UNIVERSITY OF OTAGO EXAMINATIONS 2015

COMPUTER SCIENCE

COSC342

Computer Graphics

Semester 1

(TIME ALLOWED: THREE HOURS)

This examination comprises 7 pages.

Candidates should answer questions as follows:

Candidates must answer **all** questions.
Questions are worth various marks are shown thus: (5)

The total number of marks available for this examination is 60.
You should keep your answers short.
In general, if there are two marks for an answer, you should have two things to say.

The following material is provided:

Nil.

Use of calculators:

No calculators are permitted.

Candidates are permitted copies of:

Nil.

1. Coloured light and pigments can be mixed to make new colours.

   (a) What colour would you get if you mixed blue and yellow *ink* together?

   (1)

   (b) What colour would you get if you mixed blue and yellow *light* together?

   (1)

   (c) Your answers to (a) and (b) should be different, explain why.

   (2)

2. The following questions refer to the image (left), with greyscale pixel values, and the filter kernel (right) shown below.

8	7	6	4	1
9	7	5	2	1
7	5	3	2	1
7	6	4	2	1
9	9	5	4	4

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

   (a) Describe the effect you expect the filter to have when applied to an image.

   (1)

   (b) Show how the result of the filter is computed when it is applied to the central pixel (with value 3) of the image.

   (3)

   (c) What problem arises when you try to apply the filter at the top left corner of the image (with pixel value 8)?

   (1)

   (d) Suggest one way in which this problem can be avoided.

   (1)

3. A basic line drawing algorithm is given below:

```
function sign(x) {
  if (x < 0) return -1;
  if (x > 0) return 1;
  return 0;
}

// Line from (x0, y0) to (x1,y1)
int dx = x0 - x1;
int dy = y0 - y1;
double err = 0;
double derr = abs(double(dy)/double(dx));
int y = y0;
for (int x = x0; x <= x1; ++x) {
  paint(x,y);
  err += derr;
  if (err > 0.5) {
    y += sign(dy);
    err -= 1;
  }
}
```

This code works well for a line from $(0, 0)$ to $(2, 1)$, but has problems in some other cases. These problems fall into two main categories.

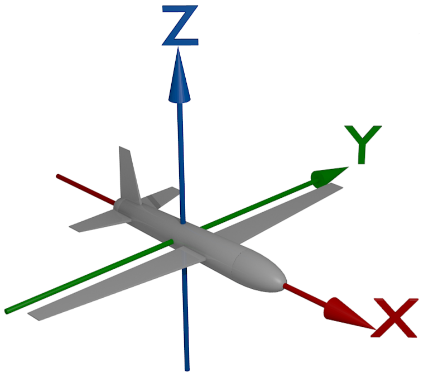
(a) What problem arises if you use this code to draw a line from $(0, 0)$ to $(-2, 1)$? (1)

(b) How could the code be modified to solve this problem? (1)

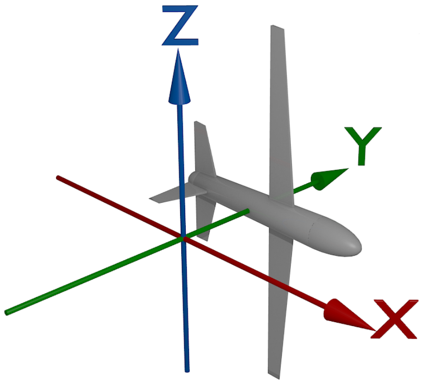
(c) What other problem arises if you use this code to draw a line from $(0, 0)$ to $(1, 2)$? (1)

(d) How could the code be modified to solve this second problem? (1)

4. The images below show a model of an aeroplane before (**A**) and after (**B**) a transformation.



(**A**)



(**B**)

(a) Describe the rotation and translation necessary to transform the aeroplane from (**A**) to (**B**). You may assume that the distance the aeroplane has moved is 1 unit. (2)

(b) Write down the homogenous matrices that represent these operations. (2)

(c) Multiply these matrices in the correct order to create a single matrix that represents the transformation from (**A**) to (**B**). (2)

5. When we solve simultaneous equations to find the intersection of the ray $\mathbf{p}(t) = \mathbf{u}+t\mathbf{v}$ with the unit sphere at the origin, the solutions for t are:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where:

$$\begin{aligned} A &= \mathbf{v}^2 \\ B &= 2\mathbf{u} \cdot \mathbf{v} \\ C &= \mathbf{u}^2 - 1.0 \end{aligned}$$

Depending on the primary ray that is chosen (i.e. for a given $\mathbf{u}$ and $\mathbf{v}$), any of the following situations can occur:

(a) no (real number) solutions for t;

(b) one negative solution for t;

(c) one non-negative solution for t;

(d) two negative solutions for t;

(e) one negative and one non-negative solution for t; or

(f) two non-negative solutions for t.

For each of these cases, describe what is happening geometrically for the interaction between the ray and the sphere. (6)

6. For a ray $\mathbf{p}(t) = \mathbf{u} + t\mathbf{v}$ intersecting with the plane with normal vector $\mathbf{n}$ that contains the point $\mathbf{q}$, the t value at the hit-point is given as:

$$t = \frac{\mathbf{n} \cdot (\mathbf{q} - \mathbf{u})}{\mathbf{n} \cdot \mathbf{v}}$$

Consider a primary ray travelling in the direction of the negative Y axis, from the point $(0, 9, 0)$. The scene contains a single object: a plane with normal vector $(1, 1, 0)$ that contains the point $(-1, 0, 0)$.

Showing your working, determine the t value at the hit-point.

(4)

7. In lectures we derived the following local illumination model:

$$I_{total} = I_a k_a + I_j \left(k_d(\hat{\boldsymbol{\ell}}_j \cdot \hat{\mathbf{n}}) + k_s(\hat{\mathbf{e}} \cdot \hat{\mathbf{r}}_j)^n \right)$$

Explain the role of the $\hat{\boldsymbol{\ell}}_j$, $\hat{\mathbf{n}}$, $\hat{\mathbf{e}}$, and $\hat{\mathbf{r}}_j$ terms within this equation.

(4)

8. It is common to approximate curved objects using a triangle mesh. In lectures we explained how Gouraud and Phong shading can be used to smooth the appearance of these sorts of mesh faces.

(a) Describe a benefit of Phong shading over Gouraud shading.

(1)

(b) Give pseudocode for the Phong shading algorithm.

(4)

(c) What is bump mapping?

(1)

(d) How would you modify your algorithm in part (b) to account for bump mapping?

(2)

9. Images in a mosaic are related by a *homography*. Given a point, (x, y) in one image, and the corresponding point (x', y') in a second image, the homography relates the points' locations as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \equiv H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix},$$

where H is a 3×3 matrix.

(a) Not all pairs of images of a scene can be related in this way. Under what circumstances does the homography relationship exist? (2)

(b) The homography relationship is an equivalence ($\equiv$), rather than an equality. What does this mean? (1)

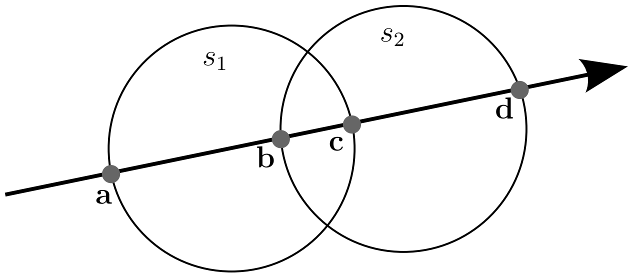
(c) Each pair of corresponding points gives 2 independent equations where the unknown values are the 9 elements of H. How many point correspondences are required to estimate the homography? (1)

10. In order to estimate a homography it is necessary to establish a correspondence between feature points in two images.

(a) Simple feature matching might use the pixel values in a small window around a feature point to describe the features. Give two reasons why this may not be sufficient to correctly match features between images. (2)

(b) When automatically matching features between images it is common for incorrect (or outlier) matches to be made. Briefly explain how RANSAC can be used to overcome this problem when computing a homography for mosaicing. (4)

11. Consider a ray passing through spheres s_1 and s_2 as shown below.



We will use constructive solid geometry (CSG) to subtract sphere s_1 from sphere s_2. The four mathematical hit-points are labeled $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$.

Progressing along the hit-points from $\mathbf{a}$, describe how a ray tracer would determine whether or not that hit-point is inside, outside, or on the border of the object created as the result of the CSG subtraction.

(4)

12. Texture mapping is often performed by mapping vertex locations into a 2D uv-space.

(a) Describe an example scene where undersampling of textures would be observed and what effect that would cause within the rendered image.

(2)

(b) Explain why it is not possible to oversample a texture defined by a continuous function $t(u, v)$.

(2)