derivation for the potential of a point charge above thin layer of uniform dielectric above an anisotropic substrate bulk material

using the furrier transform

apply boundary conditions as and as

applying the conditions ,

and ,

(2)

(1)+(5)

(1)-(5)

(4)

(8)+(3)

(3)-(8)

sub 9 in to 6

sub 10 in to 7

48

49 sub 11 in to 12

50

51

52

53

54

55 let h=0

56

57

which is the result which is expected when the layer is no longer present

let

result as expected when the layer depth goes to infinity

let

71

72

73 only issue here is that h is 2 times what is expected

74 let

75

76

77

78

79 all trivial cases return the expected results

80

81 sub 14 in to

82

83

84

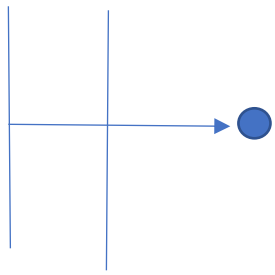
85

let

test cases
let
returns expected result
let
returns expected result
let

returnes expected result however (z+ would be normal expected result

let



returns expected result

118

119 find A

120

121

122

123

124

F

therefore integral is

contour integration and the residue theorem

letting ,

therefore

//

returns the expected results

let

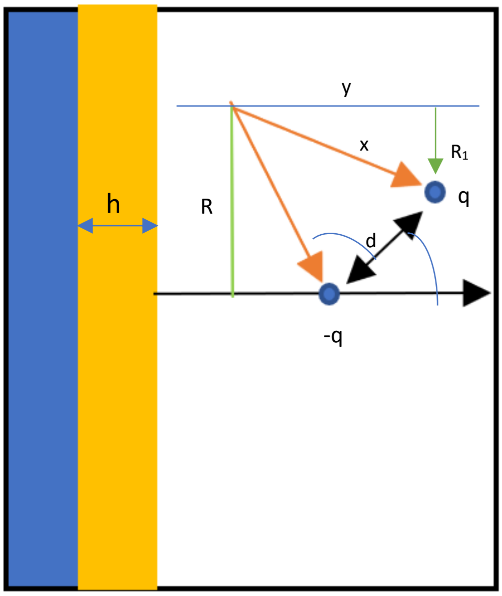
result as expected

let

first term only works for $n=0$ so

as expected for

Dipole approximation using the principle of superposition



where

–

–

as then

let

first particle

second particle

using taylor expansion where d→0, and and grater terms are ignored

first term

second term

third term

therefore

let

calculating the electric fields due to the dipole

where — and —

first term

second term

final term

first term

second term

final term

determine which contribution is due to which images we let and

first term is due to the interaction with the bottom image second term is interaction with image in the layer and the final term is due to its self

where –

first term

let

let and as

interaction with its self is given by

second term

third term

let and as this is the case where the NP are interacting with them selves

where

let and as

first term

second term

third term