



This project will apply techniques developed for studying rare fluctuations in non-quantum systems to create novel ways of understanding the effects of dissipation on the dynamics of quantum systems. In particular, we explore the trajectory ensemble approach that was developed in the study of classical glasses. In this approach, the ensemble of trajectories that a system may take is biased – the strength of the bias potentially driving transitions between jammed glassy behaviour and liquid behaviour. Dissipative effects of the environment have much in common with this trajectory biasing. At its crudest level, a binding brake on a car can bias the trajectory of the car. Similarly, dissipation in quantum systems can favour trajectories in certain directions in Hilbert space. Moreover, these may drive a transition from the presence of desired quantum properties to their absence. We will investigate the application of methods and ideas developed in classical out of equilibrium systems (where Prof. Sollich’s main expertise lies, e.g. in the area of biased trajectory ensembles) to the quantum problem (Prof. Andrew Green’s expertise, including variational approximations and tensor networks).

Trajectory Ensembles

Starting from Markovian dynamics, the trajectories are biased (as in the canonical ensemble) by the value of some time extensive observable on that trajectory. At the level of the master equation, this corresponds to a deformation of the operator $\mathbb{W} \rightarrow \mathbb{W}_A$ in the vector form (s is a biasing field, the analogue of β the inverse temperature)

$$\frac{\partial \vec{P}_A}{\partial t} = \mathbb{W}_A(s) \vec{P}_A, \quad (1)$$

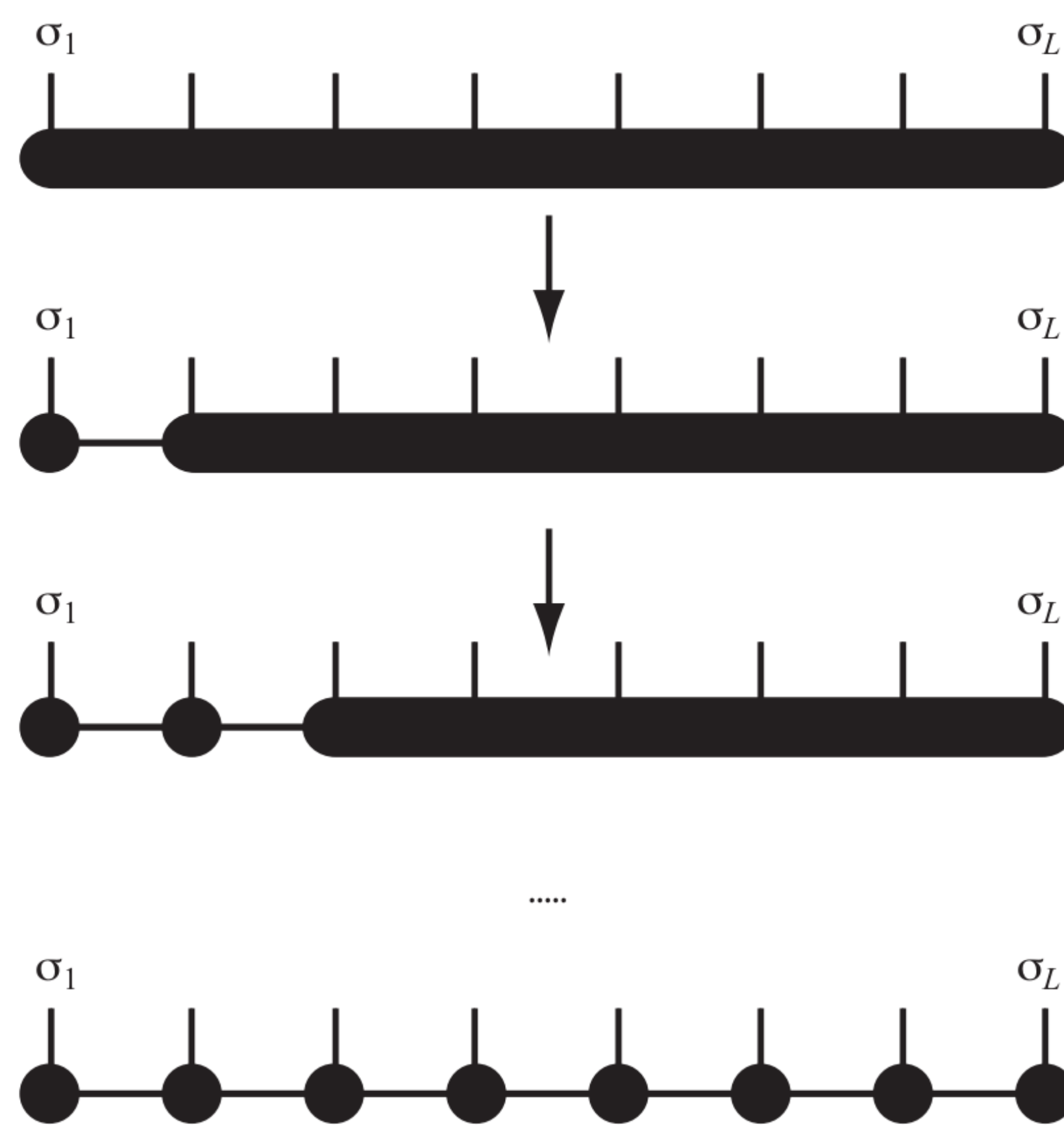
$$\mathcal{Z}_A(s, t) \sim e^{t\psi_A(s)} \quad (2)$$

where $\psi_A(s)$ is the largest eigenvalue of the matrix $\mathbb{W}_A(s)$.

Matrix Product States

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DMRG constitutes the state of the art for the study of one dimensional quantum lattices. The study of the effectiveness of the DMRG algorithm leads naturally to the concept of Matrix Product States(MPS)

The dimension of the product Hilbert space of a spin chain grows exponentially in its length ($\mathcal{O}(d^L)$ where d is the local Hilbert space dimension). This presents a problem for direct numerical methods.

An arbitrary quantum state can be written

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} C_{\sigma_1, \dots, \sigma_L} |\sigma_1, \dots, \sigma_L\rangle, \quad (3)$$

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} a_{\sigma_1} \dots c_{\sigma_L} |\sigma_1, \dots, \sigma_k\rangle \quad (4)$$

Via an iterated Singular Value Decomposition (SVD) of the original tensor reshaped into a matrix, we can express eq.3 exactly as

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} A^{\sigma_1} \dots A^{\sigma_L} |\sigma_1, \dots, \sigma_L\rangle \quad (5)$$

$$|\psi\rangle = \sum_{\sigma_1, \dots, \sigma_L} \Gamma^{\sigma_1} \Lambda_1 \dots \Lambda_{L-1} \Gamma^{\sigma_L} |\sigma_1, \dots, \sigma_L\rangle \quad (6)$$

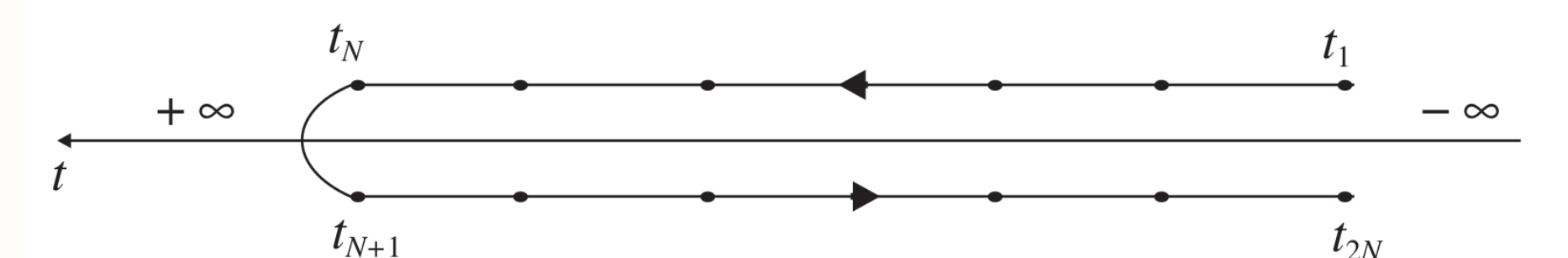
The index structure of the (A and Γ) matrices in each of these (exact) expressions goes $(1 \times d), (d \times d^2), \dots (d^2 \times d), (d \times 1)$.

Along this line, any state in the total Hilbert space can be systematically approximated by a Matrix Product state of form 6, with all but D of the singular values on each site set to zero (for Λ_i of dimension $\geq D$)

The 'bond dimension' (D) of the MPS can be used to systematically introduce entanglement structure ($D = 1$ are the product states).

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Keldysh Theory



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