Advanced Algorithms, Fall 2012

Prof. Bernard Moret

Homework Assignment #8

due Sunday night, Nov. 25

Write your solutions in LaTeX using the template provided on the Moodle and web sites and upload your PDF file through Moodle by 4:00 am Monday morning, Nov. 26.

Question 1.
Consider the following algorithm to calculate the convex hull of the given n points $P = \{p_1, p_2, \cdots, p_n\}$.

1 $K \leftarrow 2$;
2 **while** *TRUE* **do**
3     $m \leftarrow \lceil n/K \rceil$;
4   arbitrarily divide P into m disjoint subsets, $P_1, P_2, \cdots, P_m$, each of which contains at most K points;
5   **for** $i = 1 \rightarrow m$ **do**
6   compute the convex hull of P_i in time $O(K \cdot \log K)$, and denote it as H_i;
7   **end**
8   compute the point with smallest y-coordinate in $\cup H_i$, denoted as q_1;
9   let q_0 be the point whose y-axis is the same with q_1, and whose x-axis is $-\infty$;
10   **for** $k = 1 \rightarrow K$ **do**
11   **for** $i = 1 \rightarrow m$ **do**
12   compute the point $p \in H_i$ that maximize $\angle q_{k-1} q_k p$, and let it be h_i;
13   **end**
14   compute the point $p \in \{h_1, h_2, \cdots h_m\}$ that maximize $\angle q_{k-1} q_k p$, and let it be q_{k+1};
15   **if** $q_{k+1} = q_1$ **then**
16   **return** $\{q_1, q_2, \cdots, q_k\}$;
17   **end**
18   **end**
19     $K \leftarrow K^2$;
20 **end**

1. Devise an $O(\log |H_i|)$ algorithm to carry out line 12. More precisely, your algorithm is given a convex polygon with n vertices, $X = \{x_1, x_2, \cdots, x_n\}$, and two points a and b defining a line that leaves the entire polygon to one side; it outputs a vertex x of the polygon such that the angle $\angle abx$ is maximized.

2. Prove that the running time of the whole algorithm is $O(n \cdot \log s)$, where s is the number of points in the final convex hull. (Consider the value of K when the algorithm stops.)

Question 2.
Let $P = \{p_1, p_2, \cdots, p_n\}$ be the vertices of a convex polygon. Design a linear-time algorithm to find two points from P such that the distance between them is maximized.

Question 3.
Let $P = \{p_1, p_2, \cdots, p_n\}$ be a set of n points on the plane. Use the Voronoi diagram of P to devise an $O(n \log n)$ algorithm to compute, for each point in P, its closest neighbor in P.

Question 4.
You are given a subdivision of the plane into convex polygons (some finite, some infinite); the claim is that this subdivision is in fact a Voronoi diagram. Devise an efficient algorithm that will verify or contradict this claim.