Scientific Computing: F34SCO

Gravitational N-body modelling

Background

The gravitational two body problem was solved by Newton, not long after he discovered the nature of the gravitational force itself. One might expect the simple addition of one more body to the system would not unduly complicate its behaviour. In fact such a system can display incredibly complex behaviour and a usable general solution is unlikely to be found. This need not be a problem however, as given a set of initial masses, positions and velocities the subsequent orbits of the bodies can be found by numerical integration.

A direct summation N-body code calculates the gravitational force at any point by directly summing the gravitational force due to all the other particles. Hence the force on particle *i* due to all the other particles is;

$$F_i = \sum_{j=1}^{n} \frac{G m_i m_j r_{ij}}{r_{ij}^3}$$

The universal law of gravitation can be treated as a second order ODE and solved using the inbuilt Matlab solver.

Project

- Write a 2-body code in two dimensions, and check that the results lead to simple, closed, elliptical orbits.

Extend your 2-body routine to generate an N-body code capable of following 3 bodies in two dimensions. Test this code using three bodies in some simple configurations to which the analytic solution is known. For example:

- Two light bodies in orbit around a much more massive one. You should be able to reproduce simple elliptical orbits.
- Two heavy bodies in close orbit with a light body orbiting them both. You can investigate the behaviour of the light body as a function of its position relative to the heavy bodies, and identify the Lagrangian points
- Extend your two dimensional routine to three dimensions and many bodies.

Advanced options

- Study the stability of known exoplanetary systems. You will need to look up the available planetary data.
- Study the stability of the solar system under close encounters with external objects (such as an encounter with another star) or the presence of an additional large planet.
- Study the evolution of a star cluster.

Hints: You can check the accuracy of an N-body code by looking at the total energy present as this should be conserved. Be careful with close encounters, you will have to think carefully about the length of your timesteps in this case.