Transfer Report

Fergus Barratt

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1 Spin Wave calculation of the OTOC

Start with XXZ hamiltonian

$$H = -J \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \gamma S_i^z S_{i+1}^z \tag{1}$$

1.1 Holstein-Primakoff

$$S_i^+ = \sqrt{2S}\sqrt{1 - \frac{n_i}{2S}}b \tag{2}$$

$$S_i^- = \sqrt{2S}b^\dagger\sqrt{1 - \frac{n_i}{2S}} \tag{3}$$

$$S_i^z = S - n_i \tag{4}$$

The Hamiltonian is

$$\begin{aligned} H = {} & \frac{-JNS^2\gamma}{2} - \frac{JS}{2}\sum_i (b_i b_{i+1}^\dagger + b_i^\dagger b_{i+1} - \gamma(b_i^\dagger b_i + b_{i+1}^\dagger b_{i+1})) \\ & + \frac{J}{4}\sum_i (n_i b_i b_{i+1}^\dagger + b_i b_{i+1}^\dagger - n_i b_i^\dagger b_{i+1} - b_i^\dagger b_{i+1} n_{i+1} - 2\gamma n_i n_{i+1}) + \mathcal{O}(\frac{1}{S}) \end{aligned} \tag{5}$$

2 OTOC

$$C_i(t) = \left\langle [S_i^z(0), S_i^z(t)]^2 \right\rangle = 1 - \langle S_i^z(0) S_i^z(t) S_i^z(0) S_i^z(t)\rangle = 1 - K_i(t) \tag{6}$$

2.1 Spin Wave calculation

Expand around classical saddle point. Let $R(t)\vec{l}(0)$ be the solution to the classical equations of motion, and write

$$K_i(t) = R_{z\alpha}R_{z\beta}\left\langle S_i^z(0) S_i^\alpha(t) S_i^z(0) S_i^\beta(t) \right\rangle = \mathrm{tr}(M_i(t)\chi_i(t)) \tag{7}$$

where $S_i^\alpha(t)$ (and thus $\chi(t)$) is calculated using spin waves.

2.1.1 $\mathcal{O}(S)$

Free spin waves

$$H = -\frac{JNS^2\gamma}{2} + JS\sum_k (\gamma - \cos k) b_k^\dagger b_k \tag{8}$$

Compute higher order terms with mean field - then will get anomalous terms. Choose as initial state $|\psi\rangle = b_i^\dagger |\uparrow\rangle^{\otimes N}$: $\langle H \rangle - E_0 = 2J\gamma$ - can use γ to put in middle of spectrum. Then

$$\chi_i^{\alpha\beta}(t) = -\left\langle S_i^\alpha(t) S_i^z(0) S_i^\beta \right\rangle \tag{9}$$