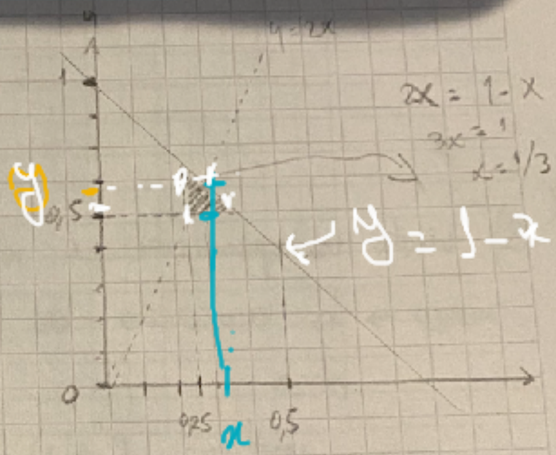
a) $f_{xy} = 96x \quad \{0,25 < x < 0,5,\ 0,5 < y < 1-x\}$

$y = 1-x$



b) $P(x > 1/2 y) = \int_{0,25}^{1/3} \int_{0,5}^{1-x} 96x \, dy \, dx \approx 0,4815$

b) $f_x(x) = \int 96x \, dy = 96xy + C$

$f_y(y) = \int 96x \, dx = 48x^2 + C$

c) $f_{x,y}(x,y) = 96x \stackrel{?}{=} 96xy \cdot 48x^2$

$96x \neq 4608x^3y$ { No son indep.

Ej 10 (Rep. cap 2)

1) • $X = \begin{cases} 1 & \text{con cara} \\ 0 & \end{cases}$ $\quad X \sim Bin(1/2)$

$$\mathbb{E}[X] = 0 \cdot 1/2 + 1 \cdot 1/2 = 1/2$$

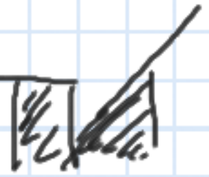
• $X =$ "# de caras en 3 tiros" $\quad X \sim Bin(3, 1/2)$

$$p_X(x) = \binom{3}{x} \underbrace{1/2^x \, 1/2^{3-x}}_{1/2^3}$$

$$\mathbb{E}[X] = \sum_{x=0}^{3} x \binom{3}{x} 1/2^3 = 0 \cdot \binom{3}{3} 1/2^3 + 1 \cdot \binom{3}{1} 1/2^3 + 2\binom{3}{2} 1/2^3 + 3\binom{3}{3} 1/2^3$$

$$= 3 \cdot 1/2$$

• $f_X(x) = 1/2 \mathbf{1}\{0 < x < 2\}$ $\quad \mathbb{E}[X] = \int_0^2 x \cdot 1/2 \, dx = 1$



• $f_X(x) = \frac{1}{2}\mathbf{1}\{0 < x < 1\} + \frac{x}{3}\mathbf{1}\{1 < x < 2\}$

$$\mathbb{E}[X] = \int_{\mathbb{R}} x f_X(x) \, dx = \int_0^1 x \, 1/2 \, dx + \int_1^2 x \cdot \frac{x}{3} \, dx = \frac{x^2}{2 \cdot 2}\Big|_0^1 + \frac{x^3}{3 \cdot 3}\Big|_1^2 = \frac{1}{4} + \frac{7}{9}$$

$$= 1{,}027$$

Ej 10 (Rep. Clase 2)

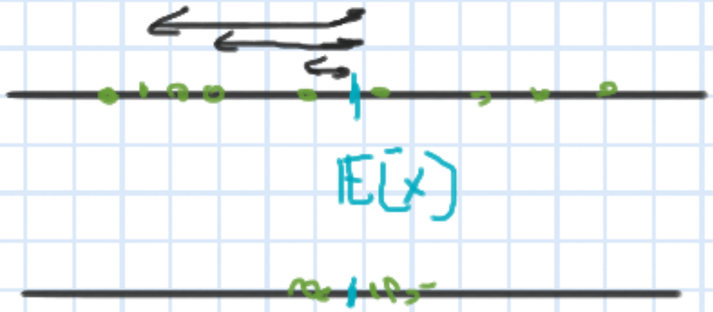
2	x \ y	1	2	3	4	5	6
	1	1/36	1/36	1/36	1/36		
	2						
	3						
	4						
	5						
	6						

$$\mathbb{E}[X+Y] \overset{\text{linealidad}}{=} \mathbb{E}[X] + \mathbb{E}[Y] = 2\,\mathbb{E}[X] = 2\sum_{x=1}^{6} x\,\frac{1}{36}$$

$\mathbb{E}[X] - \mathbb{E}[Y] = 0$

$= 2 \cdot 7/2 = 7$

x	1	2	3	4	5	6
P_X	1/6	1/6	1/6	1/6	1/6	1/6


E(X)

Es. 11

- $X \sim Ber(1/2)$ $\quad Var(X) = \mathbb{E}[X^2] - \underbrace{\mathbb{E}[X]^2}_{1/2} = 1/2 - 1/2^2 = 1/4$

$$\mathbb{E}[X^2] = 0^2\, 1/2 + 1^2\, 1/2 = 1/2$$

- $X \sim Bin(3, 1/2)$ $\quad Var(X) = \mathbb{E}[X^2] - \underbrace{\mathbb{E}[X]^2}_{3/2} = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$

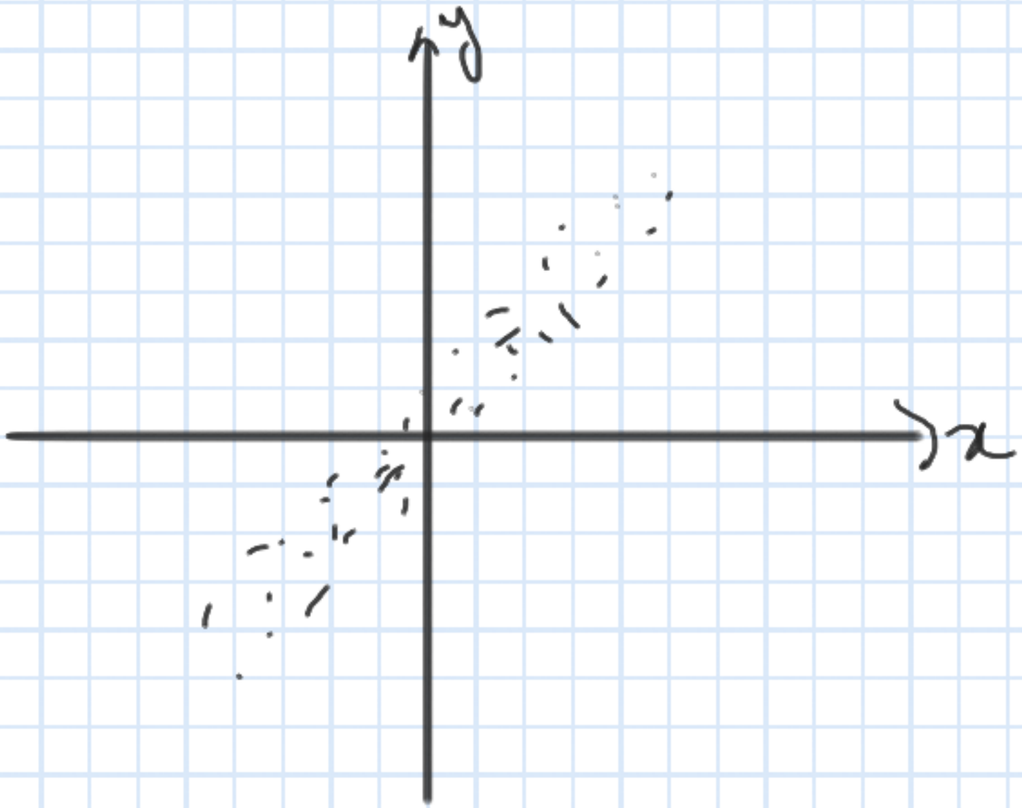
$$\mathbb{E}[X^2] = 0^2 \binom{3}{0} 1/2^3 + 1^2 \binom{3}{1} 1/2^3 + 2^2 \binom{3}{2} 1/2^3 + 3^2 \binom{3}{3} 1/2^3$$
$$= 3$$

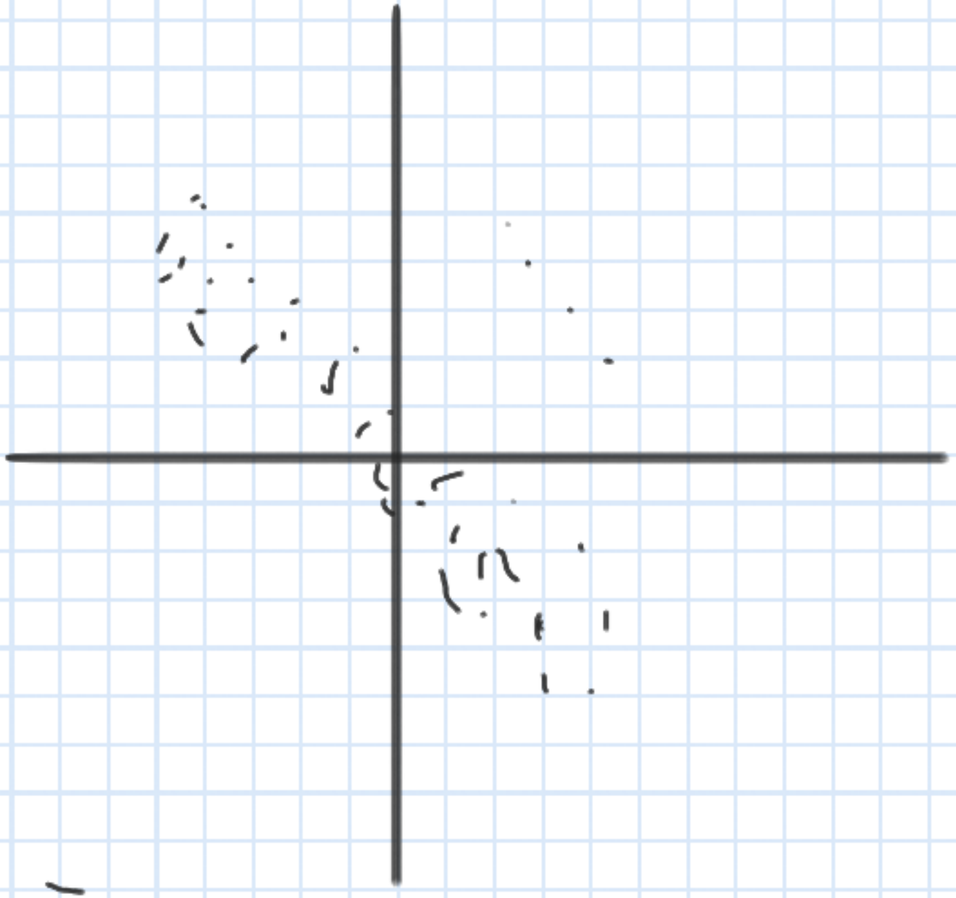
- $X \sim \mathcal{U}(0,2)$ $\quad Var(X) = \mathbb{E}[X^2] - \underbrace{\mathbb{E}[X]^2}_{1} = 4/3 - 1 = 1/3$

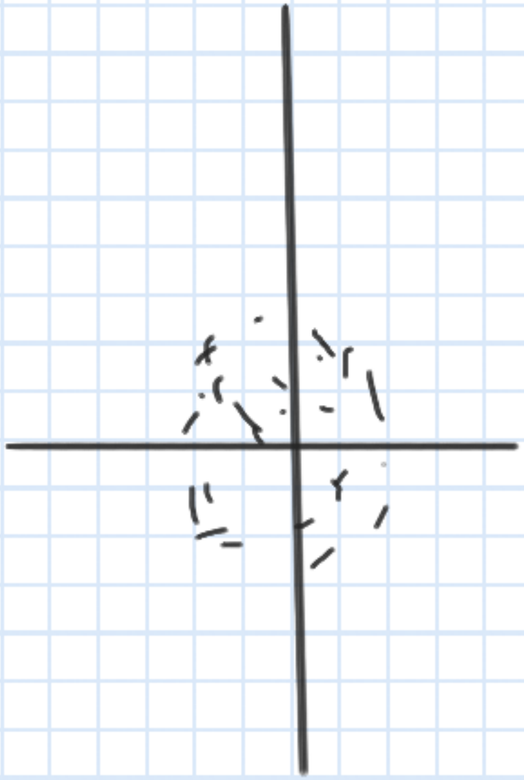
$$\mathbb{E}[X^2] = \int_0^2 x^2 \frac{1}{2}\, dx = \frac{x^3}{3}\frac{1}{2}\Big|_0^2 = 4/3$$

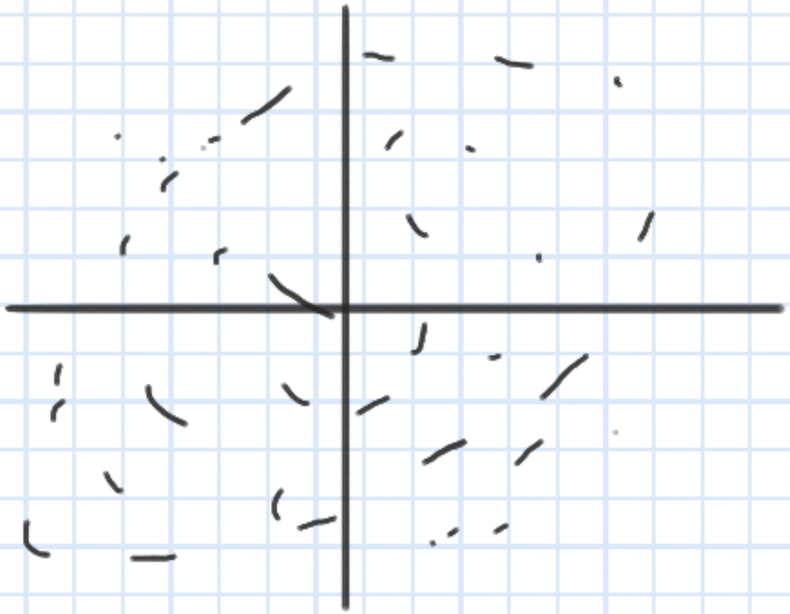
- $f_X(x) = \frac{1}{2}\mathbf{1}\{0 < x < 1\} + \frac{x}{3}\mathbf{1}\{1 < x < 2\}$ $\quad \mathbb{E}[X^2] = \int_0^1 x^2 \cdot 1/2\, dx + \int_1^2 \frac{x}{3} \cdot x^2\, dx$

$$Var(X) = \frac{17}{12} - \frac{37}{36} = 0{,}3603 = \frac{x^3}{3}\frac{1}{2}\Big|_0^1 + \frac{x^4}{4}\frac{1}{3}\Big|_1^2 = \frac{1}{6} + \frac{4}{3} - \frac{1}{12}$$


y
x







Es. 12

y \ x	0	1	2	3	P_Y
0	8/27	4/9	1/18	1/216	$\frac{125}{216}$
1	2/9	1/9	1/72	0	$\frac{25}{72}$
2	1/18	1/72	0	0	$\frac{5}{72}$
3	1/216	0	0	0	$\frac{1}{216}$
P_X	$\frac{125}{216}$ = 0,57	$\frac{25}{72}$	$\frac{5}{72}$	$\frac{1}{216}$	

$$(\mathbb{E}[X], \mathbb{E}[Y]) = (1/2, 1/2)$$

$$\mathbb{E}[X] = 0 \cdot \frac{125}{216} + 1 \cdot \frac{25}{72} + 2 \cdot \frac{5}{72} + 3 \cdot \frac{1}{216}$$

$$= 1/2$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X]\,\mathbb{E}[Y]$$

$$= \sum_{x=0}^{2} \sum_{y=0}^{3} x \cdot y \; P_{X,Y}(x,y) - 1/2 \cdot 1/2$$

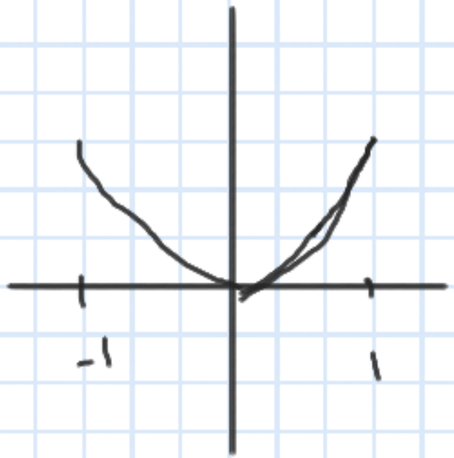
$$= 2/9 - 1/2 \cdot 1/2 = -1/36$$

Eg 12-2 → Tarea

Eg 12-3 → $X \sim \mathcal{U}(-1,1)$ $\quad Y = X^2$

$$\operatorname{cov}(X,Y) = \operatorname{cov}(X, X^2) = \mathbb{E}\left[\underbrace{X \cdot X^2}_{X^3}\right] - \mathbb{E}[X]\,\mathbb{E}[X^2]$$

$f_X(x) = \frac{1}{2} \mathbb{1}\{-1 < x < 1\}$



$$= \int_{-1}^{1} x^3 \, \frac{1}{2} \, dx - 0 \cdot \mathbb{E}[X^2]$$

$$= \frac{x^4}{4 \cdot 2}\Big|_{-1}^{1} - 0 = 0$$