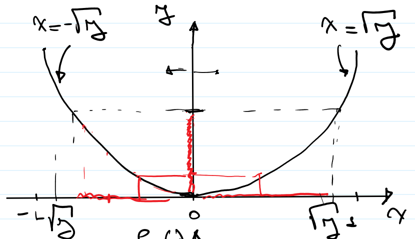
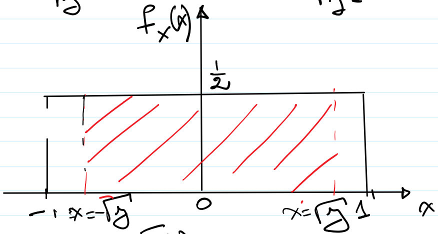
Sea X~U(-1,1), y sea Y=X². Hallar la función de densidad de Y , $g(X) = Y = X^2$



$f_X(x) = \frac{1}{2}\, \mathbb{1}\{-1 < x < 1\}$, $F_X(x) =$

$f_Y(y) = ?$ Ej: $P(Y < \frac{1}{2}) = P(X^2 < \frac{1}{2})$

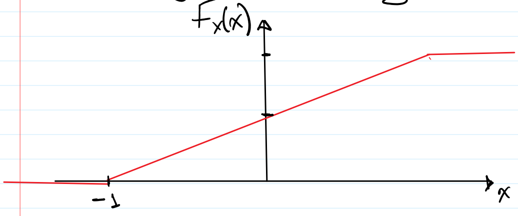
$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) =$
$= P(|X| \leq \sqrt{y}) = P(-\sqrt{y} \leq X \leq \sqrt{y}) = ⊛$



$$F_X(x) = \begin{cases} 0 & \text{si } x < -1 \\ \frac{x+1}{2} & \text{si } -1 \leq x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$$

$⊛ = F_X(\sqrt{y}) - F_X(-\sqrt{y}) =$

$= \frac{\sqrt{y}+1}{2} - \frac{-\sqrt{y}+1}{2} = \sqrt{y}$



$$F_Y(y) = \begin{cases} 0 & \text{si } y < 0 \\ \sqrt{y} & \text{si } 0 \leq y < 1 \\ 1 & \text{si } y \geq 1 \end{cases}$$

$F_Y(y) = \sqrt{y}\, \mathbb{1}\{0 \leq y < 1\} + \mathbb{1}\{y \geq 1\}$

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & \text{si } 0 \leq y < 1 \\ 0 & \text{en otro caso.} \end{cases}$$, $f_Y(y) = \frac{1}{2\sqrt{y}}\, \mathbb{1}\{0 \leq y < 1\}$

Sean X e Y dos v.a. con distribución de Poisson de parámetros μ y λ respectivamente. Hallar la función de probabilidad de W = X + Y.

$$X \sim \mathcal{P}oi(\mu) \rightarrow p_X(x) = \frac{\mu^x}{x!}e^{-\mu},\ x \in \mathbb{N}_0$$

$X \sim Po(\mu)$, $Y \sim Po(\lambda)$, $p_W(w) = ?$

$p_W(w) = P(W = w) = P(X + Y = w) = \sum_{x=0}^{w} P(X = x, Y = w - x) =$

$= \sum_{x=0}^{w} P(X = x) \cdot P(Y = w - x) = \sum_{x=0}^{w} \frac{\mu^x \cdot e^{-\mu}}{x!} \cdot \frac{\lambda^{(w-x)} \cdot e^{-\lambda}}{(w-x)!} =$

$= e^{-(\mu+\lambda)} \cdot \sum_{x=0}^{w} \underbrace{\frac{w!}{x!\,(w-x)!}}_{\binom{w}{x}} \cdot \mu^x \cdot \lambda^{w-x} \cdot \frac{1}{w!} =$

$= \frac{e^{-(\mu+\lambda)}}{w!} \underbrace{\sum_{x=0}^{w} \binom{w}{x} \cdot \left(\frac{\mu}{\mu+\lambda}\right)^x \cdot \left(\frac{\lambda}{\mu+\lambda}\right)^{w-x}}_{=1} \cdot (\mu+\lambda)^x \cdot (\mu+\lambda)^{w-x} = ⊛$, $X \sim Bi\left(w, \frac{\mu}{\mu+\lambda}\right)$

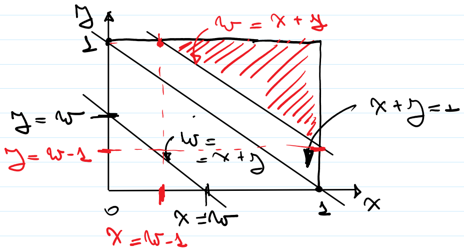
$T \sim Bi(n, p)$, $p_T(t) = \binom{n}{t} \cdot p^t \cdot (1-p)^{n-t}$

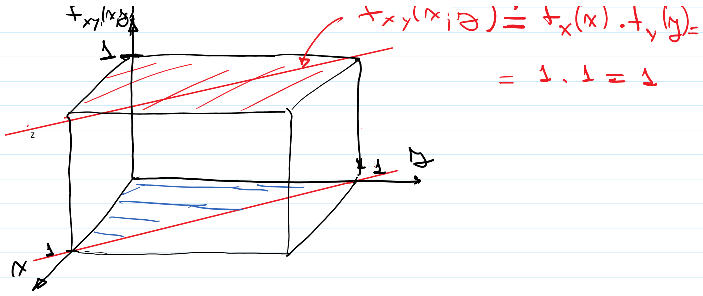
$⊛ = \frac{e^{-(\mu+\lambda)}}{w!} \cdot (\mu+\lambda)^w$, $W = X + Y \sim Po(\mu+\lambda)$

Sean X,Y ~ U(0,1) e independientes. Hallar la función de densidad de W = X+Y

si X e Y son independientes.

$f_{X,Y}(x;y) \stackrel{\downarrow}{=} f_X(x) \cdot f_Y(y) =$
$= 1 \cdot 1 = 1$





si $0 < w < 1$

$$F_W(w) = P(W \leq w) = \frac{w^2}{2}$$

si $1 \leq w < 2$

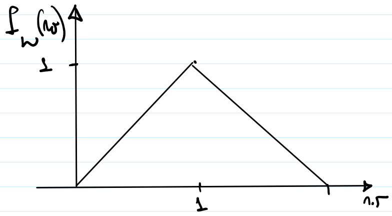
$$F_W(w) = 1 - P(W > w) = 1 - \frac{[1-(w-1)] \cdot [1-(w-1)]}{2} =$$

$$= 1 - \frac{(2-w)^2}{2}$$

$$F_W(w) = \begin{cases} 0 & \text{si } w < 0 \\ \frac{w^2}{2} & \text{si } 0 \leq w < 1 \\ 1 - \frac{(2-w)^2}{2} & \text{si } 1 \leq w < 2 \\ 1 & \text{si } w \geq 2 \end{cases} , \quad f_W(w) = \begin{cases} w & \text{si } 0 \leq w < 1 \\ 2 - w & \text{si } 1 \leq w < 2 \\ 0 & \text{en otro caso} \end{cases}$$

$$F_W(w) = \frac{w^2}{2} \mathbb{I}\{0 \leq w < 1\} + \left[1 - \frac{(2-w)^2}{2}\right] \cdot \mathbb{I}\{1 \leq w < 2\} + \mathbb{I}\{w \geq 2\}$$

$$f_W(w) = w \, \mathbb{I}\{0 \leq w < 1\} + (2-w) \, \mathbb{I}\{1 \leq w < 2\}$$



- Sea X una v.a.c. con función de densidad $f_X(x)$,
- Sea Y=g(X).
- g(x) es una función 1 a 1 (existe $g^{-1}(y)$)

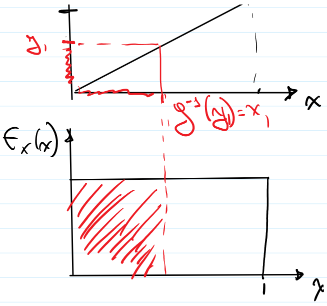
$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dg^{-1}(y)}{dy} \right|$$

$$F_Y(y) = P(Y \leq y) = P(g(X) \leq y) = P(X \leq g^{-1}(y)) =$$

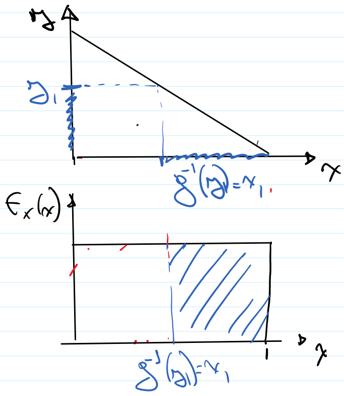
$$= F_X(g^{-1}(y))$$

$$f_Y(y) = \frac{dF_X(\overbrace{g^{-1}(y)}^{x})}{} \cdot \frac{d\,\overbrace{g^{-1}(y)}^{x}}{}$$

$$f_Y(y) = \frac{d F_X(\overbrace{g^{-1}(y)}^{x})}{dx} \cdot \frac{d\, \overbrace{g^{-1}(y)}^{x}}{dy}$$



$$f_Y(y) = f_X(g^{-1}(y)) \cdot \frac{d g^{-1}(y)}{dy}$$



$$F_Y(y) = P(Y \leq y) = P(X > g^{-1}(y)) =$$

$$= 1 - F_X(g^{-1}(y))$$

$$f_Y(y) = \frac{d(1 - F_X(\overbrace{g^{-1}(y)}^{x}))}{dx} \cdot \frac{d\, \overbrace{g^{-1}(y)}^{x}}{dy}$$

$$f_Y(y) = -f_X(g^{-1}(y)) \cdot \frac{d g^{-1}(y)}{dy}$$

Va resultar negativo por la relación inversa entre X e Y

Entonces $f_Y(y)$ siempre va resultar positivo:

Por lo tanto se lo define: $f_Y(y) = f_X(g^{-1}(y)) \cdot \left| \frac{d g^{-1}(y)}{dy} \right|$ → para cualquier tipo de relación entre X e Y.

Sean $X_1, X_2 \overset{i.i.d}{\sim} \mathcal{E}(\lambda)$ y sean $\overset{h_1(x_1;x_2)}{U} = X_1 + X_2$ y $\overset{h_2(x_1;x_2)}{V} = \frac{X_1}{X_1+X_2}$. Hallar $f_{U,V}(u,v)$. ¿Qué puede decir al respecto?

$$X_1 \sim \mathcal{E}(\lambda), \quad X_2 \sim \mathcal{E}(\lambda), \quad \begin{cases} u = x_1 + x_2 > 0 \\ v = \frac{x_1}{x_1+x_2} > 0 \end{cases}$$

$$\begin{cases} u = x_1 + x_2 \Rightarrow x_2 = u - x_1 \Rightarrow x_2 = u - uv = h_2^{-1}(u;v) \\ v = \frac{x_1}{x_1+x_2} \Rightarrow v = \frac{x_1}{x_1 + u - x_1} = \frac{x_1}{u} \Rightarrow x_1 = u \cdot v = h_1^{-1}(u;v) \end{cases}$$

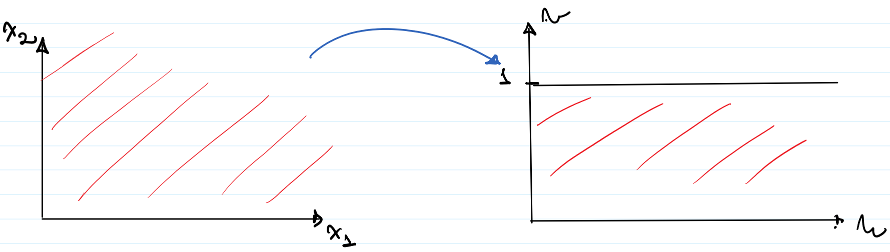
$$|J| = \begin{vmatrix} v & u \\ 1-v & -u \end{vmatrix} = -u \cdot v - (1-v)u = -u$$

Por X_1 y X_2 independientes

$$f_{Y_1,Y_2} = \overbrace{f_{X_1,X_2}(x_1,x_2)}\Big|_{h_1^{-1}(u_1,u_2),\, h_2^{-1}(u_1,u_2)} |J| \qquad , \quad f_{X_1X_2}(x_1,x_2) = \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2}\, \mathbb{1}\{x_1>0, x_2>0\} =$$

y_1, y_2 $\quad$ y_1, y_2

$$= \lambda^2 e^{-\lambda(x_1+x_2)}\, \mathbb{1}\{x_1>0, x_2>0\}$$



$$u = x_1 + x_2$$
$$v = \frac{x_1}{x_1+x_2}$$

$$f_{Y_1Y_2}(y_1,y_2) = \lambda^2 e^{-\lambda(x_1+x_2)}\, \mathbb{1}\{x_1>0, x_2>0\} \Big|_{\substack{x_1 = u \cdot v \\ x_2 = u - uv}} \cdot |-u| =$$

$$= u \cdot \lambda^2 \cdot e^{-\lambda(u\cdot v + u - u \cdot v)} \cdot \mathbb{1}\{u \cdot v > 0,\ u(1-v) > 0\} =$$

$$= \underbrace{u \cdot \lambda^2 \cdot e^{-\lambda u}} \cdot \mathbb{1}\{u > 0,\ 0 < v < 1\}$$

$$f_X(x) = \frac{\lambda^k}{\Gamma(k)} x^{k-1} \cdot e^{-\lambda x}$$

Gamma, $X \sim \Gamma(\lambda, k)$

$$= \underbrace{u \cdot \lambda^2 \cdot e^{-\lambda u}\ \mathbb{1}\{u > 0\}}_{U \sim \Gamma(\lambda, 2)} \cdot \underbrace{\mathbb{1}\{0 < v < 1\}}_{V \sim \mathcal{U}(0,1)}$$