Understanding Dissipative Control of Quantum Dynamics from Biased Trajectory Ensembles

Fergus Barratt

> Introduction

- The technological potential of quantum systems is due to the exponential scaling of the tensor product state space.
- Environmental decoupling represents a challenge to effective use of quantum technology.
- Tools from classical statistical mechanics can be used to quantify changes of dynamical behaviour in a range of systems.
- In particular, the methods of *Biased Trajectory Ensembles* can be connected to dissipation in quantum systems.
- We plan to study the transition from the presence of usable quantum resources (i.e. entanglement) to their absence.

> Keldysh Theory

- In the equilibrium formalism of quantum field theory, we rely on the adiabatic switching on and off of interactions bringing the system back into its ground state when evolved $t = -\infty \to t = \infty$
- Clearly not true in the non-equilibrium case
- Suggestion: evolve the state forward in time ($t = -\infty \to \infty$), then rewind $t = \infty \to -\infty$: fig. 5
- Convenient to separate fields on forward and backwards contours (ϕ^+, ϕ^-)
- Constructing the path integral in usual way, there is a redundancy in the Green's Function description.
- Defining

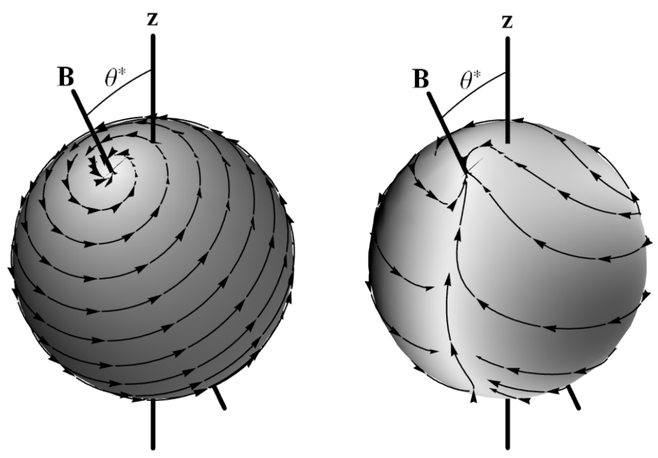
$$\phi^q = \phi^+ - \phi^- \qquad \phi^{cl} = \phi^+ - \phi^- \tag{1}$$

- leads to Green's Functions with one component always 0

$$\langle \phi^q(t)\phi^q(t')\rangle = 0 \quad \left\langle \phi^{cl}(t)\phi^{cl}(t')\right\rangle = iG^K(t,t') \tag{2}$$

$$\left\langle \phi^{cl}(t)\phi^q(t')\right\rangle = iG^R(t,t') \quad \left\langle \phi^q(t)\phi^{cl(t')}\right\rangle = iG^A(t,t') \tag{3}$$

- Introducing a bath of oscillators interacting with system degrees of freedom, and integrating out the bath degrees of freedom, and expanding to first order in the quantum fields we can generate effective langevin descriptions of quantum dynamics.



> Matrix Product States

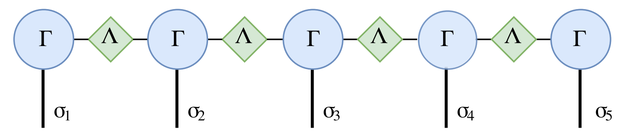


Figure 1: Open Boundary Condition MPS in $\Gamma\Lambda$ notation

- DMRG constitutes the state of the art for the study of one dimensional quantum lattices. The study of the effectiveness of the DMRG algorithm leads naturally to the concept of Matrix Product States(MPS)
- The dimension of the product Hilbert space of a spin chain grows exponentially in its length ($\mathcal{O}\left(d^L\right)$ where d is the local Hilbert space dimension). This presents a problem for direct numerical methods.
- However, by an entanglement argument [3], we can see that the interesting corner of the total Hilbert space is very small compared to the total space, and in fact this subspace permits an effective numerical description in terms of Matrix Product States.
- An arbitrary quantum state can be written

$$|\psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L} C_{\sigma_1,\ldots,\sigma_L} |\sigma_1,\ldots,\sigma_L\rangle\,, \tag{4}$$

Where $|\sigma_i\rangle$ is the local basis of the *ith* space.

- Via an iterated Singular Value Decomposition (SVD) of the tensor reshaped into a matrix, we can express eq.4 exactly as

$$|\psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L} \Gamma^{\sigma_1}\Lambda_1\ldots\Lambda_{L-1}\Gamma^{\sigma_L} |\sigma_1,\ldots,\sigma_L\rangle \tag{5}$$

Where Γ^{σ_i} denotes an indexed set of matrices, and Λ_i represents the singular values in the ith step of the decomposition.

- The index structure of the Γ matrices in each of these (exact) expressions goes $(1\times d), (d\times d^2), \ldots (d^2\times d), (d\times 1)$ i.e. the largest matrix size grows exponentially with the length of the chain L.

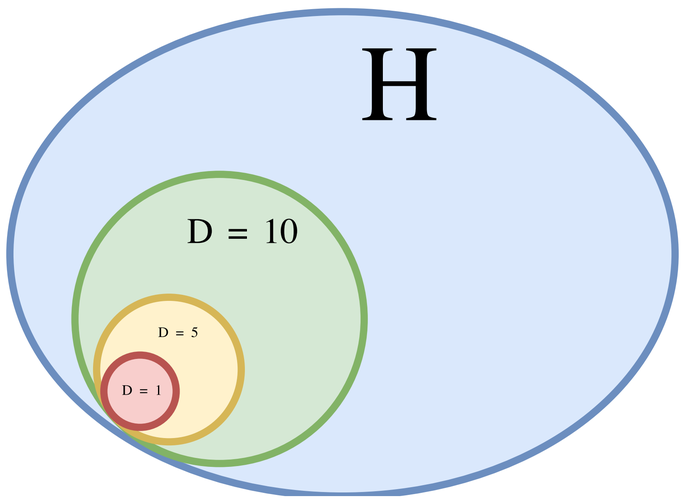


Figure 2: Size of subset of Hilbert space H with bond dimension D grows with D

- Important corner of Hilbert space is spanned by Matrix Product states of form 5, with all but D of the singular values on each site set to zero (where D is small)

> Trajectory Ensembles

- By analogy to the ensembles of microstates of equilibrium statistical mechanics, in the study of glass forming systems (and elsewhere) ensembles of system trajectories have been studied [2].

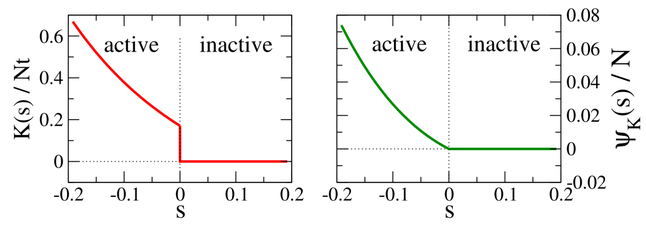


Figure 3: Dynamical phase transition in a kinetically constrained model. $\mathcal{K}(s)$ is the first derivative of $\psi_K(s)$ the dynamical free energy[1]

- Starting from Markovian dynamics, the trajectories are biased (as in the canonical ensemble) by the value of some time extensive observable on that trajectory.
- At the level of the master equation, this corresponds to a deformation of the operator $\mathbb{W} \to \mathbb{W}_A$ in the vector form (s is a biasing field, the analogue of β the inverse temperature)

$$\frac{\partial \vec{P}_A}{\partial t} = \mathbb{W}_A(s)\vec{P}_A, \tag{6}$$

and from there for the dynamical analogue of the partition function

$$\mathcal{Z}_A(s,t) \sim e^{t\psi_A(s)} \tag{7}$$

where $\psi_A(s)$ is the largest eigenvalue of the matrix $\mathbb{W}_A(s)$ and the *dynamical free energy*.

- Singular values in the dynamical free energy signal *dynamical phase transitions* from between regions of qualitatively different dynamical behaviour i.e. transition to chaos, jamming in glasses.
- An example of a class of systems for which there are results on dynamical phase transitions are *kinetically constrained models* (KCMS) of glass formers (4)

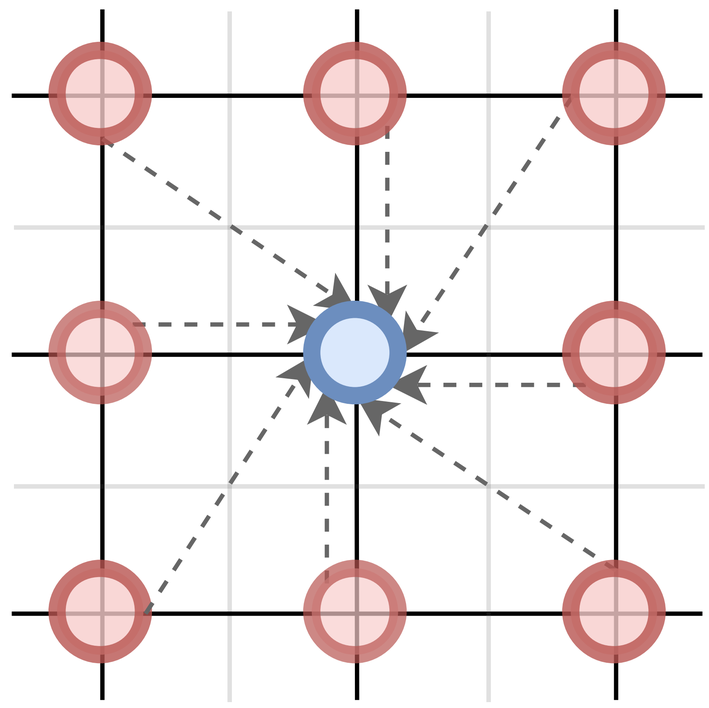


Figure 4: Schematic of site of Kinetically Constrained Model. Site i (blue) transitions from $n_i \to 1-n_i$ with rate $W(n_i \to 1-n_i) = C(\{n_j\})\frac{e^{\beta(n_i-1)}}{1+e^{-\beta}}$, where $C(\{n_j\})$ is a function only of the values of sites n_j (red)

> References

References

[1] J. P. Garrahan, R. L. Jack, V. Lecomte, E. Pitard, K. van Duijvendijk, and F. van Wijland. Dynamical first-order phase transition in kinetically constrained models of glasses. *Phys. Rev. Lett.*, 98:195702, May 2007.

[2] Juan P Garrahan, Robert L Jack, Vivien Lecomte, Estelle Pitard, Kristina van Duijvendijk, and FrÃľdÃľric van Wijland. First-order dynamical phase transition in models of glasses: an approach based on ensembles of histories. *Journal of Physics A: Mathematical and Theoretical*, 42(7):075007, 2009.

[3] Román Orús. A practical introduction to tensor networks: Matrix product states and projected entangled pair states. *Annals of Physics*, 349:117–158, oct 2014.

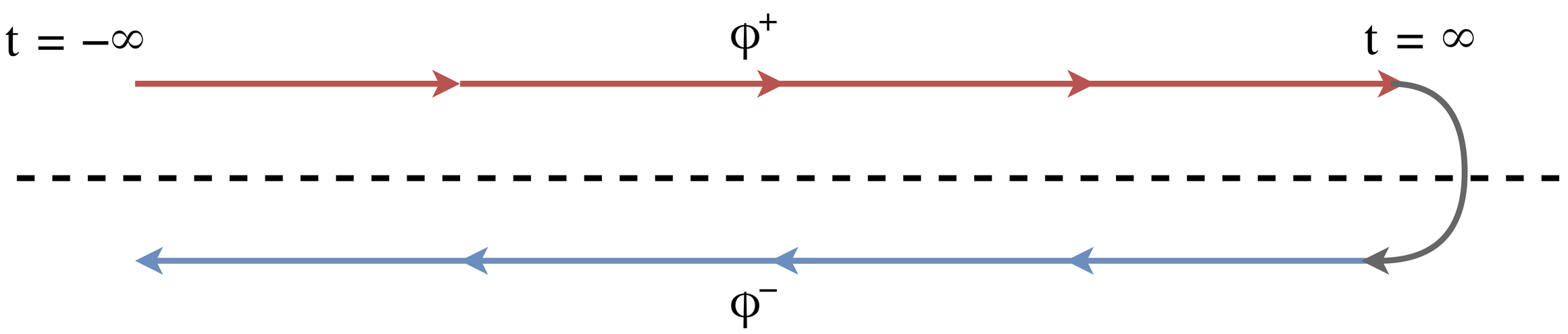


Figure 5: Keldysh closed time contour