \frac{1}{169617318218724895833424691209660845472426614100059419979674195324091498496} + \frac{1}{339234636437449791666849382419321690944853228200118839959348390648182996992} + \frac{1}{678469272874899583333698764838643381889706456400237679918696781296365993984} + \frac{1}{1356938545749799166667397529677286763779412912800475359837393562592731987968} + \frac{1}{2713877091499598333334795059354573527558825825600950719674787125185463975936} + \frac{1}{5427754182999196666669590118709147055117651651201901439349574250370927951872} + \frac{1}{10855508365998393333339180237418294110235303302403802878699148500741855903744} + \frac{1}{21711016731996786666678360474836588220470606604807605757398297001483711807488} + \frac{1}{43422033463993573333356720949673176440941213209615211514796594002967423614976} + \frac{1}{86844066927987146666713441899346352881882426419230423029593188005934847229952} + \frac{1}{173688133855974293333426883798692705763764852838460846059186376011869694459904} + \frac{1}{347376267711948586666853767597385411527529705676921692118372752023739388919808} + \frac{1}{694752535423897173333707535194770823055059411353843384236745504047478777839616} + \frac{1}{1389505070847794346667415070389541646110118822707686768473491008094957555679232} + \frac{1}{2779010141695588693334830140779083292220237645415373536946982016189915111358464} + \frac{1}{5558020283391177386669660281558166584440475290830747073893964032379830222716928} + \frac{1}{1111604056678235$$

as

test cases

let

Figure 1 illustrates the experimental setup. A participant is seated at a table, interacting with a computer monitor. The monitor displays a 'Stimulus' area with a 'Start' button and a 'Response' area with a 'Stop' button. A 'Response' box is also shown on the table. A 'Response' box is also shown on the table. A 'Response' box is also shown on the table.

Country	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	2020
Japan	12	14	16	18	20	22	24	26	27	28	28	28	28	28	28
Germany	10	11	12	13	14	15	16	17	18	19	20	21	22	23	25
Italy	8	9	10	11	12	13	14	15	16	17	18	19	20	21	24
France	7	8	9	10	11	12	13	14	15	16	17	18	19	20	23
Spain	6	7	8	9	10	11	12	13	14	15	16	17	18	19	22
Sweden	5	6	7	8	9	10	11	12	13	14	15	16	17	18	21
Canada	4	5	6	7	8	9	10	11	12	13	14	15	16	17	19
Australia	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18
United States	2	3	4	5	6	7	8	9	10	11	12	13	14	15	17
Mexico	1	2	3	4	5	6	7	8	9	10	11	12	13	14	12

taking the case where $k=0$

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third term

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therefore

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$$\frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

perpendicular and parallel components

and
