How Does Executive Effort Impact On Their Option Exercise Timing Decision? An Optimal Control-Stopping Approach

Undergraduate Research Support Scheme

The University of Warwick, Statistics
Roseanna Ferguson and Dr. Vicky Henderson

Warwick Statistics

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Introduction

Executives commonly receive stock options which can be exercised - that is, given the right to buy the stock of their company for a fixed strike price at some future time of their choosing. Typically they build up a portfolio of options over their tenure with a company. They face the optimal timing decision of when to exercise options but can also exert managerial effort to influence the underlying stock price. Here, this problem is formalized in an optimal control-stopping model.

Model

The agent receives θ_0 call options with payoff $Y_s - K$ where K is the fixed strike. Θ_t is the number of remaining options. The underlying asset Y follows exponential Brownian motion $dY/Y = \sigma dB_s + \mu ds + \delta a_s ds$. The agent can exert effort, of the form $a_s \equiv a(Y_s, \Theta_s)$, continuously for a cost, ϵ. Revenue is given by, R:

$$R = \int_0^\infty (Y_s - K)^+ d\Theta_s - \frac{\epsilon}{2}\int_0^\infty a_s^2 ds$$

The problem is to find the maximal expected utility, the value function, over all possible effort and exercise strategies:

$$V = \max_{a_t, t\geq 0} \max_{\theta_t \in S, \theta_0 = \theta} \mathbb{E}\left[U\left(x+R\right)\right]$$

The following assumptions were made in order to solve the problem explicitly:

- Exponential utility, $U(z) = -(1/\gamma)e^{-\gamma z}$, allows the initial wealth, x, to be factored out.
- Restrictions from trading in the underlying asset, the agent faces an incomplete market.
- Interest rates are zero and the claim has infinite maturity therefore the optimal strategy is of a threshold type due to time homogeneity.

General	Stock	Executive
Risk aversion, γ	Drift, μ	Skill, δ
Strike Price, K	Volatility, σ	Cost, ϵ

Table 1: The Parameters

Future Research

To consider the following questions

- Can effort help explain the fall in stock price commonly observed after executives exercise?
- What happens if the manager can partially hedge?

References

[1] Vicky Henderson and David Hobson. Perpetual american options in incomplete markets: the infinitely divisible case. *Quantitative Finance*, 8(5):461–469, 2008.

For more details contact:
fergusonroseanna@gmail.com

Methodology

- This is a stochastic control problem with singular control, the exercise of options.
- Calculus of variations is used to maximise over effort and a direct method is used to maximise over possible exercise strategies.
- The key summary parameters are: $\beta = 1 - 2\mu/\sigma^2$ and $\lambda = 1 - \delta^2/\gamma\epsilon\sigma^2$.
- β is related to the Sharpe ratio and λ summarises the parameters which, due to the presence of effort, cause changes to the results.
- The expression for effort as a function of F, the value function at the boundary, was derived by maximising F over effort strategies.
$$a_s^* = \frac{-y\delta F_y}{\gamma\epsilon F}.$$
- An ODE for the optimal boundary, h, obtained by maximising F over possible boundaries.
$$h' = \frac{-\lambda\gamma(h-K)^2 h}{(\beta+1)K - (\beta-1)h}.$$
- This reduced to the no effort case with a transformation. The optimal boundary is a transform of a key function I, $I(y/K)/(\gamma\lambda/K)$. After calculating F, an explicit formula for effort was found.
- The money, p, equivalent to θ options was found using $U(x+p) = V(x,y,\theta)$. Finally, the marginal indifference price is obtained by taking the derivative of p with respect to θ.

Results and Conclusion

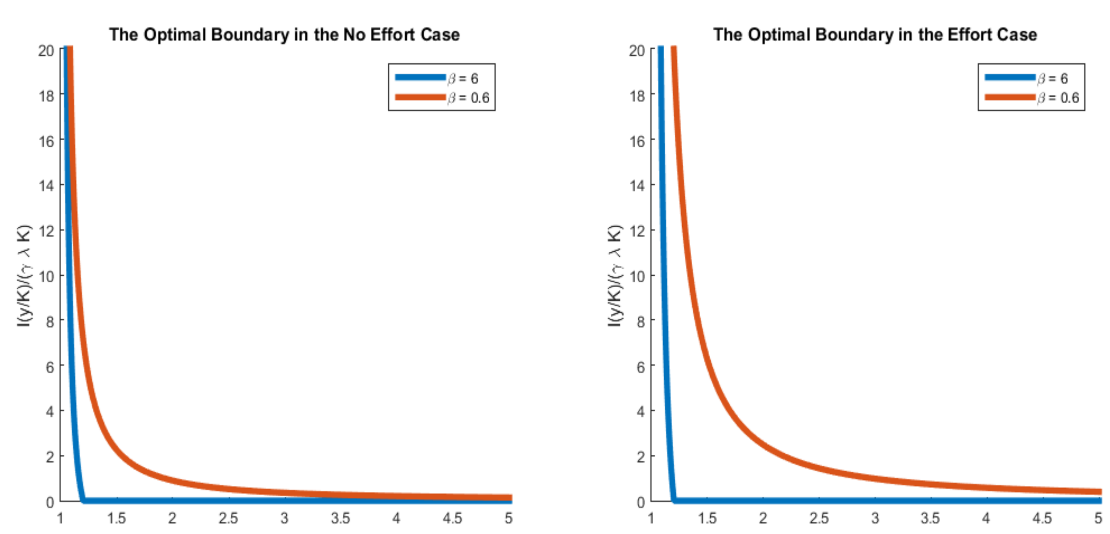

Figure 1: The Optimal Boundary

The boundary is defined on $y > K$ to reflect that it is never advantageous to exercise an out of the money option. The effort pushes the boundary outwards due to the reduction in risk caused by the executive having greater influence on the stock price. Varying parameters shows if any of ϵ, σ, γ decrease or δ, μ increase, the boundary is pushed outwards, and the executive waits till a higher price to exercise.

On the right is the effort function. It is increasing towards the boundary in y and θ and peaks at the boundary. One explanation is that given the executive follows the optimal strategy, effort is not worth the cost if far away from the boundary and so they only exert effort when the chance of exercise is more likely. Varying parameters it can be observed as, γ, ϵ decrease or δ increases the effort is higher and more spread out. Other parameters have little effect other than the previously described changes to the boundary.

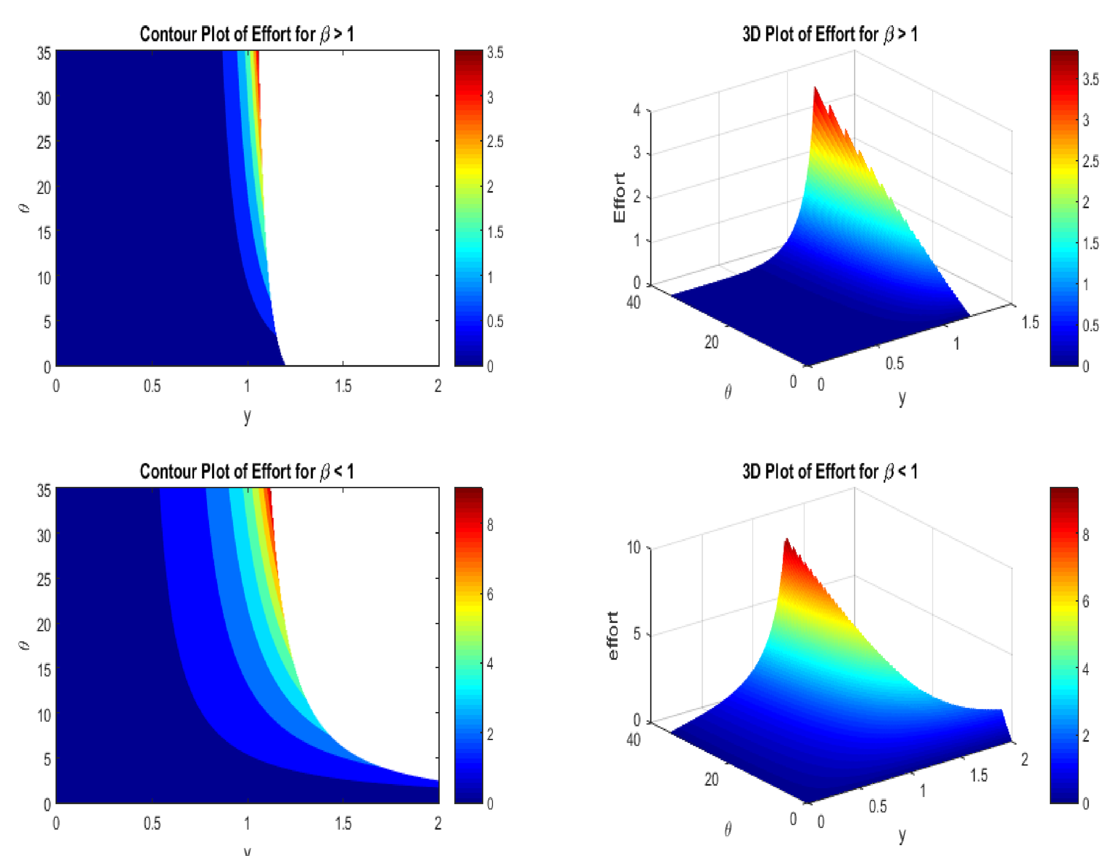

Figure 2: Executive Effort

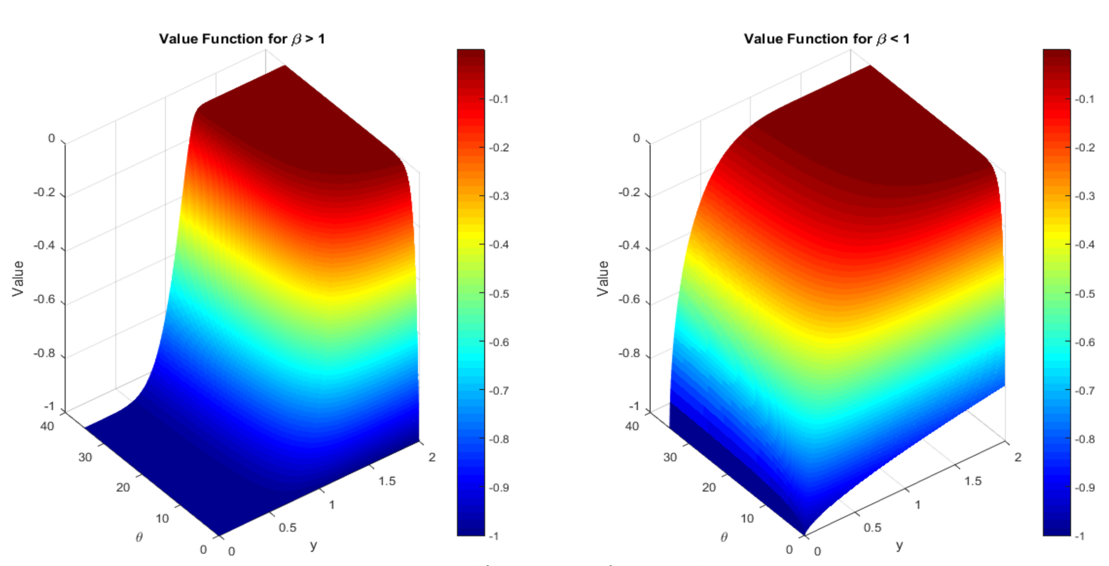

Figure 3: The Value Function

The value function has three regions. (1) Below the threshold, (2) above the threshold with either (a) $\beta > 1$ and y sufficiently large, the agent exercises in a single tranche or (b) in order to move onto the boundary, an initial exercise of options occurs. The graph shows the value to the executive of the options as increasing in both y and theta, with the change in value being greatest just below the boundary.

Shown right is the total indifference price and the difference in marginal indifference price due to effort. For $\beta < 1$ the difference due to effort is greater but impacts fewer options, in contrast to $\beta > 1$. The area around the optimal boundary is most affected by the presence of effort. If any of γ, σ increase or μ, δ decrease, the difference in the indifference price due to effort decreases. Further, for $\beta < 1$ as σ increases or any of μ, δ, γ decrease the marginal indifference price due to effort is non-zero for higher θ. For $\beta > 1$, as μ, σ increase the range of y affected by effort is greater.

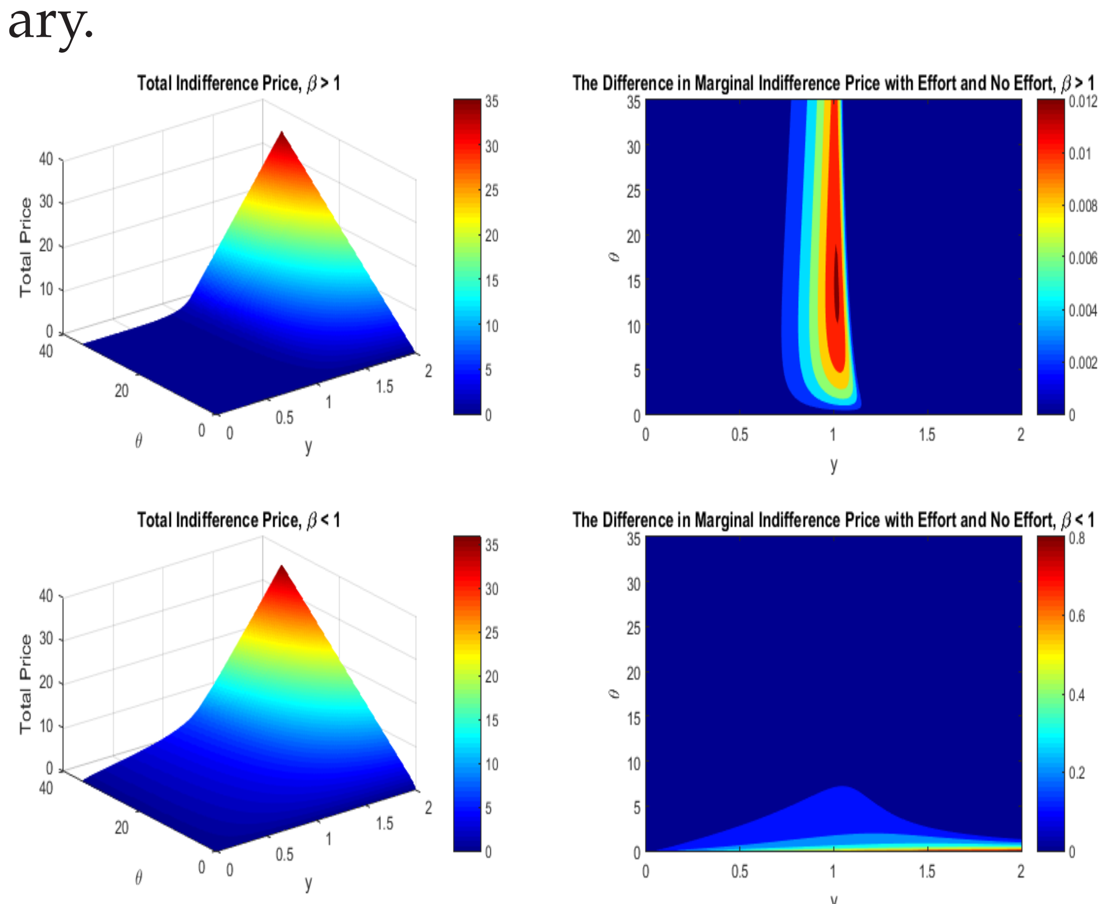

Figure 4: Indifference Price