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Assignment

Course Title : Computer Graphics
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CSE-44th Batch Section 10(E)

Answer to the question number : 01

Window-to-Viewport transformation : Is the process of transforming a 2D world-coordinate objects to device coordinates. Objects inside the world or clipping window are mapped to the viewport which is the area on the screen where world coordinates are mapped to be displayed.

Answet to the question number : 02

Mathematical calculation of Window-to-Viewport transformation : It may be possible that the size of the Viewport is much smaller or greater than the Window. In these cases, we have to increase or decrease the size of the Window according to the Viewport and for this, we need some mathematical calculations.

(Xw, Yw): A point on Window
(Xv, Yv): Corresponding point on Viewport

We have to calculate the point (Xv, Yv)

Normalized point on Window $\left(\frac{X_w - Xwmin}{X_{wmax} - Xwmin}, \frac{Y_w - Y_{wmin}}{Y_{wmax} - Y_{wmin}}\right)$

Normalized point on Viewport $\left(\frac{X_v - Xvmin}{X_{vmax} - Xvmin}, \frac{Y_v - Yvm_{in}}{Y_{vmax} - Yvm_{in}}\right)$

Now the relative postion of the object in Window and Viewport are same
For x coordinate,

$$\frac{X_w - Xwmin}{X_{wmax} - Xwmin} = \frac{X_v - Xvmin}{X_{vmax} - Xvmin}$$

For Y coordinate,

$$\frac{Y_w - Ywm_{in}}{Y_{wmax} - Ywm_{in}} = \frac{Y_v - Yvmin}{Y_{vmax} - Yvmin}$$

So, after calculating for x and y coordinate, we get

$$X_v = X_{vmin} + (X_w + X_{wmin})S_x$$

$$Y_v = Y_{vmin} + (Y_w - Y_{wmin})S_y$$

Where, s, is scaling factor of x coordinate and s, is scaling factor of y coordinate

$$S_x = \frac{X_{vmax} - Xvmin}{X_{wmax} \_ Xw_{min}} \qquad S_y = \frac{Y_{vmax} - Yvm_{in}}{Y_{wmax} - Ywmin}$$

Answer to the question number : 03

Short note on Cohen Sutherland line clipping algorithm : In the algorithm, first of all, it is detected whether line lies inside the screen or it is outside the screen. All lines come under any one of the following categories:

1. Visible
2. Not Visible
3. Clipping Case

Answet to the question number : 04

Advantages of Cohen Sutherland Line Clipping Algorithm :

1. It calculates end-points very quickly and rejects and accepts lines quickly.
2. It can clip pictures much large than screen size.

Answet to the question number : 05

Step by step algorithm of Cohen Sutherland algorithm :

Step1: Calculate positions of both endpoints of the line
Step2: Perform OR operation on both of these end-points
Step3: If the OR operation gives 0000
Then
line is considered to be visible
else
Perform AND operation on both endpoints
If And ≠ 0000
then the line is invisible
else
And=0000
Line is considered the clipped case.
Step4: If a line is clipped case, find an intersection with boundaries of the window
m=(y2-y1)(x2-x1)
(a) If bit 1 is "1" line intersects with left boundary of rectangle window
y3=y1+m(x-X1)
where X = Xwmin
where Xwminis the minimum value of X co-ordinate of window
(b) If bit 2 is "1" line intersect with right boundary
y3=y1+m(X-X1)
where X = Xwmax
where X more is maximum value of X co-ordinate of the window

(c) If bit 3 is "1" line intersects with bottom boundary

X3=X1+(y-y1)/m

where y = ywmin

ywmin is the minimum value of Y co-ordinate of the window

(d) If bit 4 is "1" line intersects with the top boundary

X3=X1+(y-y1)/m

where y = ywmax

ywmax is the maximum value of Y co-ordinate of the window

Answet to the question number : 06

Describing Liang-Barsky line clipping algorithm :

The Liang-Barsky algorithm is a line clipping algorithm. This algorithm is more efficient than Cohen–Sutherland line clipping algorithm and can be extended to 3-Dimensional clipping. This algorithm is considered to be the faster parametric line-clipping algorithm. The following concepts are used in this clipping:

1. The parametric equation of the line.
2. The inequalities describing the range of the clipping window which is used to determine the intersections between the line and the clip window.

Answet to the question number : 07

Step by step algorithm of Liang Barsky algorithm :

1. Set tmin=0, tmax=1.

2. Calculate the values of t (t(left), t(right), t(top), t(bottom)),

(i) If t < tmin ignore that and move to the next edge.
(ii) else separate the t values as entering or exiting values using the inner product.
(iii) If t is entering value, set tmin = t; if t is existing value, set tmax = t.

3. If tmin < tmax, draw a line from (x1 + tmin(x2-x1), y1 + tmin(y2-y1)) to (x1 + tmax(x2-x1), y1 + tmax(y2-y1))

4. If the line crosses over the window, (x1 + tmin(x2-x1), y1 + tmin(y2-y1)) and (x1 + tmax(x2-x1), y1 + tmax(y2-y1)) are the intersection point of line and edge.

Computer Graphics

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Answer to the question no-8

i) $I(-4,7)$, $J(-2,10)$

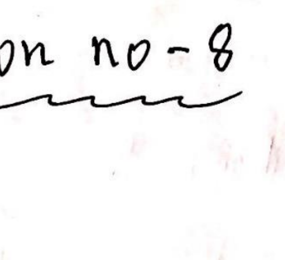
$x_{min} = -3$

$y_{min} = 1$

$x_{max} = 2$

$y_{max} = 6$

R(2,6)



L(-3,1)

Bit 1 : Sign $(y - y_{max})$

Bit 2 : Sign $(y_{min} - y)$

Bit 3 : Sign $(x - x_{max})$

Bit 4 : Sign $(x_{min} - x)$

$I(-4, 7)$

Bit 1 : $7 - 6 = 1 \longrightarrow 1$

Bit 2 : $1 - 7 = -6 \longrightarrow 0$

Bit 3 : $-4 - 2 = -6 \longrightarrow 0$

Bit 4 : $-3 + 4 = 1 \longrightarrow 1$

$\therefore I \rightarrow 1001$

$J(-2, 10)$

Bit 1 : $10 - 6 = 4 \longrightarrow 1$

Bit 2 : $1 - 10 = -9 \longrightarrow 0$

Bit 3 : $-2 - 2 = -4 \longrightarrow 0$

Bit 4 : $-3 + 2 = -1 \longrightarrow 0$

$\therefore J \rightarrow 1000$

Now,

$$
\begin{array}{r}
I \rightarrow 1001 \\
J \rightarrow 1000 \\
\hline
1000
\end{array}
\longrightarrow \text{Outside}
$$

ii) A(-4, 2), B(-1, 7)

Now,

A(-4, 2)

Bit 1 : 2 - 6 = -4 → 0

Bit 2 : 1 - 2 = -1 → 0

Bit 3 : -4 - 2 = -6 → 0

Bit 4 : -3 + 4 = 1 → 1

∴ A → 0001

Then,

B(-1, 7)

Bit 1 : 7 - 6 = 1 → 1

Bit 2 : 1 - 7 = -6 → 0

Bit 3 : -1 ~~+3~~ -2 = -3 → 0

Bit 4 : -3 + 1 = -2 → 0

∴ B → 1000

Hence,

A → 0001

B → 1000

0000 → Maybe Clippable

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{-1+4} = \frac{5}{3}$$

For, A(-4, 2)

$x = x_{min}$

$= -3$

$y = y_1 + m(x_i - x_1)$

$= 2 + \frac{5}{3}(-3+4)$

$= 2 + \frac{5}{3} \times 1$

$= \frac{11}{3}$

$\therefore A'\left(-3, \frac{11}{3}\right)$

For, B(-1, 7)

$y = y_{max} = 6$

$x = x_1 + \frac{1}{m}(y_i - y_1)$

$= -4 + \frac{3}{5}(6-2)$

$= -4 + \frac{3}{5}(4)$

$= -\frac{8}{5}$

$\therefore B'\left(-\frac{8}{5}, 6\right)$

iii) C (-1, 5), D (3, 8)

Now;
C (-1, 5)

Bit 1 : 5 - 6 = -1 ⟶ 0
Bit 2 : 1 - 5 = -4 ⟶ 0
Bit 3 : -1 - 2 = -3 ⟶ 0
Bit 4 : -3 - 1 = -4 ⟶ 0

∴ C ⟶ 0000 it is inside the window.

D (3, 8)

Bit 1 : 8 - 6 = 2 ⟶ 1
Bit 2 : 1 - 8 = -7 ⟶ 0
Bit 3 : 3 - 2 = 1 ⟶ 1
Bit 4 : -3 - 3 = -6 ⟶ 0

∴ D ⟶ 1010

Hence,

C ⟶ 0000
D ⟶ 1010
____
0000 ⟶ It is clipping candidate.

For, $C(-1, 5)$

$$x = x_{min}$$
$$= -3$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{3+1} = \frac{3}{4}$$

$$y = y_1 + m(x_i - x_1)$$
$$= 5 + \frac{3}{4}(-3+1)$$
$$= 5 + \frac{3}{4}(-2)$$
$$= \frac{14}{4} = \frac{7}{2}$$

$\therefore C'\left(-3, \frac{7}{2}\right)$

For, $D(3, 8)$

$$y = y_{max} = 6$$

$$x = x_1 + \frac{1}{m}(y_i - y_1)$$
$$= -1 + \frac{4}{3}(6-5)$$
$$= -1 + \frac{4}{3}(1)$$
$$= \frac{1}{3}$$

$\therefore D'\left(\frac{1}{3}, 6\right)$

Answer to the question no - 9

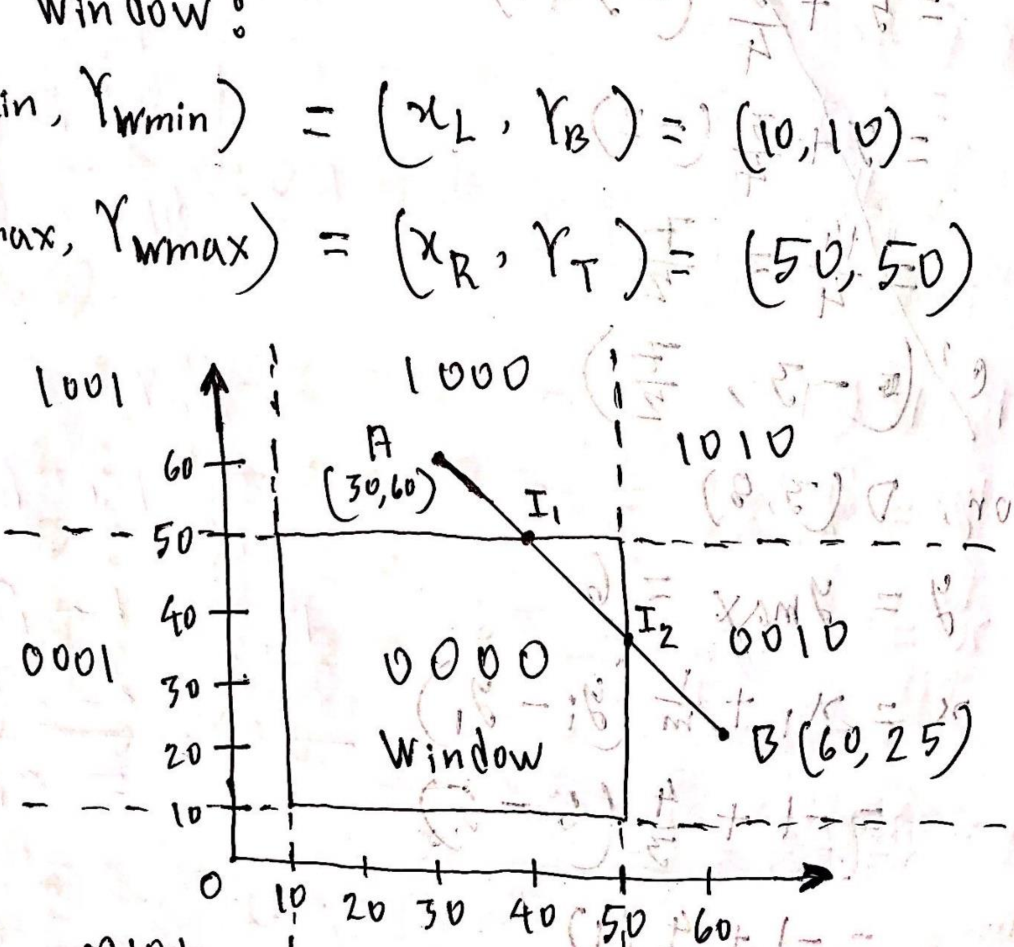
Let, line AB :

A (30 , 60) and B (60, 25)

and Window :

$(X_{wmin}, Y_{wmin}) = (x_L, Y_B) = (10, 10)$

$(X_{wmax}, Y_{wmax}) = (x_R, Y_T) = (50, 50)$



Now,

$A \rightarrow 1000$

$B \rightarrow 0100$

$AND = 0000$

Line is clipping candidate

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{25-60}{60-30}$

$= -\dfrac{7}{6}$

$Y_T,\ x = x_1 + \frac{1}{m}(y_T - y_1)$

$= 30 + \left(-\frac{6}{7}\right)(50-60)$

$= 30 - \frac{6}{7}(-10)$

$= \frac{270}{7} = 38.57$

$\therefore I_1 = (38.57, 50)$ Ans

$x_R,\ y = y_1 + m(x_R - x_1)$

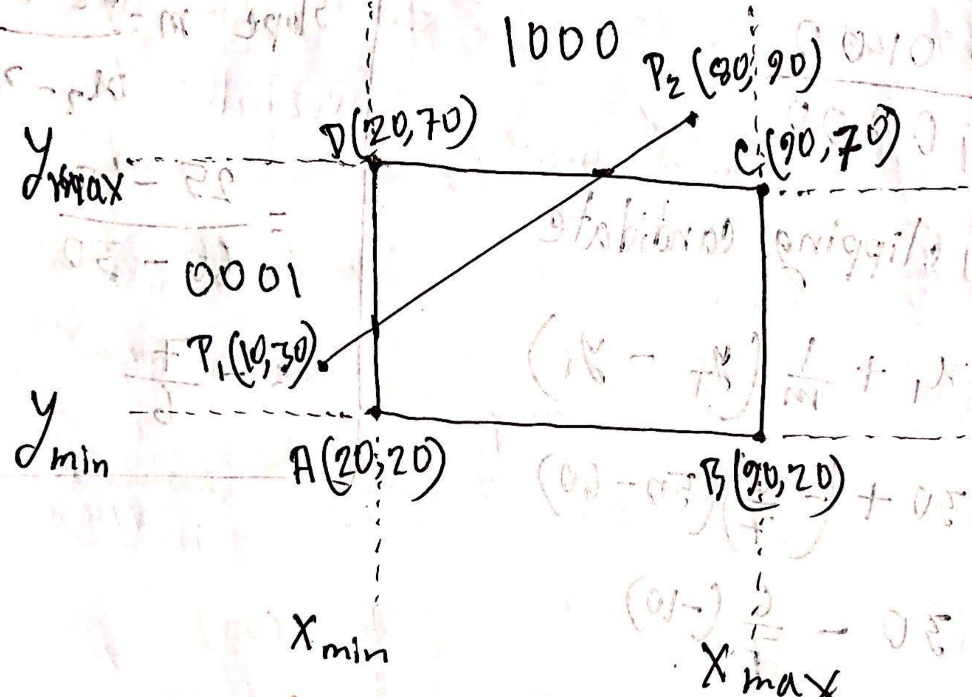
$= 60 + \left(-\frac{7}{6}\right)(50-30)$

$= 60 - \frac{7}{6}(20)$

$= \frac{220}{6} = 36.66$

$\therefore I_2 = (50, 36.66)$ Ans

Answer to the question no-10



Region codes:

$P_1 \rightarrow 0001$

$P_2 \rightarrow 1000$

AND $= 0000 \longrightarrow$ Line is clipping candidate.

slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{90-30}{80-10} = \dfrac{60}{70} = \dfrac{6}{7}$

For, $P_1(10, 30)$

$x = x_{min}$

$= 20$

$y = y_1 + m(x_i - x_1)$

$= 30 + \frac{6}{7}(20 - 10)$

$= 30 + \frac{6}{7}(10)$

$= \frac{270}{7} = 38.57$

$\therefore$ $P_1'(20, 38.57)$

For, $P_2(80, 90)$

$y = y_{max} = 70$

$x = x_1 + \frac{1}{m}(y_i - y_1)$

$= 10 + \frac{7}{6}(70 - 30)$

$= 10 + \frac{7}{6}(40) = \frac{340}{6} = 56.67$

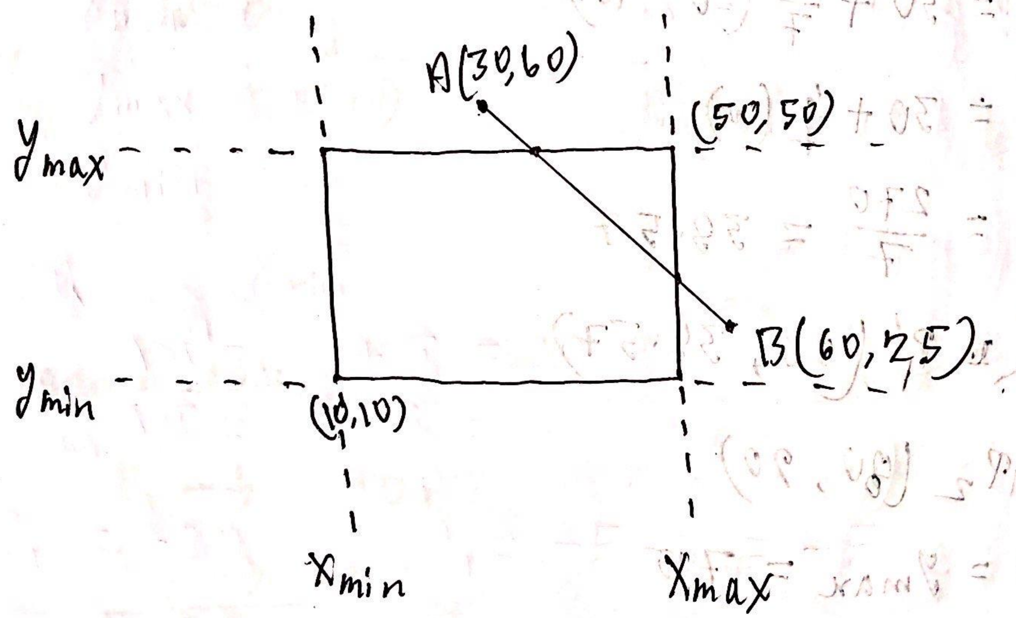
$\therefore P_2'(56.67, 70)$

Ans no - 11

Let, Line AB : A(30,60) and B(60,25)

and Window : $(X_{min}, Y_{min}) = (10,10) = (X_L, Y_B)$

$(X_{max}, Y_{max}) = (50,50) = (X_R, Y_T)$



Here,

$y_{max} = 50$

$y_{min} = 10$

$x_{max} = 50$

$x_{min} = 10$

Now,

$$\Delta x = 60 - 30 = 30$$

$$\Delta y = 25 - 60 = -35$$

$P_1 = -30$ $\quad$ $q_1 = 30 - 10 = 20$

$P_2 = 30$ $\quad$ $q_2 = 50 - 30 = 20$

$P_3 = 35$ $\quad$ $q_3 = 60 - 50 = 10$

$P_4 = -35$ $\quad$ $q_4 = 50$

Since,

$P_k < 0$

$$t_1 = MAX\left(0, -\frac{20}{30}, -\frac{50}{35}\right)$$

$$= MAX\left(0, -\frac{2}{3}, -\frac{5}{7}\right)$$

$$= 0$$

Therefore, A(30,60) doesn't need to be clipped

and $P_k > 0$

$$t_2 = MIN\left(1, \frac{20}{30}, \frac{10}{35}\right)$$

$$= MIN\left(1, \frac{2}{3}, \frac{2}{7}\right)$$

$$= \frac{2}{7}$$

B(60,25) is clipping candidate.

Here, $t_1 < t_2$ so the line is clipping candidate.

Finding B'

$$x = 30 + \frac{2}{7}(30)$$

$$= 30 + \frac{60}{7} = \frac{270}{7}$$

$$= 38.57$$

$$y = 60 + \frac{2}{7}(-35)$$

$$= 60 + \left(-\frac{70}{7}\right)$$

$$= 50$$

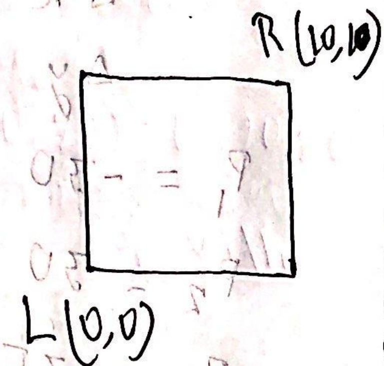
$\therefore B'(38.57, 50)$

Ans:

Ans no - 12

Given,

$P(-5, 3)$ and $Q(15, 9)$



Here,

$x_{max} = 10$

$x_{min} = 0$

$y_{max} = 10$

$y_{min} = 0$

PQ

$\Delta x = 15 + 5 = 20$

$\Delta y = 9 - 3 = 6$

$p_1 = -20$ $\qquad q_1 = -5 - 0 = -5$

$p_2 = 20$ $\qquad q_2 = 10 + 5 = 15$

$p_3 = -6$ $\qquad q_3 = 3 - 10 = -7$

$p_4 = 6$ $\qquad q_4 = 10$

$t_1 = \text{MAX}\left(0, \frac{5}{20}, \frac{7}{6}\right)$

$= \text{MAX}\left(0, \frac{1}{4}, \frac{7}{6}\right)$

$= \frac{7}{6}$

$t_2 = \text{MIN}\left(1, \frac{15}{20}, \frac{10}{6}\right)$

$= \text{MIN}\left(1, \frac{3}{4}, \frac{5}{3}\right)$

$= \frac{3}{4}$

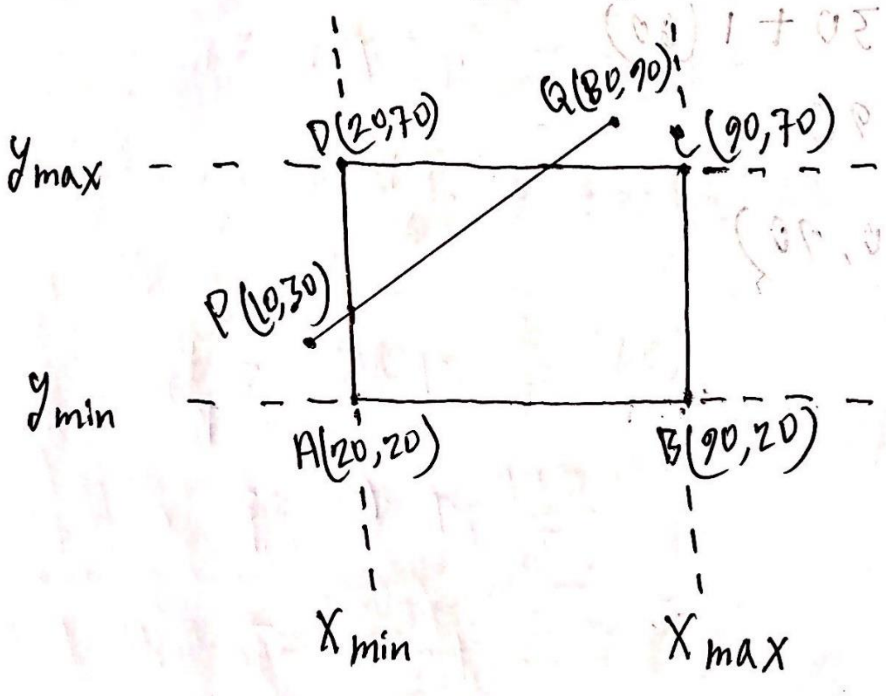
Here,

$t_1 > t_2$ so the line is outside.

it is not clipping candidate.

Ans no - 13

Given, ABCD Rectengular window with A(20, 20) B(90, 20), C(90, 70), D(20, 70) and line PQ with P(10, 30) and Q(80, 90)



Here,

$x_{min} = 20$

$y_{min} = 20$

$x_{max} = 90$

$y_{max} = 70$

Now,

$\Delta x = 80 - 10 = 70$

$\Delta y = 90 - 30 = 60$

$P_1 = -70 \qquad q_1 = 10 - 20 = -10$

$P_2 = 70 \qquad q_2 = 90 - 10 = 80$

$P_3 = -60 \qquad q_3 = 30 - 70 = -40$

$P_4 = 60 \qquad q_4 = 70$

Since, $P_k < 0$

$t_1 = MAX\left(0, \frac{10}{70}, \frac{40}{60}\right)$

$= MAX\left(0, \frac{1}{7}, \frac{2}{3}\right)$

$= \frac{2}{3}$

$t_2 = MIN\left(1, \frac{80}{70}, \frac{70}{60}\right)$ $[P_k > 0]$

$= MIN\left(1, \frac{8}{7}, \frac{7}{6}\right)$

$= 1$

Since, $t_1 < t_2$ line is clipping candidate

Now, Finding P'

$$x = 10 + \frac{2}{3}(70)$$

$$= 10 + \frac{140}{3}$$

$$= \frac{170}{3} = 56.66$$

$$y = 30 + \frac{2}{3}(60)$$

$$= 30 + \frac{120}{3}$$

$$= \frac{210}{3} = 70$$

$\therefore$ P' (56.66, 70)

Since, $t_2 = 1$, Q will not be clipped.

Ans.

Answer to the question number : 14

3D Transformaion : In Computer graphics, Transformation is a process of modifying and re-positioning the existing graphics.

Answet to the question number : 15

Different techniques of 3D Transformation are the followings :

1. Translation
2. Rotation
3. Scaling
4. Reflection
5. Shear

Answet to the question number : 16

Translation : In Computer graphics, 3D Translation is a process of moving an object from one position to another in a three dimensional plane.

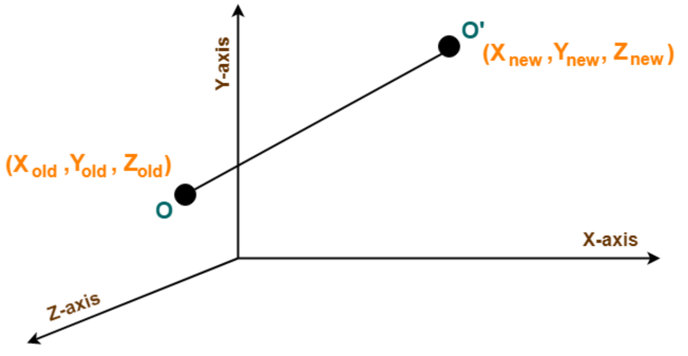
Example : Consider a point object O has to be moved from one position to another in a 3D plane.

Let-

- Initial coordinates of the object O = (Xold, Yold, Zold)
- New coordinates of the object O after translation = (Xnew, Ynew, Zold)
- Translation vector or Shift vector = (Tx, Ty, Tz)

Given a Translation vector (Tx, Ty, Tz)-

- Tx defines the distance the Xold coordinate has to be moved.
- Ty defines the distance the Yold coordinate has to be moved.
- Tz defines the distance the Zold coordinate has to be moved.



3D Translation in Computer Graphics

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- Xnew = Xold + Tx (This denotes translation towards X axis)
- Ynew = Yold + Ty (This denotes translation towards Y axis)
- Znew = Zold + Tz (This denotes translation towards Z axis)

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Translation Matrix

Rotation : In Computer graphics, 3D Rotation is a process of rotating an object with respect to an angle in a three dimensional plane.

Example : Consider a point object O has to be rotated from one angle to another in a 3D plane.

Let-
- Initial coordinates of the object O = (Xold, Yold, Zold)
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (Xnew, Ynew, Znew)

In 3 dimensions, there are 3 possible types of rotation-
- X-axis Rotation
- Y-axis Rotation
- Z-axis Rotation

<u>For X-Axis Rotation-</u>

This rotation is achieved by using the following rotation equations-
- Xnew = Xold
- Ynew = Yold x cosθ – Zold x sinθ
- Znew = Yold x sinθ + Zold x cosθ

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For X-Axis Rotation)

For Y-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- Xnew = Zold x sinθ + Xold x cosθ
- Ynew = Yold
- Znew = Yold x cosθ – Xold x sinθ

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Y-Axis Rotation)

For Z-Axis Rotation-

This rotation is achieved by using the following rotation equations-

- Xnew = Xold x cosθ – Yold x sinθ
- Ynew = Xold x sinθ + Yold x cosθ
- Znew = Zold

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

Scaling : In Computer graphics, scaling is a process of modifying or altering the size of objects.

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

Example : Consider a point object O has to be scaled in a 3D plane.

Let-
- Initial coordinates of the object O = (Xold, Yold,Zold)
- Scaling factor for X-axis = Sx
- Scaling factor for Y-axis = Sy
- Scaling factor for Z-axis = Sz
- New coordinates of the object O after scaling = (Xnew, Ynew, Znew)

This scaling is achieved by using the following scaling equations-
- Xnew = Xold x Sx
- Ynew = Yold x Sy
- Znew = Zold x Sz

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix

Reflection : In Computer graphics, It is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis. The object is rotated by180°.

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

Example : Consider a point object O has to be reflected in a 3D plane.

Let-

- Initial coordinates of the object O = (Xold, Yold, Zold)
- New coordinates of the reflected object O after reflection = (Xnew, Ynew,Znew)

In 3 dimensions, there are 3 possible types of reflection-

- Reflection relative to XY plane
- Reflection relative to YZ plane
- Reflection relative to XZ plane

<u>Reflection Relative to XY Plane:</u>

This reflection is achieved by using the following reflection equations-

- Xnew = Xold
- Ynew = Yold
- Znew = -Zold

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XY plane)

<u>Reflection Relative to YZ Plane:</u>

This reflection is achieved by using the following reflection equations-

- Xnew = -Xold
- Ynew = Yold
- Znew = Zold

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to YZ plane)

Reflection Relative to XZ Plane:

This reflection is achieved by using the following reflection equations-

- Xnew = Xold
- Ynew = -Yold
- Znew = Zold

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XZ plane)

Shearing : In Computer graphics, 3D Shearing is an ideal technique to change the shape of an existing object in a three dimensional plane.

In a three dimensional plane, the object size can be changed along X direction, Y direction as well as Z direction.
So, there are three versions of shearing-

1. Shearing in X direction
2. Shearing in Y direction
3. Shearing in Z direction

Example : Consider a point object O has to be sheared in a 3D plane.

Let-

- Initial coordinates of the object O = (Xold, Yold, Zold)
- Shearing parameter towards X direction = Shx
- Shearing parameter towards Y direction = Shy
- Shearing parameter towards Z direction = Shz
- New coordinates of the object O after shearing = (Xnew, Ynew, Znew)

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- Xnew = Xold
- Ynew = Yold + Shy x Xold
- Znew = Zold + Shz x Xold

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In X axis)

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- Xnew = Xold + Shx x Yold
- Ynew = Yold
- Znew = Zold + Shz x Yold

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Y axis)

Shearing in Z Axis-

Shearing in Z axis is achieved by using the following shearing equations-

- Xnew = Xold + Shx x Zold
- Ynew = Yold + Shy x Zold
- Znew = Zold

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Z axis)

Ans no - 17

Given,

Old coordinates of the object = $A(1,3,5)$, $B(3,5,3)$, $C(3,0,1)$, $D(2,1,5)$

Translation vector = $(T_x, T_y, T_z) = (4, 2, 5)$

For Coordinates $A(1,3,5)$

Let the new coordinates of $A = (X_{new}, Y_{new}, Z_{new})$

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 1+4 = 5$

$Y_{new} = Y_{old} + T_y = 3+2 = 5$

$Z_{new} = Z_{old} + T_z = 5+5 = 10$

Thus, New coordinates of $A = (5, 5, 10)$

Ans:

For Coordinates B (3, 5, 3)

Let the new coordinates of B = ($X_{new}, Y_{new}, Z_{new}$)

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 3 + 4 = 7$

$Y_{new} = Y_{old} + T_y = 5 + 2 = 7$

$Z_{new} = Z_{old} + T_z = 3 + 5 = 8$

Thus, New coordinates of B = (7, 7, 8)

For Coordinates C (3, 0, 1)

Let the new coordinates of C = ($X_{new}, Y_{new}, Z_{new}$)

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 3 + 4 = 7$

$Y_{new} = Y_{old} + T_y = 0 + 2 = 2$

$Z_{new} = Z_{old} + T_z = 1 + 5 = 6$

Thus, New coordinates of C = (7, 2, 6)

For coordinates D (2, 1, 5)

Let the new coordinates of c

$= (X_{new}, Y_{new}, Z_{new})$

Applying the translation equations, we have -

$X_{new} = X_{old} + T_x = 2 + 4 = 6$

$Y_{new} = Y_{old} + T_y = 1 + 2 = 3$

$Z_{new} = Z_{old} + T_z = 5 + 5 = 10$

Thus, New coordinates of D = (6, 3, 10)

Ans no - 18

Given,

Old coordinates = $(X_{old}, Y_{old}, Z_{old}) = (3, 2, 5)$

Rotation angle $= \theta = 45°$

For X - Axis rotation:

Let the new coordinates after rotation $= (X_{new}, Y_{new}, Z_{new})$

Applying the rotation equations we have -

$X_{new} = X_{old} = 3$

$Y_{new} = Y_{old} \cdot \cos\theta - Z_{old} \cdot \sin\theta = 2 \cdot \cos 45° - 5 \sin 45°$

$= 2 \times \frac{1}{\sqrt{2}} - 5 \times \frac{1}{\sqrt{2}} = -\frac{3}{\sqrt{2}}$

$Z_{new} = Y_{old} \cdot \sin\theta + Z_{old} \cdot \cos\theta = 2 \cdot \sin 45° + 5 \cos 45°$

$= 2 \times \frac{1}{\sqrt{2}} + 5 \times \frac{1}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

New coordinates after rotation $= \left(3, -\frac{3}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

For Y- Axis rotation:

Let the new coordinates after rotation = (X_{new}, Y_{new}, Z_{new})

Applying the rotation equations we have -

$X_{new} = Z_{old} \cdot \sin\theta + X_{old} \cdot \cos\theta = 5 \sin 45^\circ + 3 \cos 45^\circ$

$= 5 \times \frac{1}{\sqrt{2}} + 3 \times \frac{1}{\sqrt{2}} = \frac{8}{\sqrt{2}}$

$Y_{new} = Y_{old} = 2$

$Z_{new} = Y_{old} \cdot \cos\theta - X_{old} \cdot \sin\theta = 2 \cos 45^\circ - 3 \sin 45^\circ$

$= 2 \times \frac{1}{\sqrt{2}} - 3 \times \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$

Thus, New coordinates after rotation = $\left(\frac{8}{\sqrt{2}}, 2, -\frac{1}{\sqrt{2}}\right)$

For Z - Axis rotation:

Let the new coordinates after rotation $= (X_{new}, Y_{new}, Z_{new})$

Applying the rotation equations, we have -

$X_{new} = X_{old} \cdot \cos\theta - Y_{old} \cdot \sin\theta = 3\cos 45^\circ - 2\sin 45^\circ$

$= 3 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$Y_{new} = X_{old} \cdot \sin\theta + Y_{old} \cdot \cos\theta = 3\sin 45^\circ + 2\cos 45^\circ$

$= 3 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$Z_{new} = Z_{old} = 5$

Thus, New coordinates after rotation

$= \left(\frac{1}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 5\right)$

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Ans no - 19

Given,

Old coordinates of the object = A(1, 3, 5), B(3, 5, 3), C(3,0,1), D(2,1,5)

Scaling factor along X axis = 4

" " " Y " = 2

" " " Z " = 5

For Coordinates A(1, 3, 5)

Let the new coordinates of A after scaling = (X_{new}, Y_{new}, Z_{new})

Applying the scaling equations we have-

$X_{new} = X_{old} \times S_x = 1 \times 4 = 4$

$Y_{new} = Y_{old} \times S_y = 3 \times 2 = 6$

$Z_{new} = Z_{old} \times S_z = 5 \times 5 = 25$

The new coordinates of A = (4, 6, 25)

For Coordinates B (3, 5, 3)

Let the new coordinates of B after scaling = (X_{new}, Y_{new}, Z_{new})

Applying the scaling equations we have -

$X_{new} = X_{old} \times S_x = 3 \times 4 = 12$

$Y_{new} = Y_{old} \times S_y = 5 \times 2 = 10$

$Z_{new} = Z_{old} \times S_z = 3 \times 5 = 15$

$\therefore$ New coordinates of B = (12, 10, 15)

For Coordinates C (3, 0, 1)

Let the new coordinates of C after scaling = (X_{new}, Y_{new}, Z_{new})

Applying the scaling equations we have -

$X_{new} = X_{old} \times S_x = 3 \times 4 = 12$

$Y_{new} = Y_{old} \times S_y = 0 \times 2 = 0$

$Z_{new} = Z_{old} \times S_z = 1 \times 5 = 5$

∴ New coordinates of C = (12, 0, 5)

For Coordinates D (2, 1, 5)

Let the new coordinates of D after scaling $= (X_{new}, Y_{new}, Z_{new})$

Applying the scaling equations we have-

$X_{new} = X_{old} \times S_x = 2 \times 4 = 8$

$Y_{new} = Y_{old} \times S_y = 1 \times 2 = 2$

$Z_{new} = Z_{old} \times S_z = 5 \times 5 = 25$

∴ New coordinates of D = (8, 2, 25)

Ans no- 20

Given,

Old corner coordinates of the triangle

$= A(3,5,2),\ B(4,2,7),\ C(5,6,4)$

Reflection has to be taken on the XY and XZ plane.

For Coordinates $A(3,5,2)$

Let the new coordinates of A after reflection

$= (~~X_{max}~~ X_{new}, Y_{new}, Z_{new})$

Applying the reflection equations we have

$X_{new} = X_{old} = 3$ $\quad\quad$ $X_{new} = X_{old} = 3$

$Y_{new} = Y_{old} = 5$ $\quad\quad$ $Y_{new} = -Y_{old} = -5$

$Z_{new} = -Z_{old} = -2$ $\quad\quad$ $Z_{new} = Z_{old} = 2$

$\therefore$ New coordinates of A on XY and XZ plane

$= (3, 5, -2)$ and $(3, -5, 2)$

For Coordinates B (4, 2, 7)

Let the new coordinates of B after reflection = (X_{new}, Y_{new}, Z_{new})

Applying the reflection equations, we have -

$X_{new} = X_{old} = 4$ $\qquad$ $X_{new} = X_{old} = 4$

$Y_{new} = Y_{old} = 2$ $\qquad$ $Y_{new} = -Y_{old} = -2$

$Z_{new} = -Z_{old} = -7$ $\qquad$ $Z_{new} = Z_{old} = 7$

∴ New coordinates of B on XY and XZ plane = (4, 2, 7) and (4, -2, 7)

For Coordinates C (5, 6, 4)

Let the new coordinates of C after reflection = (X_{new}, Y_{new}, Z_{new})

Applying the reflection equations we have.

$X_{new} = X_{old} = 5$ $\quad$ $X_{new} = X_{old} = 5$

$Y_{new} = Y_{old} = 6$ $\quad$ $Y_{new} = Y_{old} = -6$

$Z_{new} = -Z_{old} = -4$ $\quad$ $Z_{new} = Z_{old} = 4$

$\therefore$ New coordinates of C on XY and XZ plane $= (5, 6, -4)$ and $(5, -6, 4)$