



THE OHIO STATE UNIVERSITY

ME7752 Final Project

Autumn 2021

*Inverse Kinematics and Trajectory Generation
of Block “O” with dVRK Robot*



Overview

- Introduction
- Forward and Inverse Kinematics Using PoE
- Trajectory Generation of the Block "O"
- Software & Hardware
- Results & Conclusions

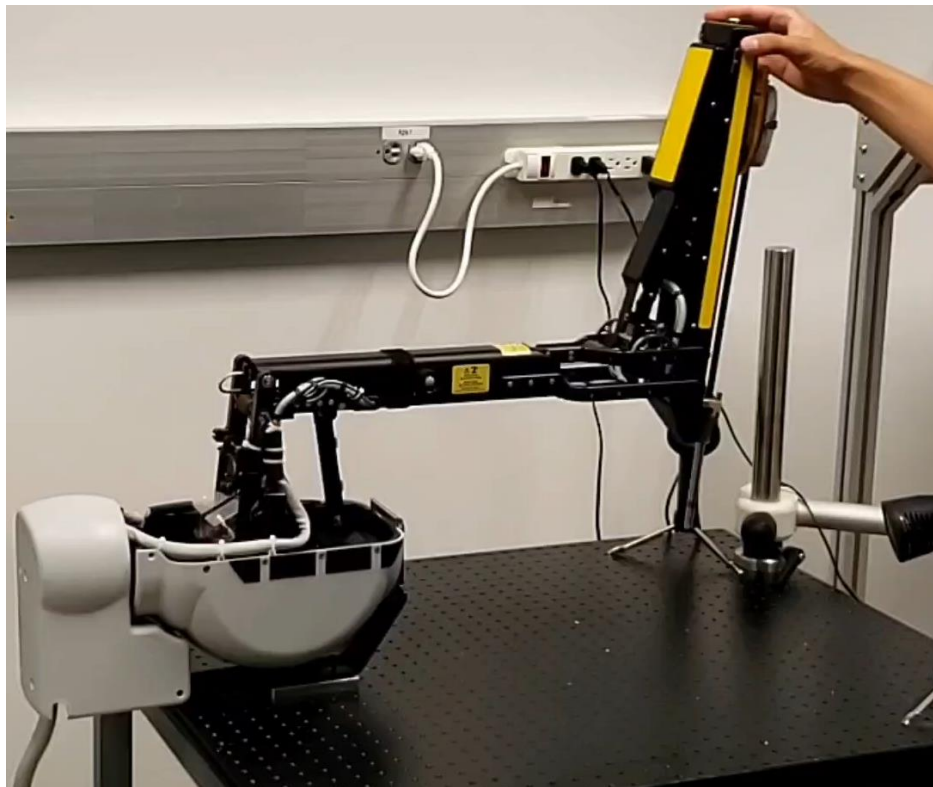


daVinci Research Kit (dVRK)

- Intuitive Surgical, Inc.
- **PSM** – “Patient Side Manipulator”
- **MTM** – “Master Tool Manipulator”

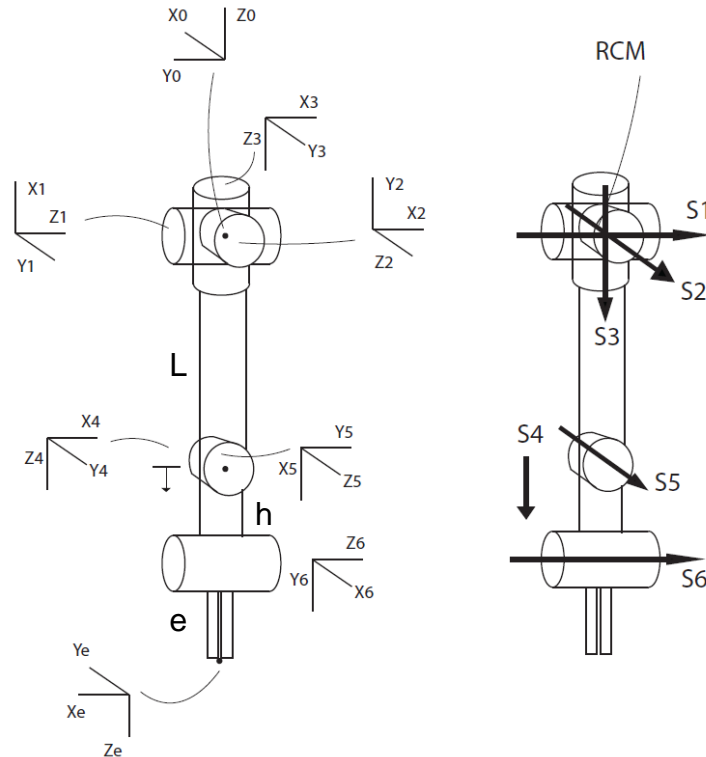








Forward Kinematics

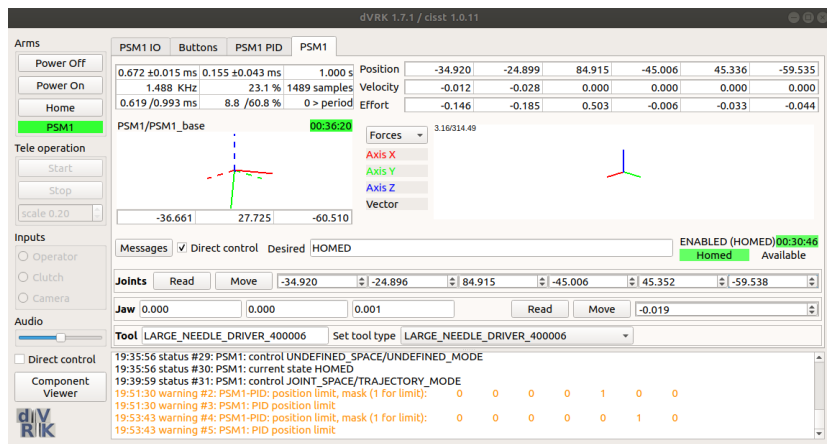


S_i	ω_{si}	v_{si}
1	(0 -1 0)	(0 0 0)
2	(-1 0 0)	(0 0 0)
3	(0 0 -1)	(0 0 0)
4	(0 0 0)	(0 0 -1)
5	(-1 0 0)	(0 L 0)
6	(0 -1 0)	$(-(h+L) 0 0)$

$$T_{0e} = e^{[S_1]\theta_1} e^{[S_2]\theta_2} \dots e^{[S_6]\theta_6} M$$

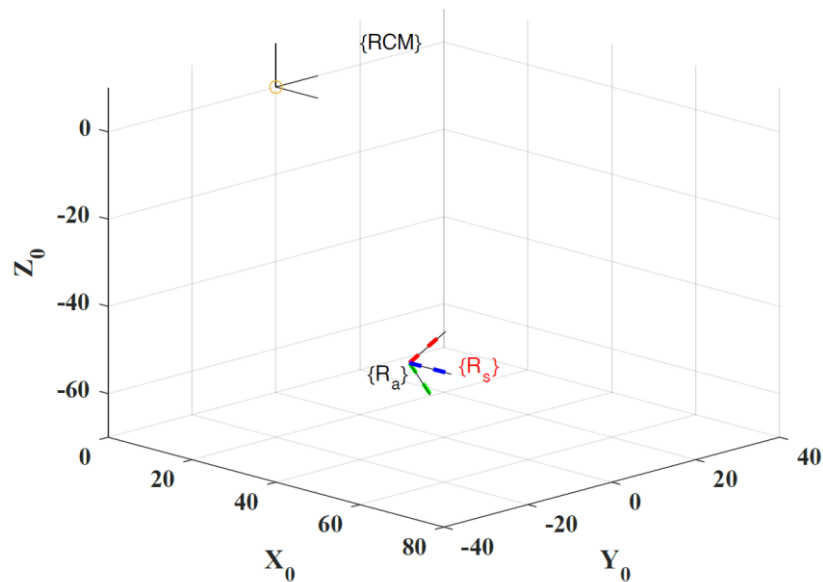
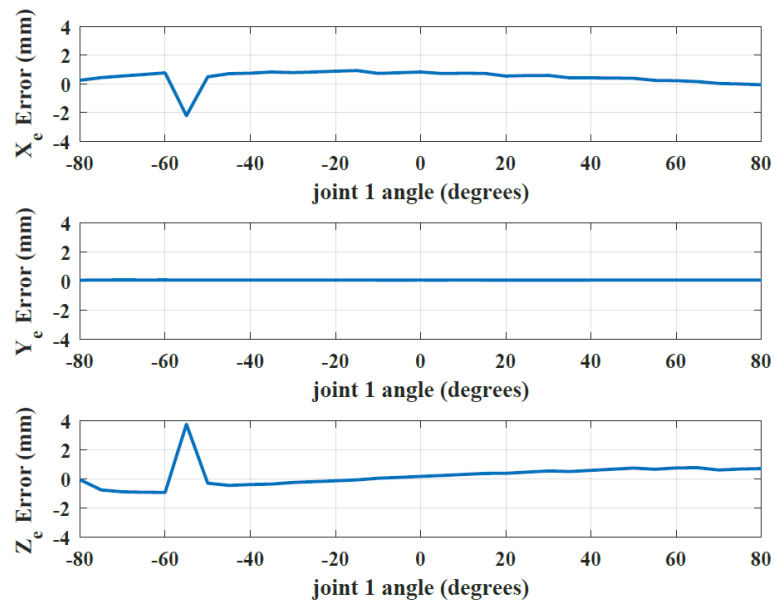


FK Validation Methods





FK Validation Results





Closed-Form Inverse Kinematics

DCM

$$\theta_1 = \text{atan2}(px, pz - e)$$

$$\theta_2 = \text{atan2}(-py, px * s1 + pz * c1 - c1 * e - h)$$

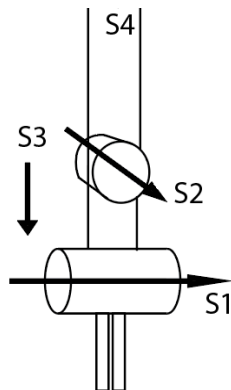
$$\theta_3 = -L - h * c2 - py * s2 + (pz - e) * c1 * c2 + px * c2 * s1$$

$$\theta_4 = \text{atan2}(-\text{Rot}(1, 2) * c1 + \text{Rot}(3, 2) * s1, \text{Rot}(2, 2) * c2 + \text{Rot}(3, 2) * c1 * s2 + \text{Rot}(1, 2) * s1 * s2)$$

$$\theta_5 = \text{atan2}(\text{Rot}(3, 1) * (s1 * s4 + c1 * c4 * s2) - \text{Rot}(1, 1) * (c1 * s4 - c4 * s1 * s2) + \text{Rot}(2, 1) * c2 * c4,$$

$$-\text{Rot}(3, 1) * c1 * c2 + \text{Rot}(2, 1) * s2 - \text{Rot}(1, 1) * c2 * s1)$$

$$\theta_6 = \text{atan2}(-\text{Rot}(3, 1) * c1 * c2 + \text{Rot}(2, 1) * s2 - \text{Rot}(1, 1) * s1 * c2, \text{Rot}(3, 3) * c1 * c2 - \text{Rot}(2, 3) * s2 + \text{Rot}(1, 3) * c2 * s1)$$



$$\theta_{IK} = [20, \ 22, \ 1.5, \ 15, \ -150, \ 25.71]$$

$$\theta_{actual} = [20, \ 22, \ 1.5, \ 15, \ 30, \ 25.71]$$



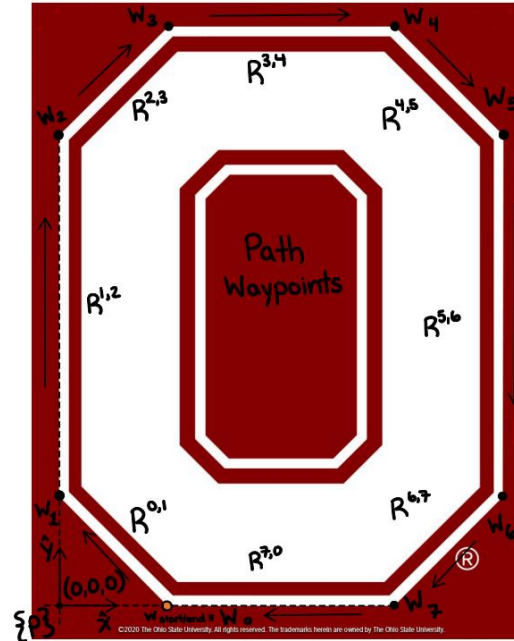
Block “O” Coordinates

- 8 positional waypoints

Block “O” Modulation

- c – scaling factor
- R_p – reorientation
- (x_f, y_f, z_f) – origin offset
- $\{W_m\}$ – set of new waypoints

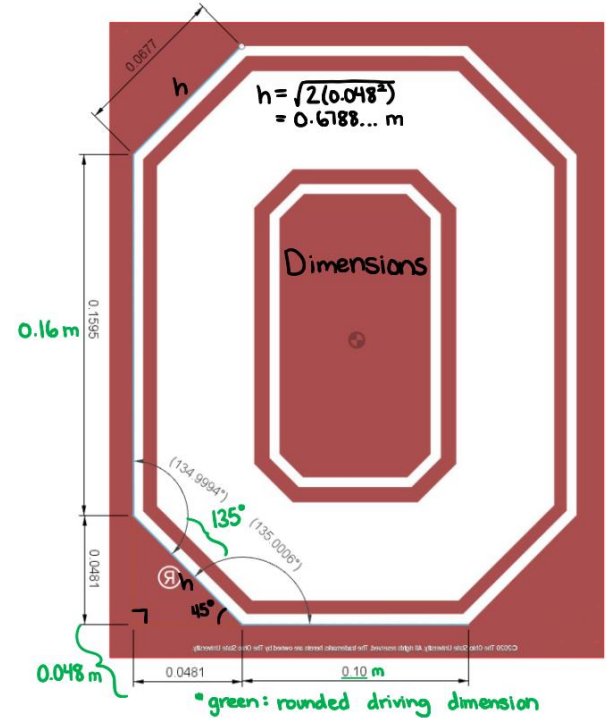
$$\{W_m\} = R_p[c * \{w_0, w_1, \dots, w_7\}] + (x_f, y_f, z_f)$$



W_i : Waypoint _{i} , $\{p3$: path plane orientation

$R^{i,j}$: orientation of EE on path from $W_i \rightarrow W_j$

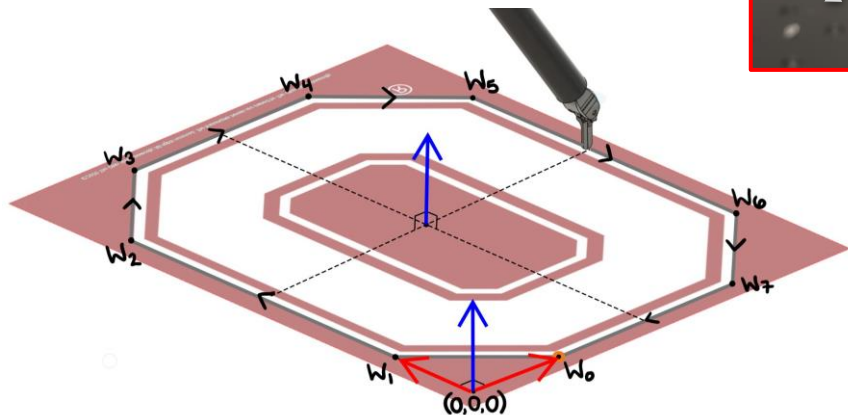
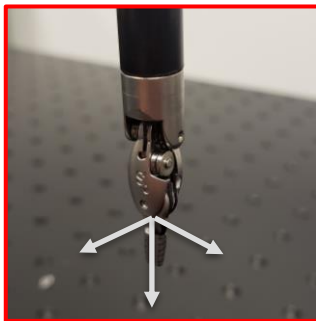
W_i	W_0	W_1	W_2	W_3	W_4	W_5	W_6	W_7
(x, y, z)	(0.048, 0, 0)	(0, 0.048, 0)	(0, 0.208, 0)	(0.048, 0.256, 0)	(0.148, 0.256, 0)	(0.196, 0.208, 0)	(0.196, 0.048, 0)	(0.148, 0, 0)
[m]								



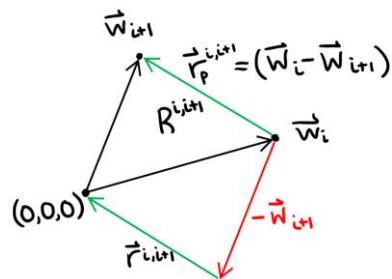
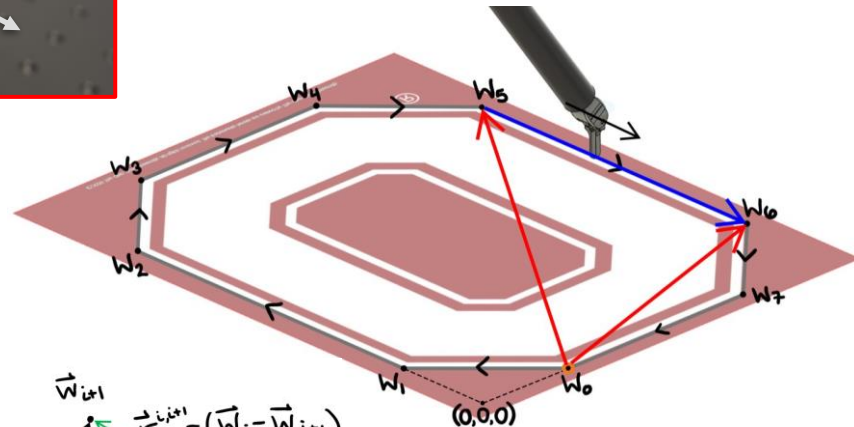


Waypoint Orientations

- Normal to plane (constant)
- Parallel to velocity



$$\hat{\underline{r}}_n = \frac{\vec{W}_0 \times \vec{W}_1}{\|\vec{W}_0 \times \vec{W}_1\|} = \frac{\vec{r}^n}{\|\vec{r}^n\|}, \text{ where } \underline{\vec{r}}^n = \vec{W}_0 \times \vec{W}_1$$



$$\underline{\hat{r}}_p^{i,i+1} = \frac{(\vec{W}_i - \vec{W}_{i+1})}{\|(\vec{W}_i - \vec{W}_{i+1})\|} = \frac{\vec{r}^{i,i+1}}{\|\vec{r}^{i,i+1}\|}$$



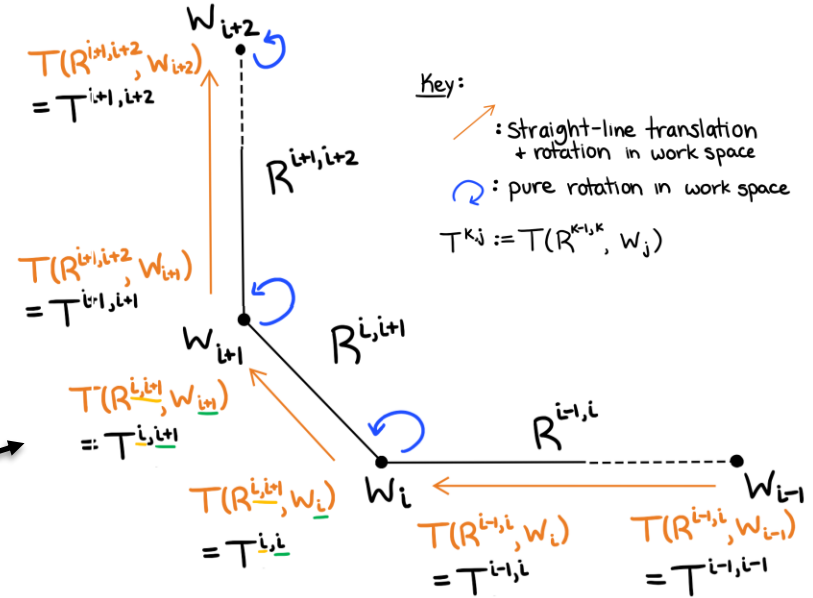
List of Waypoints

$$T^{K,j} := T(R^{K,K+1}, W_j)$$

• where path is ordered list of transformations

Block "O" Path := $[T^{0,0}, T^{0,1}, T^{1,1}, T^{1,2}, \dots, T^{7,7}, T^{7,8}]$, where

$$R^{7,8} = R^{7,0} \\ + W_8 = W_0$$



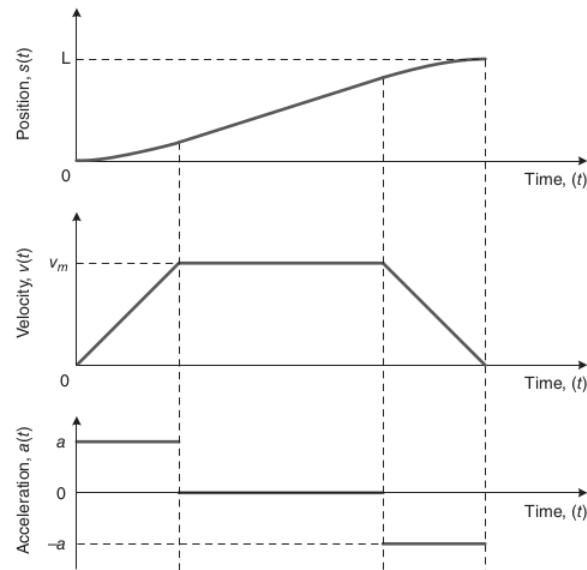
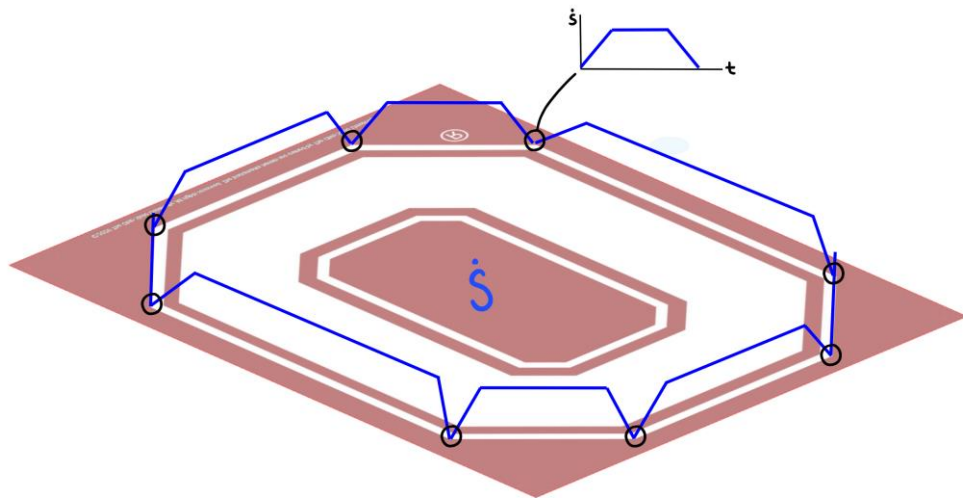
Path from W_0, W_1, \dots, W_8 :

$$\underbrace{T^{0,0}}_{\text{start}} \rightarrow T^{0,1} \curvearrowright T^{1,1} \rightarrow T^{1,2} \curvearrowright T^{2,2} \cdots \rightarrow T^{6,7} \curvearrowright T^{7,7} \rightarrow \underbrace{T^{7,8}}_{\text{end}} = T^{7,0}$$

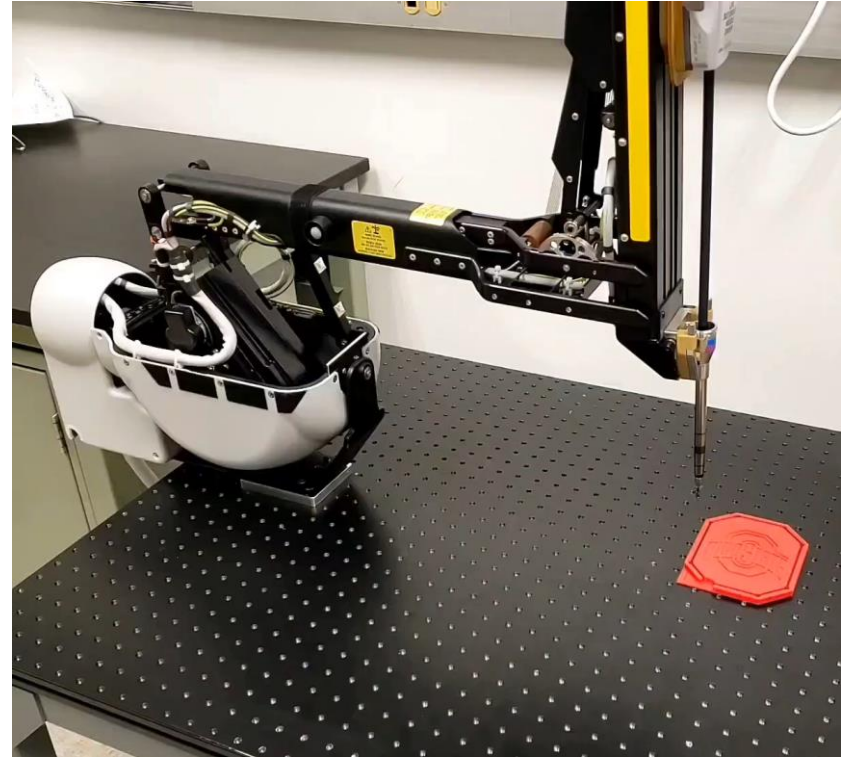
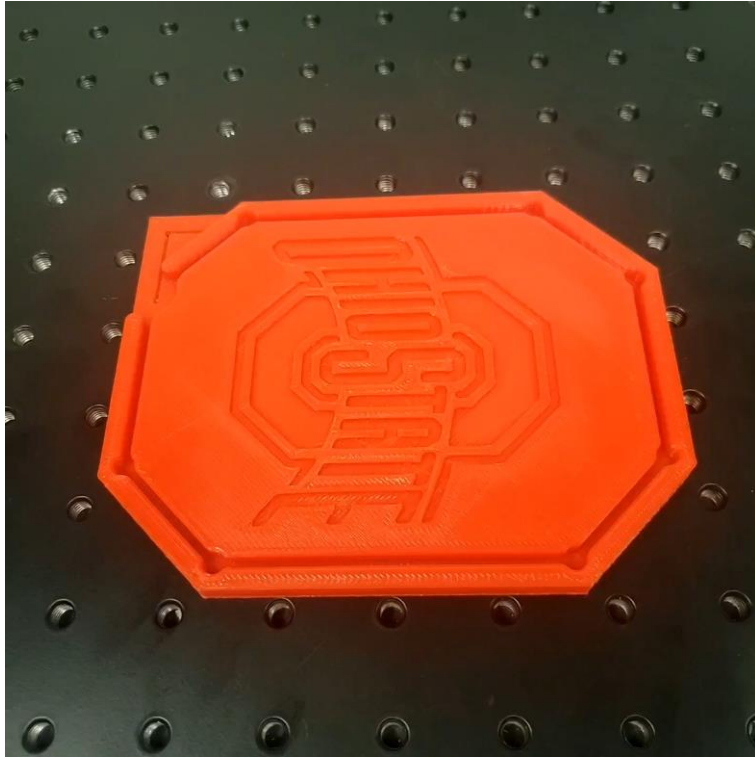


Trapezoidal Time-Scaling

- $v = 0.005$ [m/s], $a = 0.02$ [m/s²]
- $v = \pi / 12$ [rad/s], $a = \pi / 6$ [rad/s²]
- Straight-line interpolation in task space
- 200 Hz



$$p(s) = p_{\text{start}} + s(p_{\text{end}} - p_{\text{start}}),$$
$$R(s) = R_{\text{start}} \exp(\log(R_{\text{start}}^T R_{\text{end}})s)$$

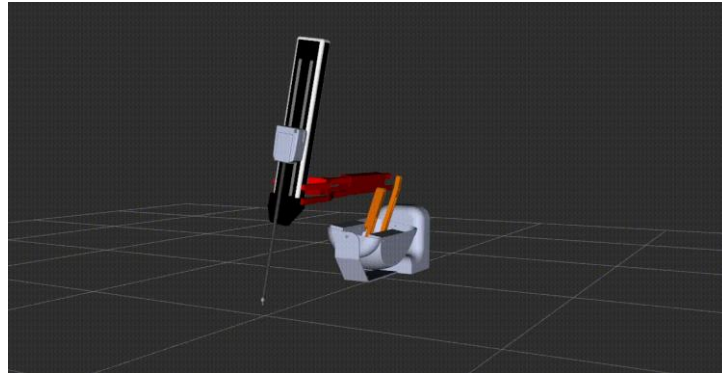




dVRK for dummies: Hardware and software integration for PSM

- Installing dVRK in personal machine:

https://github.com/ferdous-alam/DVRK_robot_project/blob/main/Documentation/Installing_dvrk.md





dVRK for dummies: Hardware and software integration for PSM

Surgical Assistant Workstation
(SAW) package
(built on *cisst* libraries)

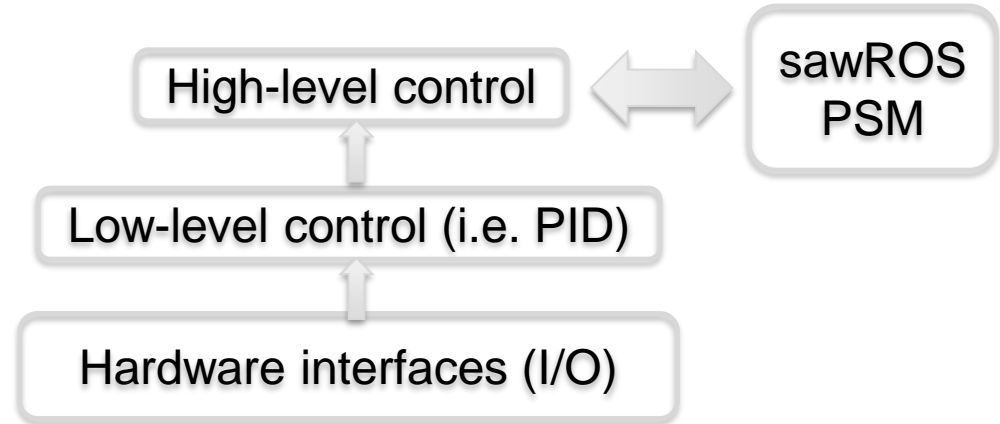


Components



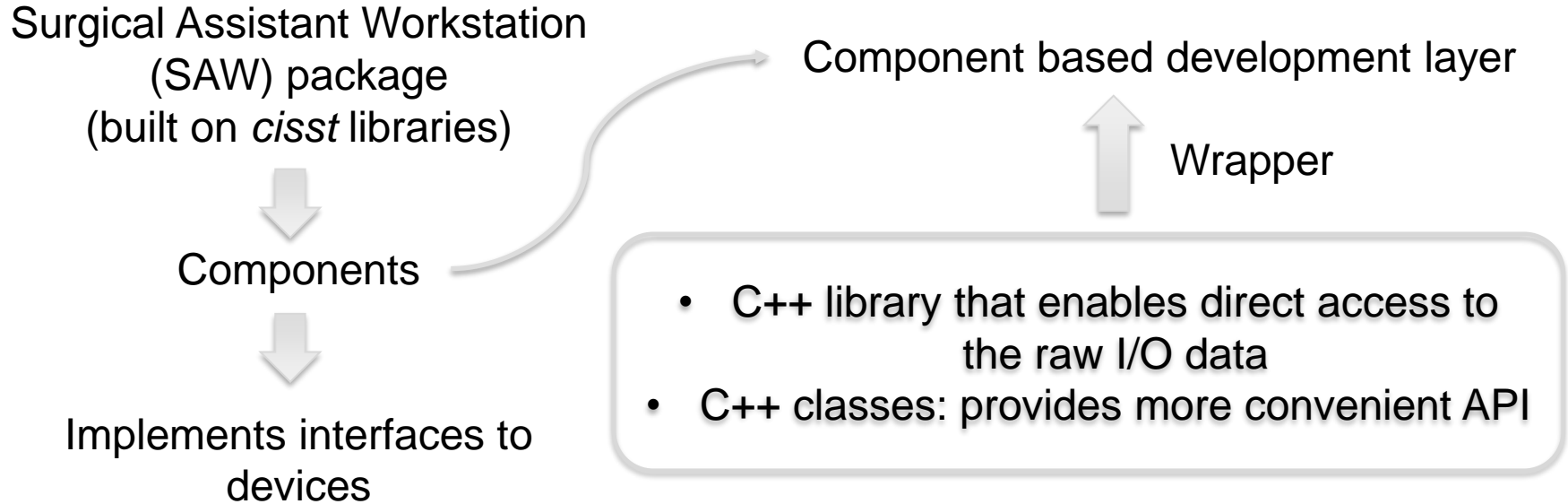
Implements interfaces to
devices

PSM software architecture





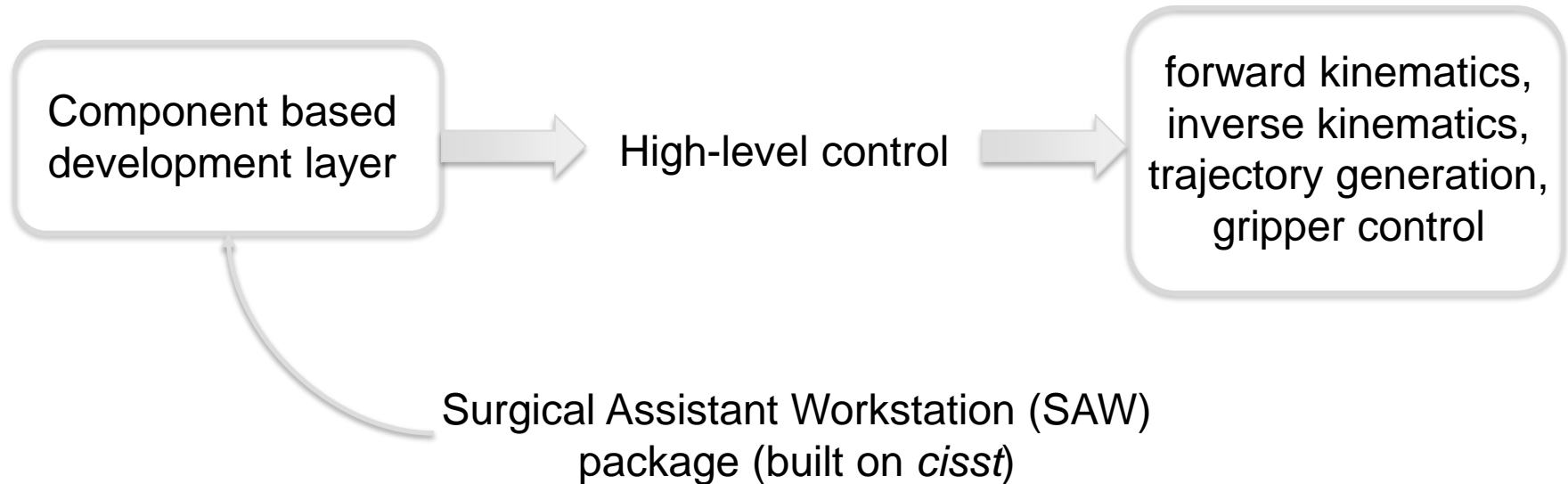
dVRK for dummies: Hardware and software integration for PSM





dVRK for dummies: Hardware and software integration for PSM

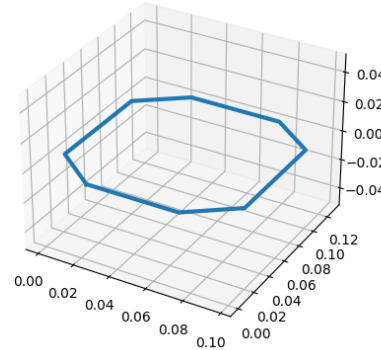
PSM software architecture





Path planning using LSPB time scaling

LSPB
time-scaling



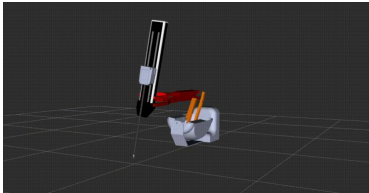
Trajectory
generation



dVRK-ROS
python



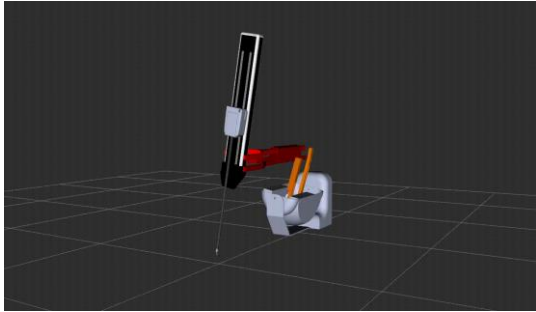
Higher-level
control



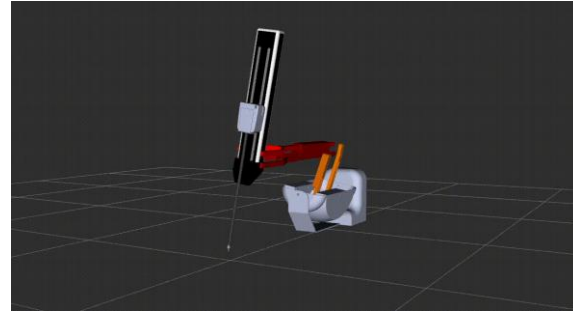


Path planning at different velocities

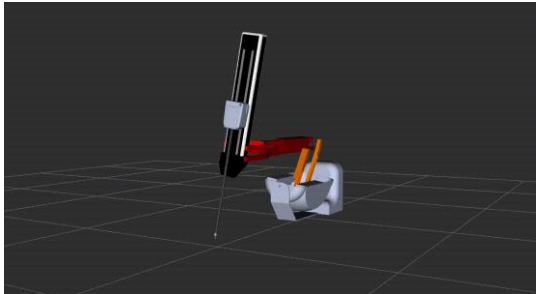
500 Hz



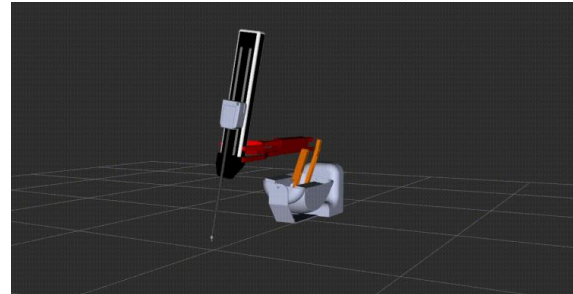
5 MHz



2 MHz



10 MHz



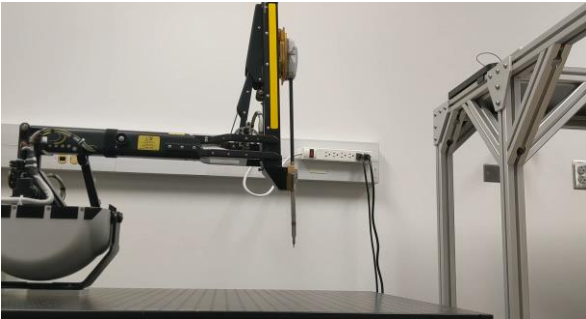


Path planning at different velocities

$$v = 5\text{mm/s}, a = 10\text{mm/s}^2$$

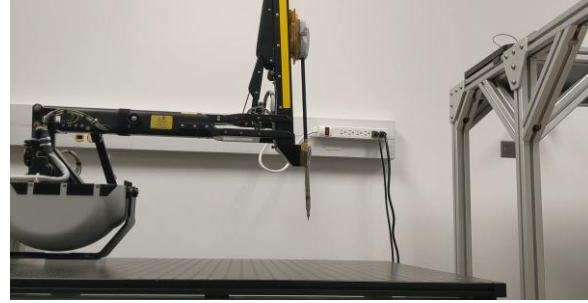


$$v = 10\text{mm/s}, a = 20\text{mm/s}^2$$



Sampling rate = 200 Hz

$$v = 7.5\text{mm/s}, a = 14\text{mm/s}^2$$



$$v = 20\text{mm/s}, a = 40\text{mm/s}^2$$





Thank you!

Questions?



Trajectory Generation

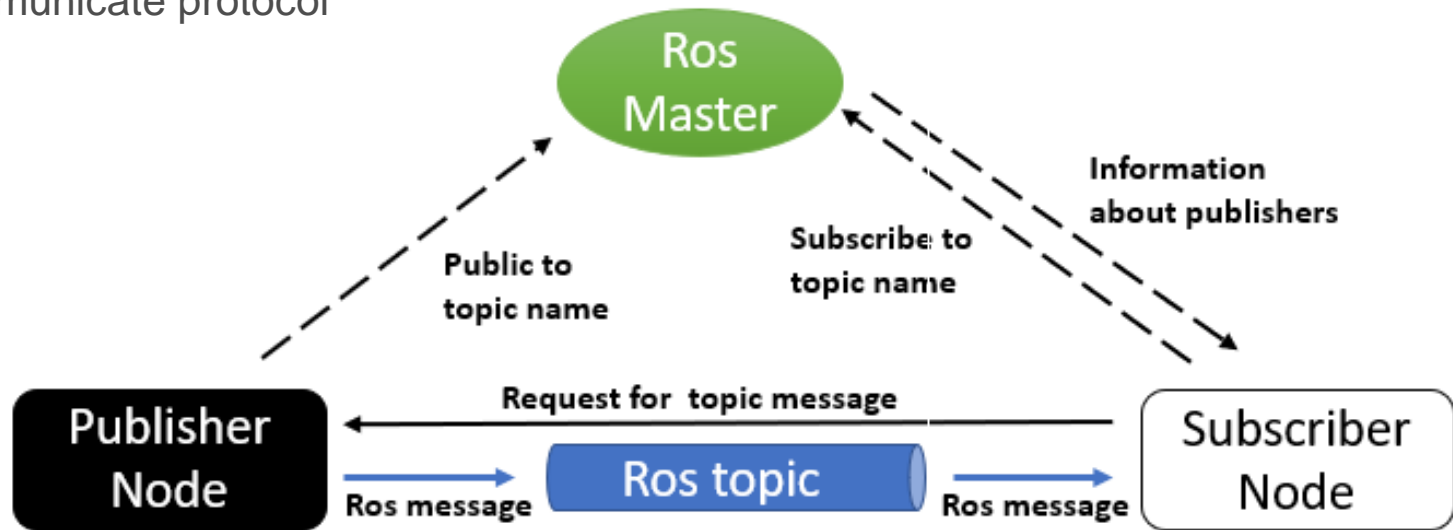
- Block "O" Modulation
- Generate corner transformations
- Time-scaling loop, 200 Hz
- Compute entire trajectory

```
def main():
    # ***** BLOCK "O" MODIFICATION *****
    modified_waypoints_ordered = modify_blockO()
    # ***** GENERATE CORNER-WAYPOINT TRANSFORMATIONS *****
    waypoint_transformations = generate_waypoint_transformations(modified_waypoints_ordered)
    # ***** GENERATE ALL TRAPEZOIDAL TIME-SCALED TRANSFORMATIONS *****
    trajectory = generate_trapezoidal_transformations(waypoint_transformations, print_data=False)
    # ***** OUTPUT TRAJECTORY TO dVRK ROBOT *****
    return modified_waypoints_ordered, waypoint_transformations, trajectory
```



ROS & Python

- "Middleware"
- Communicate protocol



[https://trojrobert.github.io/hands-on-introduction-to-robot-operating-system\(ros\)/](https://trojrobert.github.io/hands-on-introduction-to-robot-operating-system(ros)/)