QUESTİON 2 EXERCİSE 5

$f(x)=x^4-3x^2-3=0$
$x^4=3x^2+3$
$x=(3x^2+3)^{(1/4)}$
$g(x)=(3x^2+3)^{(1/4)}$
$\forall x\in[1,2] \quad \rightarrow \quad g(x)\in[1,2]$

$$g'(x)=\frac{(3x^2+3)^{(-3/4)}3x}{2}$$

$\forall x\in[1,2] \quad |g'(x)|\leq k<1$

iter		p1		p2		p2-p1		g(p)
0		1.000000		1.565085		0.565085		1.793573
1		1.565085		1.793573		0.228488		1.885944
2		1.793573		1.885944		0.092371		1.922848
3		1.885944		1.922848		0.036904		1.937508
4		1.922848		1.937508		0.014660		1.943317
5		1.937508		1.943317		0.005809		1.945617

p = 1.943317 found at 5. iteration
Theoretical number of iteration

$$|p-p_n|\leq \frac{k^n}{1-k}|p_1-p_0| < 10^{-2}$$

$|g'(x)|\leq k<1 \quad for\ x=p_0=1$
$|g'(1)|=0.3912$
$k=0.3912$
$p_1=g(p0)=g(1)=1.5650$

$$\frac{0.3912^n}{1-0.3912}|1.5650-1|<10^{-2}$$

Take ln both side of the equation
$n>4.827$
$n\approx 5$ is the theoretical iteration