Phys 512 - PS1

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P4

Use your integrator and *scipy.integrate.quad* to plot E from a spherical shell as a function of distance from center z.
Use ranges $z < R$ and $z > R$ and one of the z values equal to R.
Find out if there is a singularity, and if quad or your integrator cares about it.

The integral is

$$E_z = \frac{1}{4\pi\epsilon_0}(2\pi R^2 q)\int_0^\pi \frac{(z - R\cos\theta)\sin\theta}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}d\theta \tag{1}$$

$$= \frac{1}{4\pi\epsilon_0}(2\pi R^2 q)\int_{-1}^1 \frac{z - Ru}{(R^2 + z^2 - 2Rzu)^{3/2}}du \quad , \quad u = \cos\theta\ . \tag{2}$$

For simplicity, in my Python code *Phys512_PS1_P4_FCRM.py*, I am setting $R = 1$.

At distance $z = R$, there is a singularity. *scipy.integrate.quad* works around it, but my integrator does not, since there would be a division by zero in the way it is written if we select z-steps including the singularity (at the shell). If I include $z = R$ in my $z - steps$, I have an error and the code doesn't run. My way around it is to have z slightly different than R when crossing the R region. The result is shown in Fig. 1.

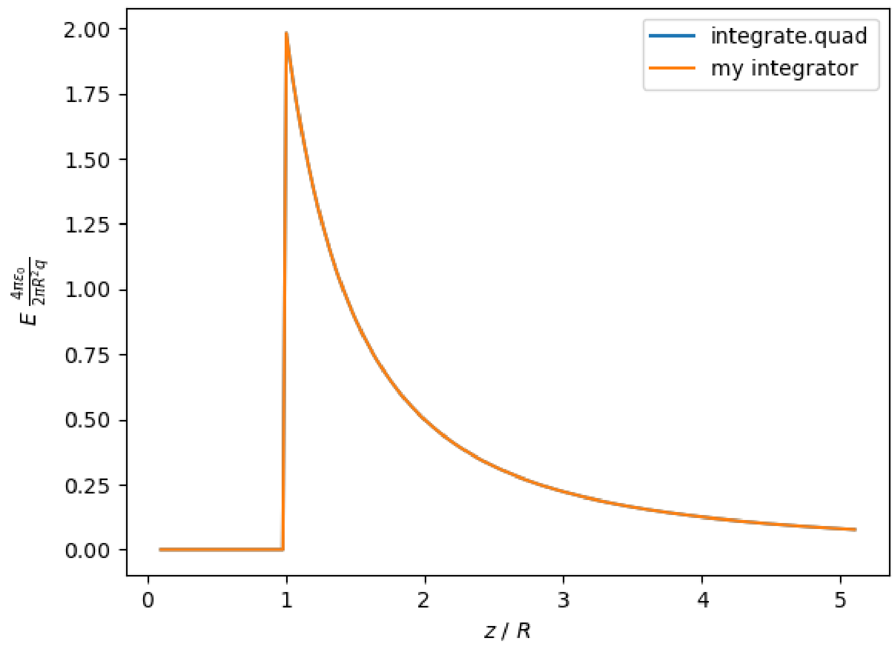

Figure 1: Electric field vs. distance from the center of a spherical shell. *Note: $z = R$ not included in the z-steps for this figure.*

As expected, the electric field is zero inside the shell, a maximum at the shell and decaying as z^2 when moving away from it.

When comparing the two integrators, the error between them is equal to 3.0994×10^{-5}.