# Advanced Algorithms, Fall 2012

Prof. Bernard Moret

## Homework Assignment #10

due Monday night, Dec. 10

*Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Tuesday morning, Dec. 11.*

*Question 1. (Computational Geometry)*
You are given $n$ points on the plane, no three of them collinear. We say $p_i$ and $p_j$ are *intimate* if and only if the circle with $\overline{p_i p_j}$ as the diameter does not contain any other points.

1. Give an $O(n \log n)$ algorithm to calculate all intimate pairs.
2. Consider the complete graph on all $n$ points, with the weight of edge $(p_i, p_j)$ set to the distance between $p_i$ and $p_j$.
   Prove that any minimum spanning tree of this graph is made of edges linking intimate pairs.

*Question 2. (Dynamic Programming)*
You are given $n$ distinct points on a line (say on the $x$ axis). Design an algorithm in $O(n^3)$ to find a path to visit all points such that the summation of the *delay* for each point is minimized. The delay for a point $p_i$, with respect to a path $P$, is the total length of the subpath in $P$ from the beginning of the path to the first arrival at $p_i$.

For example, assume you are given 5 points with coordinates 0, 9, 10, 11 and 20. If the path starts at 11, moves to 0, and then reverses direction to move to 20, then the delay is 0 for point 11, 1 for point 10, 2 for point 9, 11 for point 0, and 31 for point 20, for a total delay of 45—you can verify that this is the optimal total delay.

*Question 3. (Randomized Algorithms)*
Design a randomized algorithm that runs in $\tilde{O}(n+m)$ time and, given $n$ red points and $m$ green points on the plane (no three points are collinear), decides whether there exists a line such that all red points are on one side of this line while all green points are on the other side.

*Question 4. (Randomized Analysis)*
Consider coloring the edges of the complete graph on $n$ vertices, $K_n$, from a set of two colors, say red and blue. Thus each edge is assigned a single color; note that this is not what is termed a *coloring* of the graph, in which coincident edges must be assigned different colors. We are interested in the largest complete subgraph of $K_n$ that is entirely red or entirely blue.

Prove that, whenever we have

$$\binom{n}{m} 2^{1-\binom{m}{2}} < 1,$$

there exists a red-blue coloring of $K_n$ that does not induce a complete subgraph $K_m$ that is entirely red nor one that is entirely blue. Prove it by showing that the event [no blue $K_m$ and no red $K_m$] has nonzero probability.