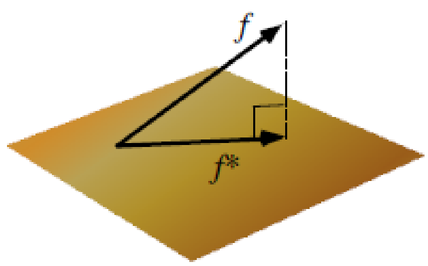
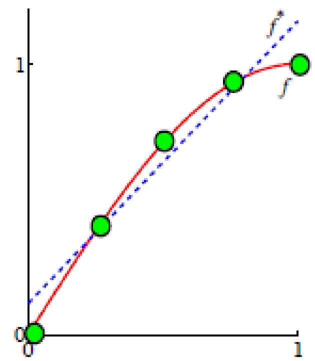
Fonksiyon Yaklaþýmý

Prof. Dr. Dursun Üstündað

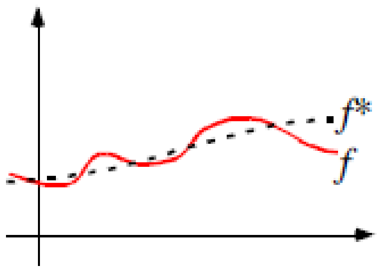
- Problem formulasyonu
- En küçük kareler



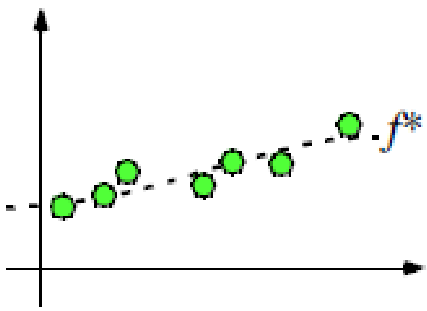


Basit sürekli bir fonksiyon ile f e nasýl yaklaþýlýr?

Ýki eðri uydurma problemleri :



sürekli f

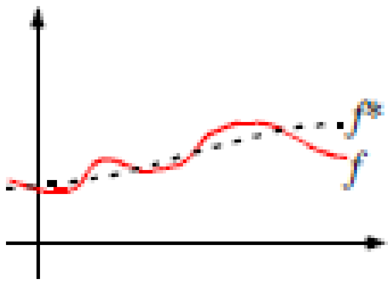


Kesikli f

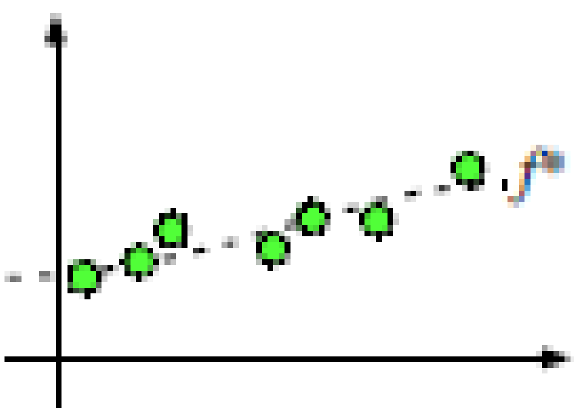
f fonksiyonuna çok yakýn f^* fonksiyonu bulmak istiyoruz.

- Bir kaç temel kavrama gereksinim duyacaðýz.

f fonksiyonu [a, b] aralýðýnda sürekli
fonksiyonlar uzayý C[a, b] ve f bu uzayýnýn bir üyesi olsun.



$\mathbf{R}^m$ m boyutlu vektör uzayý



Linear uzaylarda norm

4 aksiyom :

$$\|f\| \geq 0, \quad \|f\| = 0 \Rightarrow f = 0, \quad \|\alpha f\| = |\alpha| \cdot \|f\|, \quad \|f + g\| \leq \|f\| + \|g\|$$

$C[a,b]$ üzerinde Öklid normu:

$$\|f\| = \sqrt{\int_a^b w(x)\left(f(x)\right)^2\, dx}$$ (a,b) aralýðýnda $w > 0$

3 aksiyom:

R^m de norm :

$$\|f\| = \sqrt{\sum_{i=1}^{m} w_i f_i^2}$$

Yaklaþým problemi :

$\|f - f^*\|$ minimum yapan $f^{\times}$ bulmak

$$(f,g) = \sum_{i=1}^{m} w_i f_i g_i = f^T W g$$ (W, köþegen matris)

Öklid norm sklar çarpým ile iliþkidir

3 aksiyom :

$$(f,g)=(g,f), \quad (f,\alpha h+\beta g)=\alpha(f,h)+\beta(f,g), \quad f\neq 0 \Rightarrow (f,f)>0$$

C[a,b] da skalar carpým

$$(f,g)=\int_a^b w(x)f(x)g(x)\,dx$$

$\mathbf{R}^m$ de skalar çarpým

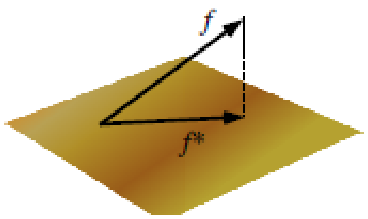
$$(f,g)=\sum_{i=1}^{m} w_i f_i g_i = f^T W g$$

iki gerçek : $\|f\| = \sqrt{(f,f)}$

$$(f,g)=0 \quad \Rightarrow \quad \|f+g\|^2=\|f\|^2+\|g\|^2$$ ((pythagorean kanun)

En uygun fonksiyon

$$g = \sum_{j=0}^{n} c_j \phi_j$$



$$g = \sum_{j=0}^{n} c_j \phi_j \quad \Rightarrow \quad (f - f^*, f^* - g) = 0$$

$$\|f - g\|^2 = \left\|f - f^* + f^* - g\right\|^2 = \left\|f - f^*\right\|^2 + \left\|f^* - g\right\|^2 \geq \left\|f - f^*\right\|^2$$

$(\Rightarrow)$

$$0 = \frac{\partial}{\partial c_k^*} \left\|f - f^*\right\|^2 = \frac{\partial}{\partial c_k^*} \left\|f - \sum_{j=0}^{n} c_j^* \phi_j\right\| = 2(f^* - f, \phi_k)$$

Lineer baðýmsýzlýk

$$\sum_{j=0}^{n} c_j \phi_j = 0 \quad \Rightarrow \quad c_0 = \cdots = c_n = 0$$

Slide

$\phi_0, \ldots, \phi_n$ lineer bagýmsýz ise o zaman öyle $[c_0, \ldots, c_n]$ bulunabilir ki

$$\left(f - \sum_{j=0}^{n} c_j \phi_j, \phi_k\right) = 0$$

olur.

$$0 = \left(f - \sum_{j=0}^{n} c_j \phi_j, \phi_k\right) = (f, \phi_k) - \sum_{j=0}^{n} c_j (\phi_j, \phi_k)$$

$$\begin{bmatrix} (\phi_n, \phi_n) & \cdots & (\phi_0, \phi_n) \\ \vdots & \ddots & \vdots \\ (\phi_n, \phi_0) & \cdots & (\phi_0, \phi_0) \end{bmatrix} \begin{bmatrix} c_n \\ \vdots \\ c_0 \end{bmatrix} = \begin{bmatrix} (f, \phi_n) \\ \vdots \\ (f, \phi_0) \end{bmatrix}$$

$$\sum_{j=0}^{n} c_j (\phi_j, \phi_k) = 0 \quad \Rightarrow \quad \underbrace{\left\| \sum_{j=0}^{n} c_j \phi_j \right\|^2 = 0}_{\sum_k c_k \sum_j c_j (\phi_j, \phi_k)} \quad \Rightarrow \quad \sum_{j=0}^{n} c_j \phi_j = 0 \quad \Rightarrow \quad c_0 = \cdots = c_n = 0$$

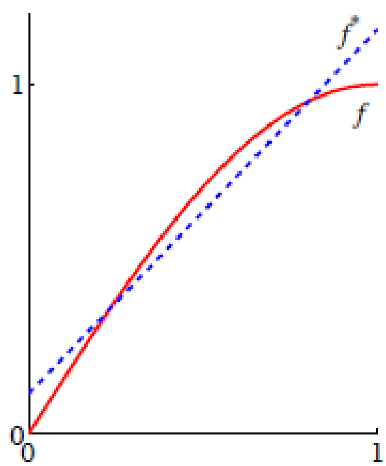
Örnek

$$f \in C[0,1],\ f(x) = \sin(\pi x/2)\ ,\ f^* \in \Pi_1,\ w \equiv 1$$

$$\underbrace{\left\| f - c_0\phi_0 - c_1\phi_1 \right\|}_{\sqrt{\int_0^1 \left(\sin(\frac{\pi x}{2}) - c_0 - c_1 x \right)^2 dx}} \Rightarrow \text{minimum } c_0,\ c_1 \text{ bulma ?}$$

$$\begin{bmatrix} 1/3 & 1/2 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} 4/\pi^2 \\ 2/\pi \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} 1.0437 \\ 0.1148 \end{bmatrix}$$

$$\therefore\ f^*(x) = 0.1148 + 1.0437x$$



Agýrlýklý En küçük kareler

$$A^T W A c = A^T W f$$ (* Normal denklem *)

$$c = \begin{bmatrix} c_n \\ \vdots \\ c_0 \end{bmatrix}, \quad f = \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_m) \end{bmatrix}, \quad W = \begin{bmatrix} w_1 & & \\ & \ddots & \\ & & w_m \end{bmatrix}, \quad A = \begin{bmatrix} \phi_n(x_1) & \cdots & \phi_0(x_1) \\ \vdots & \ddots & \vdots \\ \phi_n(x_m) & \cdots & \phi_0(x_m) \end{bmatrix}$$

$$c = (A\text{^TWA})^{-1} A^T \mathrm{W} f$$

Slide

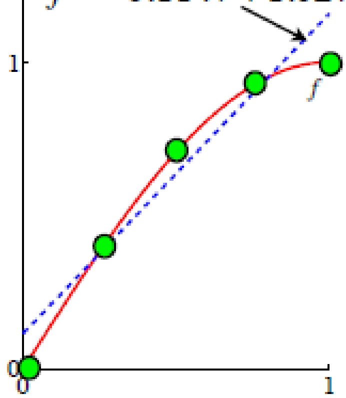
$$G=\{0,\tfrac{1}{4},\tfrac{1}{2},\tfrac{3}{4},1\},\quad f\in C[0,1]\big|_G,\quad f^*\in\Pi_1\big|_G, w_{1:5}=\frac{1}{4}\left[\tfrac{1}{2},1,1,1,\tfrac{1}{2}\right]$$

$$\sqrt{\sum_{i=1}^{5} w_i\left(\sin(\tfrac{\pi x_i}{2})-c_1x_i-c_0\right)^2}$$ $\Rightarrow$ **Minimum for c_0 ve c_1**

$$f=\begin{bmatrix}\sin(0)\\ \sin(\pi/8)\\ \sin(\pi/4)\\ \sin(3\pi/8)\\ \sin(\pi/2)\end{bmatrix},\quad W=\frac{1}{4}\mathrm{diag}\left[\tfrac{1}{2},1,1,1,\tfrac{1}{2}\right],\quad A=\begin{bmatrix}0 & 1\\ 1/4 & 1\\ 1/2 & 1\\ 3/4 & 1\\ 1 & 1\end{bmatrix}$$

$$\begin{bmatrix}0.3438 & 0.5\\ 0.5 & 1\end{bmatrix}\begin{bmatrix}c_1\\ c_0\end{bmatrix}=\begin{bmatrix}0.4105\\ 0.6284\end{bmatrix}\quad\Rightarrow\quad\begin{bmatrix}c_1\\ c_0\end{bmatrix}=\begin{bmatrix}1.0275\\ 0.1147\end{bmatrix}$$

$$f^*=0.1147+1.0275x$$



Lineer model

f[x_] := $a_0 + a_1$ x þeklinde tanýmlanýr. Burada a_0 ve a_1 bilinmiyenlerdir.

Hataların karelerinin toplamı :

$$S_r = \sum_{i=1}^{n} \varepsilon_i^{\,2} = \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i\right)^2$$

Minimum olsun :

$$\frac{\partial S_r}{\partial a_0} = -2\sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i\right)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_0} = -2\sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i\right)(-1) = 0$$

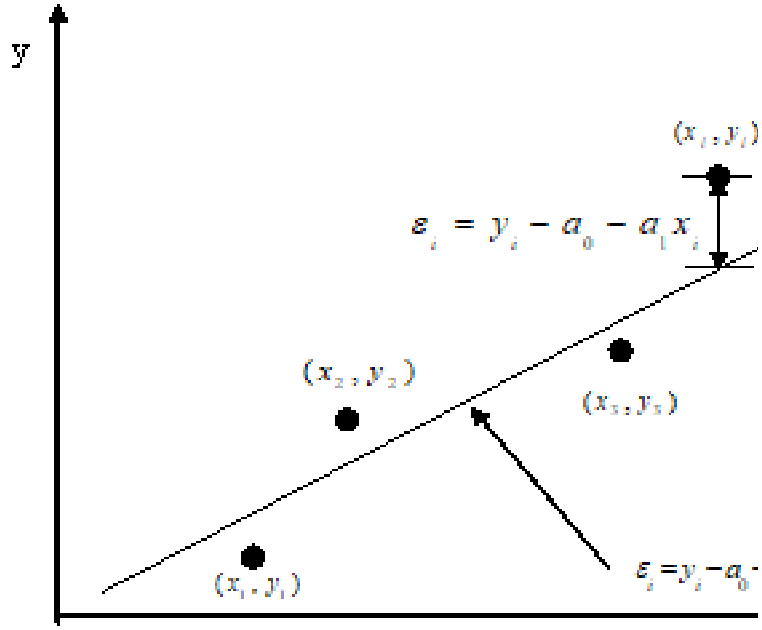
Bu denklemlerden

$$\sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = \sum_{i=1}^{n} y_i$$

$$\sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^{\,2} = \sum_{i=1}^{n} y_i x_i$$

$$(a_0 = \overline{y} - a_1 \overline{x})$$

elde edilir.



▪ özüm

Yukarýdaki (2 x2) lik denklem sistemi çözülürse

$$a_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$a_0 = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$(a_0 = \overline{y} - a_1 \overline{x})$$

elde edilir.

Örnek

Bir açý boyunca burulma yayýaçmak için gerekli olan tork aþaðýda verilmektedir.

$$T = k_1 + k_2 \theta$$

ile verilen modeldeki sabitleri bulunuz?

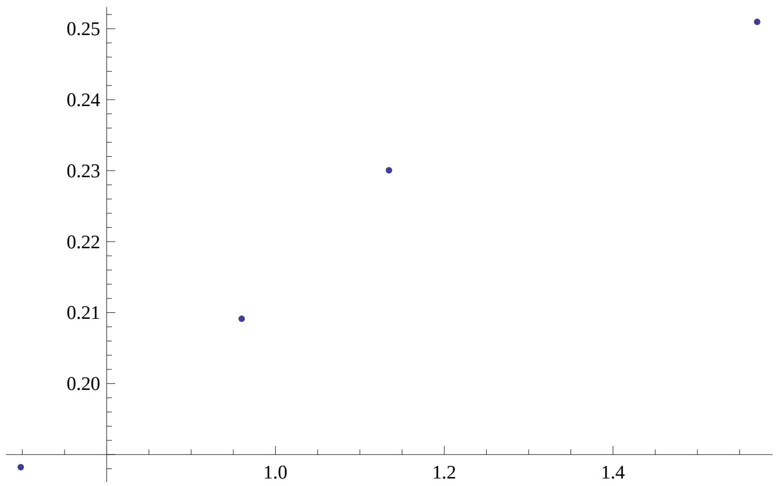
Acý , θ	Tork, T
0.698132	0.188224
0.959931	0.209138
1.134464	0.230052
1.570796	0.250965

In[1]:=
```
T[θ_] := k0 + k1 θ;
θlist = {0.698132, 0.959931, 1.134464, 1.570796}
Tlist = {0.188224, 0.209138, 0.230052, 0.250965}
ListPlot[Transpose[{θlist, Tlist}]]
denklemler = Thread[T[θlist] == Tlist]
denklemler // TableForm
NSolve[denklemler, {k0, k1}]
```

Out[2]= {0.698132, 0.959931, 1.13446, 1.5708}

Out[3]= {0.188224, 0.209138, 0.230052, 0.250965}

Out[4]=



Out[5]= {k0 + 0.698132 k1 == 0.188224, k0 + 0.959931 k1 == 0.209138, k0 + 1.13446 k1 == 0.230052, k0 + 1.5708 k1 == 0.250965}

Out[6]//TableForm=

```
k0 + 0.698132 k1 == 0.188224
k0 + 0.959931 k1 == 0.209138
k0 + 1.13446 k1 == 0.230052
k0 + 1.5708 k1 == 0.250965
```

Out[7]= { }

In[8]:=
```
veri = Transpose[{θlist, Tlist}]
doðru = LinearModelFit[veri, k, k]
```

Out[8]= {{0.698132, 0.188224}, {0.959931, 0.209138}, {1.13446, 0.230052}, {1.5708, 0.250965}}

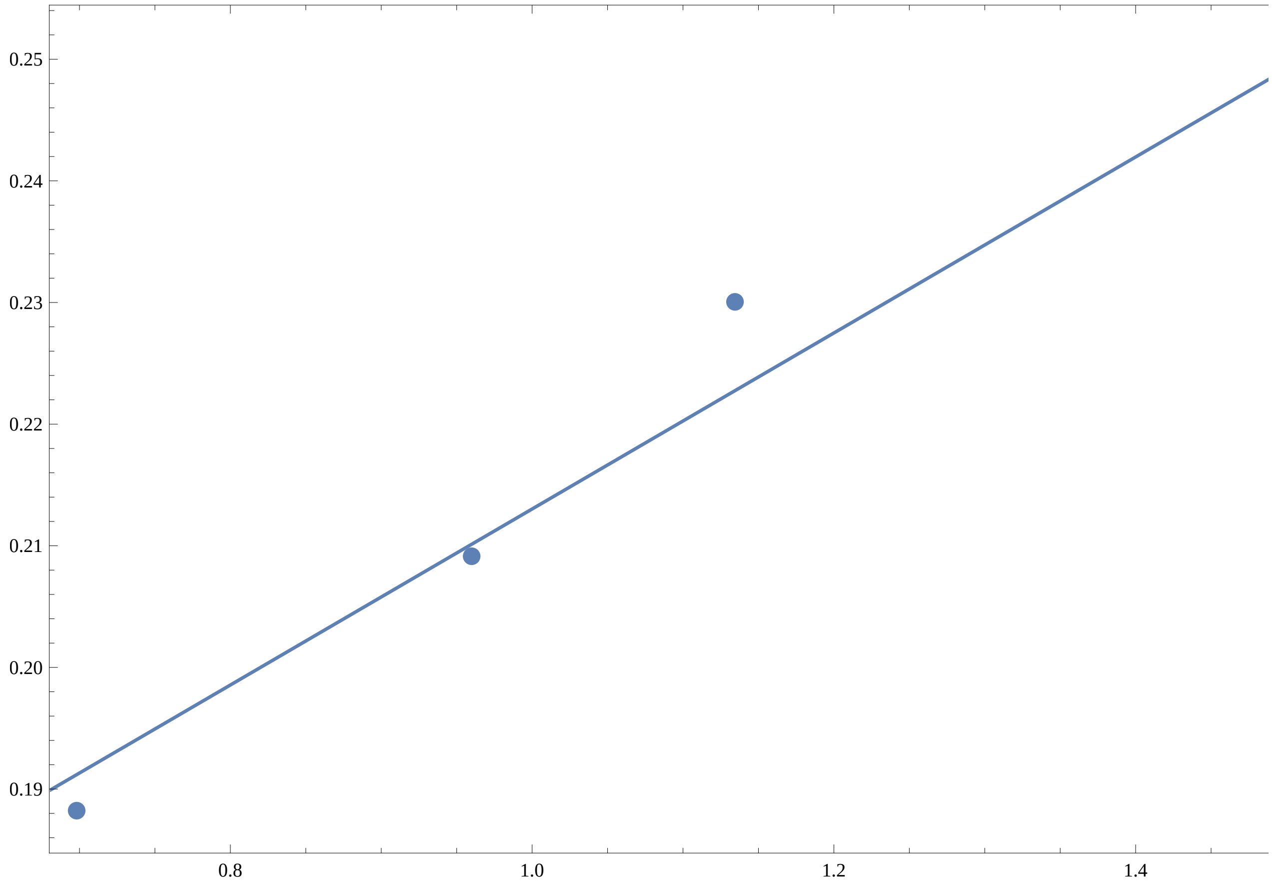
Out[9]= FittedModel[0.140675 + 0.0723481 k]

In[10]:=
```
lm = Normal[doðru]
```

Out[10]= 0.140675 + 0.0723481 k

In[38]:=
```
Show[ListPlot[veri], Plot[doðru[x], {x, 0, 5}], Frame → True]
```

Out[38]=



bulunur

Genelleþtirme

Teorem (En küçük kareler eðri uydurma). $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ verilsin. Derecesi m olan ve bu noktalara minimum uzaklýktan geçen polinom

$$P_m(x) = c_1 + c_2 x + c_3 x^2 + \ldots + c_m x^{m-1} + c_{m+1} x^m$$

þeklinde verilir. Burada katsayýlar $\{c_1, c_2, \ldots, c_m, c_{m+1}\}$ denklem sistemin çözümü ile hesaplanýr.

$$\begin{pmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^m \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \cdots & \sum_{i=1}^{n} x_i^{m+1} \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 & \cdots & \sum_{i=1}^{n} x_i^{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^m & \sum_{i=1}^{n} x_i^{m+1} & \sum_{i=1}^{n} x_i^{m+2} & \cdots & \sum_{i=1}^{n} x_i^{2m} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_{m+1} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^m y_i \end{pmatrix}$$

örnek

Aþaðýda verilen verilere en uygun eðriyi bulunuz?

In[19]:=
```
n = 4;
xlist = Table[Pi (i - 1)/n, {i, 1, n + 1}]
ylist = Cos[xlist];
Length[ylist]
Length[xlist]
```

Out[20]= $\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\right\}$

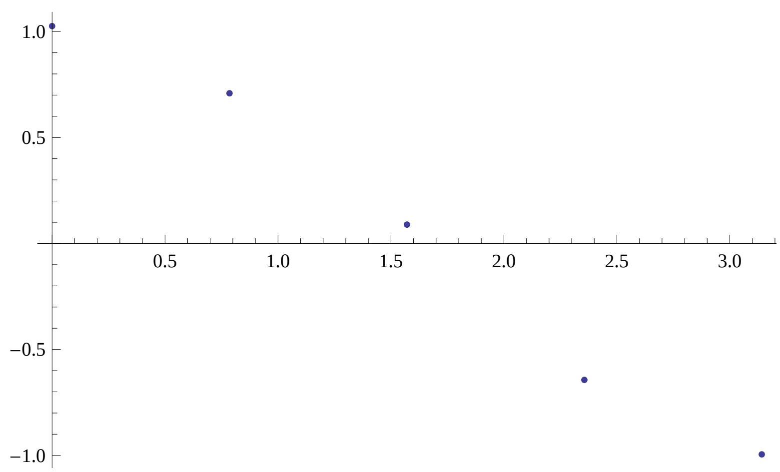
Out[22]= 5

Out[23]= 5

In[24]:=
```
ylist += 0.1 RandomReal[{0, 1}, n + 1]
XY = Transpose[{xlist, ylist}];
ListPlot[XY]
```

Out[24]= {1.02556, 0.708619, 0.0890782, -0.64375, -0.995026}

Out[26]=



In[27]:=
```
p[x_] := c0 + c1 x + c2 x^2
```

In[28]:=
```
X = Transpose[XY][[1]];
Y = Transpose[XY][[2]];
```

$$A = \begin{pmatrix} n & \sum_{k=1}^{n} X_{[[k]]} & \sum_{k=1}^{n} X_{[[k]]}^{2} \\ \sum_{k=1}^{n} X_{[[k]]} & \sum_{k=1}^{n} X_{[[k]]}^{2} & \sum_{k=1}^{n} X_{[[k]]}^{3} \\ \sum_{k=1}^{n} X_{[[k]]}^{2} & \sum_{k=1}^{n} X_{[[k]]}^{3} & \sum_{k=1}^{n} X_{[[k]]}^{4} \end{pmatrix};$$

$$B = \begin{pmatrix} \sum_{k=1}^{n} Y_{[[k]]} \\ \sum_{k=1}^{n} X_{[[k]]} Y_{[[k]]} \\ \sum_{k=1}^{n} (X_{[[k]]})^{2} Y_{[[k]]} \end{pmatrix};$$

```
Z = LinearSolve[A, B];
Z
c00 = Z[[1,1]];
c10 = Z[[2,1]];
c20 = Z[[3,1]];
pol[x_] := p[x] /. Thread[{c0, c1, c2} → {c00, c10, c20}]
pol[x]
```

Out[33]= {{1.03503}, {-0.319374}, {-0.168551}}

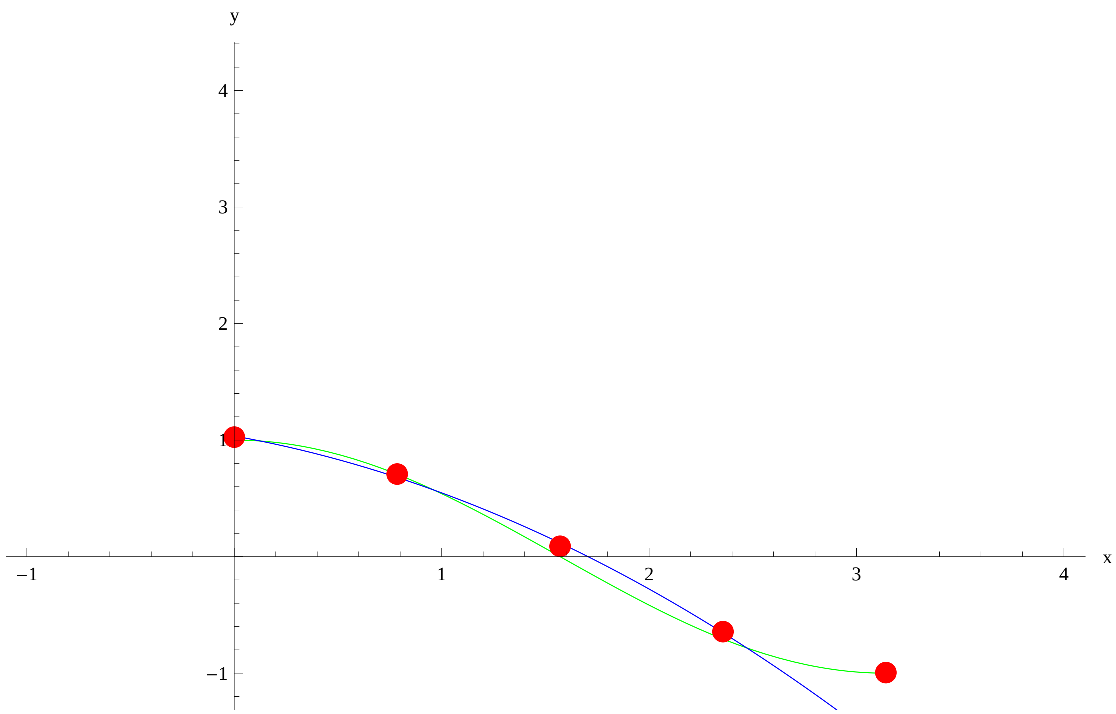
Out[38]= $1.03503 - 0.319374\,x - 0.168551\,x^2$

In[131]:= Z[[1,1]]

Out[131]= 1.08

In[39]:=
```
dots = ListPlot[XY, PlotStyle → {Red, PointSize[0.02]}];
graph1 = Plot[{Cos[x], pol[x]}, {x, 0, Pi}, PlotStyle → {Green, Blue}];
 Show[graph1, dots, PlotRange → {{-1, 4}, {-1.2, 4.3}}, AxesLabel → {"x", "y"}]
Print["Points = ", XY];
```

Out[41]=



Points = $\left\{\{0, 1.02556\}, \left\{\frac{\pi}{4}, 0.708619\right\}, \left\{\frac{\pi}{2}, 0.0890782\right\}, \left\{\frac{3\pi}{4}, -0.64375\right\}, \{\pi, -0.995026\}\right\}$

Mathematica da oluþturma

In[63]:=
```
RMS[XY0_] := Module[ {k, n, X, Y, XY = XY0 },
    n = Length[XY];
    X = Transpose[XY][[1]];
    Y = Transpose[XY][[2]];
    Return[ Sqrt[ 1.0/n Sum[(Y[[k]] - f[X[[k]]])^2, {k, 1, n}] ]; ];
```

$$\text{Return}\left[\sqrt{\frac{1.0}{n}\sum_{k=1}^{n}(Y_{[[k]]}-f[X_{[[k]]}])^2}\ ;\ \right];$$

In[64]:=
```
LSParabola[XY0_] := Module[ {k, n, XY = XY0 },
    n = Length[XY];
    X = Transpose[XY][[1]];
    Y = Transpose[XY][[2]];
```

$$A=\begin{pmatrix} n & \sum_{k=1}^{n} X_{[[k]]} & \sum_{k=1}^{n} X_{[[k]]}{}^2 \\ \sum_{k=1}^{n} X_{[[k]]} & \sum_{k=1}^{n} X_{[[k]]}{}^2 & \sum_{k=1}^{n} X_{[[k]]}{}^3 \\ \sum_{k=1}^{n} X_{[[k]]}{}^2 & \sum_{k=1}^{n} X_{[[k]]}{}^3 & \sum_{k=1}^{n} X_{[[k]]}{}^4 \end{pmatrix};$$

$$B=\begin{pmatrix} \sum_{k=1}^{n} Y_{[[k]]} \\ \sum_{k=1}^{n} X_{[[k]]}\, Y_{[[k]]} \\ \sum_{k=1}^{n} (X_{[[k]]})^2\, Y_{[[k]]} \end{pmatrix};$$

```
    Z = LinearSolve[A, B];
    a = Z[[1,1]];
    b = Z[[2,1]];
    c = Z[[3,1]];
```

$$E2=\sqrt{\left(\frac{1.0}{n}\sum_{k=1}^{n}\left(Y_{[[k]]}-a-b\,X_{[[k]]}-c\,(X_{[[k]]})^2\right)^2\right)};$$

$$\text{Return}\left[\ a+b\,x+c\,x^2\ \right];\ ];$$

Örnek

(-1,10),(0,9),(1,7),(2,5),(3,4),(4,3),(5,0),(6,-1) noktalarýndan geçen $a + b x + c x^2$ ikinçi dereceden en küçük karaler polinomunu bulunuz.

In[65]:=
```
XY = {{-1, 10}, {0, 6}, {1, 2}, {2, 1}, {3, 0}, {4, 2}, {5, 4}, {6, 7}} ;
f[x_] = LSParabola[XY];
p2[x_] = Fit[XY, {1, x, x^2}, x];
Print["Points = ", XY];
Print["Using the subroutine LSParabola"];
Print["y = f[x] = ", f[x] ];
Print["y = f[x] = ", N[f[x]], "\n"];
Print["Using Mathematica’s procedure ‘Fit‘"];
Print["y = p2[x] = ", p2[x] ];
```

```
Points = {{-1, 10}, {0, 6}, {1, 2}, {2, 1}, {3, 0}, {4, 2}, {5, 4}, {6, 7}}
Using the subroutine LSParabola
```

$$y = f[x] = \frac{118}{21} - \frac{26 x}{7} + \frac{2 x^2}{3}$$

$$y = f[x] = 5.61905 - 3.71429 x + 0.666667 x^2$$

```
Using Mathematica’s procedure ‘Fit‘
```

$$y = p_2[x] = 5.61905 - 3.71429 x + 0.666667 x^2$$

In[74]:=
```
Print["y =  a + b x + c x^2"];
Print["The normal equations for finding the coefficients a and b are:"];
Print[MatrixForm[A], MatrixForm[("a"; "b"; "c")], " = ", MatrixForm[B] ];
Print["The solution is"];
Print[MatrixForm[("a"; "b"; "c")], " = ", MatrixForm[Z] ];
Print["a = ", a];
Print["b = ", b];
Print["c = ", c];
Print[""];
Print["The ‘least squares parabola‘ is"];
Print["y = ", a + b x + c x^2, " = ", N[ a + b x + c x^2 ] ];
```

$y = a + b\,x + c\,x^2$

The normal equations for finding the coefficients a and b are:

$$\begin{pmatrix} 8 & 20 & 92 \\ 20 & 92 & 440 \\ 92 & 440 & 2276 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 32 \\ 64 \\ 400 \end{pmatrix}$$

The solution is

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{118}{21} \\ -\frac{26}{7} \\ \frac{2}{3} \end{pmatrix}$$

$$a = \frac{118}{21}$$

$$b = -\frac{26}{7}$$

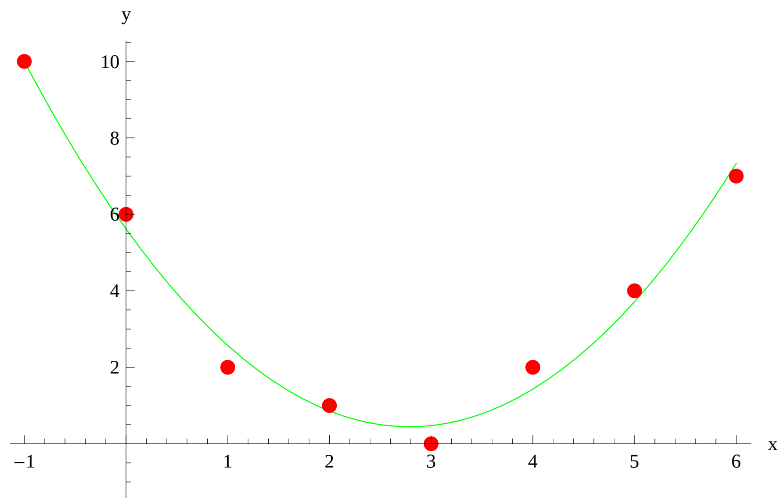
$$c = \frac{2}{3}$$

The ‘least squares parabola‘ is

$$y = \frac{118}{21} - \frac{26\,x}{7} + \frac{2\,x^2}{3} = 5.61905 - 3.71429\,x + 0.666667\,x^2$$

In[85]:=
```
dots = ListPlot[XY, PlotStyle → {Red, PointSize[0.02‘]}];
graph1 = Plot[f[x], {x, -1, 6}, PlotStyle → Green];
Show[graph1, dots, PlotRange → {{-1, 6}, {-1.2‘, 10.3‘}}, AxesLabel → {"x", "y"}]
Print["Points = ", XY]; Print["The ‘least squares parabola‘ is"];
Print["y = ", a + b x + c x^2, " = ", N[a + b x + c x^2]];
```

Out[86]=



Points = {{-1, 10}, {0, 6}, {1, 2}, {2, 1}, {3, 0}, {4, 2}, {5, 4}, {6, 7}}

The ‘least squares parabola‘ is

$$y = \frac{118}{21} - \frac{26\,x}{7} + \frac{2\,x^2}{3} = 5.61905 - 3.71429\,x + 0.666667\,x^2$$

In[88]:=
```
error1 = E2;
Print["The RMS error E₂ is: ", E2];
```

The RMS error E_2 is: 0.393398

Secme Problemler

■ Soru 1

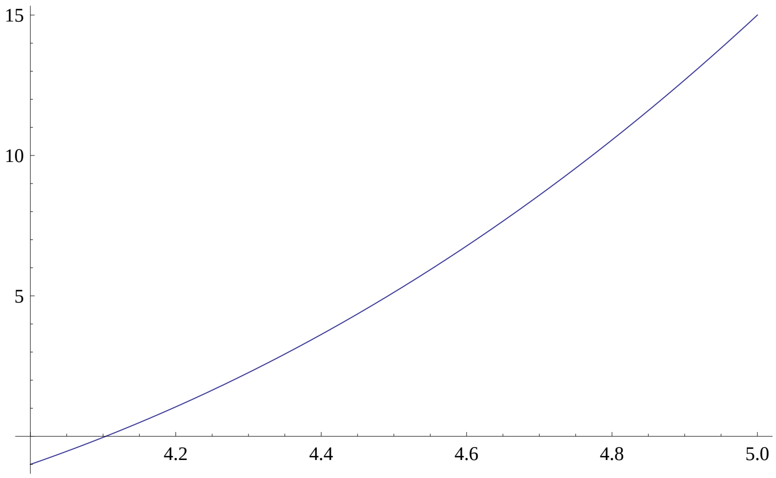
$p(x) = x^3 - 6x^2 + 9x - 5$ polinomum [4,5] aralýðýnda bir kökü olduðunu analitiksel olarak göteriniz? Bu aralýðý yarladýðýmý varsayalým. a) Hangi yarým aralýk çözümü içerdiði garanti eder? Köke yaklaþmak için kökü içeren alt aralýklarýn orta noktasý kullanýlýrsa, hata için bir üst sýnýr veriniz?

In[90]:=
```
p[x_] := x^3 - 6 x^2 + 9 x - 5
p[4] p[5] < 0
```

Out[91]= True

In[92]:=
```
Plot[p[x], {x, 4, 5}]
```

Out[92]=

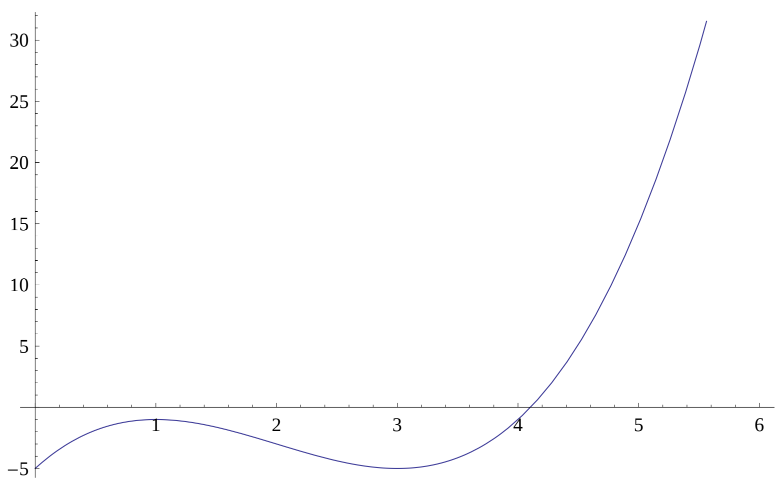


```
In[141]:= x0 = 4;
x1 = 5;
c0 = (x0 + x1)/2 // N
Abs[c0 - x0] < 10^-5
p[c0] < 10^-5
p[c0] p[x0] < 0
x1 = c0
c1 = (x0 + x1)/2 // N
```

```
Out[143]= 4.5
Out[144]= False
Out[145]= False
Out[146]= True
Out[147]= 4.5
Out[148]= 4.25
```

▪ ˙ özüm 1

```
Clear[p, x];
p[x_] := x^3 - 6 x^2 + 9 x - 5
Plot[p[x], {x, 0, 6}]
p[4] p[5] < 0
```




```
True
```

```
True
```

Dolaysýyla bu aralýkta en az bir kökü vardýr.

```
a = 4
b = 5
c1 = (a + b) / 2
p[c] p[a] < 0
p[c] p[b] < 0
```

4

5

$\frac{9}{2}$

$5 - 9c + 6c^2 - c^3 < 0$

$15\left(-5 + 9c - 6c^2 + c^3\right) < 0$

aralýk [a,c] kökü içerir

```
b = c1;
c2 = (a + b) / 2 // N
p[c2] p[a] < 0
```

4.25

True

```
b = c2;
c3 = (b + a) / 2 // N
p[c3] p[a] < 0
```

4.125

True

```
b = c3;
c4 = (b + a) / 2 // N
p[c3] p[a] < 0
```

4.0625

True

hata = $(b - a) / 2^n$

```
hata = (5 - 4) / 2^3 // N
```

0.125

```
In[149]:= bisection[f_, {x_, l_, r_}, tol_, nmax_] :=
  Module[{i, fa, fp, p, a, b},
   a = l;
   b = r;
   p = N[(a + b) / 2];
   fp = f /. x → p;
   For[i = 0, i < nmax && Abs[a - b] > tol, i++,
    p = N[(a + b) / 2];
    fa = f /. x → a;
    fp = f /. x -> p;
    Print[i, ":" , " a=", a ,
     " midpoint=", p, " b=", b, " and the value of f is ", fp];
    If[fp * fa > 0, a = p, b = p]];
   Print["The approximate solution is ", p]]

In[150]:= a = 4;
b = 5;
bisection[p[x], {x, a, b}, 0.0001, 10]
```

```
0: a=4 midpoint=4.5 b=5 and the value of f is 5.125
1: a=4 midpoint=4.25 b=4.5 and the value of f is 1.64063
2: a=4 midpoint=4.125 b=4.25 and the value of f is 0.220703
3: a=4 midpoint=4.0625 b=4.125 and the value of f is -0.413818
4: a=4.0625 midpoint=4.09375 b=4.125 and the value of f is -0.102692
5: a=4.09375 midpoint=4.10938 b=4.125 and the value of f is 0.0574608
6: a=4.09375 midpoint=4.10156 b=4.10938 and the value of f is -0.0230002
7: a=4.10156 midpoint=4.10547 b=4.10938 and the value of f is 0.0171339
8: a=4.10156 midpoint=4.10352 b=4.10547 and the value of f is -0.00295725
9: a=4.10352 midpoint=4.10449 b=4.10547 and the value of f is 0.0070823
The approximate solution is 4.10449
```

■ Soru 2

$f(x) = \exp(x) - x^2$ fonksiyonu veriliyor. f in gerçel bir kökü olduðunu gösteriniz. Newton yöntemi ile bulunuz

```
In[155]:= f[x_] := Exp[x] - x^2
Plot[f[x], {x, -2, 2}]
f[-1] f[0] < 0
```

```
In[169]:= Clear[g, x];
g[x_] := x - f[x]/f'[x]
x0 = 1.0;
NestList[g, x0, 20]

Out[172]= {1., -1.39221, -0.835088, -0.709834, -0.703483, -0.703467, -0.703467,
  -0.703467, -0.703467, -0.703467, -0.703467, -0.703467, -0.703467, -0.703467,
  -0.703467, -0.703467, -0.703467, -0.703467, -0.703467, -0.703467, -0.703467}
```

˙ özüm 2

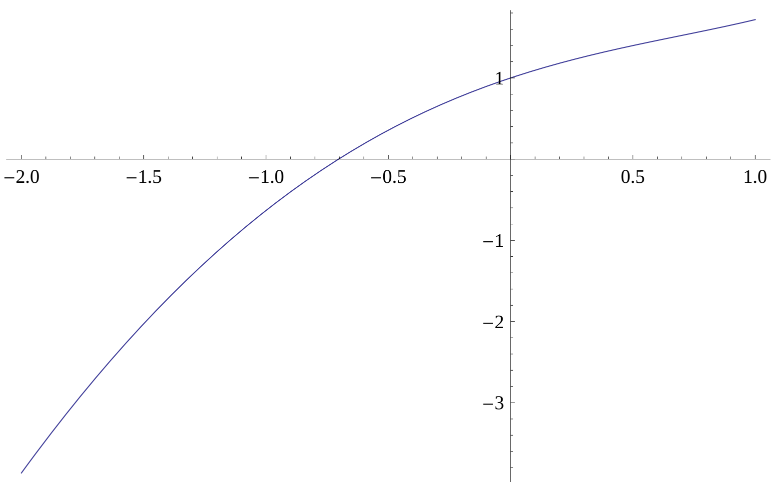
```
Clear[x, denklem];
denklem = Exp[x] - x^2 == 0
NSolve[denklem, x]
```

$e^x - x^2 == 0$

NSolve::ifun : Inverse functions are being used by NSolve, so
some solutions may not be found; use Reduce for complete solution information. ≫

{{x → -0.703467}, {x → 1.58805 - 1.54022 i}}

```
Plot[Exp[x] - x^2, {x, -2, 1}]
```




```
f[x_] := Exp[x] - x^2
f[-1] f[0] < 0
```

True

Newton Yöntemi :

```
In[173]:= a = -1
          b = 1;
          g[x_] := x - f[x]/f'[x]
          x0 = 1;
          x1 = g[x0] // N
          x2 = g[x1]
          x3 = g[x2]
          x4 = g[x3]
          x5 = g[x4]
          hata = Abs[x5 - x4]

Out[173]= -1

Out[177]= -1.39221

Out[178]= -0.835088

Out[179]= -0.709834

Out[180]= -0.703483

Out[181]= -0.703467

Out[182]= 0.0000159816

In[183]:= FindRoot[f[x] == 0, {x, 1}]

Out[183]= {x → -0.703467}
```

■ Soru 3

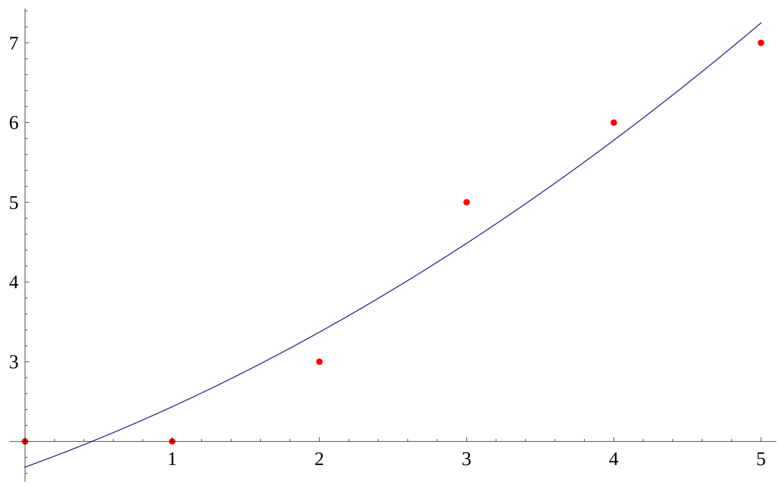
x	0	1	2	3	4	5
y	2	2	3	5	6	7

Tablo verilerini kullanarak $p(x) = a + b\,x + c\,x^{\wedge}2$ en küçük kareler polinomunu bulunuz.

```
In[191]:= xlist = {0, 1, 2, 3, 4, 5}
          ylist = {2, 2, 3, 5, 6, 7}
          XY = Transpose[{xlist, ylist}]
          parabola = Fit[XY, {1, x, x^2}, x]
          Show[ListPlot[XY, PlotStyle → Red], Plot[ parabola, {x, 0, 5}]]

Out[191]= {0, 1, 2, 3, 4, 5}

Out[192]= {2, 2, 3, 5, 6, 7}

Out[193]= {{0, 2}, {1, 2}, {2, 3}, {3, 5}, {4, 6}, {5, 7}}
```

Out[194]= $1.67857 + 0.667857\, x + 0.0892857\, x^2$

Out[195]=



˙özüm 3

```
In[198]:= Clear[a, b, c, x, A, m];
          p[x_] := a + b x + c x^2

In[200]:= denklemler = {p[0] == 2, p[1] == 2, p[2] == 3, p[3] == 5, p[5] == 7}
          Solve[Thread[denklemler], {a, b, c}]

Out[200]= {a == 2, a + b + c == 2, a + 2 b + 4 c == 3, a + 3 b + 9 c == 5, a + 5 b + 25 c == 7}

Out[201]= { }
```

In[202]:=
```
{m, A} = CoefficientArrays[denklemler, {a, b, c}] // Normal
A // MatrixForm
-m // MatrixForm
```

Out[202]= {{-2, -2, -3, -5, -7}, {{1, 0, 0}, {1, 1, 1}, {1, 2, 4}, {1, 3, 9}, {1, 5, 25}}}

Out[203]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \end{pmatrix}$$

Out[204]//MatrixForm=

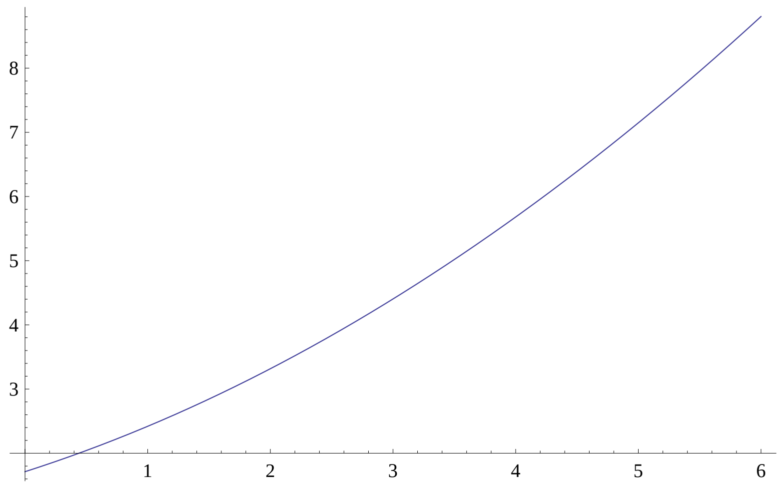
$$\begin{pmatrix} 2 \\ 2 \\ 3 \\ 5 \\ 7 \end{pmatrix}$$

```
In[205]:= çözüm = Inverse[(Transpose[A].A)].Transpose[A].(-m) // N
          p1 = p[x] /. Thread[{a, b, c} → çözüm]
          graf1 = Plot[p1, {x, 0, 6}]
          xlist = {0, 1, 2, 3, 4, 5}
          ylist = {2, 2, 3, 5, 6, 7}
          nokta = ListPlot[Transpose[{xlist, ylist}]];
          Show[{nokta, graf1}]
```

Out[205]= {1.71281, 0.611929, 0.0949926}

Out[206]= 1.71281 + 0.611929 x + 0.0949926 x^2

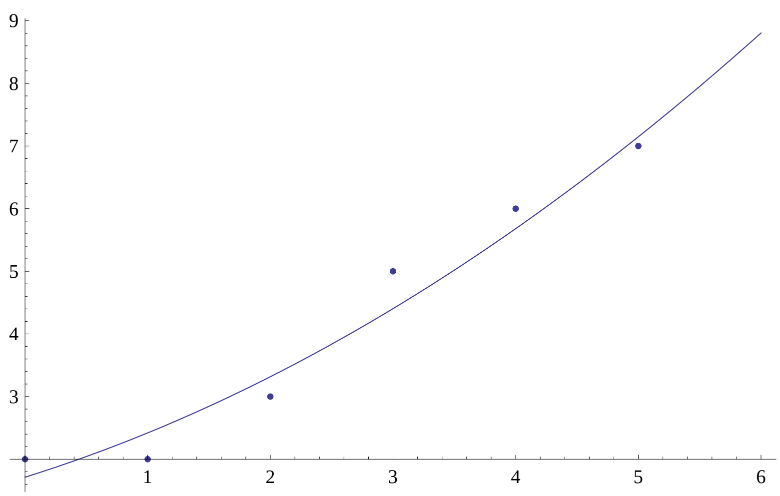
Out[207]=



Out[208]= {0, 1, 2, 3, 4, 5}

Out[209]= {2, 2, 3, 5, 6, 7}

Out[211]=




```
In[212]:= LeastSquares[A, -m] // N
```

Out[212]= {1.71281, 0.611929, 0.0949926}

Soru 4

x	1	1.05	1.08	1.1
f (x)	2.72	3.29	3.66	3.90

tablo deðerleri veriliyor.

a) $f(x)=3xe^{x}-2e^{x}$ fonksiyonuna x=1.04 de yaklaþýk deðeri için en iyi ikinci derece Lagrange enterpolasyon polinomunu oluþturunuz.

b) (a) da yapýlan yaklaþým için hata sýnýrlarýný hesaplayýnýz.

˙özüm 4

```
Remove[L, xlist, ylist, x];
```

In[213]:= $L_0[x\_] := \dfrac{(x-1.05)\,(x-1.08)\,(x-1.1)}{(1-1.05)\,(1-1.08)\,(1-1.1)}$

```
L1[x_] := ((x - 1.0) (x - 1.08) (x - 1.1)) / ((1.05 - 1) (1.05 - 1.08) (1.05 - 1.1))
L2[x_] := ((x - 1.) (x - 1.05) (x - 1.1)) / ((1.08 - 1.) (1.08 - 1.05) (1.08 - 1.1))
L3[x_] := ((x - 1.) (x - 1.05) (x - 1.08)) / ((1.1 - 1.) (1.1 - 1.05) (1.1 - 1.08))
```

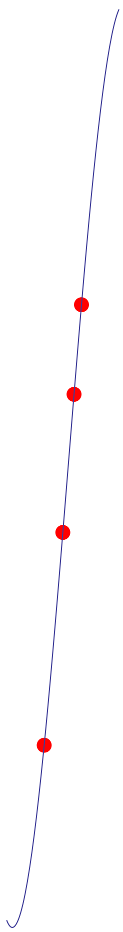
In[217]:=
```
xlist = {1, 1.05, 1.08, 1.1}
ylist = {2.72, 3.29, 3.66, 3.90}
g1 = Graphics[{PointSize[Large], Red, Point[Transpose[{xlist, ylist}]]}];
P[x_] := ylist.{L0[x], L1[x], L2[x], L3[x]}
P[x] // Expand
g2 = Plot[Expand[P[x]], {x, 0.9, 1.2}];
Show[g1, g2]
```

Out[217]= {1, 1.05, 1.08, 1.1}

Out[218]= {2.72, 3.29, 3.66, 3.9}

Out[221]= $211.47 - 610.917\,x + 585.5\,x^2 - 183.333\,x^3$

Out[223]=



In[224]:=
```
y[x_] := 3 x Exp[x] - 2 Exp[x]
hata = Abs[P[1.04] - y[1.04]];
Print["hata=", hata]
```

hata=0.000323056

■ Soru 5

$\int_0^1 x^2 e^{-x}\,dx$ integraline yaklaþmak için uygun bir integrasyon formulü kullanýnýz kullanýnýz.

In[227]:=
```
NIntegrate[x^2 Exp[-x], {x, 0, 1}]
```
Out[227]= 0.160603

˙özüm 5

In[228]:=
```
sagtoplam[f_, {x_, a_, b_}, n_] := Module[{h, toplam},
   h = (b - a) / n;
   toplam = Sum[f /. x -> a + h (i), {i, 1, n}];
   Return[N[h toplam]]
  ]
soltoplam[f_, {x_, a_, b_}, n_] := Module[{h, toplam},
   h = (b - a) / n;
   toplam = Sum[f /. x -> a + h (i), {i, 0, n - 1}];
   Return[N[h toplam]]
  ]
trap[f_, {x_, a_, b_}, n_] := Module[{},
    Return[(sagtoplam[f, {x, a, b}, n] + soltoplam[f, {x, a, b}, n]) / 2];
   ];
ortaNokta[f_, {x_, a_, b_}, n_] := Module[{h, i, toplam},
    h = (b - a) / n;
    toplam = Sum[N[f[a + (i + 0.5) h]], {i, 0, n - 1}];
    Return[h toplam];
   ];
SIMP[f_, {x_, a_, b_}, n_] := Module[{h},
    h = (b - a) / n;
    Return[(2 ortaNokta[f, {x, a, b}, n] + trap[f[x], {x, a, b}, n]
      ) / 3.0];
   ];
```

In[233]:=
```
Clear[t, f];
f[x_] := x^2 e^-x
sagtoplam[f[t], {t, 0, 1}, 1000]
soltoplam[f[t], {t, 0, 1}, 1000]
trap[f[t], {t, 0, 1}, 1000]
ortaNokta[f, {t, 0, 1}, 1000]
SIMP[f, {t, 0, 1}, 1000]
```

Out[235]= 0.160787

Out[236]= 0.160419

Out[237]= 0.160603

Out[238]= 0.160603

Out[239]= 0.160603

In[240]:=
```
Integrate[f[x], {x, 0, 1}] // N
```
Out[240]=
```
0.160603
```