Phys 512 - PS 2

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P1

I wrote an integrator based on the one we wrote in class, but with a few extra steps to store values of the function evaluated at certain points, evaluate if the function has been called before for a certain value, and then use the previously stored value.

For a Lorentzian

$$f(x) = \frac{1}{1+x^2} \ ,$$

my integrator gives $\int_{-10}^{10} f(x) = 2.942257478034349$, and saves 708 function calls.

For a sinusoid

$$f(x) = \sin(x) \ ,$$

it gives $\int_0^{\pi} f(x) = 2.000001033369413$ and saves 72 function calls.

For an exponential

$$f(x) = e^x \ ,$$

I have $\int_0^1 f(x) = 1.718284154699897$, saving 6 function calls.

P2

I fitted $\log_2(x)$ from 0.5 to 1 using a truncated Chebyshev polynomial fit based on the one we did in class, but shifting the x-axis.

With the model to generate the Chebyshev matrix T_n, I can invert the matrix equation

$$\boldsymbol{y} = \boldsymbol{T_n} \cdot \boldsymbol{c_n} \tag{1}$$

to get the coefficients as

$$\boldsymbol{c_n} = \boldsymbol{T_n^{-1}} \cdot \boldsymbol{y} \ . \tag{2}$$

Plugging Eq. 2 back on Eq. 1, I get my fit

$$\text{Cheb fit}_{\text{trunc}} = T_{n,trunc} \cdot c_{n,trunc} \ , \tag{3}$$

where the subscript *trunc* denotes the Chebyshev expansion truncated to the desired order.

To get an accuracy better than 10^{-6}, I used a Chebyshev expansion of order 8. I compared the fit to a Legendre polynomial fit of same order, and the fit results can be seen in Fig. 1.

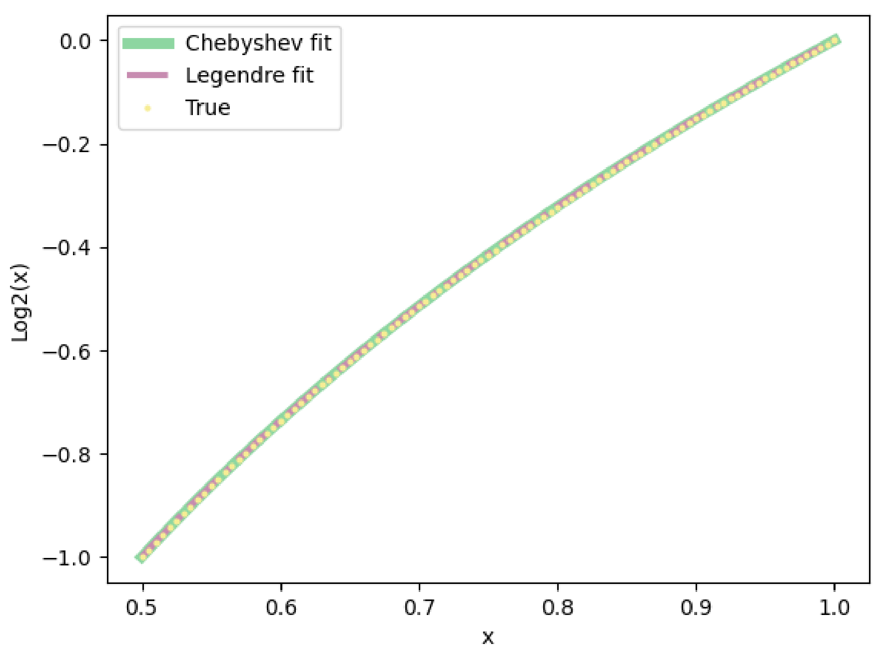

Figure 1: Chebyshev and Legendre fits to $\log_2(x)$.

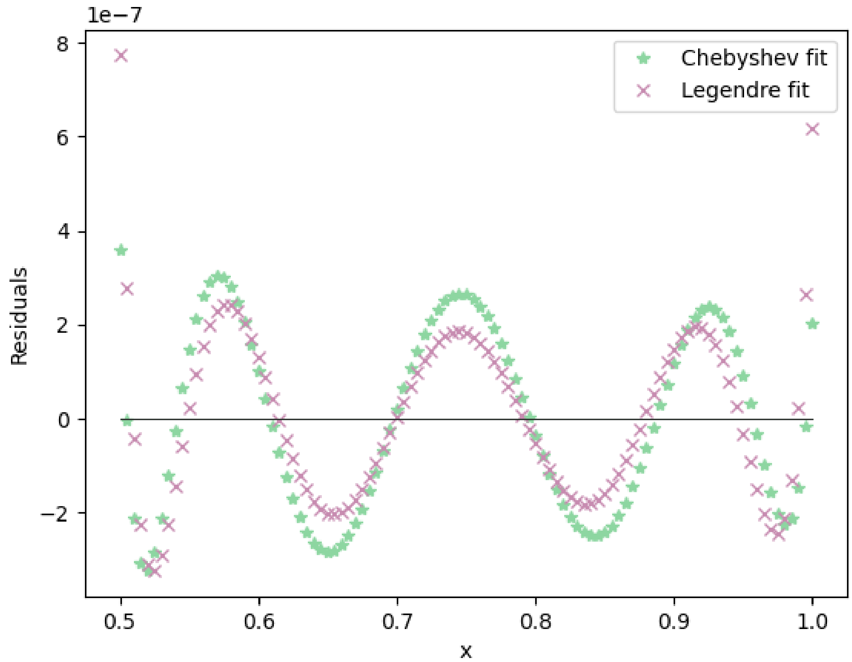

Figure 2: Residuals of Chebyshev and Legendre fits to $\log_2(x)$.

Next I looked at the residuals of each fit, as shown in Fig. 2. The Chebyshev fit RMS error was equal to $1.9319571467103043 \times 10^{-07}$ and maximum error of $3.579291045774369 \times 10^{-07}$. The RMS error of the Legendre fit was $1.8359224606319682 \times 10^{-07}$, with maximum error of $7.730558835294943 \times 10^{-07}$.

As expected, the truncated Chebyshev fit has higher RMS error and smaller maximum error when compared to a Legendre polynomial of the same order.

P3

I used the code written in class to set up the differential equations for the decays, modifying it to properly set the problem, where we have 14 decaying processes.

I used scipy *integrate.solve_ivp* to solve the system of ODEs, using the implicit Runge-Kutta method (or 'Radau'), since we have half-life times that differ from each other by huge factors. With the implicit method, it took my computer 235 evaluations and 0.04861092567443848 seconds to solve the system of ODEs.

We can better analyze the decay chain by looking at the ratio of some of the elements produced (Fig. 3. When looking at the ratios of the final product Pb206 to the first element U238, we can see the "instantaneous" decay from one to the other, as pointed in the problem, since the half-life of U238 is much larger than all others.

The ratio Th230/U234 also make sense. The half life of U234 is $\approx 0.7 \times 10^{13}$ s. So the creation of Th230 slows down after this period, as seen in the plots.

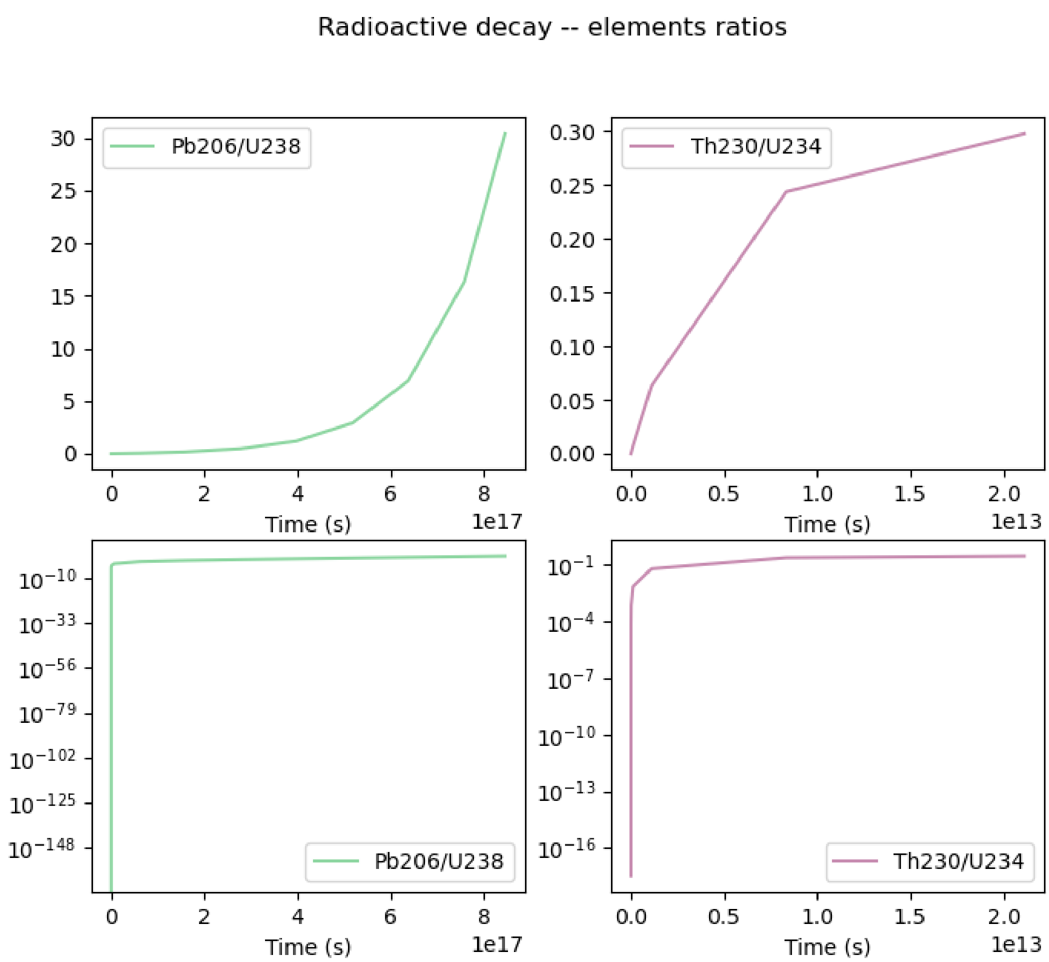

Figure 3: Ratio of products in the U238 - Pb206 radioactive decay chain.