Power supply for the Booster magnets is regulated by the Gradient Magnet Power Supply (GMPS). It comprises four sold-state power supplies that are given current on a 15Hz sampling frequency. A parameter $V \in \mathbb{R}$, along with many exogenous factors, enter into the GMPS to spur generation of a current sent to the power supplies. Before entering the power supplies, the current is sampled at 20kHz, fitted, and measured (see Figure 2). The minimum of the fitted curve is the parameter `newestimin` in the code; here we refer to it as `fit:VIMIN`[1]. The fitted value `fit:VIMIN` is then used to calculate and report to ACNET `B:IMINER`, which is then used to construct the input V_t supplied again to the GMPS system. In what follows, we report the system of equations that describes this process.

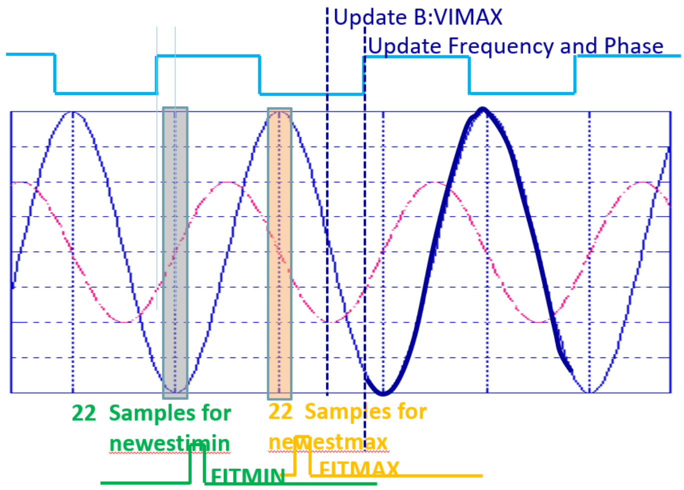

Figure 1: GMPS Schematic by Kiyomi Seiya.

First, we assume we begin at some time t_0 with input to the power supplies V_{t_0}. Let $\mathcal{G}$ be the black-box environment of the GMPS system ; $\mathcal{E} = \mathbb{R}^m$ comprise the numerical values associated with the entire accelerator system and outside influences, which we assume is some finitely-many values $m \in \mathbb{N}$; and $\vec{p}$ denote the exogenous, environmental, or system-wide influences, i.e. $\vec{p} \in \mathcal{E}$. We have $\mathcal{G} : \mathbb{R} \times \mathcal{E} \to \mathbb{R}$, where we reserve the first argument for the current $V \in \mathbb{R}$. Assume we compose its output sampled at 20kHz with some function $\mathcal{F}$ that fits the data and outputs `fit:VIMIN`, so that $\texttt{fit:VIMIN} = \mathcal{F} \circ \mathcal{G}(V, \vec{p})$. Further assume the setup that generating a new waveform at 15Hz samples moves us in time from t_0 to t_1. A series of computations are initiated to feed as input for the next cycle, in between t_0 and t_1. In this memo, we detail those calculations, which we call the GMPS system of equations. A picture of the process is shown in Figure 1. Twenty-two samples around the minimal amplitude and maximal amplitude are captured and fitted using a parabolic fit (`fit:VIMIN` and `fit:VIMAX`, respectively). The blue line is the generated V_t input into the GMPS Power Supplies using those fitted values, and again the realized current is sampled.

Assume a current has just been generated at some initial time t_0, and the fitted values $\texttt{fit:VIMIN}_{t_1}$ and $\texttt{fit:VIMAX}_{t_1}$ have been obtained. We proceed to calculate the errors $\texttt{B:IMINER}_{t_1}$ and $\texttt{B:IMAXER}_{t_1}$.

$$\texttt{B:IMINER}_{t_1} = 10\,(\texttt{fit:VIMIN}_{t_1} - \texttt{B\_VIMIN}_{t_1}). \tag{1}$$

$$\texttt{B:IMAXER}_{t_1} = 10\,(\texttt{fit:VIMAX}_{t_1} - \texttt{B\_VIMAX}_{t_1}). \tag{2}$$

Next, we calculate parameters $\texttt{B:VIMIN}_{t_1}$ and $\texttt{B:VIMAX}_{t_1}$, which are used to form V_{t_1}.

$$\begin{cases} \texttt{B:VIMIN}_{t_1} & = \texttt{B\_VIMIN}_{t_1} - \alpha\ \texttt{B:IMINER}_{t_1} - \beta_{t_1} \\ \beta_{t_1} & = \beta_{t_0} + \gamma\ \texttt{B:IMINER}_{t_1}, \end{cases} \tag{3}$$

where $\alpha = 8.5\text{e-}2$, $\gamma = 7.5350088\text{e-}5$, and β_{t_0} is some constant. The value for `B:VIMAX` is also computed:

$$\begin{cases} \texttt{B:VIMAX}_{t_1} & = \texttt{B\_VIMAX}_{t_1} - \alpha'\ \texttt{B:IMAXER}_{t_1} - \beta'_{t_1} \\ \beta'_{t_1} & = \beta'_{t_0} + \gamma' \texttt{B:IMAXER}_{t_1} \end{cases} \tag{4}$$

The parameters α', β', γ' are the analogous terms for the gains in (4). They are a scaled, in order of magnitude, version of the gains in Table 1, as the analogous parameters for (4) are. The calculated values of `B:VIMIN` and `B:VIMAX` are logged by the ACNET console system and used for the input $\vec{V}$ into GMPS at the next time step t_1. $\vec{V}_{t_1}$ is calculated as follows.

$$\begin{cases} \vec{V}_{t_1} & = c n_p\ [\ \mu_{t_1} + \nu_{t_1} \cos(2\pi \texttt{freq}_{t_1} t' + \varphi_{t_1})\ ]\ ,\ t' \in (t_1, t_2) \text{ at frequency } \texttt{freq}_{t_1} \\ \mu_{t_1} & = 0.5(\texttt{B:VIMAX}_{t_1} + \texttt{B:VIMIN}_{t_1}) \\ \nu_{t_1} & = 0.5(\texttt{B:VIMAX}_{t_1} - \texttt{B:VIMIN}_{t_1}). \end{cases} \tag{5}$$

where c some constant assumed to be 1, n_p is the number of power supplies ($n_p = 4$), and φ_{t_1} is a phase angle that changes in time and by some applied offset (which also possibly changes at each time step). The frequency `freq` is updated at each time step. Note the time value t' is supplied at frequency $\texttt{freq}_{t_1}$ over the range of time between 15Hz

[1]This choice of notation is to make the parameter consistent with ACNET notation, which uses ":" to denote paramters from measured quantites and "_" to denote parameters from settings.

Table 1: Gain Values

c code variable	value	related variable	description
`acminpropgain_set`	8.5	α	proportional gain for minimum
`acminintgrlgain_set`	0.75350088	γ	integral gain for minimum
`acmaxpropgain_set`	3.0	α'	proportional gain for maximum
`acmaxintgrlgain_set`	0.3	γ'	integral gain for maximum

samples; it is used to construct the current for the next time step (the blue line in Figure 1). We then assume this cycle for all ordered sequences in time. Specifically, for time t,

$$(\texttt{fit:VIMIN}_t, \texttt{fit:VIMAX}_t) = \mathcal{F}[\mathcal{G}(V_{t-1}; \vec{p}_{t-1})] \tag{6}$$

$$\texttt{B:IMINER}_t = 10\ (\texttt{fit:VIMIN}_t - \texttt{B\_VIMIN}_t). \tag{7}$$

$$\texttt{B:IMAXER}_t = 10\ (\texttt{fit:VIMAX}_t - \texttt{B\_VIMAX}_t) \tag{8}$$

$$\beta_t = \beta_{t-1} + \gamma\ \texttt{B:IMINER}_t \tag{9}$$

$$\texttt{B:VIMIN}_t = \texttt{B\_VIMIN}_t - \alpha\ \texttt{B:IMINER}_t - \beta_t \tag{10}$$

$$\beta'_t = \beta'_{t-1} + \gamma' \texttt{B:IMAXER}_t \tag{11}$$

$$\texttt{B:VIMAX}_t = \texttt{B\_VIMAX}_t - \alpha'\ \texttt{B:IMAXER}_t - \beta'_t \tag{12}$$

$$\vec{V}_t = cn_p\ [\ \mu_t + \nu_t \cos(2\pi \texttt{freq}_t t' + \varphi_t)\ ]\ ,\ \ t' \in (t, t+1) \text{ at frequency } \texttt{freq}_t \tag{13}$$

$$\mu_t = 0.5(\texttt{B:VIMAX}_t + \texttt{B:VIMIN}_t) \tag{14}$$

$$\nu_t = 0.5(\texttt{B:VIMAX}_t - \texttt{B:VIMIN}_t) \tag{15}$$

$$(\texttt{fit:VIMIN}_{t+1}, \texttt{fit:VIMAX}_{t+1}) = \mathcal{F}[\mathcal{G}(V_t; \vec{p}_t)], \tag{16}$$

and one can see the (16) map to (6), begin the process (7)-(15) again, and then generate (16) anew.

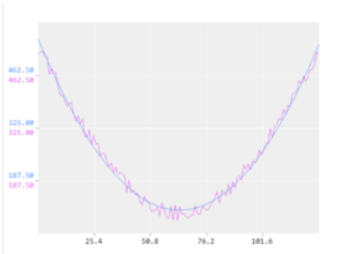

Figure 2: `fit:VIMIN` is the minimum of a fitted sequence (blue) of noisy values (magenta). Image by Brian Schupbach.