Ej 2.3) Demostrar que A es definida positiva

$$A = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \qquad A = A^T \checkmark$$

$$\hat{x} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad x^T . A . x > 0 ?$$

$$[x \quad y] \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = x^2 + x . y + \frac{1}{3} y^2$$

$$\underbrace{x^2 + 2 . x . \frac{1}{2} y + \frac{1}{3} y^2 - \frac{1}{12} y^2} + \frac{1}{12} y^2$$

$$\left(x^2 + 2 . x . \frac{1}{2} y + \frac{1}{4} y^2\right) + \frac{1}{12} y^2$$

$$\underbrace{\left(x + \frac{1}{2} y\right)^2}_{\geq 0} + \underbrace{\frac{1}{12} y^2}_{\geq 0} \qquad x^T . A . x \geq 0 \checkmark$$

b) Base canonica $\mathbb{R}^2$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Gram Schmidt

$U_1 = V_1$

$U_2 = V_2 - Proy_{U_1} V_2$

$$\langle U_1, V_2 \rangle = V_2^T . A . U_1$$

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[0 \quad 1] \, [1/2 \quad 1/3] = 1/2$$

$$U_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \frac{\langle U_1 \; V_2 \rangle}{\langle U_1 \; U_1 \rangle} . U_1$$

$$\langle U_1, U_1 \rangle = 1$$

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[1 \quad 0] \, [1 \quad 1/2] = 1$$

$$U_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\langle U_1, U_2 \rangle = [-1/2 \quad 1] \, [0 \quad 1/12] = 0$$

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$\langle U_2, U_2 \rangle = [-1/2 \quad 1] \, [0 \quad 1/12] = 1/12 \neq 1 \therefore \text{no BON}$$

$$u_2 = \frac{v_2}{\|v_2\|}$$

$$\|v_2\| = \langle v_2 ; v_2 \rangle^{1/2} = \sqrt{\frac{1}{12}}$$

$$u_2 = \begin{bmatrix} \frac{-1/2}{\sqrt{1/12}} \\ \frac{1}{\sqrt{1/12}} \end{bmatrix} = \begin{bmatrix} -\sqrt{3} \\ \sqrt{12} \end{bmatrix}$$

comprobamos $\langle u_2, u_2 \rangle = 1$?

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \qquad \begin{bmatrix} -\sqrt{3} \\ \sqrt{12} \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{3} & \sqrt{12} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{12}}{2} - \sqrt{3} & -\frac{\sqrt{3}}{2} + \frac{\sqrt{12}}{3} \end{bmatrix}$$

$$-\frac{\sqrt{12} \cdot \sqrt{3}}{2} + \sqrt{3} \cdot \sqrt{3} - \frac{\sqrt{3} \cdot \sqrt{12}}{2} + \frac{\sqrt{12} \cdot \sqrt{12}}{3} =$$

$$-3 + 3 - 3 + 4 = 1 \ \checkmark$$