"Sacar 3 cartas ≠ en 3 extrac c/R" = D

$\underline{40}\ \underline{39}\ \underline{38}$

$$P(D) = \frac{\#C}{\#\Omega} = \frac{40 \cdot 39 \cdot 38}{40\,40\,40}$$

"Sacar 3 cartas = en 3 extrac $\underline{c/R}$" = C

$\to \underline{40}\ \underline{1}\ \underline{1}$

$$P(C) = \frac{40}{40 \cdot 40\,40}$$

$\underline{40}\ \underline{40}\ \underline{40}$

"Sacar 3 palos ≠ en 3 extrac" = D

$\underline{10}\ \underline{10}\ \underline{10} \cdot \binom{4}{3} \cdot 3!$

$$P(D) = \frac{10^3\,(4)_3}{40^3}$$

$$\frac{4!}{3!\,1!}\,3! = \frac{4!}{1!} =$$

O B C

B O C

de en 4 palos tomo 3

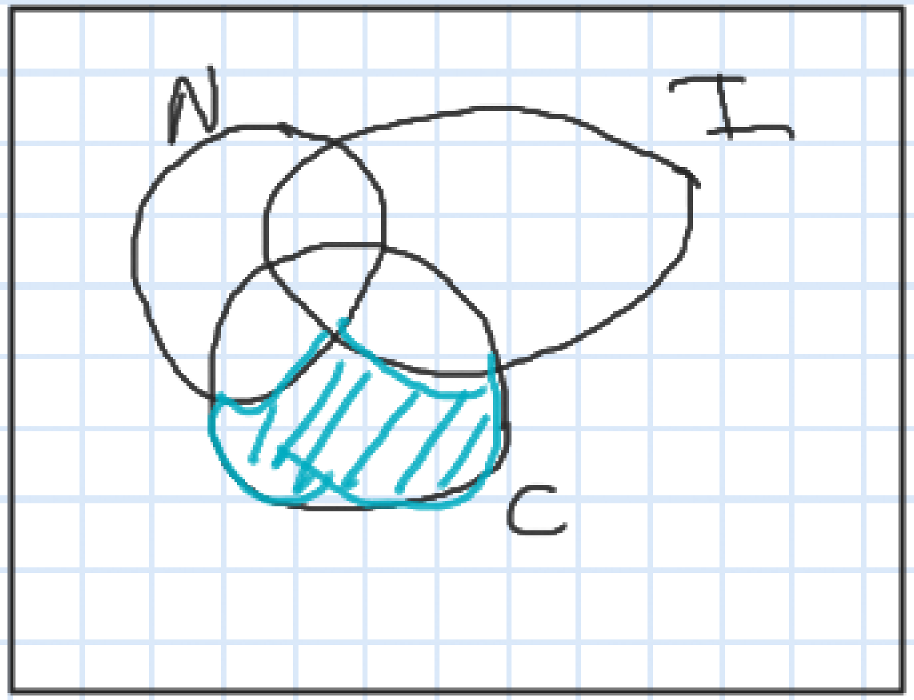
3. En una materia optativa, el 35% de los asistentes estudia ingeniería, el 67 % prefiere Netflix y el 56% toma café, el 27% estudia ingeniería y prefiere Netflix, el 29% prefiere Netflix y toma café, el 22% estudia ingeniería y toma café. El 5% no estudia Ingeniería ni prefiere Netflix ni toma café.

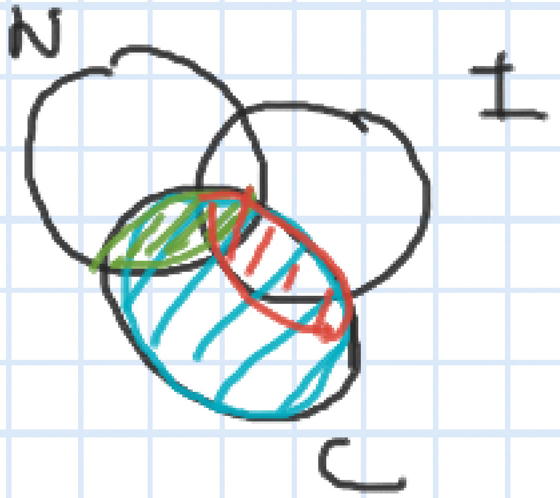
$P(I) = 0.35 \quad P(I \cap N) = 0.27$

$P(N) = 0.67 \quad P(N \cap C) = 0.29$

$P(C) = 0.56 \quad P(I \cap C) = 0.22$

$P(\bar{I} \cap \bar{N} \cap \bar{C}) = 0.05$





$$P(C \cap \bar{N} \cap \bar{I}) = P(C) - P(I \cap C) - P(N \cap C) + P(I \cap N \cap C)$$

Ej 1_a cara $\to 1$

ceca $\to 0$

$X = \begin{cases} 1 & \text{cara} \\ 0 & \text{ceca} \end{cases}$

$\Omega = \{$cara, ceca$\}$

1.b) X = "long. de un caño"

$X \in \mathbb{R}_+$

1_c) X = "cantidad de 1 observados en 6 tiros"

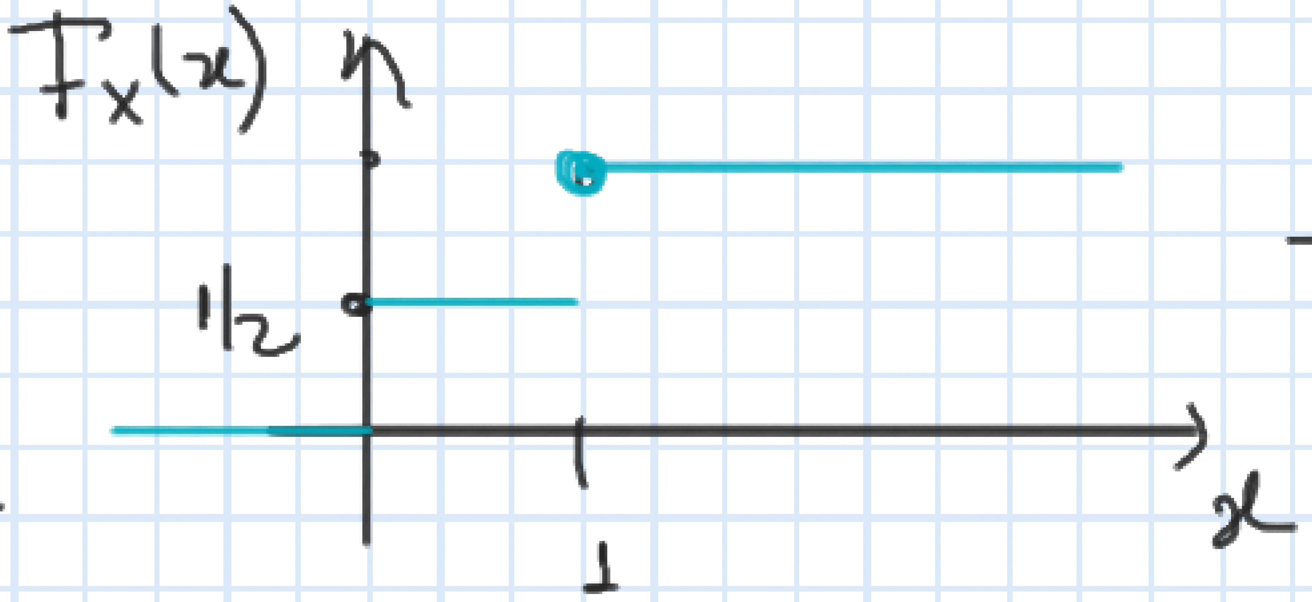
$X = \{0, 1, 2, 3, 4, 5, 6\}$

1_d) X = "cant. de llamadas en (9, 11)"

$X \in \mathbb{N}_0$

Ej. 3

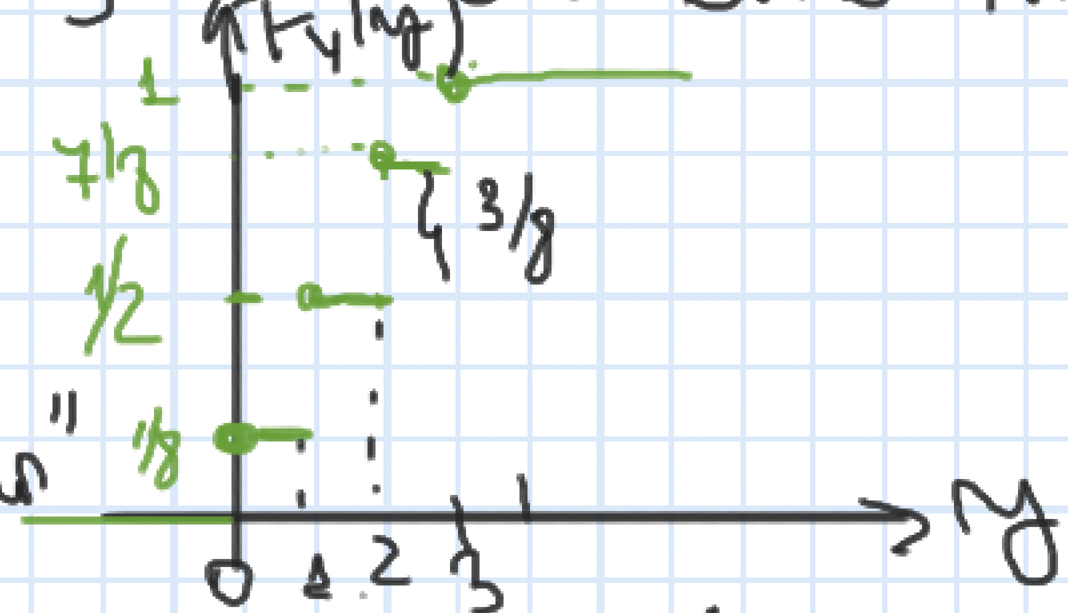
$X = \begin{cases} 1 & \text{cara} \\ 0 & \text{ceca} \end{cases}$



$$F_X(x) = \begin{cases} 0 & \text{si } x < 0 \\ 1/2 & \text{si } 0 \leq x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$$

Y: "# de caras en 3 tiros de una moneda"

$Y = \{0, 1, 2, 3\}$



$$F_Y(y) = \begin{cases} 0 & \text{si } y < 0 \\ 1/8 & \text{si } 0 \leq y < 1 \\ 1/2 & \text{si } 1 \leq y < 2 \\ 7/8 & \text{si } 2 \leq y < 3 \\ 1 & \text{si } y \geq 3 \end{cases}$$

C: "salen 2 caras en 3 tiros"

A: "No sale ninguna cara en 3 tiros"

B: "Sale 1 cara en 3 tiros"

$P(A) = (1/2)^3 = 1/8.$

$P(X \leq 1) = P(A) + P(B)$

$= 1/8 + 1/8 \cdot 3 = 1/2.$

$\underline{1}\ \underline{1}\ \underline{1}$

$\# \Omega = 2 \cdot 2 \cdot 2$

$P(X \leq 2) = P(X \leq 1) + P(C) = 1/2 + 1/8 \cdot 3$

$\underline{H}\ \underline{T}\ \underline{T}$ $\quad \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2}$ $\quad \frac{3!}{2!\,1!} = \binom{3}{1}$

T H H

$\underline{H}\ \underline{H}\ \underline{T}$ $\quad \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2} \binom{3}{2}$

$$\mathbb{P}(1{,}5 < Y \le 2) = F_Y(2) - F_Y(1{,}5) = 7/8 - 1/2 = 3/8$$

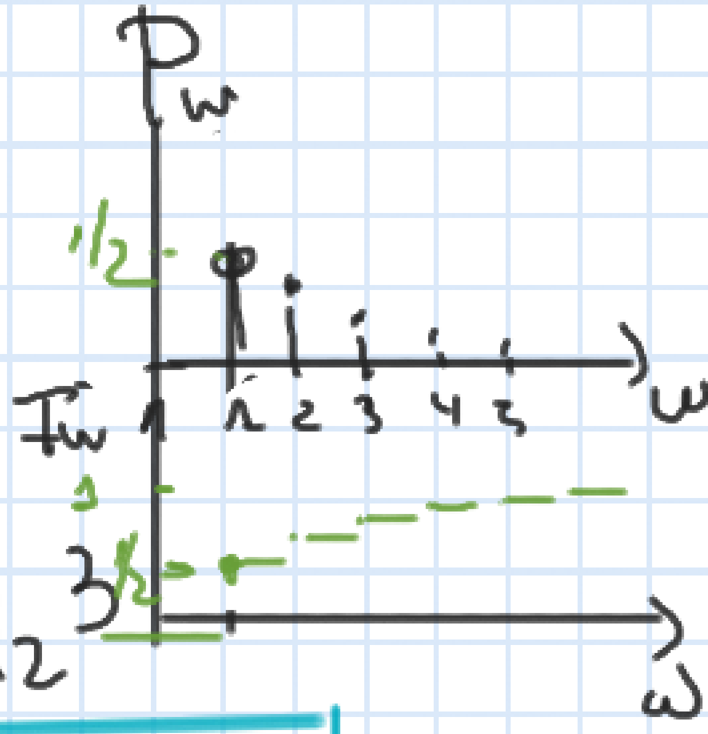
Ej 4) $p_X(x) = \begin{cases} 1/2 & \text{si } x = 0 \\ 1/2 & \text{si } x = 0 \end{cases}$

x	0	1
p_X	1/2	1/2

y	0	1	2	3
$p_Y(y)$	1/8	3/8	3/8	1/8

W = "# de tiros hasta observar la 1° cara"

$W = \mathbb{N}$



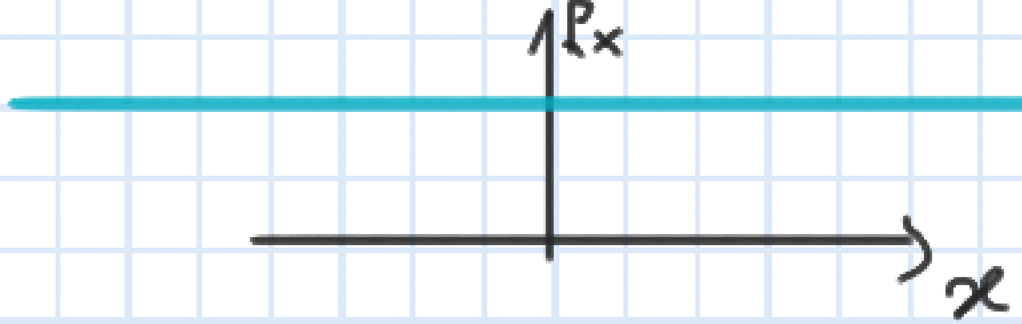
$p_W(1) = \mathbb{P}(W = 1) = 1/2$

$p_W(2) = \mathbb{P}(W = 2) = 1/2 \cdot 1/2 = 1/2^2$

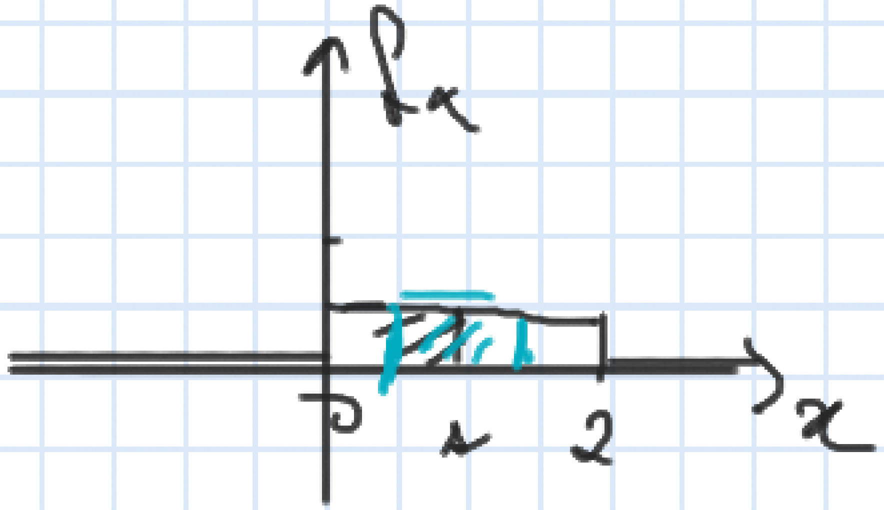
$p_W(3) = \mathbb{P}(W = 3) = 1/2^3$

$$\boxed{p_W(n) = \mathbb{P}(W = n) = (1/2)^n \quad n = 1, 2, \ldots}$$

5.1 $f_X(x) = \cancel{1/2}$



5.2 $f_X(x) = 1/2 \; \mathbb{1}\{0 < x < 2\} = \begin{cases} 1/2 & 0 < x < 2 \\ 0 & \text{eoc} \end{cases}$



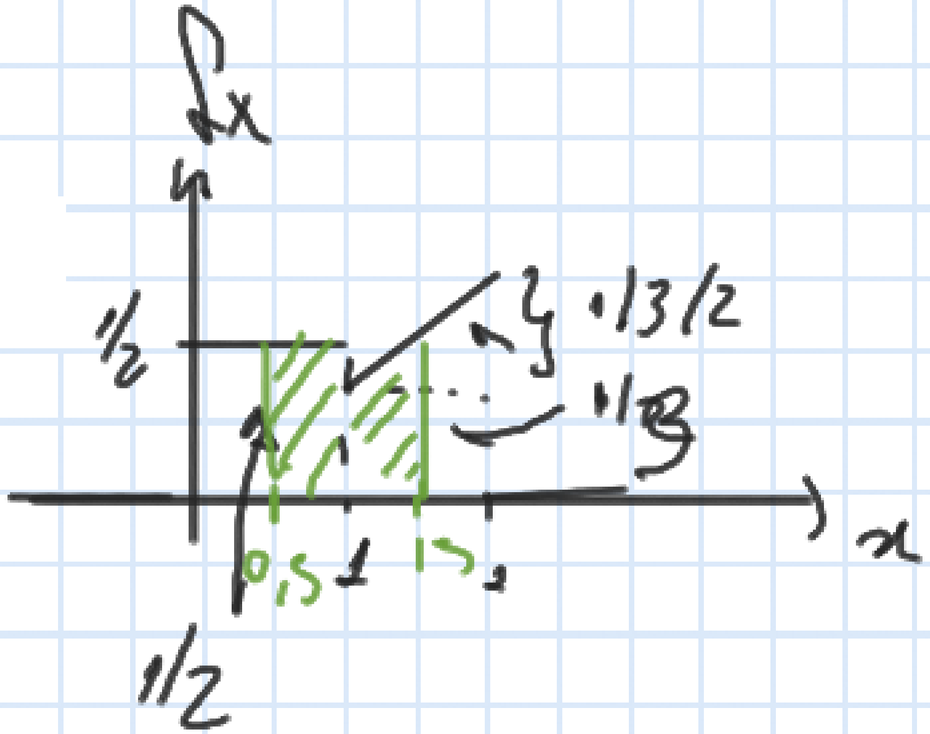
$P(X<1) = \int_{-\infty}^{1} f_X(x)\,dx = 1/2$

$P(0{,}5 < X < 1{,}5) = \int_{0{,}5}^{1{,}5} f_X(x)\,dx = 1/2$

5.3. $f_X(x) = \frac{1}{2}\mathbf{1}\{0 < x < 1\} + \frac{x}{3}\mathbf{1}\{1 < x < 2\}$

TAREA: Calcular $F_X(x)$

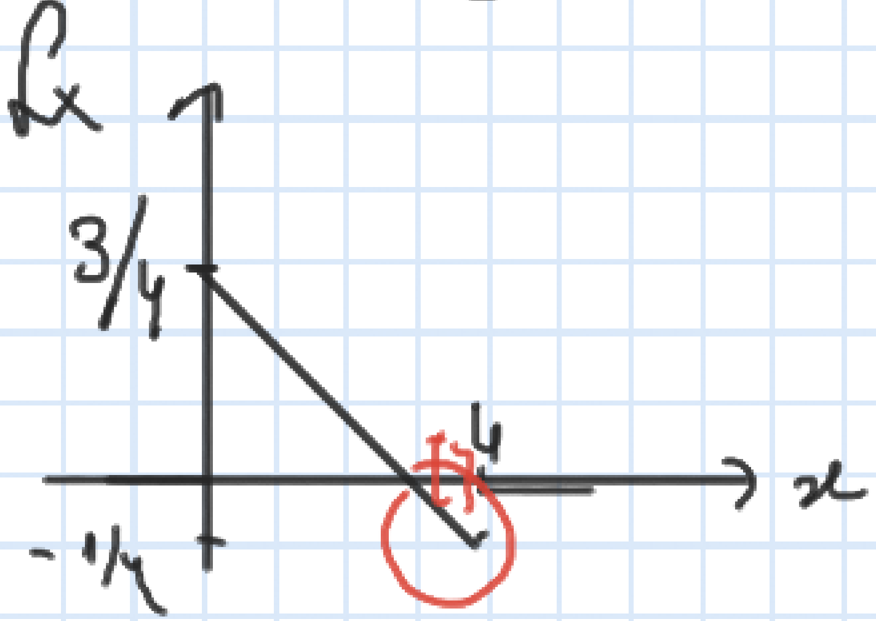
$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$



$P(X<1) = 1/2$

$P(0{,}5 < X < 1{,}5) = 5/8$

5.4 $f_X(x) = \frac{3-x}{4}\mathbf{1}\{0 < x < 4\}$



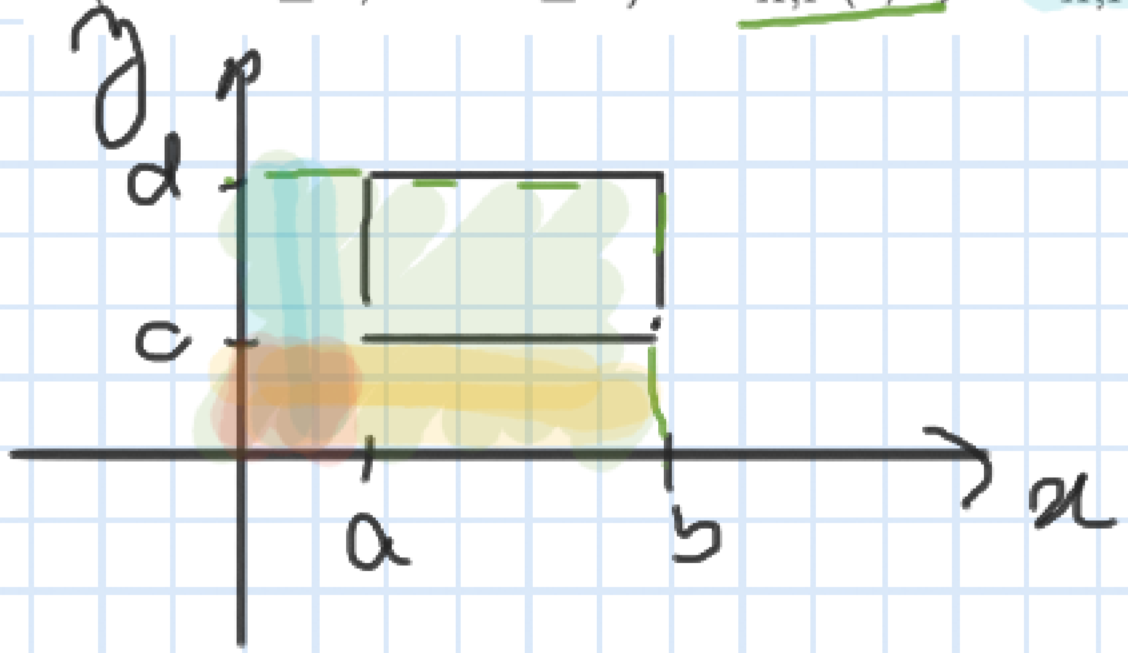
Con la densidad del Ej 5.3 hallar $f_{X|X\in(0,5;1,5)}(x)$

3. $f_X(x) = \frac{1}{2}\mathbf{1}\{0 < x < 1\} + \frac{x}{3}\mathbf{1}\{1 < x < 2\}$

$$f_{X|X\in(0,5;1,5)}(x) = \frac{1/2\,\mathbb{1}\{0,5 < x < 1\} + x/3\,\mathbb{1}\{1 < x < 1,5\}}{5/8}$$

$$\mathbb{P}(X \in (0,5;1,5)) = 5/8$$

$$\mathbb{P}(a < X \le b, c < Y \le d) = F_{X,Y}(b,d) - F_{X,Y}(a,d) - F_{X,Y}(b,c) + F_{X,Y}(a,c)$$



Ejercicio 6

X = "# de 1 en 3 tiros"

Y = "# de 2 en 3 tiros"

y \ x	0	1	2	3	p_Y
0	8/27	2/9	1/18	1/216	125/216
1	2/9	1/9	1/72	0	25/72
2	1/18	1/72	0	0	5/72
3	1/216	0	0	0	1/216
p_X	125/216	25/72	5/72	1/216	1

$p_{X,Y}(3,0) = \mathbb{P}(X=$

A: "X e Y son pares"

$\mathbb{P}(X \in \{0,2\}, Y \in \{0,2\}) =$

$= \mathbb{P}(X=0, Y=0) + \mathbb{P}(X=2, Y=0) + \dots + = \frac{11}{27}$

$\underline{4}\ \underline{4}\ \underline{4}$

$p_{X,Y}(0,0) = \mathbb{P}(X=0, Y=0) = (4/6)^3 \binom{3}{0}$

$p_{X,Y}(1,0) = \mathbb{P}(X=1, Y=0) = \frac{4^2}{6^3} \cdot 3$

$\underline{1}\ \underline{4}\ \underline{4}\ \binom{3}{1}$

$p_{X,Y}(2,0) = \mathbb{P}(X=2, Y=0) = \frac{4}{6^3} \binom{3}{2}$

$\underline{1}\ \underline{1}\ \underline{4}\ \binom{3}{2}$

$p_{X,Y}(3,0) = \mathbb{P}(X=3, 0) = 1/6^3$

$\underline{1}\ \underline{1}\ \underline{1}$

$p_{X,Y}(0,1) = \mathbb{P}(X=0, Y=1) = \frac{4^2}{6^3} \cdot 3$

$\underline{1}\ \underline{4}\ \underline{4}\ \binom{3}{1}$

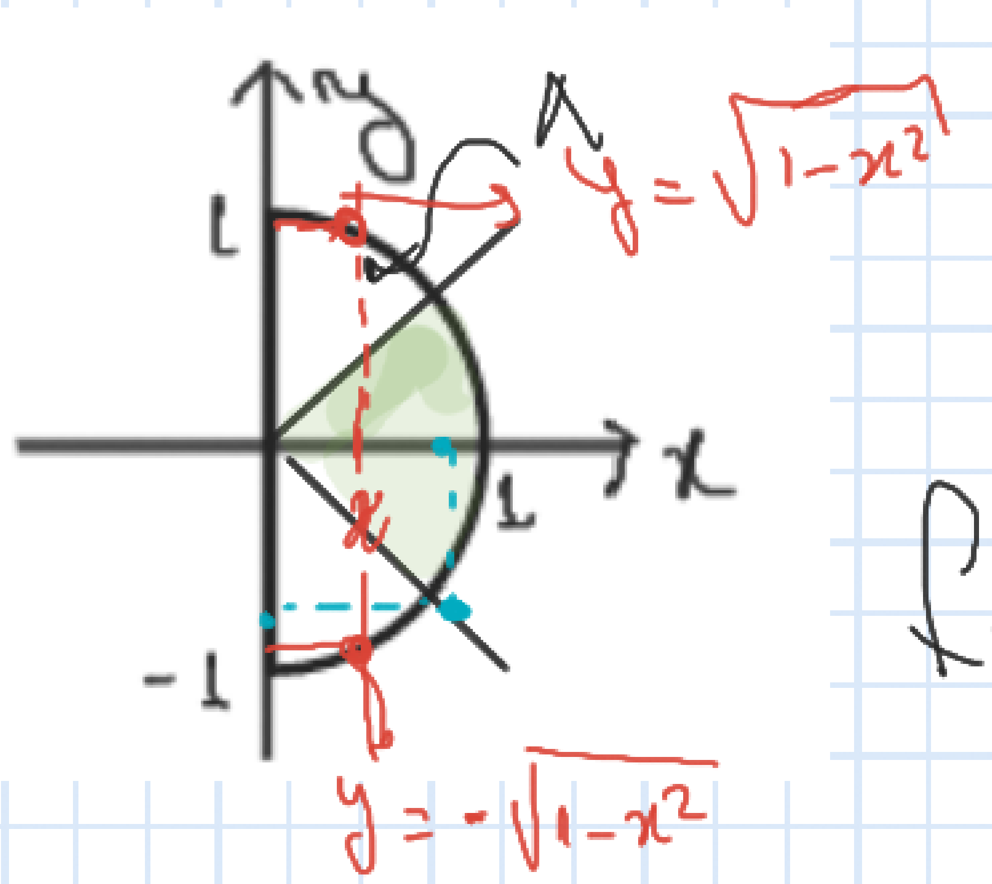
$p_{X,Y}(1,1) = \mathbb{P}(X=1, Y=1) = \frac{4}{6^3} \binom{3}{1}\binom{2}{1}$

$\underset{1}{\underline{1}}\ \underset{2}{\underline{1}}\ \underline{4}\ \underbrace{\binom{3}{1}\binom{2}{1}}_{3 \cdot 2}$

$p_{X,Y}(2,1) = \mathbb{P}(X=2, Y=1) = \frac{1}{6^3} \binom{3}{1}$

$\underline{1}\ \underline{1}\ \underline{1}\ \binom{3}{2}$

Ej 7



$$\mathbb{P}(|Y| < X) = 1/2 = \iint_{A} f_{X,Y}(x,y)\,dx\,dy$$

$$f_{X,Y}(x,y) = \frac{1}{|A|}\mathbb{1}\{(x,y)\in A\} = \frac{2}{\pi}\mathbb{1}\{(x,y)\in A\}.$$

$$f_X(x) = \int_{\mathbb{R}} f_{X,Y}(x,y)\,dy = \left[\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{\pi}\,dy\right]\mathbb{1}\{x\in(0,1)\}.$$

$$= \frac{2}{\pi}\cdot 2\sqrt{1-x^2}\;\mathbb{1}\{x\in(0,1)\}$$