Phys 512 - PS 3

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October, 2020

P1

We can expand the equation for the paraboloid and make a parameter substitution to get a function that is linear on its parameters:

Expand and change parameters

```
In[145]:= z == a ((x - x0)^2 + (y - y0)^2) + z0;
          Expand[%]
          Collect[Collect[%, -2 a], a]
          Collect[
           Expand[% /. {x0^2 + y0^2 -> (m0 - z0) / a} /. {x0 -> m1 / (-2 a)} /.
              {y0 -> m2 / (-2 a)} /. a -> m3], m3]
```

Out[146]= $z == a x^2 + a y^2 - 2 a x x_0 + a x_0^2 - 2 a y y_0 + a y_0^2 + z_0$

Out[147]= $z == a \left(x^2 + y^2 + x_0^2 + y_0^2 - 2 (x x_0 + y y_0)\right) + z_0$

Out[148]= $z == m0 + m1\, x + m2\, y + m3 \left(x^2 + y^2\right)$

The new parameters $(m0, m1, m2, m3)$ can be written in terms of the old ones (x_0, y_0, z_0, a) as

```
In[149]:= Solve[x0^2 + y0^2 == (m0 - z0) / a, m0]
          Solve[x0 == m1 / (-2 a), m1]
          Solve[y0 == m2 / (-2 a), m2]
          m3 -> a
```

Out[149]= $\{\{m0 \to a x_0^2 + a y_0^2 + z_0\}\}$

Out[150]= $\{\{m1 \to -2 a x_0\}\}$

Out[151]= $\{\{m2 \to -2 a y_0\}\}$

Out[152]= $m3 \to a$

and the old ones can be recovered by substituting:

```
In[157]:= Solve[{x0^2 + y0^2 == (m0 - z0) / a, x0 == m1 / (-2 a), y0 == m2 / (-2 a)},
          {x0, y0, z0}]

Out[157]= {{x0 -> -m1/(2 a), y0 -> -m2/(2 a), z0 -> -(-4 a m0 + m1^2 + m2^2)/(4 a)}}
```

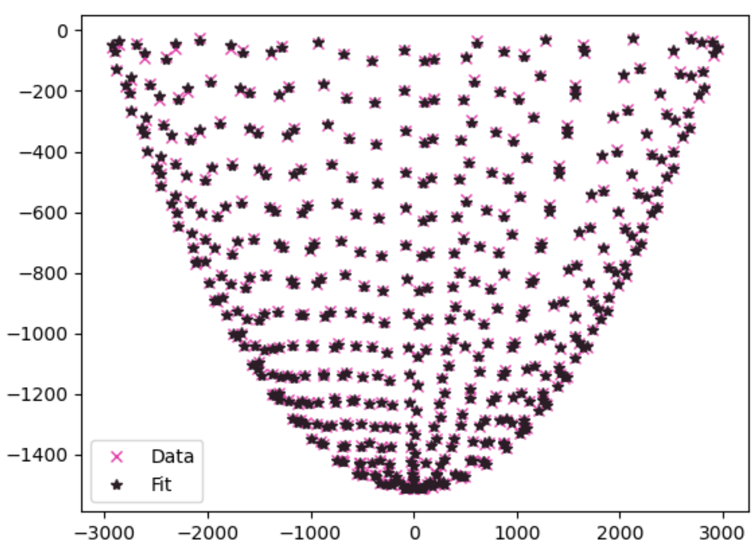
Carrying out the fit, my best-fit new parameters are:

$$
\begin{aligned}
m0 &= -1512.31182 \\
m1 &= 4.53599028 \times 10^{-4} \\
m2 &= -1.94115589 \times 10^{-2} \\
m3 &= 1.66704455 \times 10^{-4} \ .
\end{aligned}
$$

The old parameters are then given by:

$$
\begin{aligned}
x_0 &= -1.3604886221970875 \\
y_0 &= 58.22147608157978 \\
z_0 &= -1512.8772100367878 \\
a &= 0.00016670445477401347 \ .
\end{aligned}
$$

Plotting the fit and the real data, I get:



with an RMS error between fit and data equal to 3.7683386487847277.

The covariance matrix $N = (A^T N^{-1} A)^{-1}$ gives the correlated noise $\langle n_i n_j \rangle$. Since we have $a = m3$, the uncertainty in a is given by $n_3 = \sqrt{\langle n_3 n_3 \rangle}$, where $\langle n_3 n_3 \rangle$ is just the last input of the matrix N. In this way, I get the uncertainty $\delta a = 1.712133743432042 \times 10^{-8}$.

The focal length in the paraboloid is given by

$$f = \frac{1}{4a}$$

and its uncertainty (by error propagation) is

$$\delta f = \left| \frac{\delta a}{4a^2} \right| .$$

Substituting values, we get $f = 1.49966 \pm 0.00015$ m.

P2

With the given parameters, we get a $\chi^2 = 1588.2376465828454$.