

> Introduction

- Technological potential of quantum systems due to exponential state space scaling.
- Environmental decoupling challenges effective use of quantum technology.
- Tools from classical statistical mechanics?

> Trajectory Ensembles

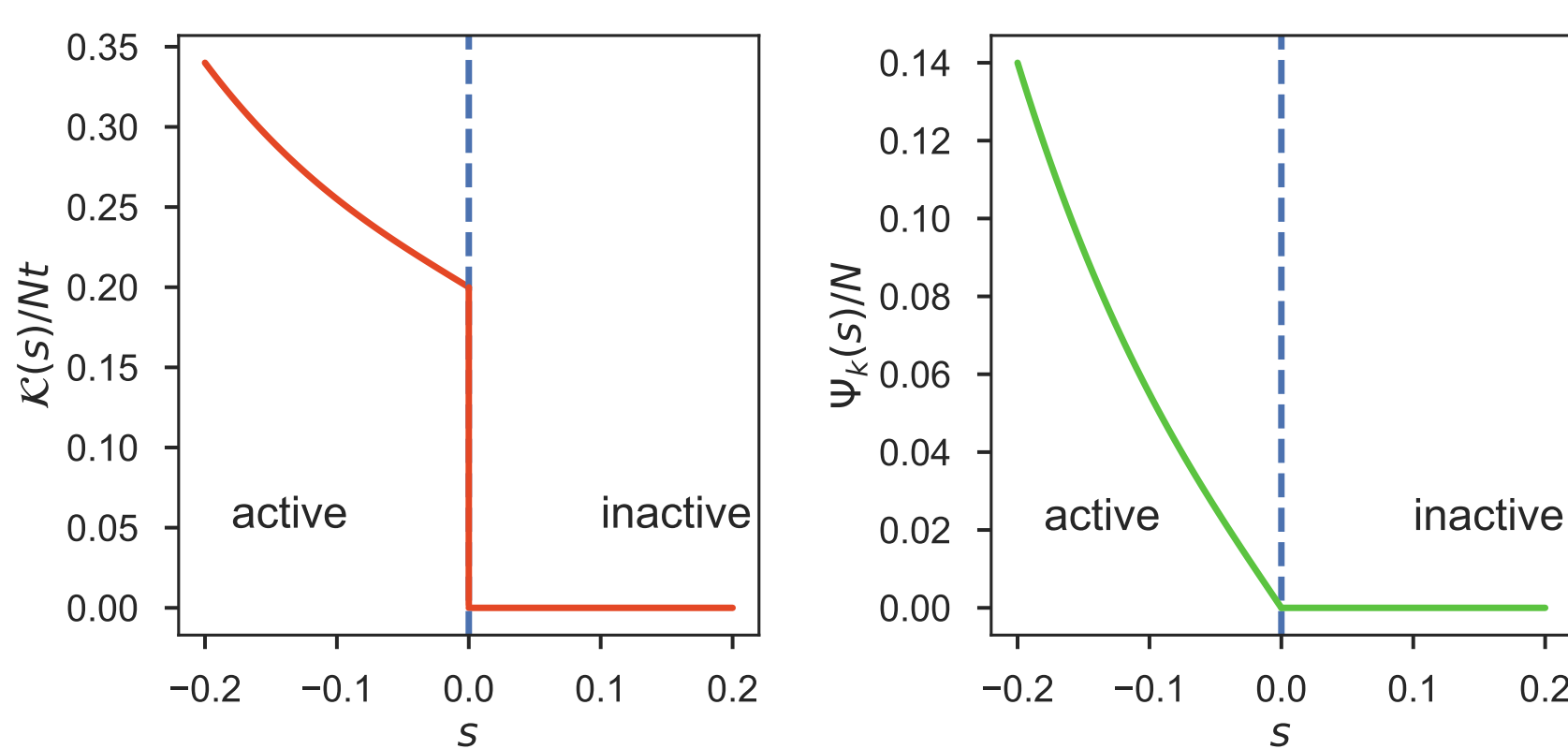


Figure 1: Dynamical phase transition in KCM. $\mathcal{K}(s)$ is the first derivative of $\psi_K(s)$ the dynamical free energy[2]

- Ensembles of trajectories - like thermal ensembles. Partition function: [2].

$$\mathcal{Z}_A(s, t) = \sum_A \Omega_{dyn}(A, t) e^{-sA} \quad (1)$$

- Markovian, paths biased (like canonical ensemble) by time extensive observable A (E) with strength s (β). $\mathbb{W} \rightarrow \mathbb{W}_A$ in

$$\frac{\partial \vec{P}_A}{\partial t} = \mathbb{W}_A(s) \vec{P}_A, \quad (2)$$

leads to

$$\mathcal{Z}_A(s, t) \sim e^{t\psi_A(s)} \quad (3)$$

with $\psi_A(s)$ largest eigenvalue of $\mathbb{W}_A(s)$: *dynamical free energy*.

- Singularities in dynamical free energy \rightarrow *dynamical phase transitions*: different dynamical behaviour i.e. transition to chaos, jamming in glasses.
- Dynamical phase transitions in *kinetically constrained models* (KCMS) of glass formers (fig. 2)

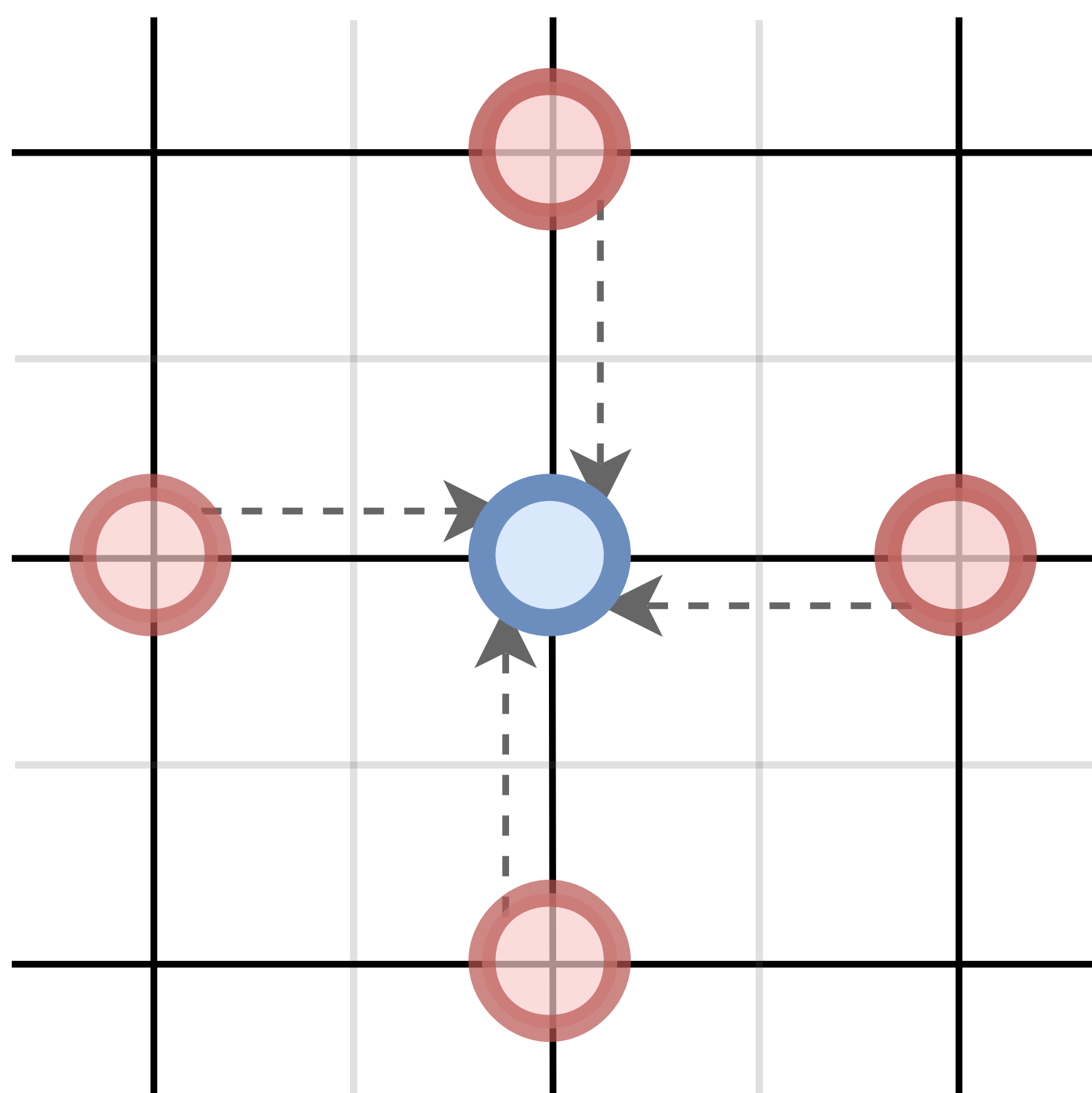


Figure 2: Schematic of site of Kinetically Constrained Model. Site i (blue) transitions from $n_i \rightarrow 1 - n_i$ with rate $W(n_i \rightarrow 1 - n_i) = C(\{n_j\}) \frac{e^{\beta(n_i-1)}}{1+e^{-\beta}}$, where $C(\{n_j\})$ is a function only of the values of sites n_j (red)

> Aims

- Classical glasses \rightarrow KCMS \rightarrow *biased trajectory ensembles*.
- Keldysh theory \rightarrow dissipation: analogous to bias in trajectory ensembles?
- Consider adiabatic quantum computation. Spin chains \rightarrow Matrix Product States
- Transition from usable quantum resources (i.e. entanglement) to absence.

> Matrix Product States

- Interesting bit of most Hilbert spaces small, permits efficient numerics.
- Many-body quantum state

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} C_{\sigma_1, \dots, \sigma_L} |\sigma_1, \dots, \sigma_L\rangle,$$

- MPS decomposition (Λ diagonal)

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} \Gamma^{\sigma_1} \Lambda_1 \dots \Lambda_{L-1} \Gamma^{\sigma_L} |\sigma_1, \dots, \sigma_L\rangle$$

- Important bit of Hilbert space spanned by MPS with Λ truncated to D dims.
- D grows with time evolution. Project back to D subspace - classical EOM for variational parameters.

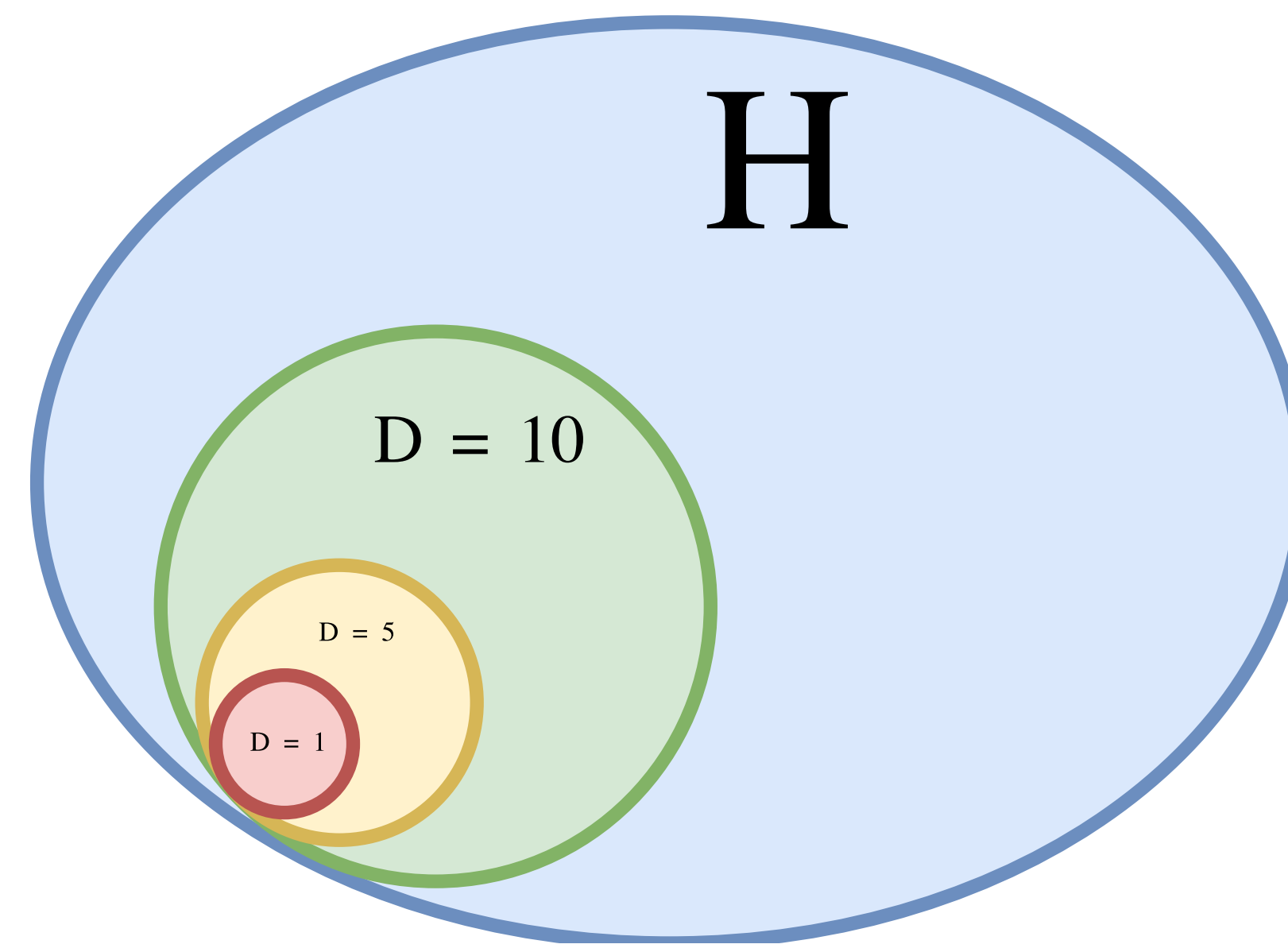


Figure 3: Size of subset of Hilbert space H with bond dimension D grows with D

> Non-Equilibrium QFT: Keldysh Theory

- Evolve the state forward in time ($-\infty \rightarrow \infty$), rewind $\infty \rightarrow -\infty$: fig. 6
- Separate fields on forward and backwards contours (ϕ^+, ϕ^-)
- Construct the path integral \rightarrow redundancy in the Green's Functions.
- Combinations such that $\langle \phi^{cl} \phi^{cl} \rangle = 0$

$$\phi^q = \phi^+ - \phi^- \quad \phi^{cl} = \phi^+ + \phi^- \quad (4)$$

- Introduce a bath interacting with system, integrate out bath, generate semi-classical path integrals for quantum systems.
- Distribution over paths. Connected to the classical trajectory ensemble?

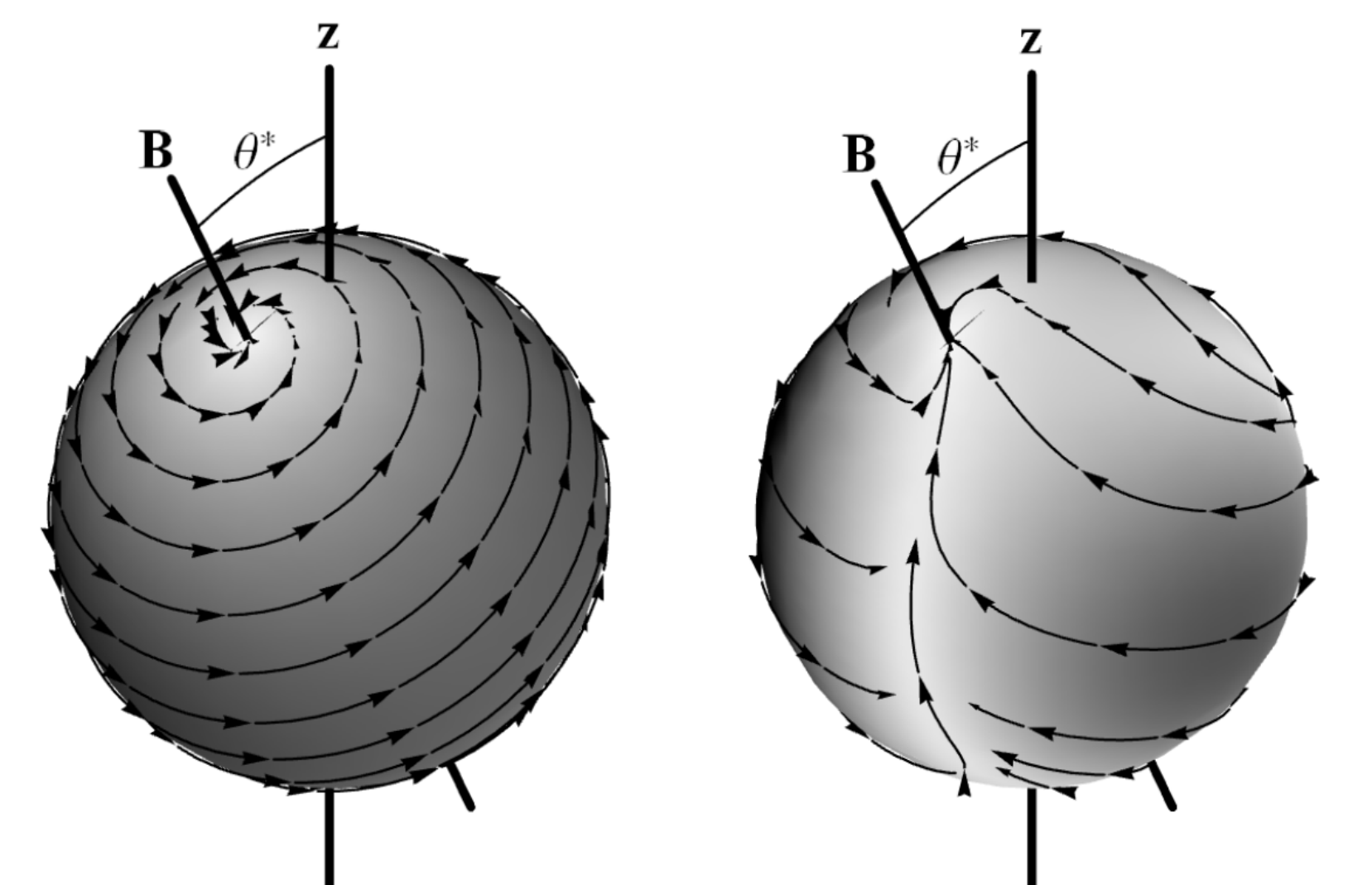


Figure 4: Trajectories of effective classical spins of qualitatively different dynamical forms [1]

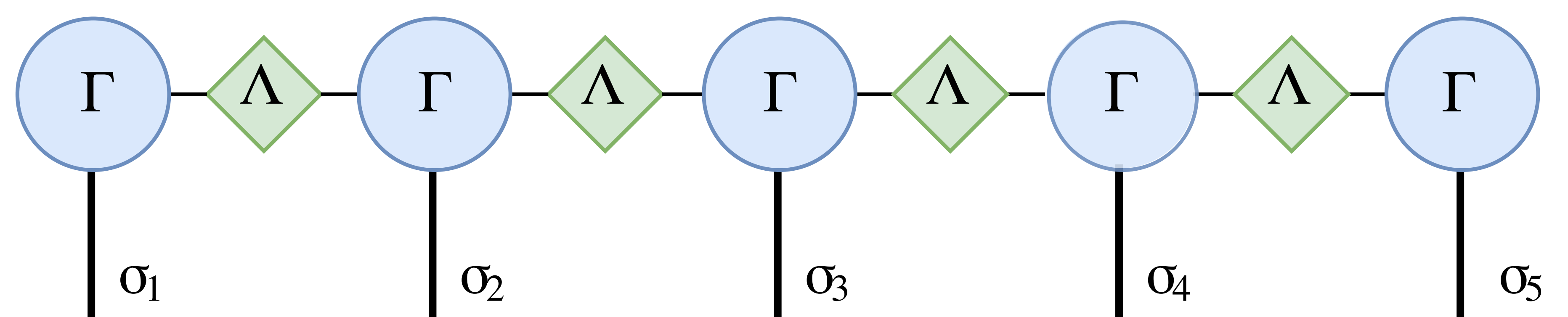


Figure 5: 5-site Open Boundary Condition MPS in $\Gamma\Lambda$ notation

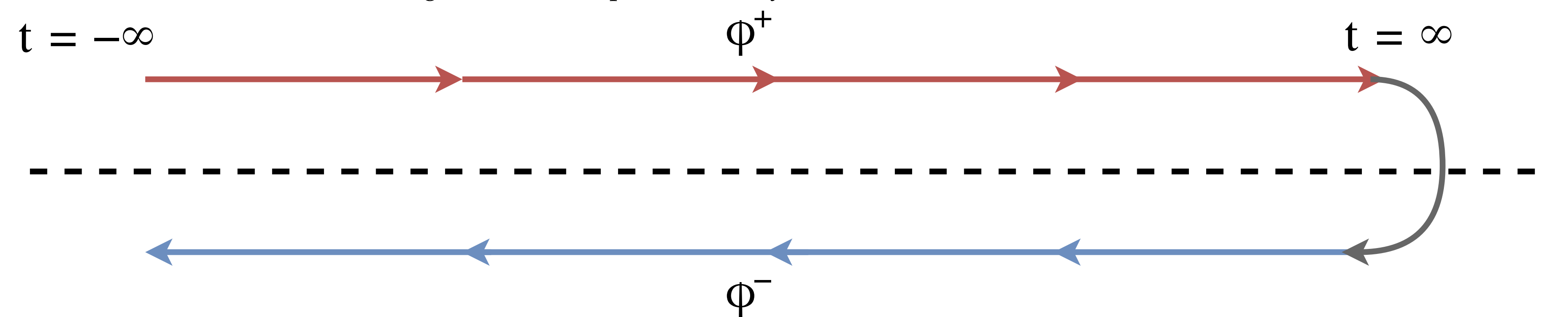


Figure 6: Keldysh Contour

References

- [1] P. J. D. Crowley and A. G. Green. Anisotropic landau-lifshitz-gilbert models of dissipation in qubits. *Phys. Rev. A*, 94:062106, Dec 2016.
- [2] J. P. G. et al. First-order dynamical phase transition in models of glasses: an approach based on ensembles of histories. *Journal of Physics A: Mathematical and Theoretical*, 42(7):075007, 2009.