accenture

3.2 a) $x^k = \begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix}$ $x^{k-1} = \begin{bmatrix} F_k \\ F_{k-1} \end{bmatrix}$ F_k: numero k de la serie de fibonacci

A?

$$\begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} F_k \\ F_{k-1} \end{bmatrix} \qquad x^k = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} x^{k-1}$$

$$F_{k+1} = \overset{1}{A_{11}} F_k + \overset{1}{A_{12}} F_{k-1}$$

$$F_k = \overset{1}{A_{21}} F_k + \overset{0}{A_{22}} F_{k-1}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

b) $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \cdot -\lambda - 1 = 0$$

$$\lambda^2 - \lambda - 1 = 0 \quad \begin{cases} \dfrac{1+\sqrt{1+4.1.1}}{2} = \dfrac{1+\sqrt{5}}{2} = \lambda_1 \\ \dfrac{1-\sqrt{1+4.1.1}}{2} = \dfrac{1-\sqrt{5}}{2} = \lambda_2 \end{cases}$$

$\lambda_1 = \frac{1}{2} + \frac{\sqrt{5}}{2}$

$$\begin{bmatrix} 1-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right) & 1 \\ 1 & -\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) x_1 + x_2 = 0 \quad \text{con } x_2 = 1$$

$$x_1 = \frac{-2}{1-\sqrt{5}}$$

Autovector 1: $\begin{bmatrix} \frac{-2}{1-\sqrt{5}} \\ 1 \end{bmatrix}$

$\lambda_2 = \frac{1}{2} - \frac{\sqrt{5}}{2}$

$$\begin{bmatrix} 1-\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right) & 1 \\ 1 & -\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) x_1 + x_2 = 0$$

$$x_1 = \frac{-2}{1+\sqrt{5}}$$

Autovector 2: $\begin{bmatrix} \frac{-2}{1+\sqrt{5}} \\ 1 \end{bmatrix}$

accenture

¿Es Diagonalizable? ¿Sus Avec. son LI?

$$\langle v_1, v_2 \rangle = \begin{bmatrix} \frac{-2}{1+\sqrt{5}} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{-2}{1-\sqrt{5}} & 1 \end{bmatrix} = \frac{-2 \cdot -2}{(1-\sqrt{5})(1+\sqrt{5})} + 1 = \frac{-4}{4} + 1 = 0 \checkmark \text{ LI}$$

Es diagonalizable porque sus autovectores son LI

c) Si $x^k = A x^{k-1} \Rightarrow x^{k+1} = A x^k = A . A . x^{k-1}$

$$x^{(1)} = A . x^{(0)} \quad , \quad x^{(2)} = A^2 x^{(0)} \Rightarrow x^{(k)} = A^k . x^{(0)}$$

$$x^k = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^k \circ \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$D = S^{-1} A . S \qquad \text{si } S \text{ es } \begin{bmatrix} \frac{-2}{1-\sqrt{5}} & \frac{-2}{1+\sqrt{5}} \\ 1 & 1 \end{bmatrix} \Rightarrow D = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{bmatrix}$$

$$A = S . D . S^{-1}$$

$$x^k = S \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{bmatrix}^k S^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\downarrow$

$$x^{(k)} = A^k . x^{(0)} \Rightarrow x^{(k)} = S . D \underbrace{S^{-1} . S}_{I} . D . \underbrace{S^{-1} . S}_{I} \ldots S . D S^{-1} . x^{(0)}$$

$$x^{(k)} = S . D^k . S^{-1} . x^{(0)}$$