REVIEW ARTICLE

Path planning and collision avoidance for autonomous surface vehicles II: a comparative study of algorithms

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Abstract

Artificial intelligence is an enabling technology for autonomous surface vehicles, with methods such as evolutionary algorithms, artificial potential fields, fast marching methods, and many others becoming increasingly popular for solving problems such as path planning and collision avoidance. However, there currently is no unified way to evaluate the performance of different algorithms, for example with regard to safety or risk. This paper is a step in that direction and offers a comparative study of current state-of-the art path planning and collision avoidance algorithms for autonomous surface vehicles. Across 45 selected papers, we compare important performance properties of the proposed algorithms related to the vessel and the environment it is operating in. We also analyse how safety is incorporated, and what components constitute the objective function in these algorithms. Finally, we focus on comparing advantages and limitations of the 45 analysed papers. A key finding is the need for a unified platform for evaluating and comparing the performance of algorithms under a large set of possible real-world scenarios.

Keywords Autonomous surface vehicle (ASV) · Path planning · Collision avoidance · Algorithms · Safety

1 Introduction

There is growing appeal for autonomous systems in multiple fields, including manufacturing, transportation, routine work, and work in dangerous environments. In the wake of progress in the domain of autonomous cars, much attention is also given to autonomous surface vehicles (ASVs). In an accompanying article in this journal [1], we present a review on theory and methods for path planning and collision avoidance of ASVs. We attempt to unify and clarify relevant terminology and concepts such as autonomy and safety, as well as models for guidance, navigation, and control. Moreover, we propose a classification scheme for distinguishing and comparing algorithms for path planning and collision avoidance.

Here, we extend this scheme to classify state-of-the-art algorithms presented in 45 different peer-reviewed scientific papers. Several kinds of algorithms are covered, including evolutionary algorithms, sampling-based algorithms, cell decomposition methods, directional approaches, and roadmap methods. We have also included some algorithms for unmanned surface vehicles (USVs).

As for any literature review paper, it is impossible to cover everything in the literature within the scope of a single paper. The number of papers studied before arriving at the shortlist of the 45 papers presented here is probably in the ballpark of several hundreds. We have carefully selected papers that we ultimately found useful to include.

Moreover, whereas much of what we present is general across vessel size, other considerations will differ whether the vessel is a small boat or a large ship. In such cases, the reader should note that larger ships are our main focus. Likewise, although some elements of path planning and collision avoidance are common across congested waters and open sea, we are mainly concerned with shorter time frames and congested waters in the papers we study here.

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Table 1 Timeline of the first time use of dominating algorithms for USV/ASV guidance applications

Year	Algorithm	References
1999	Genetic algorithm (GA)	[2]
2001	Fuzzy logic	[3]
2008	A*	[4]
2008	Rapidly-exploring random tree (RRT)	[4]
2010	Ant colony optimization (ACO)	[5]
2012	Particle swarm optimization (PSO)	[6]
2012	Dijkstra	[7]
2013	Voronoi diagram	[8]
2014	Velocity obstacles (VO)	[9]
2015	Artificial potential field (APF)	[10]
2015	Fast marching method (FMM)	[11]
2018	Deep reinforcement learning (DRL)	[12]

The rest of the paper is organised as follows: Sect. 2 provides a timeline of some of the most influential algorithms for path planning and collision avoidance for ASVs or USVs. Section 3 extracts distinguishing properties of the algorithms from the literature, and analyses and compares papers based on these properties. Section 4 analyses the proposed algorithms based on their properties whilst focusing mainly on two aspects: (1) safety and collision risk assessment (CRA), and (2) choice of objective function. Section 5 extracts the advantages and limitations of the algorithms used in the different papers. Finally, Sect. 6 presents a discussion, whilst some concluding remarks are drawn in Sect. 7.

2 Timeline of algorithms

The first use of some of the most influential algorithms used for path planning and collision avoidance for ASVs or USVs is shown in Table 1. Notably, these algorithms have also been successfully used at earlier times for guidance of autonomous underwater vehicles (AUVs), unmanned aerial vehicles (UAVs), or autonomous ground vehicles (AGVs). Note that Table 1 is by no means an exhaustive list but highlights some dominating algorithms that have been commonly employed, directly or in some derivative form, or in combination with others.

3 Properties of algorithms

Although some algorithms in the literature clearly separate the tasks of path planning and collision avoidance, others do not, and attempt to solve both problems with overlapping modules [1]. Furthermore, it is generally not easy to compare path planning and collision avoidance algorithms for ASVs due to the variety of constraints and objectives that exist. One example is the use of regulations such as the International Regulations for Preventing Collisions at Sea (COLREGs) [13]: whereas some algorithms successfully generate paths for avoiding obstacles whilst simultaneously obeying several COLREG Rules [e.g., 14–18], others fully or partially ignore these regulations [e.g., 11, 19, 20–22]. For adoption in the future, fully autonomous surface vessels must comply with all the rules of COLREGs. We appreciate, however, that algorithms that comply only with a subset of COLREGs are still a step towards this goal and a contribution towards full COLREGs compliance in the future.

The literature analysis in Vagale et al. [1] shows that there are several properties of path planning and collision avoidance algorithms that can be used for classification and analysis of the algorithms:

– *Compliance with COLREGs*: partial/full consideration of COLREGs for collision avoidance.
– *Environmental disturbances*: taking into account wind, waves, currents, and tides.
– *Planning type*: global and/or local planning.
– *Obstacle type*: whether a vessel can deal with static and/or dynamic obstacles (including single or multiple encounter situations at the same time).
– *Environment type*: discrete or continuous environment.
– *Type of avoidance action*: course change or speed change, or a combination of both.
– *Testing of algorithm*: simulation or field test.
– *Traffic category*: congested waters (areas crowded with static/dynamic obstacles, including harbour areas, lead to low own vessel speed), open waters (minimal number of static and dynamic obstacles, lead to high own vessel speed), riverines (manoeuvring is limited, current is present), and coastal areas (mostly static obstacles, such as land, islands, and shallow water, lead to varying speed).
– *Predictability of environment*: known or unknown environment.
– *Planning time*: real-time (online) or offline.
– *Control horizon*: infinite or receding horizon control.
– *Number of encountered obstacles*: single or multiple target vessel encounter situations.
– *Vessel dynamics and kinematics*: maximum ship turning rate, maximum vessel speed, other vessel's motion constraints, torque of the vessel, etc.
– *Subject of research*: type of the researched vessel or system [ASV, USV, and decision support system (DSS)].
– *Safe zones*: safety margin, virtual safety zone, ship domain, ship arena, or circle-of-rejection, around the own vessel or static/dynamic obstacles.

Note that Tsou and Hsueh [5] define ship domain as "the sea around a ship that the navigator would like to keep free of other ships and fixed objects." This criterion has been widely used in ships' collision avoidance, marine traffic simulation, calculation of encounter rates, vehicle tracking system (VTS) design, and so forth. It differs from ship arena, which is a bigger area around the vessel used to determine the time of taking collision avoidance actions [23]. Similarly, a safety zone can be assumed around each obstacle instead of the own ship, called the circle-of-rejection (COR) [24].

Based on the aforementioned properties, eight properties have been chosen for a comparative study of 45 papers containing algorithms for path planning and collision avoidance of ASVs (see Table 2). The choice of these eight properties is based on the most common available, and relevant, information in algorithm descriptions. Some other properties were neglected due to many papers excluding the very same information regarding such properties. The proposed comparison is an attempt to analyse these state-of-the art algorithms and benchmark them using the proposed criteria. Table 3 compares the ship- and environment-related properties across the chosen papers. The algorithms in the comparison of Table 3 are grouped in three groups, separated by white space, based on the "planning type" property. Each row of the table includes the paper reference ('Ref.'), the type of path planning, and/or collision avoidance algorithm(s) employed, followed by an analysis of how the 8 properties in Table 2 are taken into account.

Although the focus of this study is on methods for ASVs, papers related to USVs are also considered. The databases used for finding journal and conference papers were IEEE Xplore Digital Library and ScienceDirect. Additionally, the NTNU library was consulted using the search tool Oria, as well as suggestions from the reference organisation tool Mendeley. The keywords used for search were "ASV," "USV," "autonomous ships," "path planning," "collision avoidance," and "guidance." The papers included in the comparison are from the years 2010–2020, and the language was limited to English. The distribution of the analysed papers over the years is represented in Fig. 1. The number of papers with respect to each of the eight selected properties is represented graphically in Fig. 2.

We discuss each of the eight properties P1–P8 in turn, before making some general observations, mainly with reference to Table 3 and Fig. 2.

P1. Planning type: The analysis of the selected papers shows that 13 (29%) of the examined algorithms perform global planning and, hence, are mainly concerned with path planning; 17 (38%) algorithms perform local planning and collision avoidance; and 15 (33%) algorithms perform both global and local planning. We also found that in most of the cases, local planning is performed in real time, whereas global planning is often performed offline, prior to
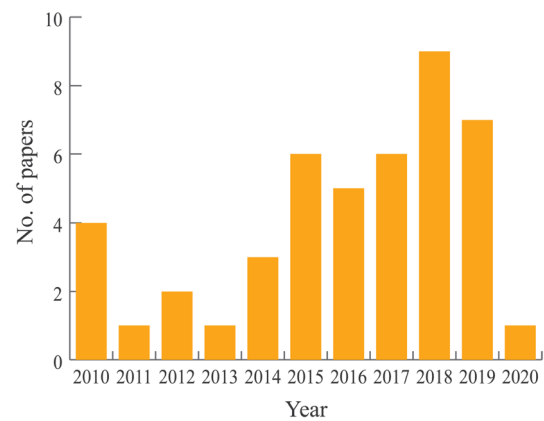

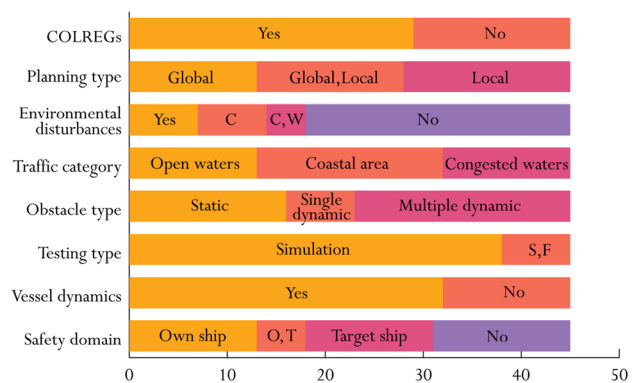**Fig. 1** Annual distribution of the papers

Fig. 2 Distribution of properties among the papers

departure. In the hybrid cases, when both local and global planning is used, the algorithm is generally a combination of both real-time and offline planning and covers both path planning and collision avoidance. Hence, with this close correlation between local/real-time planning and global/offline planning, a separate property of the algorithm being real time or offline is not considered necessary in Table 3.

P2. COLREGs: The comparison table shows that compliance with COLREGs is included only in the path planning approaches that consider local path planning and collision avoidance (algorithms with property GL and L). Most often, algorithms take into consideration only up to four of the main encounter situations, described in the three COLREG Rules 13–15 [e.g., 40, 54]. These rules are usually implemented as constraints in algorithms and indicate which collision avoidance scenario should be used in the current situation. Eriksen et al. [41], on the other hand, have implemented a cost function penalising gentle turns and small speed changes for obeying COLREG Rule 8, which states that "action taken to avoid collision should be positive, obvious and made in good time." Hence, the ASV's behaviour should be obvious and makes sense to human captains.

Table 2 Selection of algorithm properties

#	Property	Categories
P1	Planning type	Local (L), global (G), both (GL)
P2	Compliance with COLREGs	Yes, no
P3	Traffic category	Open waters (OW), coastal area (CA), congested waters (CW)
P4	Obstacle type	Static (S), single dynamic (D1), multiple dynamic (Dn)
P5	Testing type	Simulation (S), field test (F)
P6	Consideration of environmental disturbance	Current (C), wind (Wn), waves (Wv), existing but unknown (Unk), no
P7	Consideration of vessel dynamics	Yes, no
P8	Presence of safety domain	Vessel safety domain (O), target vessel/obstacles safety domain (T), no

Szlapczynski [39] has proposed an extended method that additionally focuses on COLREG Rule 19, planning the path in restricted visibility conditions. Johansen et al. [53] additionally have also implemented several other COLREG Rules, namely 8, 16, 17, and 18. These rules have been implemented as components of the cost function or as penalty functions. Some papers emphasise that, according to good seamanship practice, course change is preferred over speed change in collision avoidance scenarios [38, 49].

P3. Traffic category: Concerning the traffic categories considered in the papers, one part of the papers focuses on the "open waters" category (13 papers, or 29%), considering an area free from static obstacles such as land and islands. The same amount of papers are dealing with "congested waters" category (13 papers, or 29%) where the traffic most often is busy, such as harbour areas, where both multiple dynamic obstacles and static obstacles are present. However, most of the papers are considering the "coastal area" type of environment/traffic (19 papers, or 42%), where the environment is mostly cluttered with several static obstacles, but there is almost no presence of dynamic obstacles.

P4. Obstacle type: The analysed papers consider different types of obstacles. In the simplest cases, 16 (36%) of papers use algorithms that avoid only static obstacles, including land, islands, and underwater objects. Most of these algorithms are global path planning approaches. For dynamic obstacles, 29 (64%) of the papers consider moving target vessels, underwater vehicles, and icebergs, with 7 (16%) considering single dynamic obstacle situations and 22 (49%) considering more complicated situations involving avoidance of multiple dynamic obstacles. The high number of papers that focus on avoiding dynamic obstacles might be explained by the increased need for real-time collision avoidance solutions. The dynamic obstacle avoidance problem is more complicated, since knowledge of the target object movement is required, and therefore, the consideration of a time parameter must be included. In cases when there is no communication between the own vessel and the target vessels, the examined algorithms perform avoidance action by predicting future positions of target vessels. This can be done by assuming that the own vessel can observe and estimate the dynamics of the target object (velocity and course) and its size; inferring compliance with COLREGs; or by obtaining information from third parties, e.g., from Automatic Identification System (AIS) data.

P5. Testing type: Most of the papers, 38 (84%) in total, test the proposed algorithms only by means of simulations in a simulated test environment built for this reason, for example using simulation software and high-level programming languages such as MATLAB. The testing environment varies depending on the papers' objective, and may include the geographic area, traffic data, obstacles, and other parameters related to ship dynamics of both own and target vessels. Some papers perform tests in several scenarios for representing the flexibility of the algorithm adapting to different situations. Sometimes, the performance of an algorithm is compared with some other under the same environment. A common practice is to use real map data for simulations [e.g., 33, 35–38]. The remaining 7 (16%) papers are verified in both field tests and simulations. In these cases, small vessels, equipped with GNC systems, e.g., Springer USV [31, 24] and ARCIMS USV [48], are used. An outstanding project with thorough testing is represented in Varas et al. [48] where tests have been performed both on desktop simulations, on a bridge simulator, and on sea trials using a USV. In this paper, testing is performed using Monte Carlo simulations to detect weaknesses of the proposed method and using historical collision incident data for more realistic scenarios.

P6. Environmental disturbances: Table 3 shows that when it comes to environmental disturbances, more than half (27, or 60%) of the papers do not take any environmental disturbances into consideration. Several papers [e.g., 25, 26, 27] are focusing only on the effect of current on the vessel (7, or 16%), some consider both current and wind (4, or 9%) [47, 49, 53, 54], and only two papers consider both current, wind, and waves [29, 35]. None of the papers consider waves as the only environmental disturbance affecting the ship's movement; however, waves are included in two papers together with wind and current.

Table 3 Comparison of situation/environment and ship-related properties of different algorithms in 45 selected papers

Refs.	Algorithm	P1	P2	P3	P4	P5	P6	P7	P8
[25]	Voronoi-visibility algorithm	G	No	CA	S	S	C	No	T
[26]	Multi-layered fast marching method	G	No	CA	S	S	C	No	T
[12]	Deep Q-networks	G	No	CA	S	S	Unk	Yes	O
[27]	Pseudospectral optimal control	G	No	CA	S	S	C	Yes	T
[28]	Deep deterministic policy gradient	G	No	CA	S	S	Unk	Yes	O
[29]	Improved quantum ACO	G	No	CA	S	S	C,Wn,Wv	Yes	T
[30]	Q-learning	G	No	CA	S	S	No	Yes	No
[31]	Angle-guidance FMS	G	No	CA	S	S,F	C	Yes	No
[32]	Smoothed A*	G	No	CA	S	S,F	No	Yes	No
[33]	Finite angle A*	G	No	CW	S	S	No	Yes	O
[19]	Ant colony optimisation	G	No	CW	S	S	Unk	Yes	No
[34]	A* on border grids	G	No	CW	S	S	No	No	No
[35]	Genetic algorithm	G	No	OW	S	S	C,Wn,Wv	Yes	No
[36]	A* post smoothed + DW	GL	No	CA	S	S	No	Yes	T
[22]	Shortcut Dijkstra + APF	GL	No	CA	S	S,F	No	Yes	T
[20]	APF-ACO+Multi-layer algorithm	GL	No	CA	S	S,F	C	Yes	O
[17]	Modified artificial potential fields	GL	Yes	CA	Dn	S	No	No	No
[14]	R-RA*	GL	Yes	CA	Dn	S	No	No	O
[37]	Voronoi diagram + Fermat's spiral	GL	Yes	CA	Dn	S	C	Yes	No
[38]	Hierarchical multi-objective PSO	GL	Yes	CA	Dn	S	Unk	Yes	no
[39]	Evolutionary algorithm	GL	Yes	CA	D1	S	No	Yes	O
[16]	Fast marching square method	GL	Yes	CA	D1	S	No	No	No
[24]	Direction priority sequential selection	GL	Yes	CA	D1	S	No	Yes	T
[40]	COLREG-RRT	GL	Yes	CW	Dn	S	No	Yes	No
[21]	Bacteria foraging optimization	GL	Yes	CW	Dn	S	No	Yes	O,T
[41]	A* with OCP + MPC + BC-MPC	GL	Yes	CW	Dn	S	C	Yes	T
[42]	Path-guided hybrid APF	GL	Yes	CW	Dn	S	No	Yes	T
[43]	Adaptive wolf colony search	GL	Yes	OW	D1	S	No	Yes	O
[44]	Artificial potential fields	L	Yes	CW	Dn	S	No	Yes	T
[45]	Deep reinforcement learning	L	Yes	CW	Dn	S	No	Yes	O,T
[46]	COLREGs-constrained APF	L	Yes	CW	Dn	S	No	Yes	O,T
[47]	Dynamic reciprocal velocity obstacles	L	Yes	CW	Dn	S	C,Wn	Yes	O,T
[48]	Multi-objective PSO	L	Yes	CW	Dn	S,F	Unk	Yes	O
[49]	Deep Q-learning	L	Yes	CW	Dn	S,F	C,Wn	Yes	O,T
[50]	Fuzzy relational products	L	yes	OW	Dn	S	no	no	T
[15]	Optimal reciprocal collision avoidance	L	Yes	OW	Dn	S	No	No	No
[5]	Ant colony optimisation	L	Yes	OW	Dn	S	No	No	O
[51]	Deterministic algorithm	L	Yes	OW	Dn	S	no	yes	no
[52]	Probabilistic obstacle handling + A*	L	Yes	OW	Dn	S	No	Yes	No
[53]	Model predictive control	L	Yes	OW	Dn	S	C,Wn	Yes	O
[54]	Evolutionary algorithm	L	Yes	OW	Dn	S	C,Wn	Yes	T
[9]	Velocity obstacles	L	Yes	OW	Dn	S,F	No	No	T
[55]	Fuzzy membership function	L	Yes	OW	D1	S	No	No	O
[18]	Genetic algorithm	L	Yes	OW	D1	S	No	No	O
[23]	NSGA-II	L	Yes	OW	D1	S	No	No	O
	Sums of sub-properties	G: 13	No: 16	CA: 19	S: 16	S: 38	C,Wn,Wv: 2	Yes: 32	O,T: 5
		GL: 15	Yes: 29	CW: 13	Dn: 22	S,F: 7	C,Wn: 4	No: 13	O: 13
		L: 17		OW: 13	D1: 7		C: 7		T: 13
							Unk: 5		No: 14
							No: 27		

Out of all of the environmental disturbances, current is the most often included one (13 papers), followed by wind (6 papers) and waves (2 papers).

P7. Vessel dynamics: Vessel dynamics have been considered in most of the cases (32 papers, or 71%). Some of the ship's parameters included in the papers are dynamics of the vessel, a manoeuvring model, a kinetic model, turning ability, maximum steering angle or speed, and other vessel motion constraints or limitations. The remaining 13 papers (29%) do not consider vessel dynamics.

P8. Safety domain: To enhance safety, a safety zone (domain) is required for ensuring the respect of the closest area around the own vessel, target vessels, or obstacles. Across the applied algorithms, safety zones take a variety of shapes, including circle, ellipse, rectangle, shipshape, and inverted cone. An own ship domain has been implemented using various parameters in 13 (29%) papers. A safety domain around target vessels or a safety zone around obstacles has been implemented in the same number of papers (29%). Finally, 5 (11%) of the algorithms have implemented both an own ship domain and a domain, whereas 14 (31%) algorithms do not include a safety domain.

Hybrid approaches: The study shows that most of the algorithms are using a hybrid approach for path planning and collision avoidance that combines two or more methods to improve the performance and cover different sides of real-life situations. For example, Niu et al. [25] combine Voronoi diagram with visibility graph and Dijkstra's search, creating a hybrid Voronoi-visibility algorithm; Wu et al. [20] combine artificial potential field method with ACO algorithm for global planning and uses a multi-layer obstacle-avoidance framework for local planning; Xie et al. [22] combine Dijkstra's algorithm with APF method; and Candeloro et al. [37] merge Voronoi diagram with Fermat's spiral (FS) to ensure curvature-continuous paths. In most cases, the purpose of the hybrid approach is to be able to solve both local and global path planning.

Single- vs. multiple-vessel control: Most papers are focusing on single-vessel path planning methods, whereas a few authors are considering path planning of a formation or a fleet of more than one vessel [e.g., 16, 56–58]. Notably, for formation path planning in a static environment, conflicting collision avoidance situations between formation members also need to be considered, turning the environment into a dynamic one.

4 Safety and objective functions

A crucial aspect of ASVs is the ability to navigate safely in open waters, coastal areas, and congested waters like harbours. To achieve safe manoeuvring, multiple components should be considered, such as COLREGs, situational awareness (consideration of both dynamic and static obstacles), dynamic properties and limitations of the vessel, environmental disturbances, and safety domain [59]. One way of ensuring the safety of the own vessel considering the dynamic target vessels in the vicinity is to include some safety aspects when searching for collision-free paths, thus evaluating risk of collision. Hence, safety of the own and target vessels should be incorporated, or at least considered, when generating paths based on optimisation of an objective function.

In Fig. 3, we highlight what we have identified as being the four most often used safety components across the examined literature, namely (1) safety conventions, (2) collision risk assessment (CRA), (3) safety domain, and (4) environmental disturbances.

In the following subsections, however, we limit our study to analysing the employment of (1) collision risk assessment (CRA) and (2) objective function in the algorithms proposed in the selection of literature.

4.1 Collision risk assessment

CRA is one of the key factors that aids in evaluating the safety of the path to be taken. It is an assessment tool that may include several safety criteria based on the current and predicted situation, own or target vessels' parameters, and their mutual relationship.

An often used risk evaluation criterion for CRA in the literature is the closest point of approach (CPA), which can be measured both in time and distance, as illustrated in Fig. 4.

The CPA is the position at which two dynamically moving vessels will have the shortest distance between them at a specific time. The time to the closest point of approach (TCPA) is the time when this position is reached. The
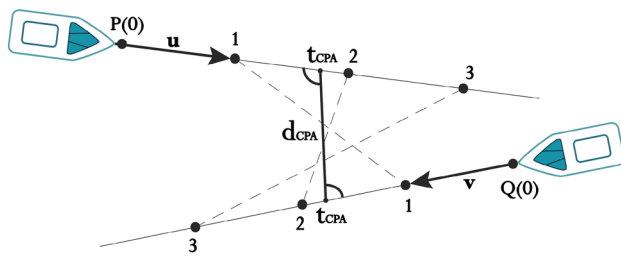
Fig. 3 Safety components

Fig. 4 The concept of time and distance of CPA

distance of the closest point of approach (DCPA) is the distance between both CPA points on the trajectory of each vessel.

Both TCPA and DCPA are proposed for the maritime field by Kearon [60], and they are used mainly for collision risk assessment and navigation safety enhancement. The TCPA and DCPA parameters, however, have a drawback. As noted by the authors in Nguyen et al. [61], both parameters do not adequately represent the danger of a collision when moving into head-on situations and overtaking situations.

CRA parameters are not limited only to TCPA and DCPA, although these are the most commonly used ones. Other papers also consider parameters such as the distance of the last-minute avoidance, distance to the target vessel, ratio of speed, relative bearing, safe passage circle, and distance of adopt avoidance action [15, 47, 62].

4.2 Objective function

There are many possible criteria for path evaluation using an objective function. Some of the most often used criteria which we have identified are:

- *Path length*: length of the obtained path (either before or after smoothing of the path).
- *Voyage time*: time required to reach the target position when traversing the obtained path.
- *Smoothness*: connection of waypoints in an optimal way taking into consideration limited curvature or turning radius of the ship. This property partly reflects whether the path is feasible from the ship's perspective. Reduced number of sharp turns or a path smoothing module are some examples of a smoothness component.
- *Tractability*: the practicality of the path, especially in dynamic environments when some waypoints have to be relocated during the journey [63].
- *Energy consumption*: a criterion that might be influenced by several other factors, including path length, vessel's speed, or the effect of sea currents on the vessel, in terms of economy.
- *Path precision*: how precisely does the designed path pass through waypoints [63].

The comparison of (1) CRA components and (2) objective function criteria included in papers is presented in Table 4. Here, CRA analysis includes only the most often used criteria, namely TCPA and DCPA. The analysis of the objective function considers only the four most often implemented components: length, time, smoothness, and energy efficiency. For all columns, the presence/absence of the criteria is indicated with '+'/'−', respectively. The analysis is performed for the same 45 papers that were chosen and analysed in Sect. 3 with the same sequence of papers and the division based on "planning type" property. The last row of the table summarises the number of papers that have included each of the criteria.

4.3 Analysis

Table 4 shows that the most often used CRA criterion is DCPA, used in 21 (47%) papers, whereas TCPA was used in 15 (33%) papers. 14 (31%) papers use both TCPA and DCPA, whereas half the papers (23, or 51%) use neither TCPA nor DCPA as a CRA criterion. Most of these 23 papers are dealing with static obstacles; therefore, there is no need for calculating CPA. Instead, authors in Tam and Bucknall [54] use a two-step CRA process by (1) determining the type of encounter, and (2) calculating the dimensions of the safety area. The rest of the 23 papers that do consider dynamic encounters use other ways to ensure safety, and collision-free paths, such as considering COLREGs [16, 24, 40, 53], applying a safety domain around own or target vessels [24, 53, 54], or calculating the probability of collision [52].

Regarding the objective function, path length (27 papers, or 60%) is the component taken into account the most, followed by smoothness (13 papers, or 29%), time (12 papers, or 27%), and lastly energy efficiency (8 papers, or 18%). 10 papers use none of the four objective function components, and no paper uses all four. In most of these cases, the papers are dealing with collision avoidance [9, 15, 47, 51, 52, 53, 55]; therefore, authors do not prioritize optimization of the path's length, energy efficiency, or other parameters but instead focus on safety of the collision-free path. Other components included in objective functions by some authors are tractability [31]; cost on deviating from the relative nominal trajectory, and on control input [41]; and navigation restoration time and angle during collision avoidance manoeuvre as well as optimal safe avoidance turning angle [18].

Algorithms based on reinforcement learning (RL) [e.g., 49] do not use a standard objective function but rather a reward function. This means that standard objective parameters are not optimised directly. Instead, the reward function helps the agent to learn and improve based on the dynamics of an agent and the practicality and safety of the path. Therefore, even though RL algorithms do not optimise smoothness directly, they might generate a path that is smooth.

Table 4 The use of CRA and objective function components in 45 selected papers

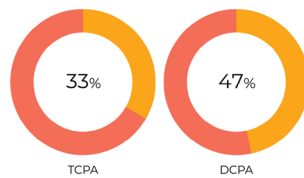| Reference	CRA		Objective function			
	TCPA	DCPA	Length	Time	Smoothness	Energy
[25]	–	–	+	–	–	+
[26]	–	–	+	–	–	+
[12]	–	–	+	–	–	–
[27]	–	–	–	–	–	+
[28]	–	–	–	–	–	+
[29]	–	–	+	-	+	+
[30]	–	–	+	–	+	–
[31]	–	–	+	+	-	–
[32]	–	–	+	+	+	–
[33]	–	–	+	–	+	–
[19]	–	–	+	–	-	–
[34]	–	–	+	–	–	–
[35]	–	–	+	+	–	+
[36]	–	–	+	–	+	–
[22]	–	–	+	–	–	–
[20]	–	–	+	–	–	–
[17]	–	+	–	–	–	–
[14]	–	+	+	–	+	–
[37]	+	+	+	–	+	–
[38]	+	+	+	–	+	–
[39]	–	+	+	+	–	–
[16]	–	–	–	–	–	–
[24]	–	–	+	–	+	–
[40]	–	–	–	+	-	–
[21]	+	+	–	+	–	–
[41]	+	+	+	–	-	+
[42]	+	+	+	+	–	–
[43]	+	–	+	+	–	–
[44]	–	+	–	–	+	–
[45]	+	+	–	+	–	–
[46]	–	+	–	–	+	–
[47]	+	+	–	–	–	–
[48]	–	+	+	–	+	–
[49]	+	+	–	–	–	–
[50]	–	–	–	–	–	+
[15]	+	+	–	–	–	–
[5]	+	+	+	+	–	–
[51]	+	+	–	–	–	–
[52]	–	–	–	–	–	–
[53]	–	–	–	–	–	–
[54]	–	–	+	+	–	–
[9]	+	+	–	–	–	–
[55]	–	+	–	–	–	–
[18]	+	+	+	+	–	–
[23]	+	+	+	–	+	–
Total	15	21	27	12	13	8
%	33	47	60	27	29	18

Fig. 5 Usage of the CRA components TCPA and DCPA in 45 selected papers
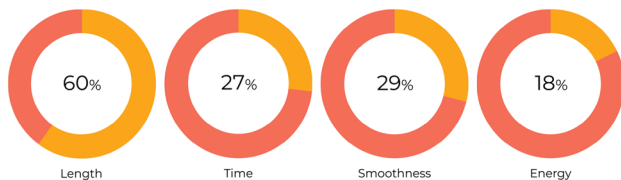

Fig. 6 Usage of the objective function components Length, Time, Smoothness, and Energy in 45 selected papers

Statistics of CRA and objective function components included in the papers are summarised in Figs. 5 and 6, respectively.

5 Advantages and limitations

To further enhance our comparative study of path planning and collision avoidance algorithms for ASVs and USVs, we summarise the advantages and limitations (room for improvement) of the algorithms proposed in the 45 selected papers, as shown in Table 5. The criteria of the analysis include computational complexity, convergence, planned path features (particularly optimality and smoothness), the ability to re-plan, operation in real time, the complexity of the environment, consideration of the local minima trap, and others.

The analysis of the advantages and limitations of the proposed algorithms is based purely on the information provided by the authors of each one of the analysed papers. Therefore, this evaluation is inherently subjective, and in most cases, the authors have not stated the limitations of the algorithms at all even if they exist (noted in the table as 'N/D') or they have been extracted from the future work section.

The analysis of the algorithms in Table 5 shows that in many papers, authors do not state their limitations in a straightforward manner. In many cases, the limitations of the proposed algorithms have been extracted from the future work section of the paper. This section often gives a better comprehension of the current state of the proposed method and its limitations and parts that have to be improved.

In some cases, the conventional version of an algorithm has been extended and improved to form promising derived algorithms that avoid limitations of the conventional algorithm. For example, a well-known limitation of the conventional APF algorithm is the local minima problem. However, for derived algorithms that are based on the conventional APF, authors often state avoiding local minima trap as their advantage, additionally to other improvements.

To sum up, many of the proposed algorithms are trying to overcome different problems connected with developing an autonomous system that performs well in real-life applications. However, the analysis shows that even when the limitations of the algorithms are not stated clearly by the authors, they still exist. That is, although researchers demonstrate knowledge about which components should be included in an ASV path planning and collision avoidance system, there inevitably will still be difficulties in implementing the system in real life.

Finally, we wish to point out that, according to our knowledge, several other path planning algorithms used for mobile robots, ground vehicles, aerial vehicles, or underwater vessels have not been applied to surface vessels yet, e.g., bug algorithm [64], Voronoi fast marching method [65], symbolic wavefront expansion [66], probabilistic roadmaps [67], and fast marching* (FM*) [68]. Even though these algorithms have been applied for path planning in various other fields, it would be possible to adapt these algorithms also to applications for ASVs. Moreover, interested readers should note that additionally to our own comparison of algorithms, and a comparison of performance of the A* algorithm and derivative algorithms (A*PS, Theta*, and A*GB) used for path planning for autonomous inland vessels is provided by Chen et al. [34].

6 Discussion

The timeline of algorithms for the latest decade shows an increased interest of researchers for solving path planning and collision avoidance problems for surface vessels by experimenting with, and developing new, methods and algorithms from the AI domain. However, this comparative study shows that there is still no unambiguous model of how "the ultimate" autonomous ship should be designed, which components it should include, and how it should act. The analysed papers offer various solutions to example problems, but these solutions are often limited to perform well under specific and restrictive conditions.

Through the analysis, we have identified a number of limitations in recent solutions for path planning and collision avoidance of ASVs (some of these limitations have also been pointed out in other review papers in the field, as described in our accompanying paper [1]):

Table 5 Advantages and limitations of the analysed algorithms

Method	Refs.	Advantages	Limitations
Fuzzy membership function	[55]	The actions of experienced helmsman in ocean navigation are simulated The algorithm avoids close-quarter situations	N/D
Fuzzy relational products	[50]	The path to the goal is reasonable, optimal, and safe Compliance with COLREGs System is a practical and effective candidate for a real-time path planning	N/D
Modified artificial potential fields	[17]	Local minima avoidance It effectively solves the online path planning problem	The resulting path exhibits some jaggedness It is difficult to tune gains to achieve specific clearances
Finite Angle A*	[33]	The vessel's dimensions and its turning ability are considered It keeps safe distance from the path to obstacles Can be used in real time The resulting path is more pragmatic and can be utilized by a USV directly The algorithm can be used on binary satellite images	N/D
A* Post-Smoothed + DW	[36]	Reduced number of waypoints by eliminating redundant turns Resulting path is the shortest path The motion dynamics of the USV are considered Remarkable performance and superiority in path planning with obstacle avoidance The USV can avoid unknown obstacles via selecting optimal velocities	N/D
Optimal reciprocal collision avoidance	[15]	Autonomous detection of a collision occurs in real time The proposed approach is both valid and efficient It takes into account the reactive avoidance action of the threatening vessel	The environmental conditions are not considered
Voronoi-visibility algorithm	[25]	The generated path is feasible and takes the energy consumption into account based on sea current data The USV keeps a safe distance from all islands and coastlines The path is energy-efficient The proposed algorithm integrates the advantages of the Voronoi diagram and Visibility graph	The sea current data are time-invariant The speed of the USV does not change depending on the sea current state
Multi-layered fast marching method	[26]	Generates practical trajectories in dynamic environment Keeps a safe distance away from obstacles Saves on the energy cost by following counter-flow areas	Does not take into account wind Does not follow COLREGs
Shortcut Dijkstra + modified APF	[22]	The navigation is efficient both in a changeable and unchangeable environment The algorithm avoids falling into local minimum	Little work is done on trajectory tracking
AFMS	[31]	Generates feasible and practical waypoints Calculates the optimal path according to the vehicle's constraints	N/D
FMS	[16]	The algorithm avoids falling into local minimum trap The path generated is safer than the one of FMM Compliance with COLREGs	N/D

Table 5 (continued)

Method	Refs.	Advantages	Limitations
Adaptive-BFOA	[21]	The algorithm is applicable in real time; The path does not get trapped in local optimum; The rules of the road can be properly taken into account; The algorithm is very robust and reliable, given the diversity of marine traffic environment	It is assumed that target ships do not change their courses and speeds
Multi-objective PSO	[48]	Compliance with COLREGs; The system demonstrates its robustness	The algorithm need improvements when dealing with conflicting rules, interaction of autonomous and manned vessels, poor or degraded sensor picture, and manoeuvring in restricted waters
ACO	[5]	The algorithm outperforms GA with respect to both execution efficiency and execution results; The planned path considers both economy and safety, while being the safety critical, shortest collision avoidance route	No real-time decision support
GA	[18]	Saves calculation space and time; Provides a practical and meaningful application to the navigator; Finds a theoretically safety-critical recommendation for the shortest route of collision avoidance from an economic viewpoint	Only circular shape ship domain is considered; The algorithm is not customized for parallel processing
ACO	[19]	The algorithm converges quickly	N/D
Direction priority sequential selection	[24]	Viable and realistic trajectories; Compliance with COLREGs; Consideration of dynamics of an actual USV; Improved trajectories and computational costs over A*; A path is smoother and with less jagged segments than A*; Avoids both static and dynamic obstacles	COR for obstacle detection might fail in some cases; An edge detection system for obstacles would be more appropriate
R-RA*	[14]	The system automatically detects and avoids multiple, dynamic, and pop-up obstacles in compliance with COLREGs; Safe navigation is ensured without the need for operator intervention and decision making; It is a fast, online solution without dependence on AIS data; The computational time is saved by doing only local replanning	Vessel dynamics are not considered; The current system could be improved by incorporating optimal speed assignment in addition to the recommended spatial path
APF-ACO + multi-layer algorithm	[20]	Faster convergence than that of the ACO algorithm; The method overcomes the problem of premature convergence; It offers a solution in a complex environment	The environmental disturbance during the obstacle avoidance should be reduced; Precision should be improved
Deterministic algorithm	[51]	The algorithm is algorithmically complete (the outputs from the algorithm are entirely predictable); Practical and COLREGs compliant navigation path; The path planning is based on good seamanship practice; The algorithm will not return the vessel to its initial course unnecessarily after the avoidance manoeuvre	The avoidance of area-based obstructions has not been implemented; The reactive planning subroutine is included only partially; The energy management and mission planning modules have not been implemented yet
A* on border grids	[34]	The algorithm takes advantage of grid search and visibility check; The algorithm reduces unnecessary heading changes, it is faster than traditional A* and proposes shorter path for inland path planning	The impact of infrastructures and real-time information are not included in this paper

Table 5 (continued)

Method	Refs.	Advantages	Limitations
Artificial potential fields	[44]	The algorithm is effective in complex navigational situations Real-time collision avoidance Compliance with COLREGs	Local minima problem Oscillations in narrow passages Weather conditions are not taken into consideration The simulation does not have any optimisation or prediction ability per se Extreme encounter cases are not considered
Voronoi diagrams + Fermat's spiral	[37]	Clearance constraints are satisfied with respect to both land and shallow waters The path is produced real time; it is safe, practical, and intuitive Compliance with COLREGs Low computational cost	N/D
Hierarchical multi-objective PSO	[38]	The method generates safe and COLREGs compliant paths A collision-free path is always guaranteed It can deal with multiple-vessel encounters simultaneously	Choosing the optimal safety objective function is still an open problem
Probabilistic obstacle handling + A*	[52]	The produced evasive paths keep distances to other vessels The path planning is triggered less frequently (unnecessary manoeuvres due to imprecise measurements are avoided) The collision risk of any path is known	Dynamic adaptations of the parameters should be investigated to suit narrow environments, such as rivers, equally well The accuracy of the algorithm can be improved by approximating the integral that is used to calculate the occupancy probabilities
Model predictive control	[53]	The method is conceptually and computationally simple It accounts for the dynamics of the ship, its steering and propulsion system, forces due to wind and ocean current, and any number of obstacles The method is effective and can safely manage complex scenarios with multiple dynamic obstacles and uncertainty	N/D
NSGA-II	[23]	Existence of elitism Fast convergence Computationally not complex The resulting path is safe, economical, and considers COLREGs	In practice, there will be a complex sea conditions
Evolutionary algorithm	[54]	It deals with close-range encounters, using known and predicted data about the traffic and environment The navigation path is optimal, collision free, COLREGs compliant and practical The algorithm outputs are consistent	The algorithm needs more extensive testing before practical implementation to validate its completeness
Deep Q-networks	[12]	Deals with unknown environmental disturbances The algorithm is concise, effective, and extendable The analytic control law is not required to manoeuvre the vessel	N/D
COLREG-RRT	[40]	Higher navigation success rate and COLREGs compliance compared to MPC-based and APF based methods More efficient in identifying long-term trajectories given a limited amount of forward simulation calls	The algorithm does not deal with disturbances such as waves and ocean currents

Table 5 (continued)

Method	Refs.	Advantages	Limitations
Velocity obstacles	[9]	The rule hysteresis ensures that each COLREGS manoeuvre is obvious to other drivers The algorithm navigates safely in dynamic, cluttered environments COLREGs are encoded in the velocity space in a natural way	N/D
Evolutionary algorithm	[39]	The algorithm works in restricted visibility The method is relatively economical It is capable of finding collision avoidance manoeuvres quickly	The paper addresses ship-to-ship encounters only
Pseudospectral optimal control	[27]	The method has proven successful in real-world applications The method is not as sensitive to dimensionality compared to Hamilton–Jacobi–Bellman methods The path is feasible and energy-efficient	The method is not integrated with the complete collision avoidance system
Deep deterministic policy gradient	[28]	The reward system is highly customisable and can be changed according to the task and the control model The algorithm automatically generates the perfect path under unknown environmental disturbance	N/D
Adaptive wolf colony search	[43]	High speed of global convergence Excellent calculation robustness High-solution accuracy The algorithm can be realized easily and is very suitable to solve the optimization problem With slight modifications, the algorithm can be used in multi-ship collision avoidance path planning	N/D
Deep Q-learning	[49]	The approach has great possibility for automatic collision avoidance in highly complicated navigational situations The algorithm is able to avoid collisions in severely congested and restricted waters	The algorithm needs to be enhanced more for realistic applications It does not consider the change of speed as an action for collision avoidance
Deep reinforcement learning	[45]	The approach has an excellent adaptability to unknown complex environments with various encountered ships	High sample complexity, difficult to use in learning in the real world
Improved quantum ACO	[29]	The algorithm can plan a path considering multiple objectives simultaneously The number of iterations required to converge to the minimum was 11.2–24.5% lower than for the quantum ACO and ACO The obtained minimum was 2.1–6.5% lower than for the quantum ACO and ACO The optimized path for the USV was obtained effectively and efficiently	The kinetic and kinematic constraints of the USV should be added to the cost function More practical environmental loads should be applied to calculate their effects on the path energy consumption of the USV
COLREGs-constrained APF	[46]	The method is fast, effective, and deterministic for path planning in complex situations with multiple moving target ships and stationary obstacles The method can account for the unpredictable strategies of other ships A smoother path is achieved by considering the dynamics of the OS	Speed reduction behaviours are not considered Ship dynamics could be more accurate The uncertainty of environmental disturbances and area-based obstructions are not considered

Table 5 (continued)

Method	Refs.	Advantages	Limitations
Genetic algorithm	[35]	The optimized routes have an advantage in towing tension and satisfy motion constraints	There are drawbacks with regard to solution precision
A* with OCP + MPC + BC-MPC	[41]	The hybrid algorithm deals with multi-obstacle situations with multiple simultaneously active COLREG rules, and situations where obstacles violate the COLREGs The ship follows an energy-optimized trajectory unless the moving obstacles interfere	The hybrid collision avoidance system has to be validated in full-scale experiments
Path-guided hybrid APF	[42]	The algorithm provides fast feedback in a changeable environment The method has the potential to rapidly generate adaptive, collision-free, and COLREGs-constrained trajectories in restricted waters This method has the potential to perform path planning on an electronic chart platform The algorithm avoids local minima problem, oscillations in narrow passages, and "goals non-reachable with obstacles nearby" problem The method can be used in the fields of USVs, AUVs, and robots	The collision avoidance actions taken by ships are limited to course change only The method should be validated in a more real-life experiment
Q-learning	[30]	The approach is more effective in self-learning and continuous optimisation, and therefore closer to human manoeuvring A rational path can always be found	Dynamic obstacles in waterways are not considered Ship collision avoidance rules are not considered in the ship agent model reward function during the process of learning
Smoothed A*	[32]	The algorithm reduces unnecessary 'jags', has no redundant waypoints, and offers a more continuous route compared to A* The proposed algorithm outperforms the A* algorithm in terms of turning and distance cost The algorithm can be integrated and applied for real applications	The effect of a complete loss of navigational data needs to be investigated The hydrodynamic forces are not considered
Dynamic reciprocal velocity obstacles	[47]	The method is proactive in dealing with the uncertainty of the future behaviour of obstacles The ASV behaviour is predictable compared with both velocity obstacles and reciprocal velocity obstacles methods, especially when the obstacles are following COLREGs	The effect of uncertain decision variables and unknown disturbances should be studied more

– The variety of algorithms used for solving path planning issue is wide, with researchers continuously exploring different options and trying to find better and more general solutions.
– Many developed algorithms that appear to be efficient theoretically have not been tested in a real environment or with real traffic data; hence, it is not possible to evaluate their efficiency in handling real-world issues.
– Some algorithms deal only with static obstacles, excluding dynamic ones.
– In many cases, the developed algorithms do not take into account external disturbances such as wind, waves, or current, which means that the modelled environment is not complete and the performance of the algorithms under realistic conditions would differ.
– Some algorithms assume that the velocity of target ships (that need to be avoided) is constant, and/or that target vessels do not follow COLREGs, meaning that the controlled vessel is not observed and is ignored by other vessels, which is not very realistic.
– Although many researchers agree that safety is the top priority when navigating vessels, not all solutions are considering COLREGs as part of their safe collision avoidance or path planning algorithm.
– Collision risk assessment is typically based only on one or two factors that do not represent the full comprehension of the safety situation of the own vessel in the environment.

Several of these shortcomings lead to the consideration of non-realistic testing environments for vessels, which, in turn, might cause situations where the behaviour of the vessel at sea will differ from the one in simulation tests.

Regarding the limitations of this comparative study, we wish to highlight the following:

– It could be argued that the algorithms in the selected papers should be sorted depending on whether they are solving a path planning (on the global level) or a collision avoidance (on the local, reactive level) problem. The reason for not doing this is the difficulty in distinguishing the algorithms based on this division, as some algorithms are used both for solving path planning and collision avoidance issues.
– Another limitation is that the comparison of the considered properties only gives a partial understanding of the performance of different algorithms in action.
– Finally, it is difficult to extract sufficient details about the properties of the algorithms because of the incomplete or vague descriptions in some of the papers, thus requiring interpretation by the reader.

Future work should try to address these limitations, and examine in more depth some of the properties in Sect. 3 left out in this study, especially "predictability of environment" and planning with uncertainty.

7 Conclusions

The extent of this research is large and fills in some gaps in the field by comparing existing path planning and collision avoidance algorithms of ASVs in a manner they have not been compared before.

ASVs clearly have a big potential in future maritime transportation, but their limitations should also be considered and treated with caution. In this study, we extracted a set of defined properties and characteristics that was used for comparison of the proposed algorithms in 45 carefully selected papers. These properties can be used later by other researchers for benchmarking and for comparing their own algorithm to others'. With respect to the analysis of the 45 papers, the main contribution is threefold and consists of: (1) a comparison of the usage of eight important ship- and environment-related properties; (2) an analysis of how safety has been incorporated, and what components constitute the objective function; and (3) an analysis of advantages and limitations of the proposed algorithms. We consider this comparative study a good attempt at comparing the current state-of-the-art and believe that it can serve as the basis for a deeper performance evaluation system of path planning and collision avoidance algorithms for ASVs.

Future research should be dedicated to simulation as well as real-world field tests that evaluate the actual performance of algorithms in various scenarios under different conditions for a more precise comparison of the developed methods. Such testing systems might aid in evaluating the reliability, durability, and the flexibility of the methods, and in designing appropriate algorithms for specific applications and needs. Testing a large number of different scenarios might be performed using Monte Carlo simulation methods.

Another interesting direction of future research is the evaluation of safety and collision risk assessment of the own ship navigating realistic environments. Components that should be considered when evaluating safety and collision risk are obedience to COLREGs, environmental disturbances, static and dynamic obstacles, and safety domain.

Finally, quantitative and objective evaluation of ASV behaviour should be supplemented by qualitative and subjective evaluation by domain experts such as pilots that could observe ASV behaviour in simulated and real environments. This would lead to improved safety evaluation and could

help with designing new quantitative performance measures for evaluating safety and risk in ASV operations.

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