1 2	derivation for the potential of a point charge above thin layer of uniform dielectric above an anisotropic substrate bulk material
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15	using the furrier transform
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20	apply boundary conditions as and as
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24	applying the conditions ,
25	and — , — — —
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47	sub 10 in to 7	

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61	which is the result which is expecte	ed when the layer	is no longer presen	t
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68	result as expected when the layer	depth goes to infi	nity	
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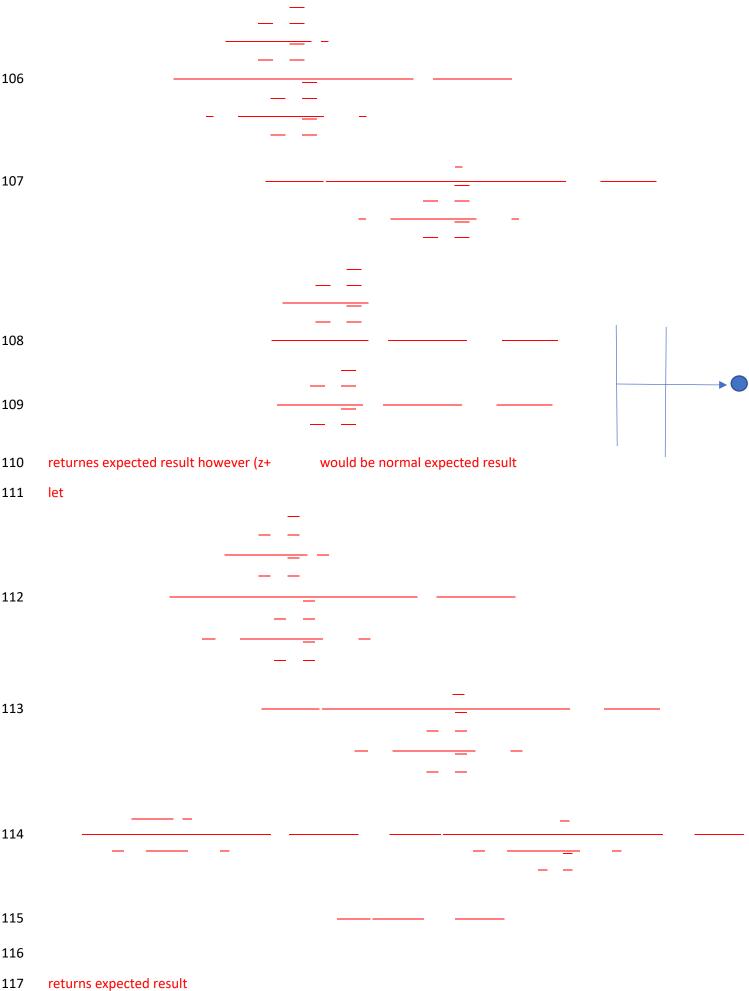
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73	only issue here is that h	is 2 times what is ex	pected	
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79	all trivial cases return th	e expected results		
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let

90	test cases
91	let
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99	returns expected result
100	let
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104	returns expected result
105	let



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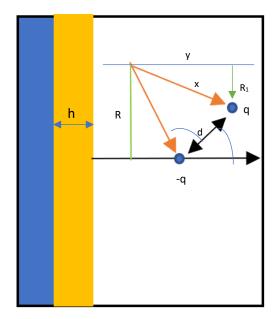
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204	first term only works for n=0 so
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## Dipole approximation using the principle of superposition



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