Phys 512 - PS 3

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P1

We can expand the equation for the paraboloid and make a parameter substitution to get a function that is linear on its parameters:

Expand and change parameters

```
In[145]:= z == a ((x - x0)^2 + (y - y0)^2) + z0;
          Expand[%]
          Collect[Collect[%, -2 a], a]
          Collect[
           Expand[% /. {x0^2 + y0^2 → (m0 - z0) / a} /. {x0 → m1 / (-2 a)} /.
              {y0 → m2 / (-2 a)} /. a → m3], m3]

Out[146]= z == a x^2 + a y^2 - 2 a x x0 + a x0^2 - 2 a y y0 + a y0^2 + z0

Out[147]= z == a (x^2 + y^2 + x0^2 + y0^2 - 2 (x x0 + y y0)) + z0

Out[148]= z == m0 + m1 x + m2 y + m3 (x^2 + y^2)
```

The new parameters $(m0, m1, m2, m3)$ can be written in terms of the old ones (x_0, y_0, z_0, a) as

```
In[149]:= Solve[x0^2 + y0^2 == (m0 - z0) / a, m0]
          Solve[x0 == m1 / (-2 a), m1]
          Solve[y0 == m2 / (-2 a), m2]
          m3 → a

Out[149]= {{m0 → a x0^2 + a y0^2 + z0}}

Out[150]= {{m1 → -2 a x0}}

Out[151]= {{m2 → -2 a y0}}

Out[152]= m3 → a
```

and the old ones can be recovered by substituting:

In[157]:= $\text{Solve}\left[\left\{x_0^2 + y_0^2 == (m0 - z_0)\,/\,a,\ x_0 == m1\,/\,(-2\,a),\ y_0 == m2\,/\,(-2\,a)\right\},\ \{x_0, y_0, z_0\}\right]$

Out[157]= $\left\{\left\{x_0 \to -\frac{m1}{2\,a},\ y_0 \to -\frac{m2}{2\,a},\ z_0 \to -\frac{-4\,a\,m0 + m1^2 + m2^2}{4\,a}\right\}\right\}$

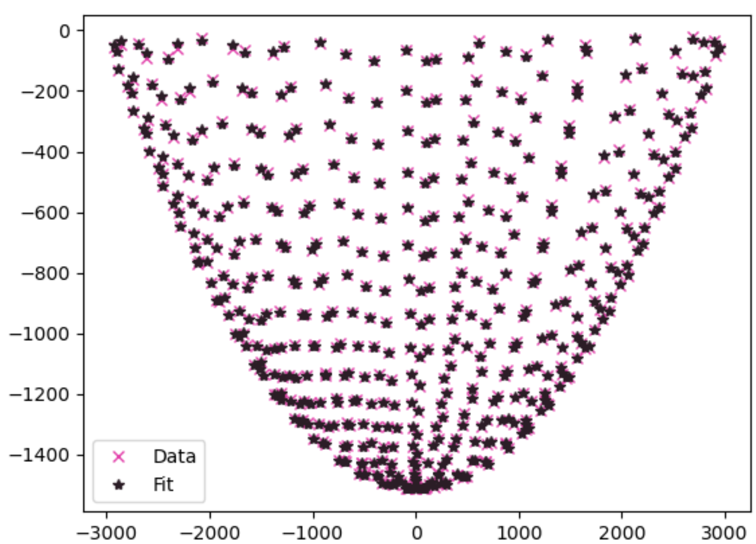
Carrying out the fit, my best-fit new parameters are:

$$
\begin{aligned}
m0 &= -1512.31182 \\
m1 &= 4.53599028 \times 10^{-4} \\
m2 &= -1.94115589 \times 10^{-2} \\
m3 &= 1.66704455 \times 10^{-4}\ .
\end{aligned}
$$

The old parameters are then given by:

$$
\begin{aligned}
x_0 &= -1.3604886221970875 \\
y_0 &= 58.22147608157978 \\
z_0 &= -1512.8772100367878 \\
a &= 0.00016670445477401347\ .
\end{aligned}
$$

Plotting the fit and the real data, I get:



with an RMS error between fit and data equal to 3.7683386487847277.

The covariance matrix $N = (A^T N^{-1} A)^{-1}$ gives the correlated noise $\langle n_i n_j \rangle$. Since we have $a = m3$, the uncertainty in a is given by $n_3 = \sqrt{\langle n_3 n_3 \rangle}$, where $\langle n_3 n_3 \rangle$ is just the last input of the matrix N. In this way, I get the uncertainty $\delta a = 1.712133743432042 \times 10^{-8}$.

The focal length in the paraboloid is given by

$$f = \frac{1}{4a}$$

and its uncertainty (by error propagation) is

$$\delta f = \left| \frac{\delta a}{4a^2} \right| .$$

Substituting values, we get $f = 1.49966 \pm 0.00015$ m.