Advanced Algorithms, Fall 2012

Prof. Bernard Moret

Homework Assignment #3

due Sunday night, Oct. 14

Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Monday morning, Oct. 15.

Question 1. Recall that the *rank* of a node v, denoted as $r(v)$, is defined as $\log w(v)$, where $w(v)$ is the number of nodes in the subtree rooted at v. In the splay trees, we define its potential function as $\sum_v r(v)$. Prove that among all possible binary trees of n nodes, the linked list has the maximum potential and the complete binary tree has the minimum potential.

Question 2. You are given a list of n distinct items and a request sequence of accesses. Find an offline static ordering of those n items to minimize the total access cost and prove the optimality.

Question 3. Show that the competitive analysis for the move-to-front algorithm against the static optimal algorithm is asymptotically tight. (Hint: You need to give a general instance for which the cost of the move-to-front algorithm is twice as that of the static optimal algorithm in the limit.)

Question 4. You are given an array of n items with distinct values (e.g. non-negative integers) and you would like to pick up the most valuable item. You can check the array only once by scanning from the beginning to the end. When you check the value of the i-th item during the scanning, you have to decide whether to pick it up or not immediately based on your experience (e.g. based on the values of all previous items that you have checked). If you pick it up, then it is your final choice and you cannot pick up other item; if you do not pick it up, then this item cannot be picked up in the following process. All the $n!$ permutations are equally distributed. You are asked to design an deterministic algorithm that it returns the most valuable item with the probability strictly greater than $1/4$.