Advanced Algorithms, Fall 2012

Prof. Bernard Moret

Homework Assignment #7

due Sunday night, Nov. 11

Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Monday morning, Nov. 12.

Question 1. (Network Flow)
Let $G = (V, E)$ be a directed graph with source s and sink t. Let k be a positive integer.

1. Propose a polynomial-time algorithm to decide whether there exist k edge-disjoint directed paths from s to t.
2. Propose a polynomial-time algorithm to decide whether there exist k node-disjoint directed paths from s to t.
3. If there exist k edge-disjoint directed paths from s to node u and there exist k edge-disjoint directed paths from u to t, do there exist k edge-disjoint directed paths from s to t? Give a proof or a counterexample.
4. If there exist k node-disjoint directed paths from s to node u and there exist k node-disjoint directed paths from u to t, do there exist k node-disjoint directed paths from s to t? Give a proof or a counterexample.

Question 2. (Network Flow)
A group of n people share a flat, in which there is only one kitchen. On any given day, not all of them will use the kitchen; of those who do use it, one (chosen uniformly at random) will clean it. Thus, during a period of m days, the expected number of days on which the i-th person should clean the kitchen is $D_i = \sum_{j=1}^{m} I_{\{i \in S_j\}} |S_j|^{-1}$, where S_j is the set of people who use the kitchen on the j-th day, and $I_{\{\cdot\}}$ is the indicator function. Prove that we can always assign one person from S_j to clean the kitchen and make sure that the i-th person will clean the kitchen on at most $\lceil D_i \rceil$ days, and give a polynomial-time algorithm to compute such an assignment.

Question 3. (Network Flow)
A group of n people share a washing machine. In each week, totally there are m disjoint available time-slots for washing. Each person choose a subset of them as his or her candidate washing time. A valid assignment is to choose at least l time-slots and at most h time-slots for each person from his or her candidate list, and make sure that these chosen time-slots are distinct. Propose a polynomial-time algorithm to decide whether there exist a valid assignment such that the total number of chosen time-slots is exactly a given integer k, $k \leq m$.

Question 4. (Computational Geometry)
You are given n distinct points in the plane, no three of them collinear. Devise a $O(n \log n)$ algorithm to produce a simple polygon that has these n points as its n vertices.