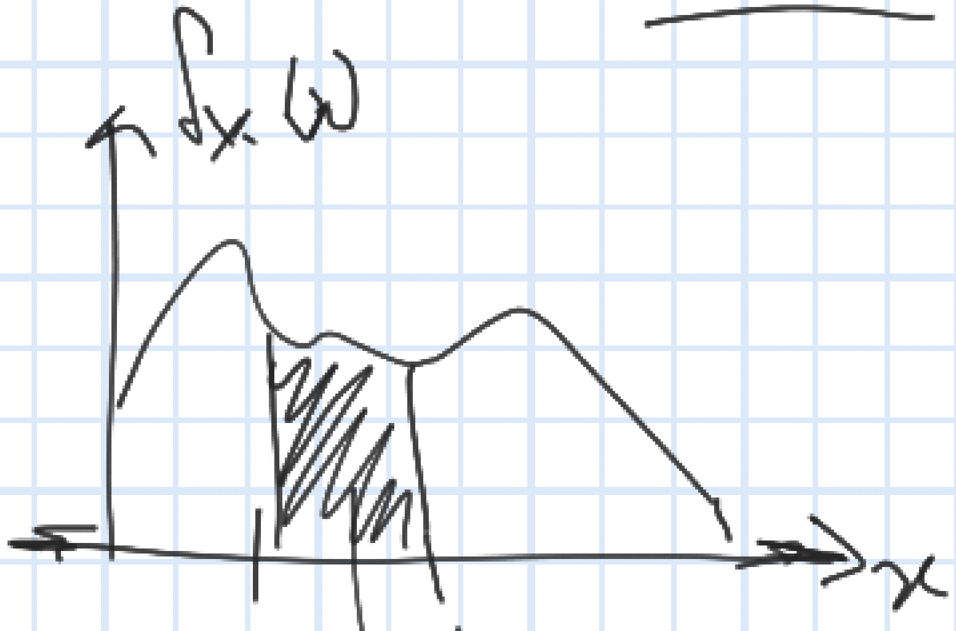
Si X v.a.c. $\quad \mathbb{P}(X = x) = 0 \leftarrow$



$\mathbb{P}(a < X < b)$

$$\mathbb{P}(a < X < b) = \int_a^b f_X(x)\,dx$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \qquad f_X(x) \geq 0$$

Si X es v.a.d. $\quad \mathbb{P}(a < X < b) = \sum_{x \in (a,b)} p_X(x)$

$$p_X(x) \geq 0$$

$$\sum_x p_X(x) = 1$$

Si X es v.a.d.

$$\mathbb{E}[X] = \sum_x x \underbrace{\mathbb{P}(X=x)}_{p_X(x)}$$

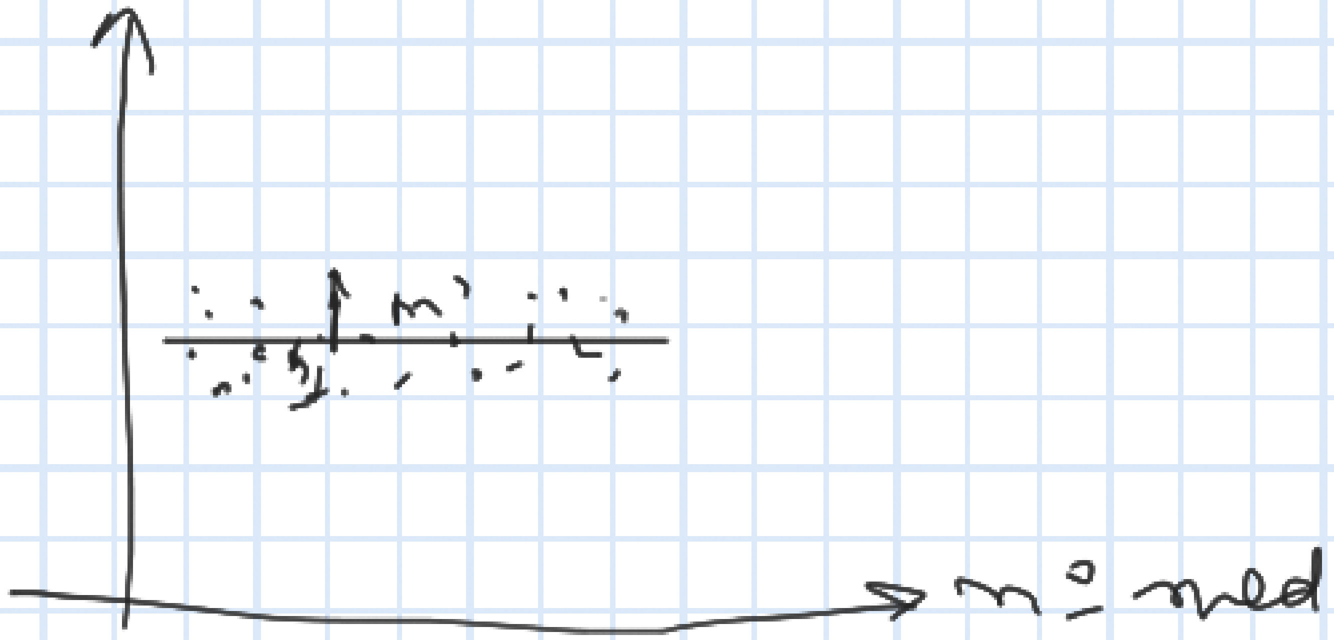
$$E[X]$$

Si X es v.a.c.

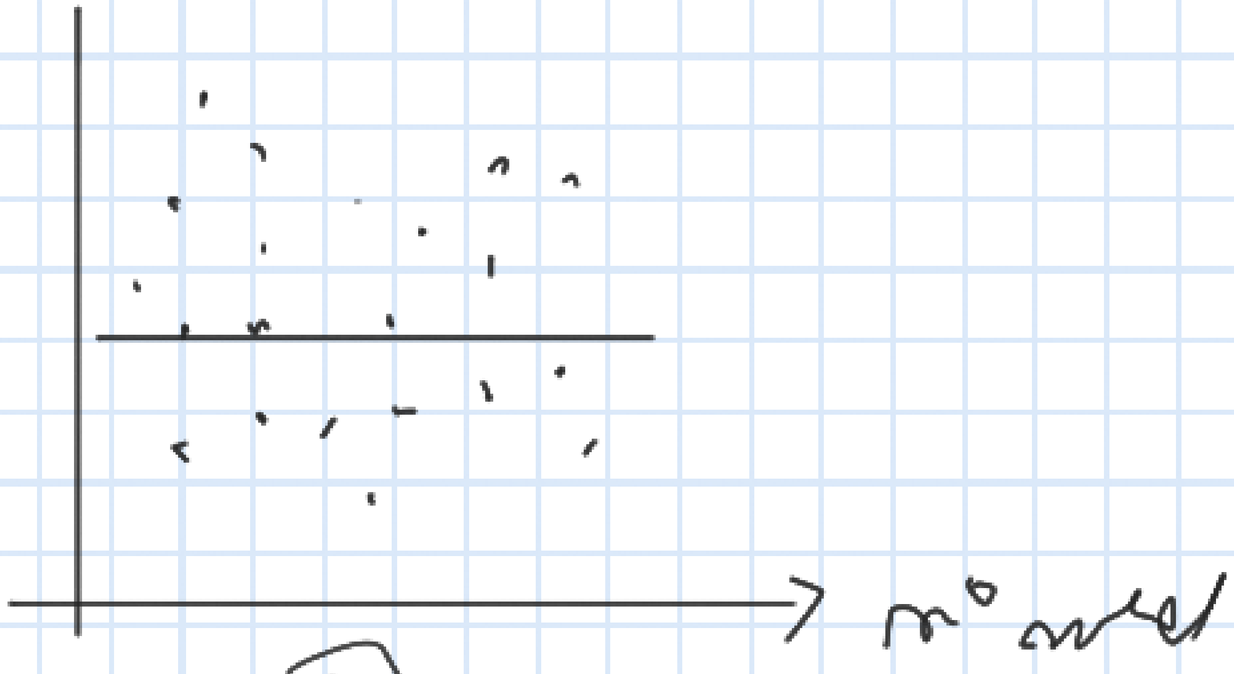
$$\mathbb{E}[X] = \int_{\mathbb{R}} x \cdot f_X(x)\, dx$$

$$Y = g(X) \qquad \mathbb{E}[Y] = \mathbb{E}[g(X)] = \sum_x g(x)\, \mathbb{P}(X=x) \quad (\text{vad})$$

$$= \int g(x)\, f_X(x)\, dx \quad (\text{vac})$$



1



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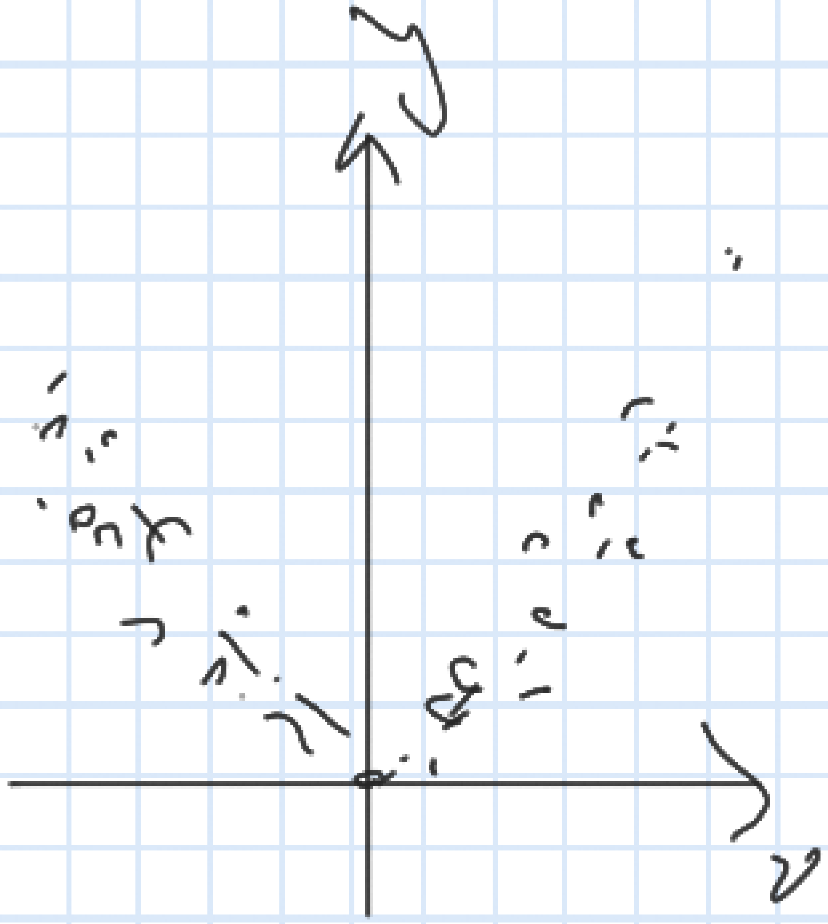
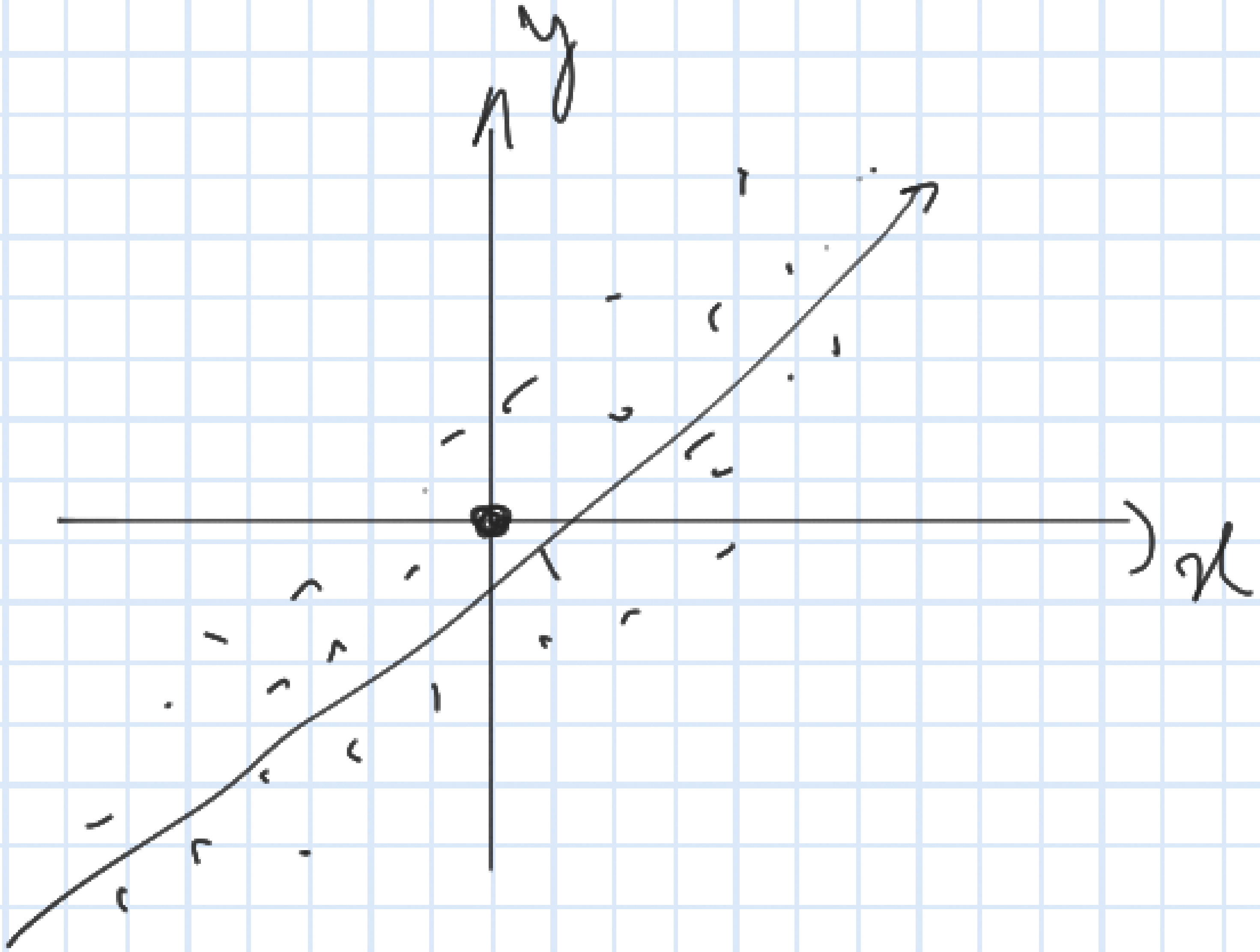
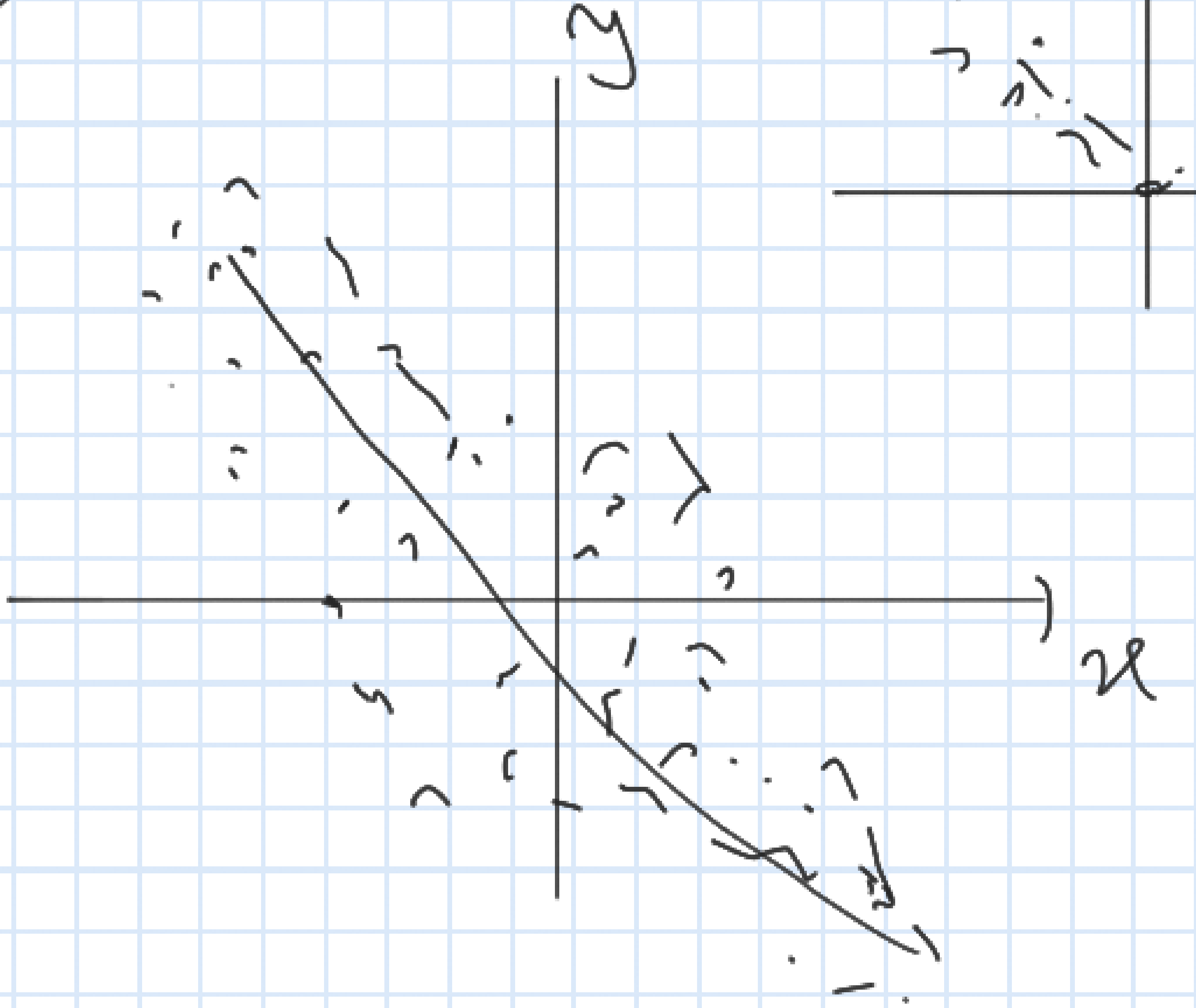
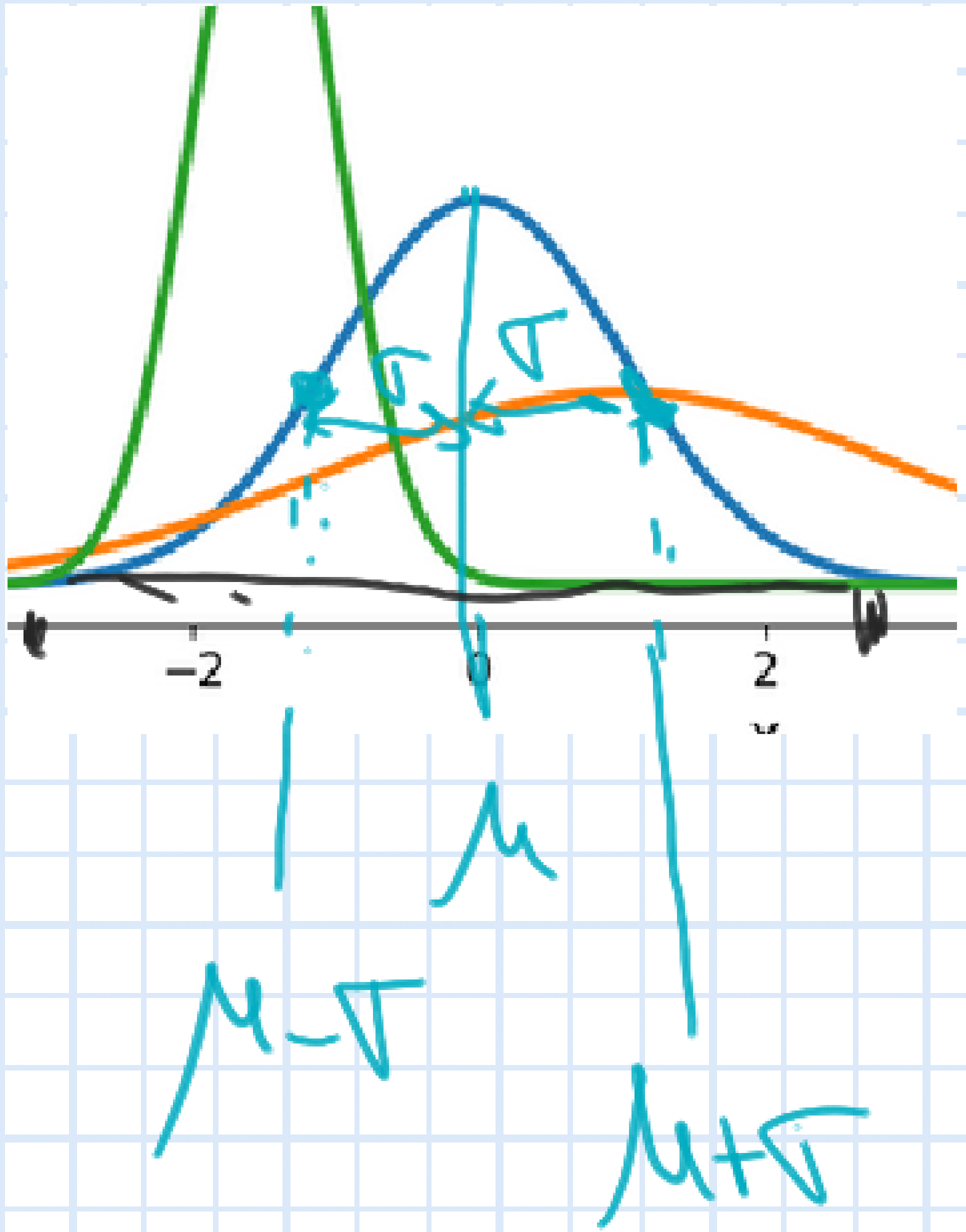








- $\mathbb{P}(\mu-\sigma < X < \mu+\sigma) \approx 0.68$
- $\mathbb{P}(\mu-2\sigma < X < \mu+2\sigma) \approx 0,9544.$

$\sigma = \sqrt{Var(X)}$ = desvío estándar

- $\mathbb{P}(\mu-3\sigma < X < \mu+3\sigma) \approx 0,9973$

$X \sim \mathcal{N}(0,1)$

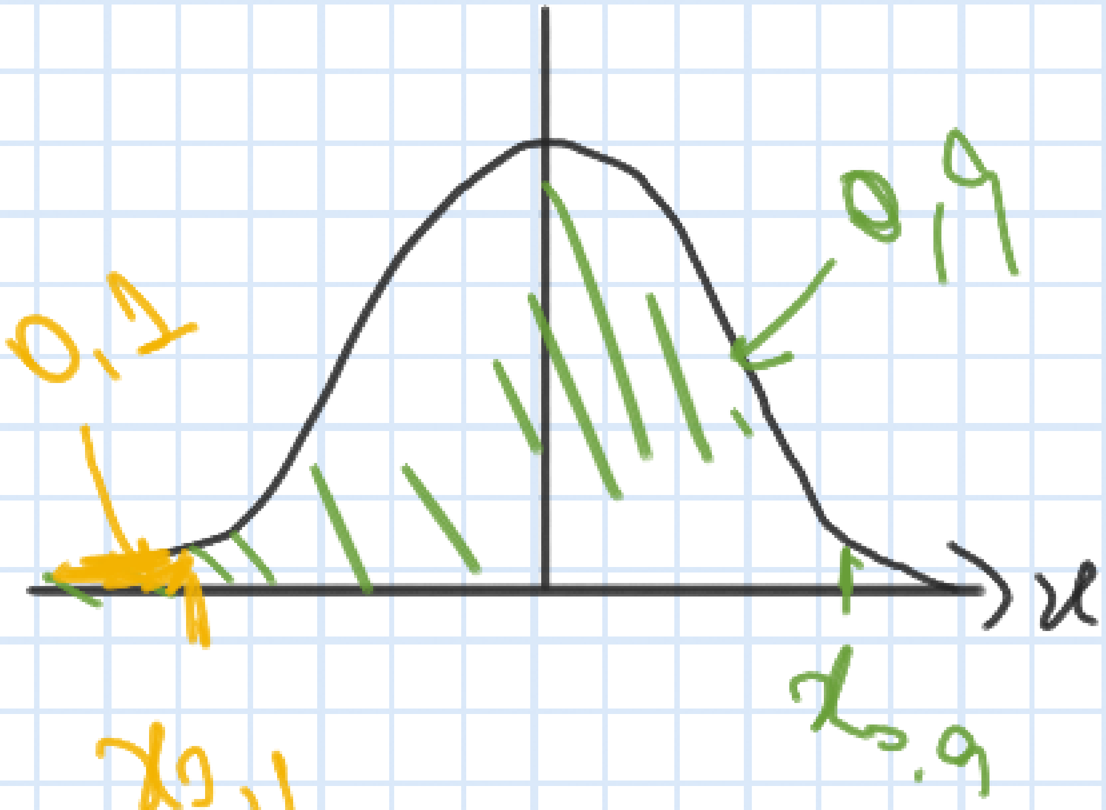
$\mathbb{P}(X > 1) = 0{,}1586$

$\mathbb{P}(X < -1) = 0{,}1586$

$\mathbb{P}(|X| < 2) = \mathbb{P}(-2\sigma + \mu < X < 2\sigma + \mu)$

$= 0{,}9544$

- pdf → func. de prob.
- cdf → func. de dist $F_X(x) = \mathbb{P}(X \leq x)$
- ppf → quantiles $x_\alpha \mid \mathbb{P}(X \leq x_\alpha) = \alpha$



$x_{0,1} = -x_{0,9}$

$x_{0,9} : \mathbb{P}(X < x_{0,9}) = 0,9$

$= 1,2813$

$x_{0,1} = -1,2815$

$X \sim N(0,1) \quad Y \sim N(2,9) \quad V = 2X + Y \sim N\left(\underbrace{2 \cdot 0 + 2}_{2}, \underbrace{2^2 \cdot 1 + 9}_{13}\right)$

$\mathbb{P}(V < 5) = 0,792$

$= \mathbb{P}\left(\overbrace{\frac{V-2}{\sqrt{13}}}^{N(0,1)} < \frac{5-2}{\sqrt{13}}\right) =$

Ej. 2) X = "Tiempo hasta el 1° llamado"

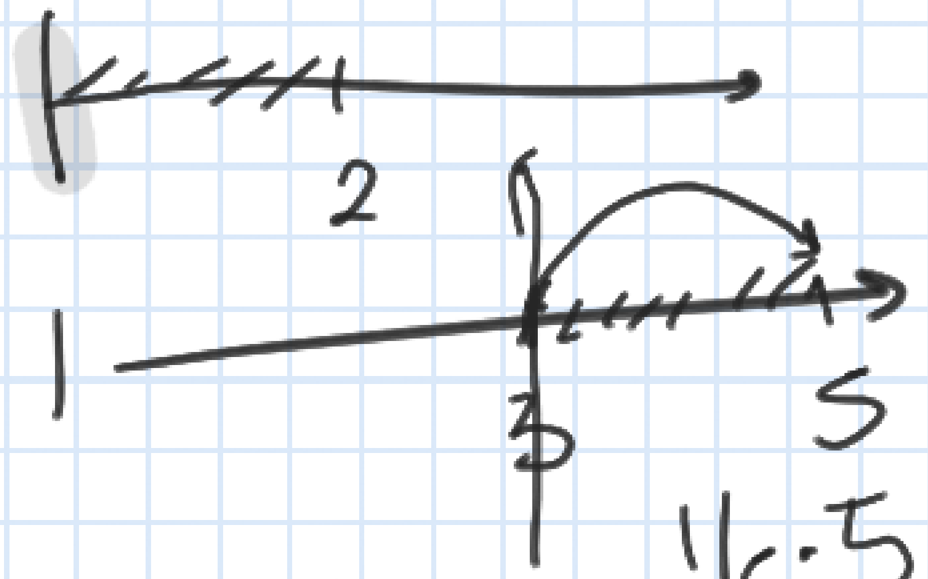
$$X \sim \mathcal{E}(1/5)$$

$$\boxed{\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

$$\mathbb{P}(X > 2) = e^{-1/5 \cdot 2}$$

$$S_X(x) = \mathbb{P}(X > x) = e^{-1/5\, x}$$

$$\hookrightarrow 1 - \mathbb{P}(X \leq 2) = 0{,}67032$$



$$\mathbb{P}(X > 5 \mid X > 3) = \frac{\mathbb{P}(X > 5, X > 3)}{\mathbb{P}(X > 3)} = \frac{\mathbb{P}(X > 5)}{\mathbb{P}(X > 3)} = \frac{e^{-1/5 \cdot 5}}{e^{-1/5 \cdot 3}}$$

$$= e^{-1/5\,(5-3)} = e^{-1/5 \cdot 2}$$

pérdida de memoria

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\underline{x}-\underline{\mu})^T \Sigma^{-1} (\underline{x}-\underline{\mu})}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2 \frac{(x-\mu)^2}{\sigma^2}}$$

$$-0.8 = \operatorname{cov}(X, Y)$$

$$\sigma_X^2 = 1 \qquad \mu_X = 0$$

$$\sigma_Y^2 = 1 \qquad \mu_Y = 0$$

$$f_{X,Y}(x, y) = \frac{1}{2\pi 0.6} e^{-\frac{1}{2}\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \end{bmatrix}}$$

$$\begin{bmatrix} x-0 & y-0 \end{bmatrix}$$

$$X \sim N(0, 1)$$

$$Y \sim N(0, 1)$$

Si X e Y son indep $\Rightarrow$ $cov(X,Y)=0$

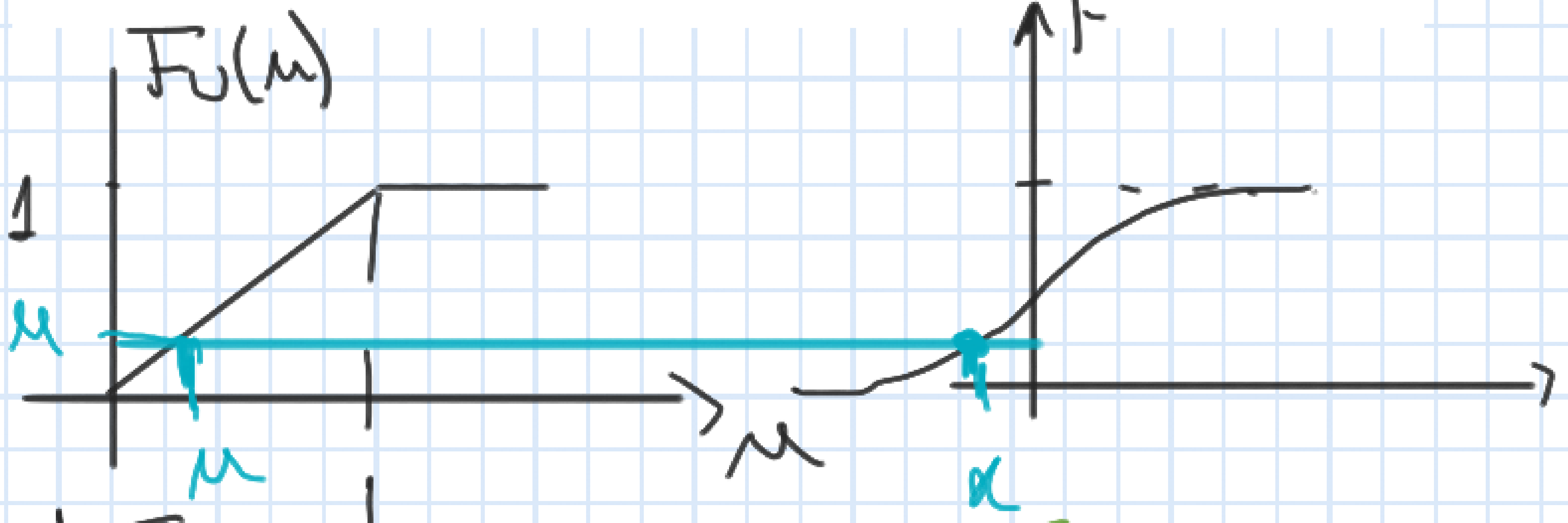
$\not\Leftarrow$

Si X e Y son normales $\quad X,Y$ indep $\Longleftrightarrow cov(X,Y)=0$

$\mathbb{P}(X<2, Y<-1) = 0,1378$

$$F_X^{-1}(u) = \min\{x \in \mathbb{R} : F_X(x) \geq u\},\ u \in (0,1)$$

$U \sim \mathcal{U}(0,1)$

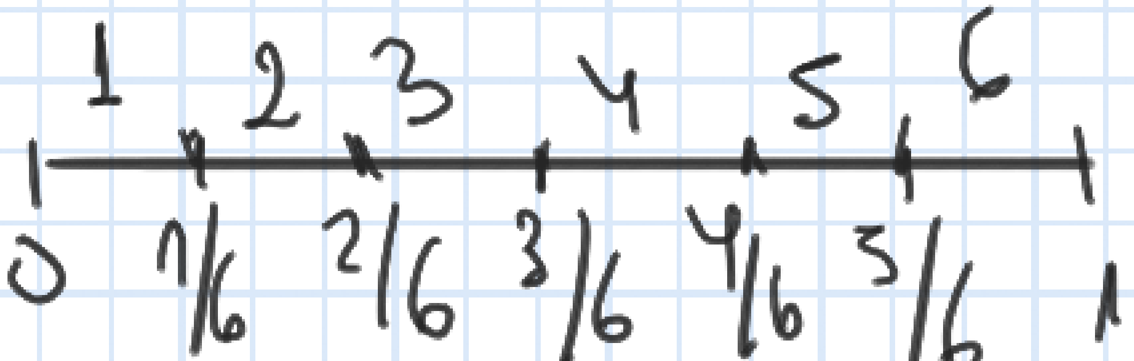


$F(x) = \underbrace{F_U(u)}_{u}$

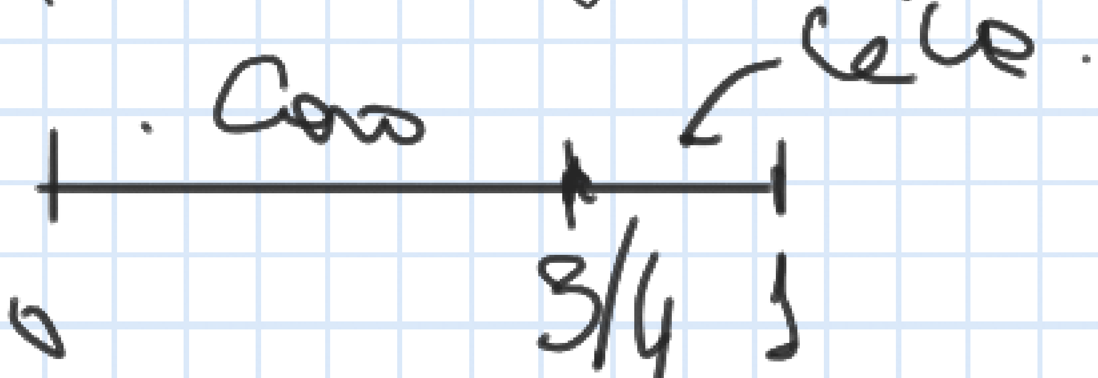
$\vdots$

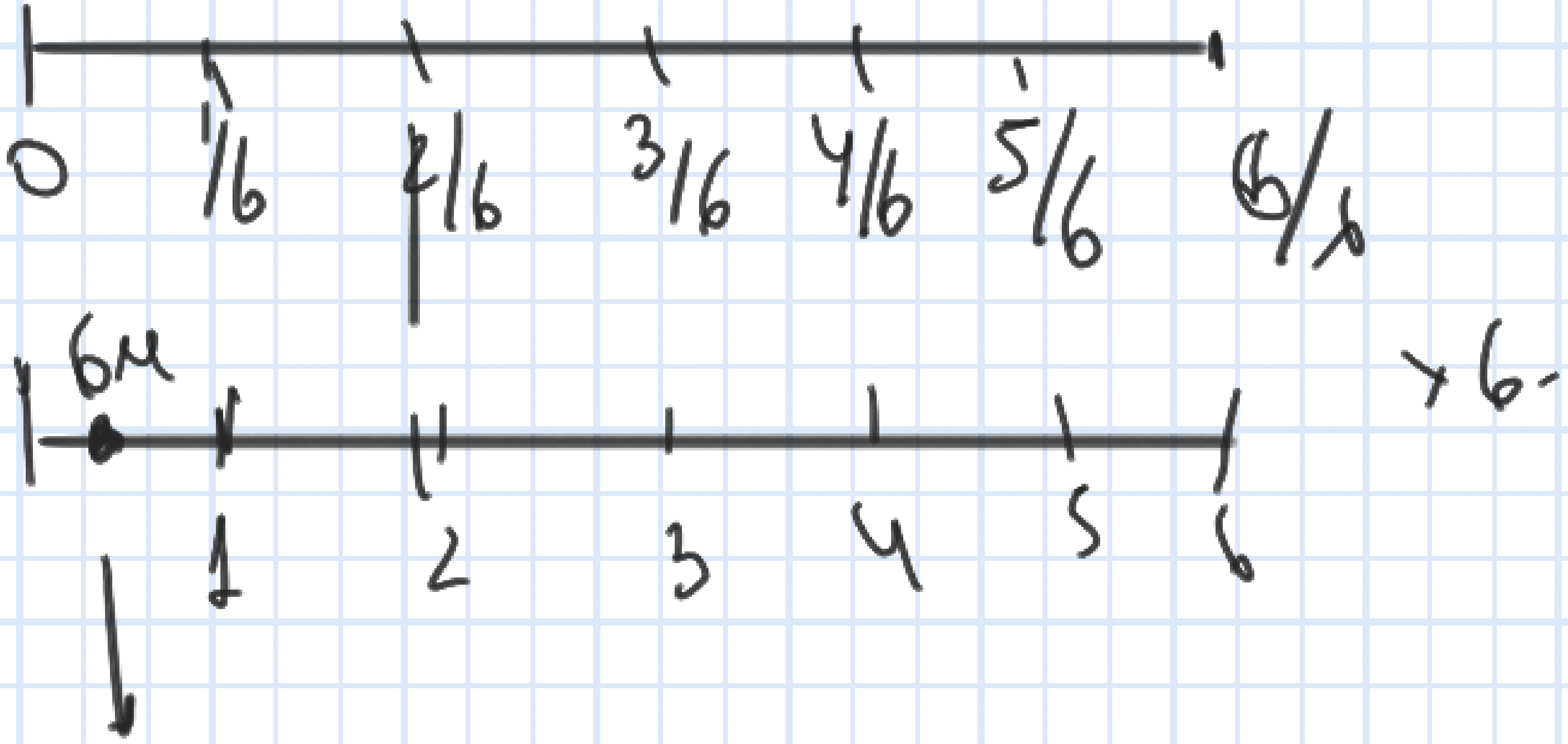
$x = g(u)$

$= F^{-1}(u)$



$X \sim \mathrm{Ber}(3/4)$

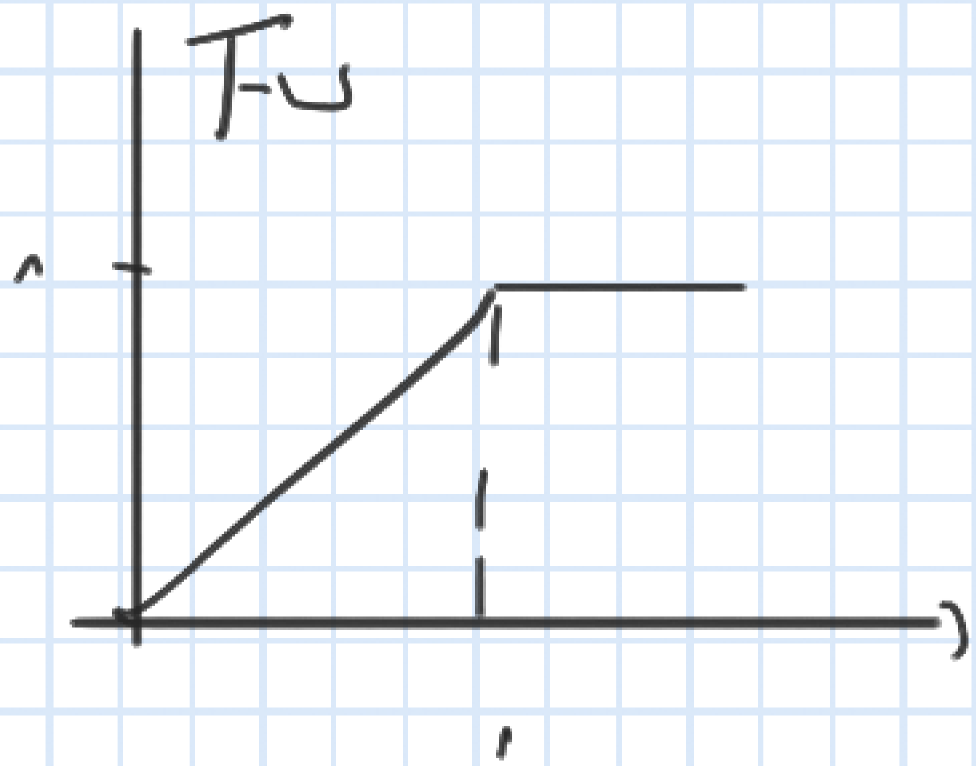


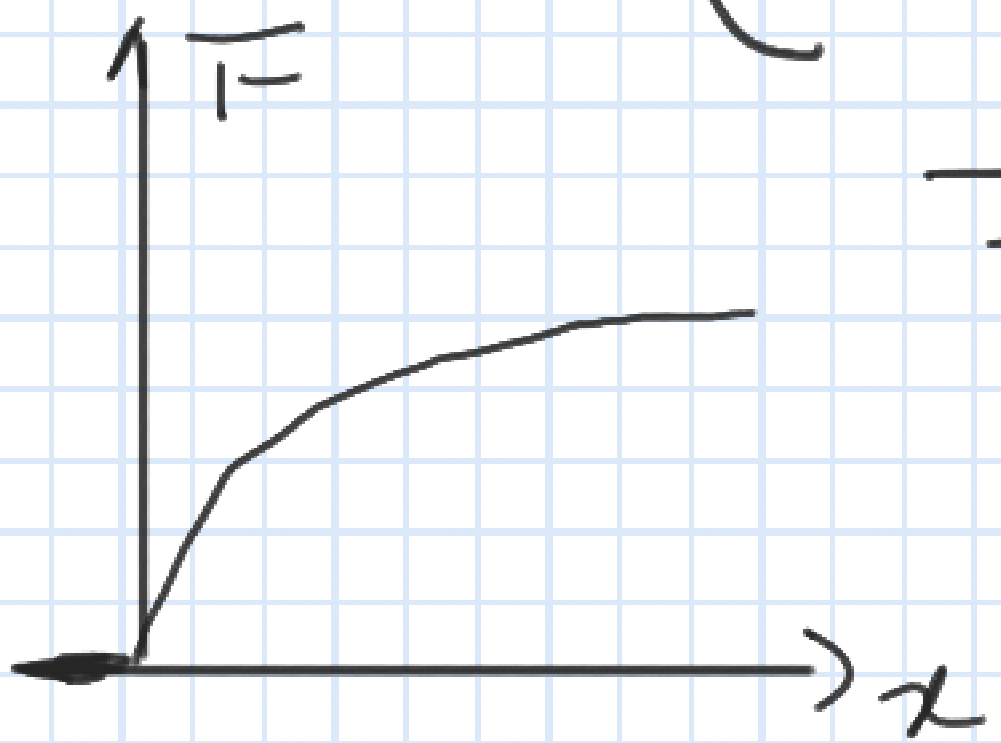


[6м] :0

Ejer 5

$F(x) = \left(1 - e^{-1/5\, x}\right) I\{x > 0\}$





$F_U(u) = F(x)$

$u = 1 - e^{-1/5\, x}$

$\underline{-5 \ln(1-u) = x}$

$I\{x \in A\} = \begin{cases} 1 & \text{si } x \in A \\ 0 & \text{si no} \end{cases}$