# The Aggregate-Demand Doom Loop: Precautionary Motives and the Welfare Costs of Sovereign Risk*

Francisco Roldán†
IMF

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**Abstract**

Sovereign debt crises coincide with deep recessions. In the conventional view, poor economic conditions increase default incentives and bond spreads. I provide evidence suggesting that the reaction of consumption demand creates feedback from sovereign spreads to output even while the government is in good standing with creditors. Because they ignore the savings behavior of private agents, existing models cannot capture this empirical feature of crises. I study the implications of this feedback mechanism in a model where the government of a small open economy borrows from foreign lenders but some of the debt is held by heterogeneous domestic savers. Because of this heterogeneity in wealth, potential sovereign defaults carry redistributive effects besides aggregate income losses. Both effects introduce risk in private agents' expectations upon bad news for repayment. Default risk then exacerbates the precautionary motive of households and depresses aggregate spending. In a calibration to Spain in the 2000s, I find that about 30% of the output contraction is attributable to default risk. More generally, sovereign risk exacerbates volatility in consumption, creating large welfare losses even if default does not materialize.

**JEL Classification** E2, F3, G2

**Keywords** Sovereign risk, default, heterogeneous agents, precautionary motives, aggregate demand

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†e-mail: *froldan@imf.org*

## Introduction

Sovereign debt crises coincide with pronounced output contractions. During the Eurozone crisis, when spreads increased steeply for European countries, Spanish output fell by about 10% of its 2008 peak. While Spain did not ultimately default on its debts, unemployment doubled. The drop in private consumption was steeper than output, by up to 15% of the pre-crisis maximum. The resulting increase in the aggregate saving rate was not a unique feature of the Spanish experience. I find that increases in sovereign spreads are associated with declines in consumption and output for EU countries during the crisis.

In this paper I propose a model of sovereign debt that rationalizes these large output and consumption contractions in response to sovereign risk. The main mechanism relies on the interaction between two well-understood features that so far been studied separately: default risk and the precautionary savings motive of households. When economic conditions worsen and sovereign default becomes more likely, households anticipate its negative consequences. Each of them finds it optimal to cut consumption in favor of higher savings, leading to low aggregate spending. Thus, the model generates a vicious cycle of high spreads causing demand shortages, causing further increases in debt and spreads. This vicious cycle amplifies underlying shocks to create deep recessions when sovereign risk emerges. Even if default does not materialize, its possibility endogenously exacerbates the volatility of consumption and output, creating large welfare costs of sovereign risk. I calibrate the model to Spain's debt crisis and use it to quantify the importance of this amplification mechanism.

The welfare costs of sovereign risk I emphasize yield a clear policy implication: debt crises should be resolved quickly. The model illustrates how prolonged periods of heightened sovereign risk carry costs in terms of lost output and employment through an aggregate-demand doom loop. I find that swift resolution mechanisms that avoid delays would enable significant welfare gains.

Domestic demand conditions are typically overlooked in the sovereign debt literature. Most studies assume either households that are effectively in financial autarky or one-sector small open economy models in which domestic demand is irrelevant as all production can be exported. As a result, it cannot account for the role of households' savings decisions in the unfolding of a crisis. When households and the government are explicitly separate actors, their interactions can create crisis events in which aggregate spending reacts to the anticipation of the government's actions differently than if those decisions were taken in a coordinated way. In the canonical model, output and spreads are correlated because recessions increase default incentives: given debt prices, the presence of sovereign risk does not affect the economy unless a default actually happens.

To investigate the interaction between precautionary motives and sovereign risk, I consider a small open economy in which heterogeneous households, subject to uninsurable idiosyncratic income risk, can save as well as choose their exposure to government debt. This Bewley setup results in distributions of wealth and exposures to sovereign risk that interact with the government's decision to repay its debts. In the model, when the government defaults, it bears a cost in terms of lost TFP and exclusion from capital markets, as is common in the literature. From a household's point of view, however, sovereign risk means volatility in future income, which boosts precautionary motives. Households fail to internalize the demand externality and cut consumption more than a planner would. In addition to this effect, as some of the debt is held domestically, defaults redistribute wealth. They transfer from bondholders (including foreigners), who receive lower payments, to domestic households, who face lower taxes. My setup with endogenous distributions of wealth and exposures to sovereign debt shapes the potential redistribution, along with the (fixed, exogenous) distribution of the tax burden. Finally, nominal wage rigidity enables the transmission of insufficient aggregate demand to increases in unemployment and contractions in output.

In the model, the government's decision to repay its debts takes into account distributional considerations. When it chooses between default and repayment, the government understands that defaults free up fiscal resources but harm the economy's productivity (and employment) and cut access to international capital markets. The government also observes the distribution of wealth and exposures to sovereign risk and anticipates the redistribution that defaults bring about. In line with both the literature and the data, the model implies higher sovereign risk when debt is higher and when productivity is lower. But default incentives are also heightened when the distribution of wealth is more unequal (Ferriere, 2016; Deng, 2020) as well as when the private sector is poorer as a whole.

The model ties the presence of sovereign risk to a high volatility of consumption relative to output. Standard models of sovereign debt share this feature on the surface: the real interest rate is countercyclical, which makes consumption procyclical. However, these models refer to the government's borrowing rate, which is not necessarily the one faced by households. While private borrowing rates tend to correlate with the rate on government securities, this is typically not true of saving rates (see Martínez Pagés, 2017; Arnold and de Vries-van Ewijk, 2014, and Figure 21). In the 2000s, Spanish households held about 94% of GDP in net worth (Figure 22), which makes the claim that only borrowing rates matter problematic.

The heterogeneous-agents setup creates non-Ricardian features which are central to my analysis. In the model, Ricardian equivalence fails because of incomplete markets and borrowing constraints. A potential default would redistribute from agents who are exposed to sovereign risk to agents who are not. When the default probability increases, however, those who stand to gain fail to increase their consumption. This happens because these agents are poorer and close to their borrowing limit. Future potential transfers do not enter their relevant measure of permanent income. Indeed, these agents have low marginal propensities to consume (MPCs) out of future income for the same reason that they have high MPCs out of current income.

Another, direct effect occurs through the government's borrowing costs. In a crisis, conditional on no default, the present value of surpluses increases as more resources are needed to service the debt. When a default is likely but has not yet occurred, high spreads amount to a transfer from taxpayers to bondholders (who tend to be richer or foreigners), which by the same MPC logic also tends to hurt aggregate spending.

To quantitatively assess the welfare costs of sovereign risk, I calibrate the model to Spain in the 2000s. In addition to standard targets in the sovereign debt literature, I match the wealth-to-GDP ratio as well as the Gini index for wealth and the share of sovereign debt held domestically. I then simulate a long time-series from the calibrated model and isolate episodes of crisis. I define a crisis in the simulated data as an episode that resembles the Spanish experience of 2010–2013: no default accompanied by an elevated default probability. The model, solved without the possibility of sovereign default but simulated with the same underlying shocks, features much milder recessions at the times in which the benchmark model is in crisis: about 30% of the output contraction during a typical simulated crisis can be directly attributed to the presence of sovereign risk. The behavior of consumption is also markedly different across models during these crisis episodes: while the benchmark model broadly matches the Spanish experience, the model without default exhibits much more smoothing in aggregate consumption.

The comparison with the model without default reveals that almost all of the volatility in aggregate consumption is caused by sovereign risk. The extra volatility is costly for the economy: on average, households would give up about 1.76% of consumption to make defaults impossible. This is a large number considering that the frequency of default in the model is only 1.1%. The default episodes themselves are extremely costly: at the height of a crisis, moving to a world without default is worth between 4.5% and 7% of permanent consumption to the average household.

The model also allows me to examine the impact of sovereign risk across the wealth distribution. I find substantial heterogeneity in welfare costs of sovereign risk, which range between 10.2% of permanent consumption for the bottom 10% of the wealth distribution to 5.6% for the top 10%.

I consider intermediate comparison models to decompose how sovereign risk affects the economy. I construct them by removing some consequences of default. I consider versions in which the government's policy for default is kept the same as in the benchmark, but either the TFP costs of default are removed or the redistribution is shut down by forbidding domestic agents from holding sovereign debt in the first place. An interesting aspect is that the anticipation of income losses and redistribution interact to create amplification. A version of the model without redistributive effects produces marginal changes in the volatilities. However, the version without income losses does not remove all of the volatility created by sovereign risk, suggesting that the potential for redistribution matters when combined with the potential income losses. A clue is that government debt is a bad hedge only if income is lower in default.

A common argument during debt crises is that a lack of 'confidence' causes aggregate demand to fall short of levels consistent with full employment. This paper addresses this argument as a rational, although inefficient, response to the evolution of fundamentals in the economy. Without commitment to future policies, the government's ex-post default incentives act during crises as large-scale ex-ante increases in uncertainty (Bloom, 2009). From a broader perspective, the amplification I emphasize helps explain why emerging economies exhibit high volatility of consumption relative to output 'as if' they were subject to trend shocks, especially on the downside (Aguiar and Gopinath, 2007).

The setup with heterogeneous households allows for a clean separation of the debts and assets of the government and the private sector. In canonical models of sovereign debt, allowing households access to risk-free borrowing and saving unravels the equilibrium. The reason is simple: if the government has access to lump-sum taxes and the representative household can commit to repay loans, then the government can use its tax policy to effectively have the household borrow on its behalf at the risk-free rate. This has naturally led researchers to study models in which the private sector's financial choices are constrained. An alternative is to constrain the tax instruments at the government's disposal. In my model, even though the government can collect lump-sum taxes and agents can save, it cannot make those taxes agent-specific. This provides a natural constraint on the government's ability to sidestep its lack of commitment.

**Discussion of the Literature** This paper relates to several strands of literature. I build on canonical models of sovereign debt (Eaton and Gersovitz, 1982; Aguiar and Gopinath, 2006; Arellano, 2008) by considering a benevolent government borrowing without commitment from international creditors. Recent papers have emphasized internal costs of sovereign default. Mendoza and Yue (2012) assume that domestic firms lose access to some imported inputs after a default, which reduces aggregate productivity. Dovis (2018) rationalizes these costs of defaults as a descentralization of the optimal contract between the country and its lenders subject to lack of commitment and information frictions. From these papers I take the shape and size of default costs, which are exogenous in my model.

Others such as Gennaioli, Martin, and Rossi (2014), Pérez (2018), and Mallucci (2015) argue that the presence of domestic debt creates default costs through the disruption of financial intermediation. These papers assume households are able to save and provide deposits to the financial sector. However, because they use one-sector models in which the law of one price holds, they effectively abstract from the aggregate demand effects I emphasize.

I build on models in which nominal rigidities in wage setting combined with an exchange rate peg create an aggregate demand externality (Schmitt-Grohé and Uribe, 2016, and a large literature). Anzoategui (2020) combines wage rigidities and default risk to estimate counterfactual series for Spain had it not imposed austerity measures in the crisis. The tradeoff emphasized in that paper is that austerity depresses aggregate demand but endogenously decreases the probability of a debt crisis. Bianchi, Ottonello, and Presno (2016) also think about fiscal multipliers in the presence of sovereign risk and they characterize the optimal policy in the presence of wage rigidities, where the government can affect the real exchange rate via the relative demand for traded and nontraded goods. Both papers abstract from the precautionary effects that are at the core of my argument by assuming that domestic households are unable to save.

In a similar line to mine, Arellano, Bai, and Mihalache (2020) consider a New Keynesian small open economy model with an aggregate demand externality where the government chooses its fiscal and default policy. They focus on the case when the Central Bank follows a Taylor rule, the currency floats freely and the economy undergoes a real devaluation at the time of default. These differences in assumptions change the conclusions in interesting and complementary ways.

Some studies, like Bocola (2016); Arellano, Bai, and Bocola (2017); Arellano, Bai, and Mihalache (2018); Balke (2017), explicitly consider anticipation effects in investment from sovereign risk. In the first two papers, when the probability of default is high, banks attach a higher value to safe assets. They lose appetite for risk and charge firms a higher interest rate. Investment drops which depresses growth in a complementary way to the one explored here. Because it works through the supply side of the economy, this mechanism cannot by itself account for the savings pattern of households in the crisis. Moreover, this mechanism requires that banks be unable to raise equity, which is correct in the short run but less likely as time passes. I take the opposite stand that the financial sector acts as a veil for the nonfinancial private sector. This also highlights inequality within the private sector as a driver of the output response to sovereign risk.

Arellano, Bai, and Mihalache (2018) consider the consequences for investment of an assumed correlation between private borrowing costs and sovereign spreads. In a centralized economy, they find that negative TFP shocks that endogenously increase sovereign risk also induce lower investment, and that the fall is concentrated in the nontradable sector. The reason is that, even if the planner wants to invest less because returns are low, it tries to tilt investment towards traded goods, which are needed to service the debt. The authors show that the allocation can be descentralized using a rich enough taxation scheme. In two-digit sectoral data for Spain in the 2011 crisis, they find that investment did fall by more in sectors that rank lower in a measure of tradedness. In Balke (2017), the same public-private correlation in borrowing costs constrains firms' ability to obtain working capital loans, which depresses vacancy posting and job creation.

Philippon and Roldán (2018) describe in a related but more stylized setting the possibility of expansionary austerity. There, austerity can ease the fears of unconstrained 'savers' and boost aggregate demand. In a calibration to the Eurozone, the direct contractionary impact negates expansionary austerity. They then focus on the optimal design of the sovereign deleveraging plan. Romei (2015) considers the distributional impact of different speeds of fiscal consolidation in the absence of aggregate demand effects. In a flexible-price model, Cuadra, Sánchez, and Sapriza (2010) argue that governments constrained by their own lack of commitment to future actions find it optimal to follow procyclical fiscal policies.

Part of how sovereign risk affects demand is because of redistribution. In this sense, I build on models such as Eggertsson and Krugman (2012), Auclert (2017), or Korinek and Simsek (2016), where shocks contract demand because they redistribute from high-MPC to low-MPC agents. This paper features this idea prominently, except that the timing of transfers reverses the identities of low- and high-MPC agents.

I also relate to studies in which sovereign debt policy responds to distributional concerns, as has been emphasized since Woodford (1990). While distributional concerns are featured in its objective function, the government in my model does not issue debt to help domestic agents, who can save in the international risk-free bond, with their self-insurance (as in Aiyagari and McGrattan, 1998; Shin, 2006). D'Erasmo and Mendoza (2016) build a heterogeneous-agents model of sovereign default and find that levels of debt like those of present day Spain suggest a government with a bias towards favoring its creditors. Ferriere (2016), Ferriere and Navarro (2017), and Deng (2020) argue for a positive link between progressive taxation on the one hand and incentives to repay sovereign debt and fiscal multipliers on the other. Guembel and Sussman (2009) and Andreasen, Sandleris, and van der Ghote (2011) study political economy considerations in sovereign debt policy, while Dovis, Golosov, and Shourideh (2016) find that, in an overlapping-generations economy, the tension between the ex-ante desire to promote savings and the ex-post temptation to redistribute by taxing capital can lead to 'populist cycles' of austerity and external debt-financed expansions.

**Layout** The remainder of the paper is organized as follows. Section 2 presents some motivating evidence. Section 3 describes the model while Section 4 defines the equilibrium and provides some intuition on the inner workings of the model. Section 5 discusses the calibration and Section 6 summarizes results from the model solution. Section 7 focuses on crises and present the main results. Finally, Section 8 concludes.

## 2. Motivating Evidence

Figure 1 plots total GDP and households' consumption for Spain in the 2000s. To show each series in as raw a form as possible, I plot them relative to the value at the start of 2008. Output and consumption strongly contract during the crisis years. Moreover, consumption contracts more than output as the crisis unfolds. Figures 25 and 26 in the Appendix reinforce this point by showing that the same pattern appears in HP-detrended data and that it corresponds to an increase in the trade balance. Peak to trough, the declines in output and consumption are of about 10% and 15%. These numbers can be misleading as they include the effect of the Global Financial Crisis and the Spanish housing bust. Comparing the trough of the crisis to early 2011 to isolate the effect of sovereign risk as much as possible, the output and consumption contractions are of the order of 5% and 9%.

Table 6 in the Appendix replicates the volatility calculations from Aguiar and Gopinath (2007) using the Eurozone crisis data. For Spain, Italy, and Portugal (and the Netherlands), the volatility of consumption is greater than the volatility of output, and the ratio of those volatilities is larger now than it was in Aguiar and Gopinath's data from the late 20th century. On the other hand, for countries that were mostly unscathed by the crisis (such as Austria, Belgium, Denmark, and Finland), the relative volatility of consumption remains low.

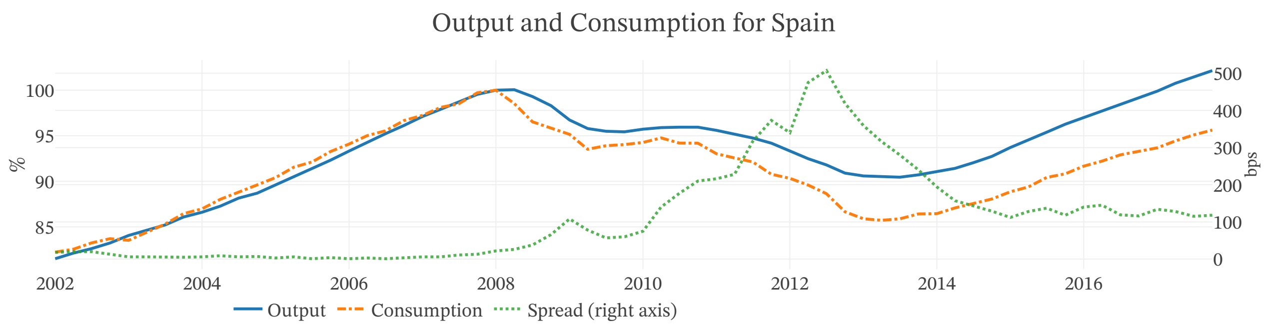


Figure 1: Spanish Output and Consumption in the 2000s

The case of Spain in the Eurozone crisis is of particular interest, as a default did not actually happen even though full repayment of government was uncertain during the period: Figure 1 reveals that a 10-year Spanish government bond paid a significant interest rate spread over a comparable German Bund. Figure 19 shows that measures of slack in the Spanish economy increase significantly during the crisis in what looks like two phases, consistent with the private deleveraging followed by sudden stop interpretation of Martin and Philippon (2017). The two phases are also noticeable as two separate instances of output contraction in Figure 1. Finally, Figure 20 in the Appendix contains results from a Eurostat survey of Spanish firms who are asked about the reasons why they produce what they do and not more. The proportion of firms reporting 'demand' as their main limiting factor is elevated during the crisis.

### 2.1 *Feedback*

The open-economy macroeconomics literature has thought about different ways in which sovereign spreads hurt the economy (see Neumeyer and Perri, 2005, among others). I contribute evidence based on quarterly data for 11 European countries. Appendix C details the data sources. I am interested in estimating an equation like

$$Q_{jt} = \beta \Delta Spread_{jt} + \gamma X_{jt} + \mu_j + \delta_t + \epsilon_{jt} \tag{1}$$

for the sovereign debt crisis period of 2010Q1:2013Q1, for $Q_{jt} = \log Y_{jt}, \log C_{jt}$, where the $X_{jt}$'s are controls and $\mu_j$ and $\delta_t$ are time and country fixed effects. Clearly, shocks that tend to make output or consumption drop also may push up spreads. This is, spreads are endogenous in equation (1). A source of exogenous variation is needed to estimate the causal effect.

I construct an instrument following Martin and Philippon (2017). I start by estimating the following auxiliary regression

$$\Delta Spread_{jt} = \underbrace{\phi B_{j0} + \delta_t}_{Z_{jt}} + \eta_{jt} \tag{2}$$

where the increase in spreads of country $j$ in quarter $t$ is predicted using time fixed-effects $\delta_t$ and the debt-to-GDP ratio $B_{j0}$ measured in the first quarter of 2008. I run this regression including all countries during the sovereign debt crisis period. The fitted values $\hat{Z}_{jt}$ are then used as instruments for the spreads in equation (1).

A negative coefficient in this IV estimation of (1) reflects that countries that saw their spreads increase more because of their earlier fiscal situation saw bigger output or consumption contractions in the crisis. Table 1 summarizes the estimation of equation (1)

| | *Dependent variable:* | | | |
|---|---|---|---|---|
| | OLS | | IV | |
| | $\log Y_{jt}$ | $\log C_{jt}$ | $\log Y_{jt}$ | $\log C_{jt}$ |
| | (1) | (2) | (3) | (4) |
| $\Delta Spread_{jt}$ | $-0.008^{***}$ | $-0.013^{***}$ | $-0.006^{**}$ | $-0.010^{***}$ |
| | (0.001) | (0.001) | (0.002) | (0.003) |
| Country + Time FE | ✓ | ✓ | ✓ | ✓ |
| Observations | 143 | 143 | 143 | 143 |
| Adj. $R^2$ | 0.772 | 0.784 | 0.765 | 0.776 |

Standard errors in parentheses. $^{***}$ $p < 0.01$, $^{**}$ $p < 0.05$, $^{*}$ $p < 0.1$.

Table 1: Feedback of Spreads and Macro Outcomes

Columns (3) and (4) report the benchmark IV estimates, the first two columns provide OLS estimates for comparison. An increase of 100bps in government spreads is associated with falls in output and consumption of about half and one percentage points, respectively. In both specifications, the response of consumption is significantly larger than that of output. In principle, this seems at odds with consumption smoothing in advanced economies where the private sector manages considerable net worth.

## 3. Model

I consider a small open economy populated by a continuum of heterogeneous households and firms that produce tradable and nontradable goods. A government runs an exogenous, estimated fiscal rule for spending and debt issuance, but chooses between default and repayment with discretion. The key ingredients of the economy are an endogenous distribution of wealth that interacts with default risk and wage rigidities. There are incomplete markets and only two assets are traded: a one-period, risk-free private security and a long-term, non-contingent, defaultable government bond.

### 3.1 Households

There is a continuum of heterogeneous households who differ in the realization of an uninsurable idiosyncratic shock to their effective labor supply, $\epsilon$, as well as in their asset holdings. Let $a$ and $b$ denote holdings of the risk-free asset and of government debt, respectively. Households are limited in their ability to hold negative positions in these assets: it is impossible to short the government, and there is an ad-hoc lower bound $\bar{a}$ on the risk-free asset. Respecting these restrictions, both assets trade at prices $q^h$ and $q^g$.

Households value the consumption of traded and nontraded goods according to a CES aggregator

$$c = \left[ \varpi^{\frac{1}{\eta}} c_N^{\frac{\eta-1}{\eta}} + (1-\varpi)^{\frac{1}{\eta}} c_T^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $\eta$ is the elasticity of substition among the two goods. I assume an inelastic labor supply. Households have Epstein-Zin preferences over streams of consumption represented by the value function

$$v_t = \left( (1-\beta) c_t^{\frac{\psi-1}{\psi}} + \beta \mathbb{E}_t \left[ {v_{t+1}}^{1-\gamma} \right]^{\frac{\psi-1}{\psi(1-\gamma)}} \right)^{\frac{\psi}{\psi-1}}$$

where $\beta$ is the discount factor, $\gamma$ is the coefficient of risk-aversion and $\psi$ is the inverse elasticity of intertemporal substitution.

In period $t$, households observe the aggregate state of the economy $\mathbf{S}_t = (B_t, \lambda_t, \xi_t, \zeta_t, z_t)$, comprised of total government debt outstanding $B_t$, the current distribution of households over their idiosyncratic states $\lambda_t$, the current value of a shock to sovereign spreads $\xi_t$, the current state of the country in international credit markets $\zeta_t$, and the current level of a productivity shock $z_t$. In equilibrium, this information is enough to recover the relative price of nontraded goods $p_N(\mathbf{S}_t)$ and the current wage rate $w(\mathbf{S}_t)$, as well as the price of government debt $q^g(\mathbf{S}_t)$, lump-sum taxes $T(\mathbf{S}_t)$, firms' profits $\Pi(\mathbf{S}_t)$, and the price of consumption (CPI)

$$p_C = p_C(p_N) = \left[ \varpi p_N^{1-\eta} + (1-\varpi) \right]^{\frac{1}{1-\eta}}$$

where the price of traded goods is normalized to $p_T = 1$.

Government debt is a long-term asset which promises a geometrically-decaying coupon payment denominated in the traded good, as in Leland (1998) and Chatterjee and Eyigungor (2012). While the government is not in default, holders of debt purchased $t$ periods ago receive $\kappa(1-\rho)^{t-1}$. This standard setup makes one unit of debt issued $j$ periods ago a perfect substitute of $1-\rho$ units of debt issued $j-1$ periods ago. When the government defaults, a haircut $\hbar$ is applied to all outstanding debt and coupon payments are suspended until the government regains market access.

The household's idiosyncratic states are the current level of its labor productivity $\epsilon$ as well as the total value of its asset portfolio $\omega_t = a'_{t-1} + R^b_{t-1,t} b'_{t-1}$. I adopt the convention that the risk-free asset pays one unit of the traded good while the government bond yields $R^b_{t-1,t}$. Let $\mathbf{s} = (\omega, \epsilon)$ denote the idiosyncratic state vector. Individual labor productivity follows an AR(1) process in logs so $\log \epsilon_{t+1} = \rho_\epsilon \log \epsilon_t + \sigma_\epsilon \nu^\epsilon_{t+1}$, where $\nu^\epsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0,1)$.

Because of nominal rigidities (see below), there can be rationing in the labor market when labor demand falls short of supply. In that case, I assume that households are rationed proportionally so that everyone works the same amount of hours. These assumptions mean that a household with current shock $\epsilon$ receives (pre-tax) labor income equal to $y^L(\mathbf{s}, \mathbf{S}) = w(\mathbf{S})L(\mathbf{S})\epsilon$ at wage $w$ and employment $L$ in state $\mathbf{S}$, of which a fraction $\tau$ is paid to the government as labor income taxes.

Households also receive income from ownership of the firms. I assume that this income is rebated lump-sum in proportion to the current value of the shock $\epsilon$. Because the integral of $\epsilon$ is normalized to 1, households receive income $y^\Pi(\mathbf{s}, \mathbf{S}) = \Pi(\mathbf{S})\epsilon$.

The household's problem (3) summarizes the discussion above.

$$
v(\omega, \epsilon, \mathbf{S}) = \max_{a', b', c} \left( (1-\beta) c^{\frac{\psi-1}{\psi}} + \beta \mathbb{E}\left[ \left( v(a' + R_b(\mathbf{S}, \mathbf{S}')b', \epsilon', \mathbf{S}') \right)^{1-\gamma} | \omega, \epsilon, \mathbf{S} \right]^{\frac{\psi-1}{\psi(1-\gamma)}} \right)^{\frac{\psi}{\psi-1}} \tag{3}
$$

$$
\begin{aligned}
\text{subject to } & p_C(p_N(\mathbf{S}))c + q^h(\mathbf{S})a' + q^g(\mathbf{S})b' = \omega + \ell(\mathbf{S})\epsilon - T(\mathbf{S}) \\
& \ell(\mathbf{S}) = w(\mathbf{S})L(\mathbf{S})(1-\tau) + \Pi(\mathbf{S}) \\
& R_b(\mathbf{S}, \mathbf{S}') = 1_{(\zeta'=1)}\kappa + (1-\rho)\left(1 - \hbar 1_{(\zeta=1)\cap(\zeta'\neq 1)}\right) q^g(\mathbf{S}') \\
& b' \geq 0; \qquad a' \geq \bar{a} \\
& \mathbf{S}' = \Psi(\mathbf{S}, \xi', z', \zeta')
\end{aligned}
$$

This problem is affected by the presence of sovereign risk in at least three distinct ways. An increase in default risk depresses expected future income, generates capital losses through movements in realized $R_b$, and worsens the savings technology by making expected $R_b$ more negatively correlated with future income. Section 4.4 discusses these effects in detail.

The solution to the household's problem consists of policy function $\phi_a, \phi_b, \phi_c : \mathbf{s} \times \mathcal{S} \rightarrow \mathbb{R}$. It is important to notice that the value function $v(\mathbf{s}, \mathbf{S})$ describes a household *after* the government's default decision.

### 3.2 *Relative prices and the real exchange rate*

I now turn to the determination of the relative price of the nontraded good (the real exchange rate). Because of the homotheticity of CES demand, each household consumes both goods in the same proportions. The first-order condition for the composition of consumption, summing over all agents, then reads

$$
p_N(\mathbf{S}) = \frac{\varpi^{1/\eta}}{1-\varpi^{1/\eta}} \left( \frac{C_T(\mathbf{S})}{C_N(\mathbf{S})} \right)^{\frac{1}{\eta}} \tag{4}
$$

### 3.3 *Firms*

There are two types of firms that produce traded and nontraded goods. Their technologies are concave in labor and are given by

$$Y_{Nt} = f_N(z_t, \zeta_t) L_{Nt}^{\alpha_N} \tag{5}$$

$$Y_{Tt} = f_T(z_t, \zeta_t) L_{Tt}^{\alpha_T} \tag{6}$$

The functions $f_i$ for $i \in \{N, T\}$ describe productivity in both sectors. TFP depends on the productivity shock $z_t$ and is reduced when the economy is in default. As a benchmark, I consider the case where the shock $z_t$ only affects the production of traded goods

$$f_N(z, \zeta) = 1 - \Delta 1_{(\zeta \neq 1)}$$
$$f_T(z, \zeta) = z \left(1 - \Delta 1_{(\zeta \neq 1)}\right)$$

where $\Delta$ is the output cost of default and $\zeta = 1$ denotes good standing in international markets.

In equilibrium, firms in both sectors must pay the same wage. However, because of nominal rigidities, the wage $w_t$ cannot fall below $\bar{w}$, as in Bianchi, Ottonello, and Presno (2016). When the constraint does not bind, the economy operates at full employment; otherwise, workers are rationed. I discuss this way of introducing nominal rigidities in more detail in Section 4.5.

### 3.4 *Fiscal policy*

The government's policy determines three actions: whether to repay its current debt obligations in full, how much new debt to issue, and the level of lump-sum transfers it gives to households.

The government's policy consists of repayment and issuance strategies $h'(\mathrm{S}, \xi', z'), B'(\mathrm{S})$. The issuance strategy simply states how much new debt the government is issuing. On the other hand, the repayment strategy maps an aggregate state in $t$ and a realization of shocks in $t+1$ into a probability of repayment.

When the government is in default (denoted by $\zeta = 0$), coupon payments are interrupted. Holders of the debt can still trade it in the secondary market. Defaulted debt is still valuable as the government recovers access to markets with constant probability $\theta$ each period. While in default, new debt cannot be issued (even if it would command a positive price), which restricts $B'(\mathrm{S}_t) = B(\mathrm{S}_t)$ in default states.

The government's budget constraint (7) equates resources from (net) debt issuance and labor income taxes to expenditures given by coupon payments, government spending, and lump-sum transfers

$$\underbrace{q^g(\mathrm{S}_t)}_{\textit{debt price}} \underbrace{\left(B'(\mathrm{S}_t) - (1-\rho)B(\mathrm{S}_t)\right)}_{\textit{new debt issued}} + \underbrace{\tau w(\mathrm{S}_t) L(\mathrm{S}_t)}_{\textit{income tax}} = \underbrace{\kappa 1_{(\zeta=1)} B(\mathrm{S}_t)}_{\textit{coupon}} + \underbrace{g(\mathrm{S}_t)}_{\textit{spending}} - \underbrace{T(\mathrm{S}_t)}_{\textit{lump-sum}} \tag{7}$$

This budget constraint means that, given $q^g(\mathbf{S}_t)$, the government's choice of transfers $T(\mathbf{S}_t)$ can be obtained as a residual from its issuance policy.

I assume that the government follows exogenous fiscal rules to determine consumption $g_t$ and debt issuances $B'_t$. These are allowed to be a function of the whole state vector, and I estimate them to match observed correlations with key business cycles statistics (see Section 5). Finally, I assume that the government spends a constant fraction $\vartheta_N$ of its expenditures on the nontraded good.

### 3.4.1 Defaults and the evolution of debt

The repayment strategy of the government $h'(\mathbf{S}_t, \xi_{t+1}, z_{t+1})$ specifies a repayment probability in each state of the following period. The government makes its default choice in period $t+1$ having observed the exogenous states $(\xi_{t+1}, z_{t+1})$ and understanding which aggregate states $\mathbf{S}_{t+1}$ result from repayment and from default. The government also receives an *iid* preference shock $\xi^{def}$ orthogonal to all other variables, which plays the role of smoothing out the policy for numerical tractability.

If there is a default in period $t+1$, a haircut of $\hbar$ applies to the debt of the government. This means that $B(\mathbf{S}_{t+1}) = (1-\hbar)B'(\mathbf{S}_t)$, whereas $B(\mathbf{S}_{t+1}) = B'(\mathbf{S}_t)$ otherwise. When in default, there is a constant probability $\theta$ of reentering financial markets.

The budget constraint (7) captures a particular tradeoff. When resources from tax collections and debt issuance are low (for instance, when spreads are high), the government chooses between default or low lump-sum transfers. In this context, one could interpret the second option as a regressive austerity plan.

## 3.5 Monetary policy

The small open economy defends a pegged exchange rate. Everywhere in the model, this assumption amounts to a normalization of the (constant) price of nontraded goods $p_T \equiv 1$. Importantly, I assume that the economy does not abandon the peg upon default, as Na, Schmitt-Grohé, Uribe, and Yue (2018) argue is a relevant case.

Relaxing my assumption to have devaluations accompany defaults would not be innocuous. It would certainly reduce the aggregate income losses from default by allowing real wages to fall. On the other hand, it would create wealth effects from the currency of denomination of contracts and assets that households own. The first consequence can be captured in this model by making the bound on wages depend on the default state $\zeta$. However, addressing the second, probably more interesting consequence requires a rich model of currency choice and is beyond the scope of this paper.

### *3.6 Foreign borrowing and the external sector*

I assume a large quantity of foreigners who have access to funds at a fixed international risk-free rate $r^\star$. Immediately, this implies that

$$q^h(\mathrm{S}) = \frac{1}{1+r^\star} \tag{8}$$

Furthermore, if foreigners hold the government's debt in state S, then by no arbitrage it has to be the case that

$$q^g(\mathrm{S}) = \frac{1}{1+r^\star}\mathbb{E}\left[\underbrace{1_{(\zeta'=1)}\left(1-\xi'\right)\kappa}_{coupon} + \underbrace{\left(1-\rho 1_{(\zeta'=1)}\right)}_{depreciation}\underbrace{\left(1-\hbar 1_{(\zeta=1\cap\zeta'\neq 1)}\right)}_{potential\ haircut}\underbrace{q^g(\mathrm{S}')}_{resale\ price}\ \middle|\ \mathrm{S}\right] \tag{9}$$

which reflects that debt is a claim to coupon payments while there is no default, that a default entails the haircut $\hbar$, and that the unmatured fraction $(1-\rho)$ of the bond can be resold in secondary markets. With respect to the coupon payments, I assume that foreigners price debt as if the coupon payment was $(1-\xi')\kappa$ where the stochastic process for $\xi$ is constrained to remain within the interval $(0,1)$. This assumption artificially depresses the price of government debt in order to match the home bias in holdings (see Section 4.3 for a discussion).

If the government was already in default at state $\mathrm{S}_t$, equation (9) specializes to

$$q^g(\mathrm{S}\mid\zeta\neq 1) = \frac{1}{1+r^\star}\left(\theta\mathbb{E}\left[1-\xi'\mid\mathrm{S}\right]\kappa + (1-\rho\theta)\,\mathbb{E}\left[q^g(\mathrm{S}')\mid\mathrm{S}\right]\right)$$

as the government reenters international markets with constant hazard $\theta$ and it cannot default again while in the default state.

Equation (9) only holds when foreigners hold some of the debt. I assume that, as in the data, domestic demand for government debt always falls short of the total amount outstanding. I then check in simulation that this is the case.

When the small open economy is indebted with the rest of world, its consolidated intertemporal budget constraint states that the value of debt obligations must equal the expected discounted value of trade surpluses. If $A$ denotes the total amount of risk-free debt and $A^f$, $A^h$ that in hands of foreigners and domestic agents, respectively, and the same convention applies to government debt $B$, net foreign inflows are given by

$$\mathtt{NFI}_t = \underbrace{q_t^h A_{t+1}^f + q_t^g\left(B_t'^f - (1-\rho)B_t^f\right)}_{\text{Capital inflows}} - \underbrace{\left(\kappa B_t^f + A_t^f\right)}_{\text{Capital outflows}} \tag{10}$$

where resources flow into the small open economy when domestic agents borrow from foreigners and when foreigners purchase debt. On the other hand, resources flow out when the government makes coupon payments to foreigners and when domestic agents repays their debts.

Because the distribution $\lambda$ does not distinguish holdings of both assets separately, neither $A_t$ nor its components are a function of the state variables $\mathrm{S}_t$. However, some manipulation allows to recast (10) in terms of flows as

$$\begin{aligned} \mathtt{NFI}_t &= q_t^g B_t'^f - \left(A_t^f + (\kappa + (1-\rho) q_t^g) B_t^f\right) + q_t^h A_{t+1}^f \\ &= \int \left(\omega - q_t^h \phi_a - q_t^g \phi_b\right) d\lambda_t - \kappa B_t + q_t^g (B_t' - (1-\rho) B_t) \end{aligned}$$

where government debt held by foreigners equals $B_t'^f = B_t' - \int \phi_b d\lambda_t$, private debt held by foreigners equals $A_{t+1}^f = -\int \phi_a d\lambda_t$, and $\int \omega d\lambda_t = A_t^h + (\kappa + (1-\rho) q_t^g) B_t^h$.

Finally, market clearing requires that

$$Y_{Nt} = C_{Nt} + \frac{\vartheta_N}{p_{Nt}} G_t \qquad \text{and} \qquad Y_{Tt} + \mathtt{NFI}_t = C_{Tt} + (1 - \vartheta_N) G_t \tag{11}$$

as net foreign inflows must equal the trade deficit.

### 3.7 *Timing*

Figure 2 summarizes the unfolding of events within a period. The past state is carried from the previous period. Then nature chooses the current level of TFP and risk premia. Observing these, the government decides its repayment if it is not in default already. If the country was already in default, then nature chooses whether there is reentry to financial markets. Only then do foreign lenders set asset prices. At this point the distribution of wealth across households is determined. The government then implements its issuance and transfer policies. Finally, firms choose employment and prices, the households make their consumption and savings choices, and the period ends.

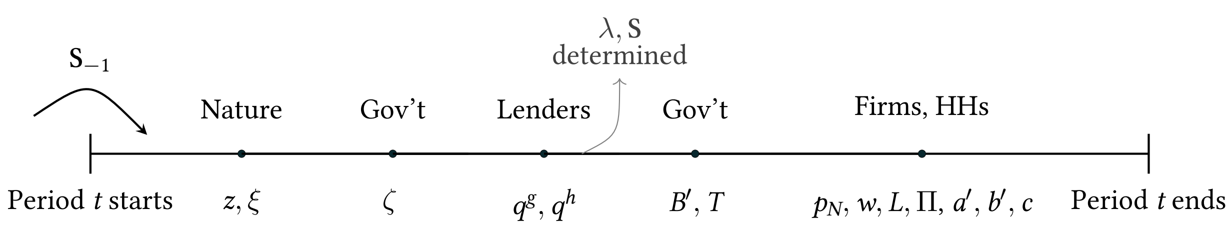


Figure 2: Timeline

### 3.8 *Evolution of the distribution*

Before defining the equilibrium, I discuss a particular assumption that allows me to solve the model parsimoniously. In the exposition above, the state vector S contains the whole distribution of agents across their idiosyncratic states, which is an infinitely-dimensional object. As is usual in heterogeneous-agents models, I proceed by solving for a bounded rationality equilibrium where agents only have limited knowledge of the distribution $\lambda$.

Specifically, I assume that agents believe the distribution of wealth and individual labor productivity to be jointly lognormal with

$$\begin{pmatrix} \omega_t \\ \epsilon_t \end{pmatrix} \sim \log \mathcal{N}\left( \begin{bmatrix} \mu_t \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_t & \rho_t \\ \rho_t & \sigma_\epsilon \end{bmatrix} \right) \tag{12}$$

where as of this writing, I further assume that $\rho_t = 0$. $\mu_t$ and $\sigma_t$, however, vary over time and become state variables.

This assumption allows me to summarize $\lambda_t$ with $(\mu_t, \sigma_t, \rho_t)$. As for the law of motion of the distribution, the household's policy functions need not imply that $\lambda_{t+1}$ is exactly lognormal even if $\lambda_t$ is. This is the key place where the approximation is taken, as I assume laws of motion for the distribution parameters.

Two approximations happen at once. The first is that I only solve for the equilibrium on a subset (the lognormal distributions) of the infinite dimensional space of all possible distributions. Bounded rationality really happens along the second approximation: whenever the policy functions imply a distribution that is not lognormal for the following period, I project it back onto the $(\mu, \sigma, \rho)$ space by making all agents in the model expect a lognormal distribution with the same mean and variance than the one implied by the policies.

These approximations allow me to solve for the equilibrium of the model without the usual simulation step. Instead, I check in simulation that the agents' forecasting rule accurately predicts the dynamics of relevant variables.

Given all functions of the state (including the households' policy functions) and the current distribution, substituting $\lambda_t$ for the corresponding lognormal in (13) yields a system for the joint evolution of the parameters of the distribution as well as the price of debt (which depends on the future distribution through the government's default incentives)

$$\begin{cases} R_b(\mathbf{S}_{t+1}) & = & 1_{(\zeta_{t+1}=1)}\kappa + (1-\rho)q^{g}(\mathbf{S}_{t+1}) \\ \int \omega \, d\lambda_{t+1} & = & \int \phi_a(\mathbf{s}_t, \mathbf{S}_t) + R_b(\mathbf{S}_{t+1})\phi_b(\mathbf{s}_t, \mathbf{S}_t) d\lambda_t \\ \int \omega^2 d\lambda_{t+1} & = & \int \left[\phi_a(\mathbf{s}_t, \mathbf{S}_t) + R_b(\mathbf{S}_{t+1})\phi_b(\mathbf{s}_t, \mathbf{S}_t)\right]^2 d\lambda_t \\ \int \omega\epsilon \, d\lambda_{t+1} & = & \int \left[\phi_a(\mathbf{s}_t, \mathbf{S}_t) + R_b(\mathbf{S}_{t+1})\phi_b(\mathbf{s}_t, \mathbf{S}_t)\right] \epsilon' f(\epsilon_t, \epsilon') d\lambda_t \end{cases} \tag{13}$$

# 4. Equilibrium

## 4.1 *Competitive equilibrium*

**Definition** Given government policies $h'(\mathbf{S}, \xi', z'), B'(\mathbf{S}), and g(\mathbf{S})$, a *competitive equilibrium* consists of value and policy functions $\{v, \phi_a, \phi_b, \phi_c\}(\mathbf{s}, \mathbf{S})$, aggregates $L_T(\mathbf{S})$, $L_N(\mathbf{S})$, $\Pi(\mathbf{S})$, $Y_N(\mathbf{S})$, $Y_T(\mathbf{S})$, prices $p_C(\mathbf{S})$, $p_N(\mathbf{S})$, $w(\mathbf{S})$, $q^g(\mathbf{S})$, $q^h(\mathbf{S})$, taxes $T(\mathbf{S})$ and laws of motion for the distribution parameters $\{\mu', \sigma'\}(\mathbf{S}, \xi', z', \zeta')$ such that

- The policy functions solve the household's problem (3) given prices, aggregates, and the law of motion for the distribution.
- The relative price of nontraded goods $p_N(\mathbf{S})$ satisfies the intratemporal first-order condition (4).
- The aggregates $L_T(\mathbf{S}), L_N(\mathbf{S})$ maximize the firms profits given prices $w(\mathbf{S}), p_N(\mathbf{S})$ and the quantities produced $Y_N(\mathbf{S})$, $Y_T(\mathbf{S})$ satisfy the production functions (5, 6).
- The lump-sum taxes $T(\mathbf{S})$ satisfy the government's budget constraint (7).
- Asset prices $q^h(\mathbf{S})$ and $q^g(\mathbf{S})$ satisfy the no-arbitrage conditions (8, 9).
- Market clearing
  1. in traded and nontraded goods (11).
  2. in labor: either $w(\mathbf{S}) = \bar{w}$ or $L_T(\mathbf{S}) + L_N(\mathbf{S}) = \int \epsilon d\lambda_{\mathbf{S}}$.
- The laws of motion for the distribution parameters satisfy the consistency requirement (13).

## 4.2 *The government's strategy*

The government's objective is to maximize current welfare in the economy. I assume that it places equal weights on every agent. Without commitment, the government maximizes

$$\mathcal{W}(\mathbf{S}, h') = \int v(\mathbf{s}, \mathbf{S}) d\lambda_{\mathbf{S}}(\mathbf{s}) + 1_{(\zeta=1)} \sigma_g \xi^{def} \tag{14}$$

where $\xi^{def} \overset{iid}{\sim} \mathcal{N}(0, 1)$ is a preference shock that serves the numerical purpose of smoothing the default policy. The government is subject to equilibrium conditions and its budget constraint, where the notation $\lambda_{\mathbf{S}}$ emphasizes that the distribution is a part of the state $\mathbf{S}$. Importantly, the value function and the distribution correspond to the competitive equilibrium that results under the policy $h'$.

A policy $h'$ for repayment is a part of an *equilibrium* if, at each $(S, z')$, the probability of repayment satisfies

$$h'(S, z') = \mathbb{P}\left(\sigma_g \xi^{def} \leq \underbrace{\mathcal{W}\left(\Psi(S, \xi', z', \zeta' = 1), h'\right)}_{\textit{value under repayment}} - \underbrace{\mathcal{W}\left(\Psi(S, \xi', z', \zeta' \neq 1, h')\right)}_{\textit{value under default}}\right) \tag{15}$$

where $\Psi(S, \xi', z', \zeta') = S'$ is the state that ensues when $(\xi', z')$ are realized after $S$ and the government chooses a default state $\zeta'$. Equation (15) makes it clear that, after observing the realization of $\xi'$ and $z'$, the government understands which state $S'$ results if it decides to default or to repay. This includes the level of debt remaining to be paid as well as the distribution induced in each case.

Condition (15) is a rational-expectations restriction: the policy that households, foreigners, and the current government expect of future governments, $h'$, must coincide with the policy that the government would choose if allowed a deviation that did not alter future expectations. In other words, condition (15) insists that the policy $h'$ be part of a Nash equilibrium. The restriction that all policies depend only on the current state $S$ (and not on the whole history of play) further refines the solution concept to that of recursive equilibrium.

Section A.2 in the Appendix describes the computation of a solution in detail.

### 4.3 *Euler equations and coupon payments*

The Euler equation (16) determines a household's purchases of government bonds:

$$q^g(S) \geq \beta \mathbb{E}\left[\underbrace{R_b(S, S')\frac{p_C(S)}{p_C(S')}}_{\textit{real repayment}} \underbrace{\underbrace{\left(\frac{\phi_c(\omega', \epsilon', S')}{\phi_c(\omega, \epsilon, S)}\right)^{-\frac{1}{\psi}}}_{\textit{Intertemp. subs.}} \underbrace{\left(\frac{v(\omega', \epsilon', S')}{\mathbb{E}\left[v(\omega', \epsilon', S')^{1-\gamma} \mid S\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}}_{\textit{Risk aversion}}}_{\textit{SDF}} \;\middle|\; S\right] \tag{16}$$

with equality if the household is purchasing a positive amount of bonds.

Comparing this equation with the pricing equation (9), it is immediate to infer that if $\xi$ were equal to zero then the household would not buy too many government bonds. Being risk-averse, the household demands a risk premium to expose itself to the risk of the government. The shock $\xi$ plays the role of creating a risk premium in the return of the government bond when compared to the return of the risk-free asset. This allows the model to match the high proportion of sovereign debt held by domestic agents in the data. Moreover, introducing $\xi$ as a shock allows me to study the economy's response to increases in spreads that are not driven by changes in fundamentals of the domestic economy.

### 4.4 The household's reaction to sovereign risk

There are at least three main ways in which sovereign risk affects the household's problem (3). The first effect concerns the aggregate income losses that happen in case of default. Conditional on default, TFP drops by $\Delta$ is both sectors for a random amount of periods, which puts downward pressure on the market-clearing wage. If the constraint binds, unemployment increases. In any case, other things equal labor income $w(\mathbf{S})L(\mathbf{S})\epsilon$ is lower in default than in repayment. In states with a higher default probability, the household consequently feels poorer and reduces consumption.

Figure 3 shows expected labor income (integrating out heterogeneity in $\epsilon$) as a function of next period's TFP for default and repayment. Various panels condition on current levels of debt and the risk-premium shock $\xi$. Labor income is clearly increasing in TFP and higher in repayment than in default. Both the current level of government debt and the risk-premium shock tend to close the gap.

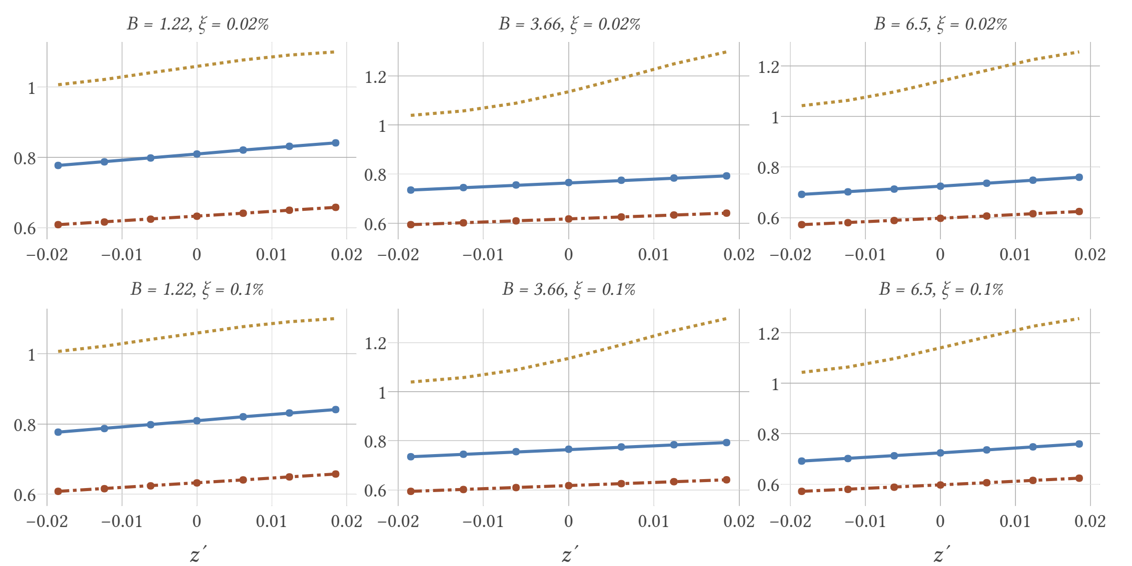


Figure 3: Labor Income and Expected Returns

Note: Blue lines plot income in repayment, red dashed lines plot income in default, orange dotted lines plot the return of holding government debt

A second effect goes through the price of government bonds. $q^g(\mathbf{S})$ reflects the default probability and shocks that make it increase also decrease the resale value of bonds. Households who purchased these bonds in the past make an inmediate capital loss when $q^g$ drops. In the aggregate, a distributional effect shifts the wealth distribution to the left when the default probability increases. The strength of this channel depends critically on the proportion of bonds held by domestic agents, as well as in the level of inequality in domestic bondholdings. Figure 9 below shows the price of debt as function of the economy's state variables. Variables that predict future default risk have a significant effect on sovereign spreads.

Finally, the household cares about the insurance properties of the government bond. Sovereign risk makes those very different in normal times and in crisis times. Recall the return of a government bond

$$R_b(\mathbf{S}, \mathbf{S}') = 1_{(\zeta'=1)}\kappa + (1-\rho)\left(1 - \hbar 1_{(\zeta=1)\cap(\zeta'\neq 1)}\right) q^g(\mathbf{S}')$$

In normal times, the variance of $R_b$ is relatively low. Its variation comes mostly from variation in the future resale price $q^g(\mathbf{S}')$. However, as the default probability increases more and more of the variance of $R_b$ becomes driven by variation in the repayment probability. Moreover, repayment correlates with aggregate income as the government's incentives to default are stronger in bad times. Hence, the conditional covariance between the bond return and the stochastic discount factor of households tends to be larger in crises. This feature makes the bond a bad hedge always but an even worse one when spreads are high.

Figure 3 shows realized bond returns in addition to labor income. Both the aggregate income losses and the savings technology effects are evident in the picture. The variance of returns, as well as its covariance with income, increases when default becomes more *uncertain*. When default is very unlikely (left panels) or very likely (right panels), next period's shocks do not influence the default probability (and hence return on holding the debt) very much. When debt is intermediate, holding it bears a very volatile return which also comoves significantly with income.

#### 4.4.1 *Who fears sovereign default? A robustness-based perspective*

The household's Euler equation offers insights into how sovereign risk affects different types of households. I calibrate the model with a unitary elasticity of intertemporal substitution. With this parameterization, households act 'as if' they had logarithmic preferences combined with concerns about model misspecification (Maenhout, 2004; Hansen and Sargent, 2001; Tallarini, 2000). In this reinterpretation, the risk aversion parameter maps into a robustness parameter. I define the *subjective expectation*, taken by an agent in state $(\mathbf{s}, \mathbf{S})$, of a random variable $X$ as

$$\tilde{\mathbb{E}}\left[X \mid \mathbf{s}, \mathbf{S}\right] = \mathbb{E}\left[X \frac{v(\omega', \epsilon', \mathbf{S}')^{1-\gamma}}{\mathbb{E}\left[v(\omega', \epsilon', \mathbf{S}')^{1-\gamma} \mid \mathbf{S}\right]} \mid \mathbf{s}, \mathbf{S}\right] \tag{17}$$

The subjective expectation twists expectations by attaching more weight to states in which the household's value function is lower. It overstates events feared by the household. Figure 4 shows the twisted probability of default for each household, computed setting $X$ to the indicator of a default in the next period in (17). The computation is conditional on a state of crisis. Figure 4 also shows the actual probability of default for comparison. Richer and higher-income households fear default, while poorer and low-income households fear the prolongation of the crisis.

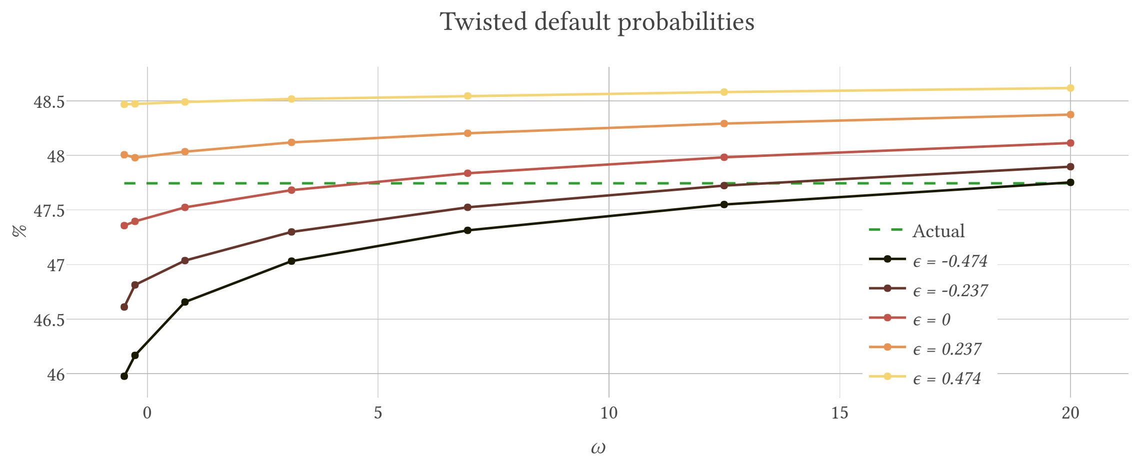


FIGURE 4: SUBJECTIVE PROBABILITIES OF DEFAULT

## 4.5 *Wage rigidities and aggregate demand*

When sovereign risk increases, the demand for consumption is likely to fall. This feeds back to the rest of the economy mainly through the market for nontraded goods.

In the market for traded goods, firms can supply whatever quantities they produce at the international price. Therefore, for a given wage rate $w_t$ prevailing in the economy, traded goods-producing firms observe the current level of TFP and choose employment accordingly.

The market for nontraded goods features more action, which is summarized by its supply curve. To trace it out, suppose a decrease in the relative price of nontraded goods. According to their first-order condition (18), firms respond to this decrease by cutting down production.

$$L_N^d = \left(\alpha_N \frac{p_N}{\max\{w, \bar{w}\}}\right)^{\frac{1}{1-\alpha_N}} \tag{18}$$

When firms in the nontraded sector retract their production they expell workers. This pushes down wages. In normal times, wages fall so some of these workers reallocate to the traded goods sector. At the same time, some others 'return' to work in the nontraded sector. When the constraint is binding, however, these second-round effects cannot happen: the fall in the price of nontradables results in an increase in unemployment and in a larger fall in the production of nontraded goods.

Figure 5 makes this point by showing that the supply curve is flatter when the constraint on wages is binding. This means that when demand falls, quantities fall more and prices fall less than in normal times. Wage rigidities create price stickiness.

The introduction of wage rigidities in this paper departs from the traditional approach of Schmitt-Grohé and Uribe (2016). In it, the wage in period $t$ is constrained be no less than $\gamma_w w_{t-1}$, where $\gamma_w \leq 1$ is a parameter. I follow instead Bianchi, Ottonello, and Presno (2016) and set a constant lower bound $\bar{w}$

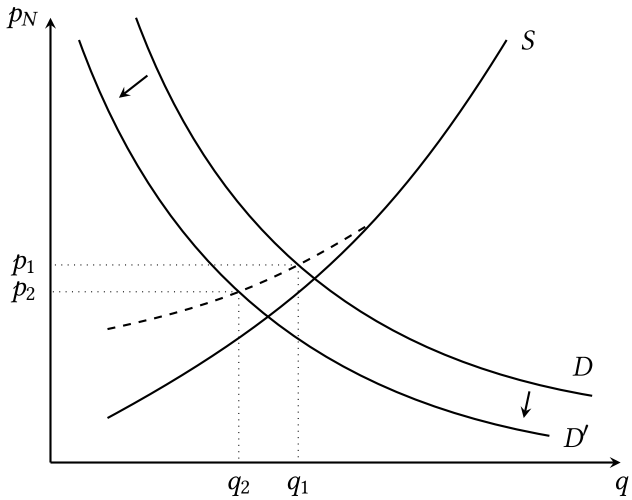


Figure 5: Market Clearing in the Nontradable Sector

on nominal wages. While both assumptions are similar, in this context there are some advantages to the second formulation.

The first obvious advantage is that not having to carry the previous period wage saves one state variable. But there is a second advantage: in the traditional formulation good TFP shocks can be welfare-decreasing if they push the current wage rate too high and generate future unemployment. This is the 'overborrowing' externality emphasized by Schmitt-Grohé and Uribe (2016): individual households do not internalize that their consumption pushes up wages. In a scenario like this, where defaults also artificially depress TFP, a benevolent government might want to default on its debt only to suppress the overconsumption externality. This would lead to counterfactually many defaults in good times. If the government was allowed to choose spending and debt issuances, it could use fiscal policy to curtail the boom instead of defaulting. However, I am constraining the government to follow the estimated fiscal rules. Hence, the admittedly less realistic constant lower bound on wages is preferred.

## 5. Calibration

### 5.1 *Fiscal rules*

I estimate fiscal rules for government spending and issuances of new debt, using quarterly data for the Eurozone. Data are taken from Eurostat and cover the period 1999Q1 to 2017Q4. In the model, government consumption and net issuances as fractions of GDP depend on the whole state vector, so in the data I regress those against endogenous variables.

Table 2 summarizes the results across various specifications. For each dependent variable, the first column contains the preferred specification. The second column contains simpler versions of the same regression. Country as well as time fixed effects are included.

| | $G_t/Y_t$ | | $(B'_t - (1-\rho)B_t)/Y_t$ | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Unemployment$_t$ | 0.031 | 0.073$^{***}$ | 0.334$^{**}$ | 0.346$^{***}$ |
| | (0.039) | (0.015) | (0.158) | (0.059) |
| Unemployment$_t^2$ | 0.002 | | 0.0001 | |
| | (0.001) | | (0.006) | |
| $B_t/Y_t$ | 0.010$^{*}$ | $-0.017^{***}$ | $-0.010$ | 0.009 |
| | (0.005) | (0.002) | (0.020) | (0.007) |
| $(B_t/Y_t)^2$ | $-0.0002^{***}$ | | 0.0001 | |
| | (0.00004) | | (0.0001) | |
| Net Exports$_t$ | 0.009 | 0.007 | 0.046 | 0.019 |
| | (0.019) | (0.012) | (0.075) | (0.046) |
| Net Exports$_t^2$ | $-0.0001$ | | $-0.001$ | |
| | (0.001) | | (0.003) | |
| Mean FE | 20.675 | 21.085 | 1.079 | 0.571 |
| Country + Time FE | ✓ | ✓ | ✓ | ✓ |
| Observations | 968 | 968 | 957 | 957 |
| Adj. $R^2$ | 0.904 | 0.901 | 0.697 | 0.698 |

Standard errors in parentheses. $^{***}$ $p < 0.01$, $^{**}$ $p < 0.05$, $^{*}$ $p < 0.1$.

Table 2: Estimated Fiscal Rules

The fit for both government consumption and debt issuances is good, with adjusted $R^2$s at 90% and 70%, respectively. Fiscal policy is countercyclical, with positive responses to unemployment for both spending and issuances. New issuances respond negatively to the debt-to-GDP ratio, consistent with debt stabilization.

Figure 6 shows the fitted values for Spain from the preferred specification. The predicted rules track the observed series closely. Clearly some of the fit is due to the inclusion of time fixed effects, which in the model version of the fiscal rule are absent. In the model, I do include Spain's fixed-effect coefficient as the intercept in the calibrated fiscal rules.

## 5.2 *Model parameters*

The current calibration of the model is able to generate a good match to some critical standard targets in the literature. Table 3 reports some critical parameter values. Because of the numerical complexity of the model, I rely on external calibration as much as possible. For the supply side of the economy, I closely follow Anzoategui (2020) and Stockman and Tesar (1995) and set preference parameters $\varpi$ and $\vartheta_N$ to match

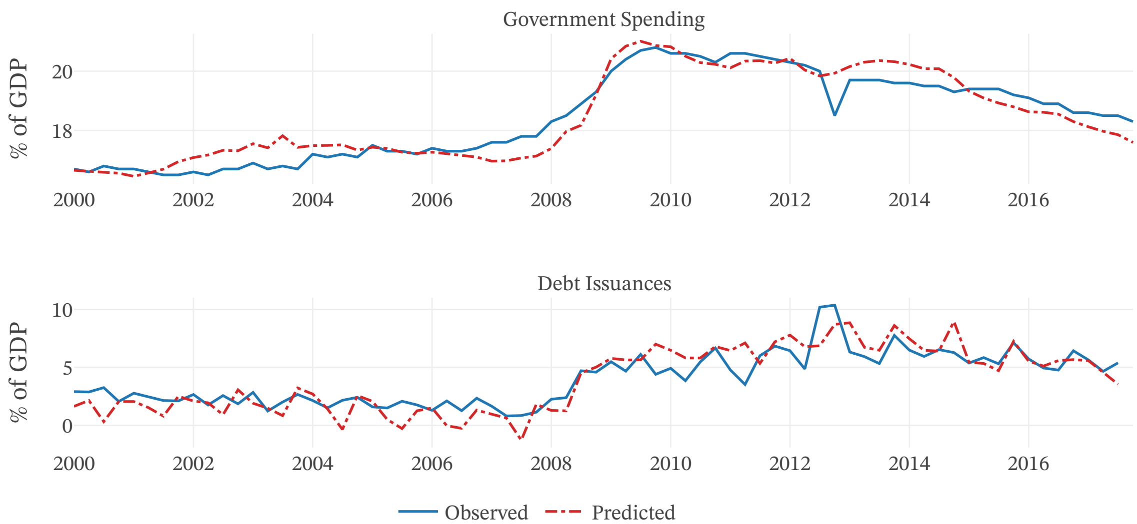


FIGURE 6: ESTIMATED FISCAL RULES

the shares of traded and nontraded goods in both private and public consumption, as well as the elasticity $\eta$ to the elasticity of relative consumption demand. Because of CES demand, every household consumes traded and nontraded goods in the same proportions. Therefore, $\varpi$ and $\eta$ only link prices and aggregate quantities through

$$p_{Nt} = \frac{\varpi^{1/\eta}}{1-\varpi^{1/\eta}} \left( \frac{C_{Tt}}{C_{Nt}} \right)^{\frac{1}{\eta}}$$

so the estimation from Stockman and Tesar (1995) is valid in this context as well.

The risk-free interest rate is set at a standard value in the literature. For the costs of default, I follow Philippon and Roldán (2018) and set the haircut and conditional TFP losses to a Greek-style default. The probability of reentry is set to give an expected duration of default of 25 quarters, on the lower end of the Cruces and Trebesch (2013) estimation for large haircuts.

As for the household idiosyncratic income shocks process, I follow the estimation of D'Erasmo and Mendoza (2016) based on the Spanish income distribution for the same period that I study.

Table 4 provides details on the fit of the model. Statistics in the Model column are computed on a simulation of the benchmark model over 10000 years. Even with substantial amplification (see Table 5), the model predicts a volatility of consumption slightly smaller than the volatility of output.

| Description | Parameter | Value | Source / Target |
|---|---|---|---|
| **Debts and defaults** | | | |
| Risk-free rate | $r^{\star}$ | 4% ann. | Anzoategui (2020) |
| Haircut in case of default | $\hbar$ | 45% | Philippon and Roldán (2018) |
| TFP loss in case of default | $\Delta$ | 10% | Philippon and Roldán (2018) |
| Reentry probability | $\theta$ | 0.04167 | Cruces and Trebesch (2013) |
| **Traded and nontraded goods** | | | |
| Share of nontraded in prod | $\varpi$ | 0.7397 | Anzoategui (2020) |
| Labor share in prod | $\alpha_N, \alpha_T$ | 0.67 | Anzoategui (2020) |
| Share of nontraded in $G$ | $\vartheta_N$ | 88% | Anzoategui (2020) |
| Elasticity of nontraded consumption | $\eta$ | 0.74 | Anzoategui (2020) |
| **Idiosyncratic income and preferences** | | | |
| Persistence log $\epsilon_{it}$ | $\rho_\epsilon$ | 0.978 | D'Erasmo and Mendoza (2016) |
| Std. deviation log $\epsilon_{it}$ | $\sigma_\epsilon$ | 0.022 | D'Erasmo and Mendoza (2016) |
| Elasticity of intertemporal substitution | $\psi$ | 1 | Epstein-Zin = robustness |
| **Internally calibrated** | | | |
| Discount rate of HHs | $1/\beta - 1$ | 8.445% ann. | Moments in Table 4 |
| Risk aversion | $\gamma$ | 6 | Moments in Table 4 |
| Progressivity of tax schedule | $\tau$ | 22% | Moments in Table 4 |
| Wage minimum | $\bar{w}$ | 0.8787 | Moments in Table 4 |
| TFP process | $\rho_z, \sigma_z$ | (0.97, 0.003) | Moments in Table 4 |
| Mean risk premium | $\bar{\xi}$ | 0.06% | Moments in Table 4 |
| Risk premium AR(1) | $\rho_\xi, \sigma_\xi$ | (0.95, 0.00025) | Moments in Table 4 |

TABLE 3: PARAMETER VALUES

| Target | Model | Data |
|---|---|---|
| AR(1) coef $\log(Y_t)$ | 0.976 | 0.966 |
| Std coef $\log(Y_t)$ | 0.0168 | 0.0129 |
| AR(1) coef $\log(C_t)$ | 0.977 | 0.962 |
| Std coef $\log(C_t)$ | 0.0141 | 0.0166 |
| AR(1) coef spread | 0.983 | 0.967 |
| Std coef spread | 0.0161 | 0.103 |
| Avg Debt-to-GDP | 31.6% | 64.6% |
| Std Debt-to-GDP | 12.8% | 23.5% |
| Avg unemployment | 7.01% | 15.9% |
| Std unemployment | 5.84% | 6.09% |
| Median dom holdings | 39.2% | 56.5% |
| Avg wealth-to-GDP | 63.8% | 94.5% |
| Avg wealth Gini | 57.2% | 57.5% |

Table 4: Model Fit

All data from Eurostat 2000Q1:2017Q4, except Gini index from Eurostat 2010, private consumption from OECD 2000Q1:2017Q4, domestic holdings from Banco de España, 2004Q1:2017Q4

# 6. Analysis

## 6.1 Government policy

Figure 7 shows the government's value function $\mathcal{W}(\Psi(\mathbf{S}, \xi', z', \zeta'), h')$ of the following period as function of the realization of shocks. Each panel shows welfare as a function of next period's TFP realization $z'$. The first row corresponds to a low realization of $\xi'$, while in the second row the risk premium is high. Finally, the columns consider different $\mathbf{S}$ for the *current* period: initial debt increases from left to right. Higher debt levels increase the value of default relative to repayment. For intermediate amounts of initial debt, moreover, a higher realization of future TFP raises the relative value of repayment. Higher spreads also marginally raise the relative value of default.

Figure 8 shows the lump-sum transfers that would clear the government's budget constraint in states $\Psi(\mathbf{S}, \xi', z', \zeta')$ of next period, as a function of $(\xi', z', \zeta')$. Unsurprisingly, high levels of indebtedness shrink the government budget constraint and force it to collect high lump-sum taxes. In repayment, a higher level of future TFP induces high lump-sum taxes as well. This effect is driven by relatively low unemployment in those states, which leaves room for the government to reduce its leverage.

Figure 9 shows the price of debt $q^g(\mathbf{S})$ at different states $\mathbf{S}$. The left panel shows that spreads rise (the price of debt falls) steeply when debt reaches a certain threshold. The effect of TFP is less stark but higher values of $z$ are associated with lower spreads. Both of these are taken for a fixed distribution when the

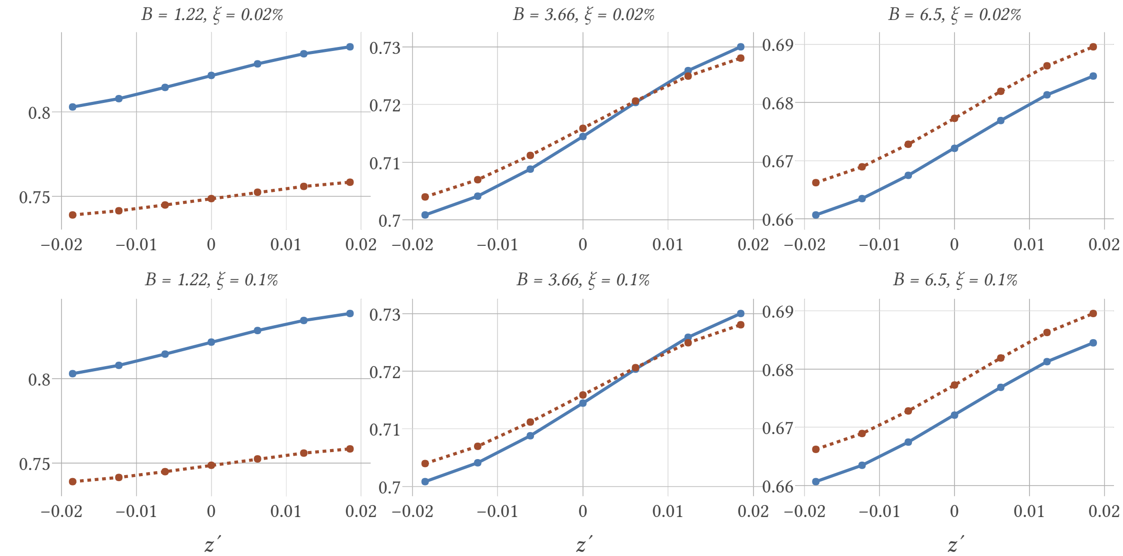


FIGURE 7: WELFARE FUNCTIONS

Note: Blue lines plot the value of repayment, red dashed lines plot the value of default

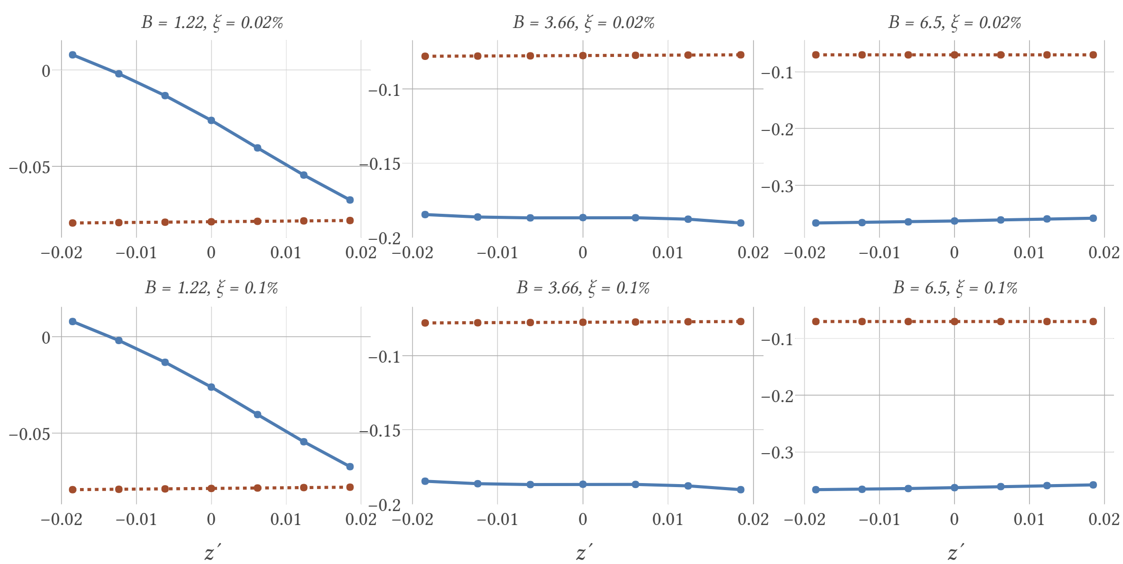


FIGURE 8: TRANSFERS

Note: Blue lines plot transfers in repayment, red dashed lines plot transfers in default economy is not in default.

The right panel shows the impact of the distribution (for fixed values of $B$ and $z$ when the economy is not in default). Higher spreads occur when the economy is poorer and more unequal. This is because the value of autarky depends strongly on how rich the economy is: with lower aggregate wealth, more agents are close to their borrowing limit and would suffer from the loss in TFP (and hence wages and employment) that follows a default. The effect of variance also goes through the value of autarky but through a different channel: when inequality is greater, defaults become a better way to redistribute.

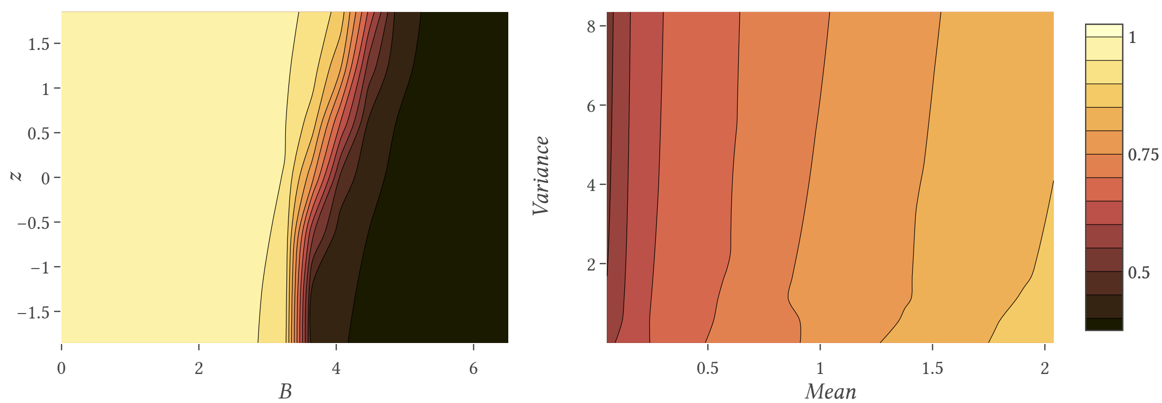


Figure 9: Price of Debt

### 6.2 Macroeconomic conditions

The unemployment rate is shown in Figure 10. Unemployment decreases with productivity and increases with government debt. The right panel shows that unemployment is related negatively to total wealth and positively to inequality.

### 6.3 Ergodic distributions

Figure 11 plots estimated densities for output and consumption along the simulated path. To compute these, I subtract the mean and divide by the standard deviation of each variable. The estimated densities reveal a left skew in consumption.

Figure 17 in the Appendix recomputes the density of normalized output and consumption conditioning on a positive default probability (but still normalizing as before). This exercise reveals that the extra mass on the left tail of consumption comes from crisis episodes. Figure 18 shows the fears of savers: conditional on default, output and (especially) consumption fall to extremely negative levels. Most of the mass of the consumption distribution is between 2 and 4 standard deviations below the unconditional mean.

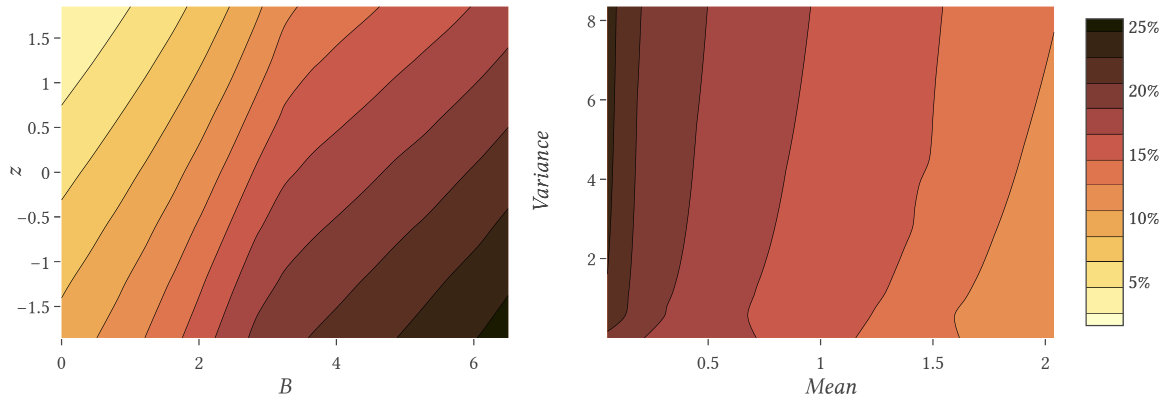


Figure 10: Unemployment

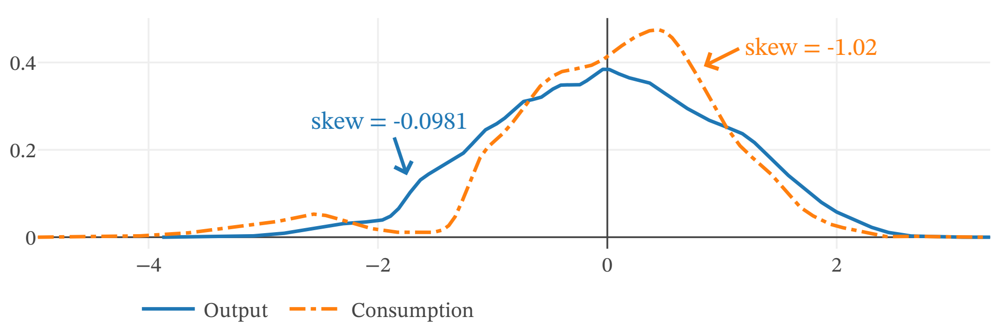


Figure 11: Ergodic Densities for Normalized Output and Consumption

# 7. Crises

Using the simulated series, I focus on episodes of crisis. I define an episode of high spreads as a period when the default probability has been above 6% for 11 quarters (to match the Spanish experience of 2010-mid 2012) but a default did not take place during that time. I choose a default probability threshold of 6% to match the Spanish output contraction of about 5% with respect to 2010. Figure 12 plots endogenous variables around episodes of high spreads, projecting about a year into past for context. Time is measured in years I condition on high spreads between periods -1.25 and 1.25.

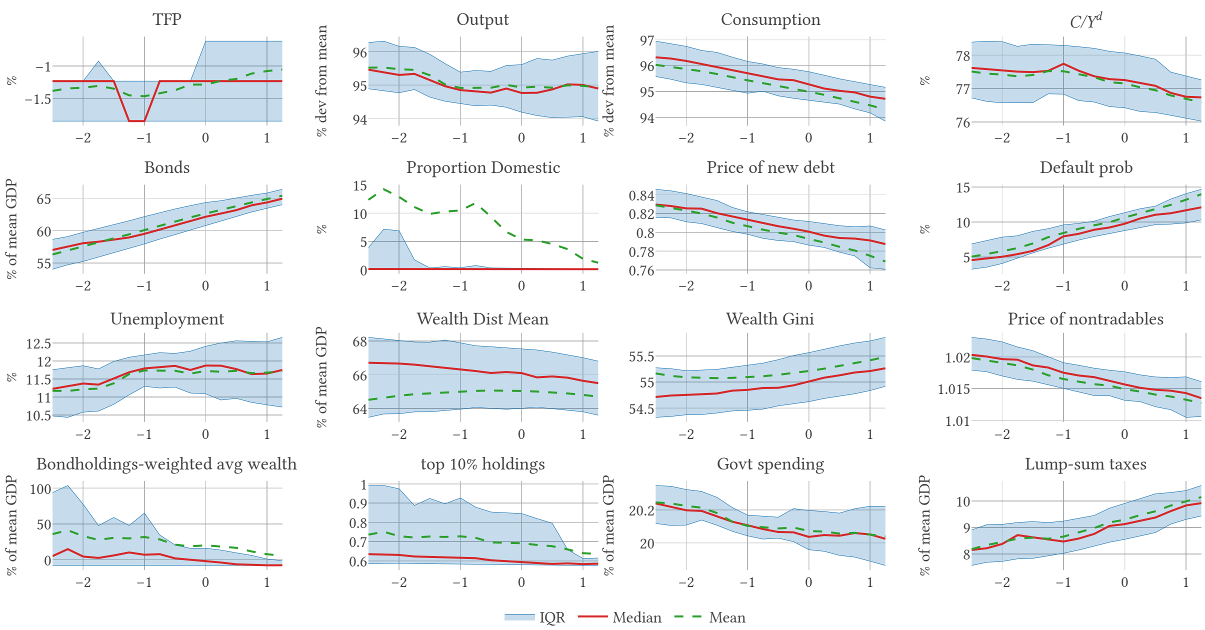


Figure 12: Times of High Spreads

TFP, output, and consumption are significantly below their normal times values. The government's finances deteriorate and lump-sum taxes increase and unemployment increases, further compromising tax collections. At the same time, inequality increases as measured by the Gini index for wealth. Total private wealth falls, especially when the default probability becomes higher at the end of the episode. Output hits a minimum of about 5% below its long-run mean, while TFP varies between -2% and -1% at the trough of the episode. As TFP really only affects the traded sector (which is about 25% of the economy), this is a sizeable amplification. Consumption also drops significantly below its long-run mean by about 5%. The average propensity to save also increases during the crisis: the typical household cuts consumption by about 1.5pp. of disposable income.

Figure 12 shows some of the dynamics at play. In the buildup to the crisis consumption falls both in levels and as a fraction of a measure of disposable income. There is also a significant fiscal contraction: there is a sustained increase in taxes along with a slight fall in government spending. Government spending is the result of two forces: in the fiscal rule, the government spends more when unemployment increases but also has the stabilize the level of debt. The level of government accumulates rapidly during the crisis as a consequence of high spreads.

Figure 13 shows in the same type of windows as before, but conditioning on a default happening at $t = 0$.

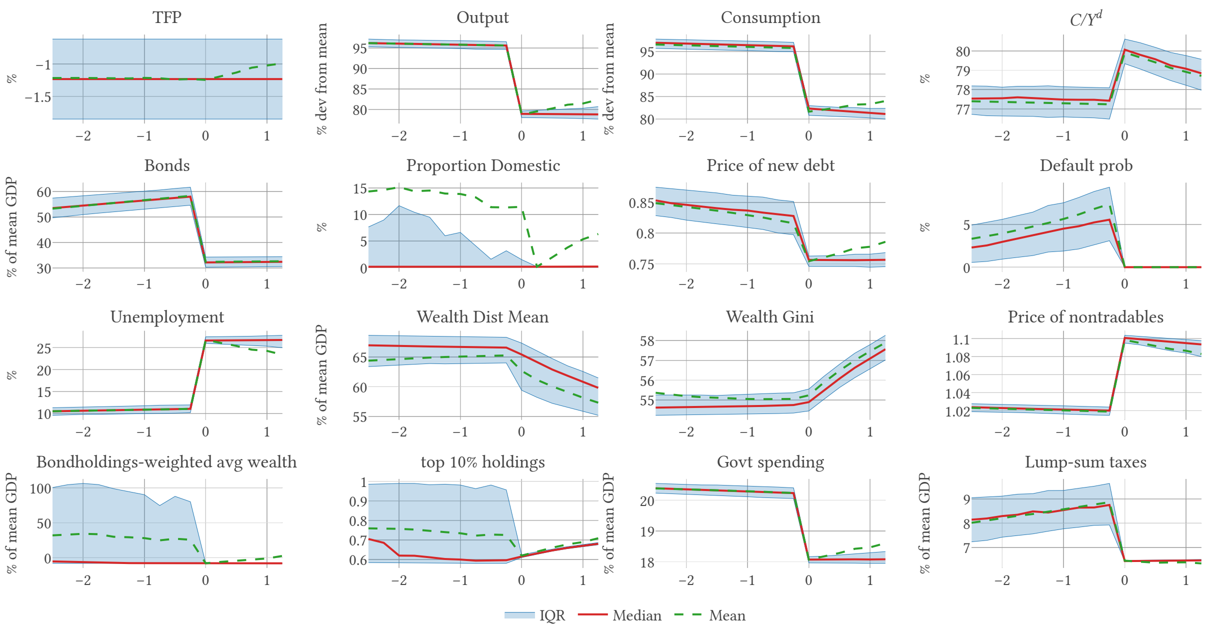


Figure 13: Outcomes around Defaults

Most variables exhibit a jump at the moment of default: output falls driven both directly and indirectly (through unemployment) by effective TFP, the level of debt reflects the haircut applied at default, etc. A striking feature is that the propensity to consume increases dramatically at default, as households start depleting their wealth buffers to weather this temporary (but large) fall in income. The mean of the wealth distribution starts drifts down during default. Inequality also shoots up as wealthy households are better able to withstand the shock. The rich also start to buy more of the defaulted debt.

## 7.1 *Amplification forces in the crisis*

Figure 14 takes the benchmark economy and its episodes of high spreads and compares it with an alternate economy in which the government followed a policy of always repaying the debt. The contractions in output and consumption are muted in the no-default economy. Fiscal policy also differs across the models. While government spending has a marginally sharper fall in the benchmark economy, lump-sum taxes (the residual in the government's budget constraint) have a markedly different evolution, increasing much less rapidly in the no-default economy, reflecting the destruction of fiscal space in the benchmark.

The differences in welfare are substantial. The specification of preferences is such that the value function equals the level of permanent consumption (a constant amount which would yield the same utility). At the height of a crisis, the average household would give up the equivalent of as much as 7% of consumption to be able to move to the economy with no default. At the onset of the crisis, this difference is markedly lower, standing at about 4%.

Comparing the benchmark economy with the version where default is not possible, crisis events are associated sharper contractions in output and especially in consumption. These events are also associated with an acceleration of debt as the price of debt deteriorates. Unemployment also diverges during crises, as the government is forced to implement what looks like a fiscal adjustment, by increasing lump-sum taxes and cutting on government spending.

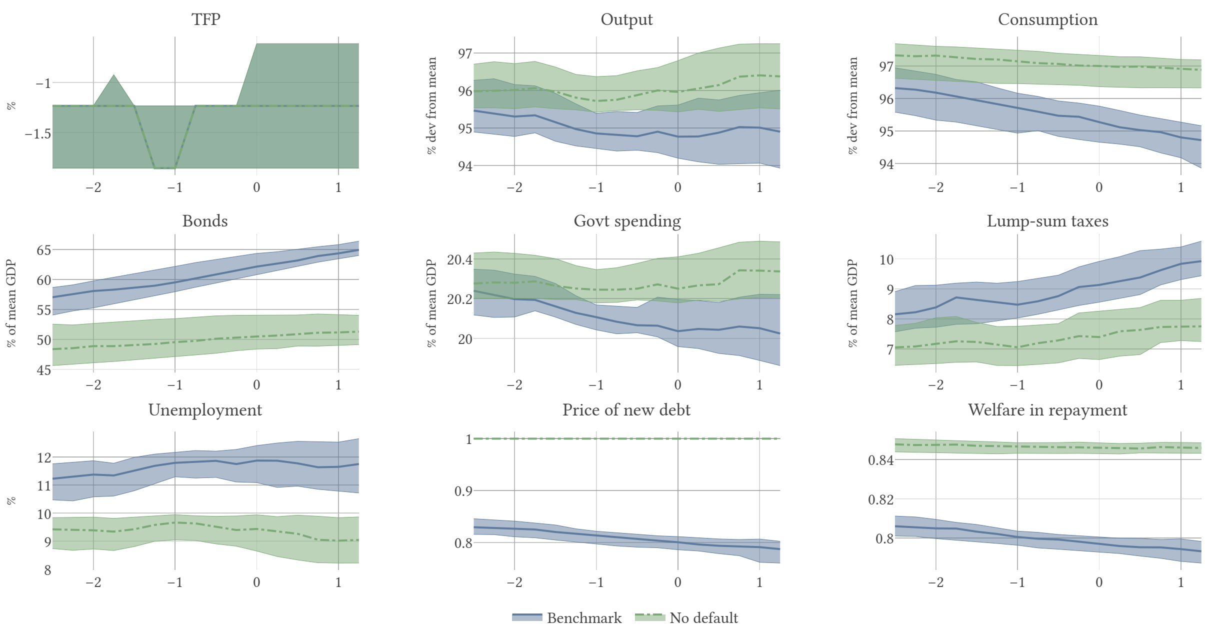


Figure 14: Crises

Note: Interquartile ranges shaded

Figure 14 shows that output falls to about 94.9% of its long-run mean in the benchmark economy. When default is not possible (and therefore not expected), the fundamental shocks of the economy, amplified by nominal rigidities, decrease output to about 96.4% of its long-run mean. This means that about 29% of the output contraction is due to sovereign risk. For consumption, these numbers are even starker: in the benchmark economy, aggregate consumption stands at about 94.7% of its long-run mean, compared to 96.9% for the no-default economy. Sovereign risk then explains 41% of the fall in private consumption.

Table 5 shows statistics from simulation of the benchmark model alongside some comparison models. Statistics from the no-default model are on the rightmost column. Between them are two intermediate models. The first comparison model I consider is one in which defaults carry no aggregate losses. By setting $\Delta$ to zero, TFP in case of default does not fall and neither does expected income when sovereign risk increases. In the second one, defaults do not have distributional consequences: to achieve this I solve the model forcing domestic households to save only in the risk-free asset. In both cases, I keep the government's default policy from the benchmark and allow agents to re-optimize and prices to re-adjust.

Table 5 also reveals that, averaging across periods, the average household obtains a gain of about 1.76% of permanent consumption from moving to the economy without default.

| **Moment** | **Benchmark** | $\Delta = 0$ | **No dom. holdings** | **No default** |
|---|---|---|---|---|
| AR(1) coef $\log(Y_t)$ | 0.976 | 0.973 | 0.976 | 0.979 |
| Std coef $\log(Y_t)$ | 0.0168 | 0.00665 | 0.0171 | 0.00561 |
| AR(1) coef $\log(C_t)$ | 0.976 | 0.983 | 0.979 | 0.998 |
| Std coef $\log(C_t)$ | 0.0141 | 0.00404 | 0.0135 | 0.00107 |
| AR(1) coef spread | 0.983 | 0.965 | 0.977 | 1 |
| Std coef spread | 0.0161 | 0.0521 | 0.0199 | 0 |
| Avg Debt-to-GDP | 31.6% | 38% | 32.7% | 31.7% |
| Std Debt-to-GDP | 12.8% | 9.15% | 13.2% | 11.8% |
| Avg unemployment | 7.01% | 6.59% | 7.32% | 5.63% |
| Std unemployment | 5.83% | 2.42% | 6.06% | 2.29% |
| Median dom holdings | 38.5% | 0.723% | 0% | 184% |
| Avg wealth-to-GDP | 63.8% | 56.3% | 64.6% | 56.4% |
| Avg wealth Gini | 57.2% | 60.5% | 56.7% | 60.5% |
| Default frequency | 1.11% | 2.94% | 1.27% | 0% |
| Welfare in repayment | 0.854 | 0.853 | 0.84 | 0.869 |

Table 5: Models

The volatility of output is two-thirds lower in the no-default case, while that of consumption shrinks by an order of magnitude. The presence of sovereign risk has an enormous impact on the ability of households to smooth consumption. The intermediate comparison models shed more light into the mechanism. In the version without TFP costs of default, the volatility of consumption falls to about a third of its value in the benchmark, suggesting an important role for the high correlation between income and repayment.

An interesting point is that banning domestic holdings of sovereign does not seem to affect the economy much. As long as there are costs of default, households anticipate losses in income when sovereign risk is present. This is enough to create volatility in consumption.

### 7.2 The distributional impact of crises

Figure 15 plots the welfare costs of sovereign risk across the wealth distribution. It reveals substantial heterogeneity in the costs of sovereign risk. The gains from removing the possibility of default are decreasing in wealth and range from 10.2% (7.3%) for the bottom 10% (25%) of the distribution to 5.6% (6.2%) for the top 10% (25%). The median household would pay 6.6% of permanent consumption to abolish default, compared to about 6.8% for the average, which enters the government's objective function.

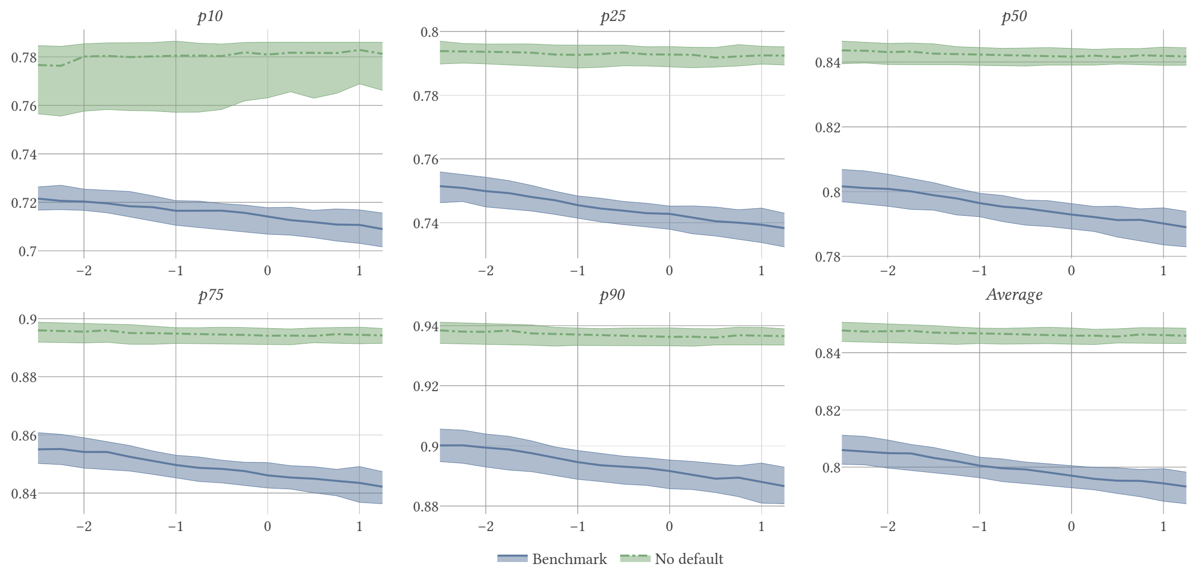


Figure 15: Value functions in crises

Note: Interquartile ranges shaded

## 8. Concluding Remarks

Inspired by events in the Eurozone crisis, this paper analyzes a model in which households' consumption demand is negatively affected by the presence of sovereign risk. The mechanisms in the model generate substantial amplification of underlying shocks even if the risk of default does not materialize.

The amplification mechanism relies on the precautionary motives of households, which are magnified by sovereign risk. Sovereign risk creates endogenous shifts in demand conditions which exacerbate the equilibrium volatility of aggregate consumption. I find large welfare costs of sovereign risk, which range from about 1.6% of permanent consumption in normal times to as much as 7% at the height of a crisis.

Households increase their savings in response to sovereign risk because they anticipate income losses and redistribution in case of default. The anticipation of income losses in case of default explains most of the amplification. However, this effect interacts substantially with the anticipation of redistribution.

While I calibrate the model to Spain, the mechanism can help explain patterns in emerging-market business cycles, which also exhibit sovereign risk as a feature. Both the relative volatility of consumption to output and the volatility of output itself are typical calibration targets in the sovereign debt literature when applied to emerging-market economies. The setup presented here offers a more complete explanation of these phenomena by explicitly considering the saving behavior of private agents as well as the interest rate they face. The amplification mechanism and the welfare costs of sovereign risk are both natural consequences of this more granular description when private agents are net savers. This is the case of Spain in the 2000s but also of salient episodes in emerging markets, when the private sector's international investment position is positive even as the government is in debt.

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# A. Appendix: Solution Details

## *A.1 The household's problem*

To make the problem more tractable, I rewrite the controls in a way that eliminates the budget constraint and replaces it with inequalities that are easier to handle. Let

$$\begin{aligned} s(a', b', \mathrm{S}) &= \frac{q^h(\mathrm{S})a' + q^g(\mathrm{S})b'}{q^h(\mathrm{S})} && \geq \bar{a} \\ \theta(a', b') &= \frac{a' - \bar{a}}{s - \bar{a}} && \in [0, 1] \end{aligned}$$

where the constraint set (now a rectangle) is highlighted in red and

$$\begin{aligned} a'(s, \theta) &= \bar{a} + \theta_a(s - \bar{a}) \\ b'(s, \theta, \mathrm{S}) &= (1 - \theta_a)(s - \bar{a})\frac{q^h(\mathrm{S})}{q^g(\mathrm{S})} \end{aligned}$$

I further decompose the household's problem into two value functions, $v$ and $w$. The first value function reflects the consumption-savings decision, while the second value function focuses on the portfolio allocation of those savings. We have

$$\begin{aligned} v(\omega, \epsilon, \mathrm{S}) = \max_{s,c} & \left((1 - \beta)c^{\frac{\psi-1}{\psi}} + \beta w(s, \epsilon, \mathrm{S})^{\frac{\psi-1}{\psi}}\right)^{\frac{\psi}{\psi-1}} \\ \text{subject to } & p_C(p_N)c + q^h(\mathrm{S})s = \omega + \ell(\mathrm{S}, p_N)\epsilon - T(\mathrm{S}, p_N) \\ & s \geq \bar{a} \end{aligned}$$

and

$$\begin{aligned} w(s, \epsilon, \mathrm{S}) = \max_{\theta} & \, \mathbb{E}\left[\left(v(a' + R_b(\mathrm{S}')b', \epsilon', \mathrm{S}')\right)^{1-\gamma} | \omega, \epsilon, \mathrm{S}\right]^{\frac{1}{1-\gamma}} \\ \text{subject to } & R_b(\mathrm{S}) = 1_{(\zeta=1)}\kappa + (1 - \rho)q^g(\mathrm{S}) \\ & \theta \in [0, 1] \\ & a' = a'(s, \theta) \\ & b' = b'(s, \theta, \mathrm{S}) \end{aligned}$$

What makes this transformation so tractable is that $\omega$ and $s$ naturally belong in the same domain, so one can use the same grid for both.

## *A.2 Solution method*

The algorithm follows closely the definition of equilibrium: to solve for an equilibrium, I solve a series of nested problems. Given government policies, I find a competitive equilibrium by finding functions of the aggregate state that describe the aggregates in the economy, in such a way that they are consistent with the household problem and policy functions.

**Algorithm** Given a policy for the government, I

1. Guess a law of motion for the distribution.
2. For each state S
   (a) Compute $q^g(\mathrm{S})$ from the foreigners' sdf (9).
   (b) Guess a relative price of nontradables $p_N$
      - Get the wage rate $w$ as well as total labor demand $L^d$ and profits of the firms $\Pi$.
      - Compute lump-sum taxes $T$ from gov't budget constraint (with $\tau wL$ in hand).
      - Solve the household's problem at prices $w$, $p_N$, profits $\Pi$, and transfers $T$.
      - Check market clearing (11) for nontraded goods.

   (c) Iterate on the function $p_N(\mathrm{S})$ to convergence
3. Iterate on the law of motion for the distribution using the households' policy functions.

Finally, I update the government's policy according to (15) and iterate until a policy that respects it is found.

### *A.3 More model results*


FIGURE 16: A SIMULATED PATH

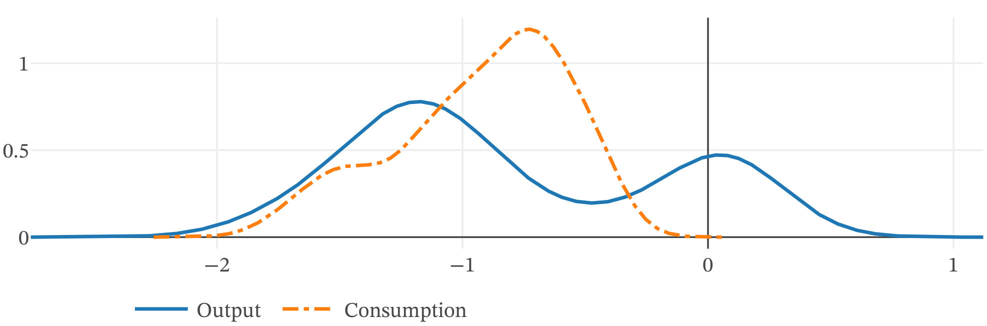



FIGURE 17: DENSITIES FOR OUTPUT AND CONSUMPTION DURING CRISES

Densities conditional on a default probability above 15%

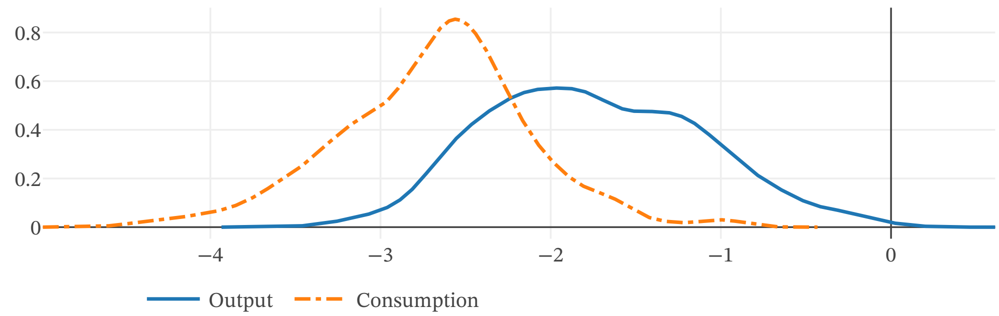



Figure 18: Ergodic Densities for Output and Consumption during Defaults

## B. Appendix: Evidence

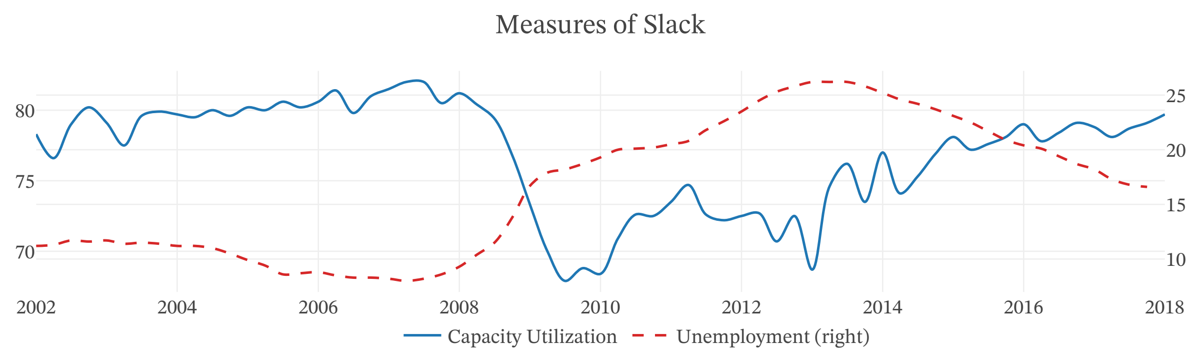



Figure 19: Slack in the Spanish economy

Source: Eurostat

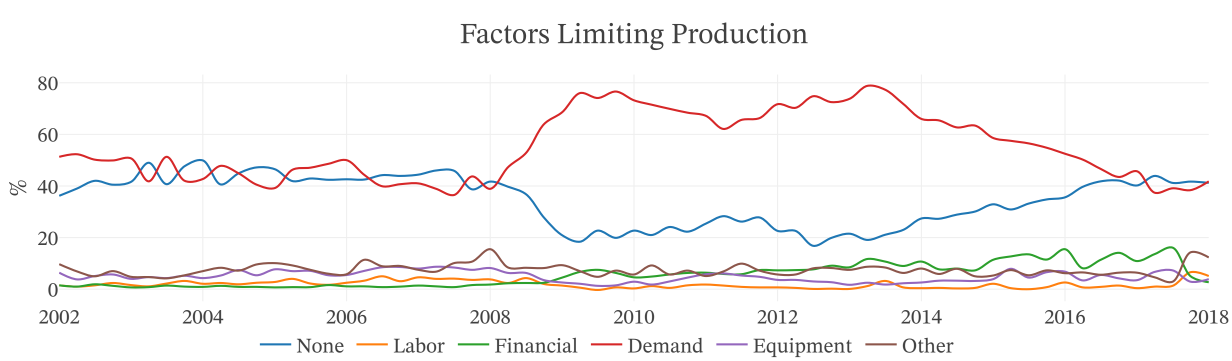
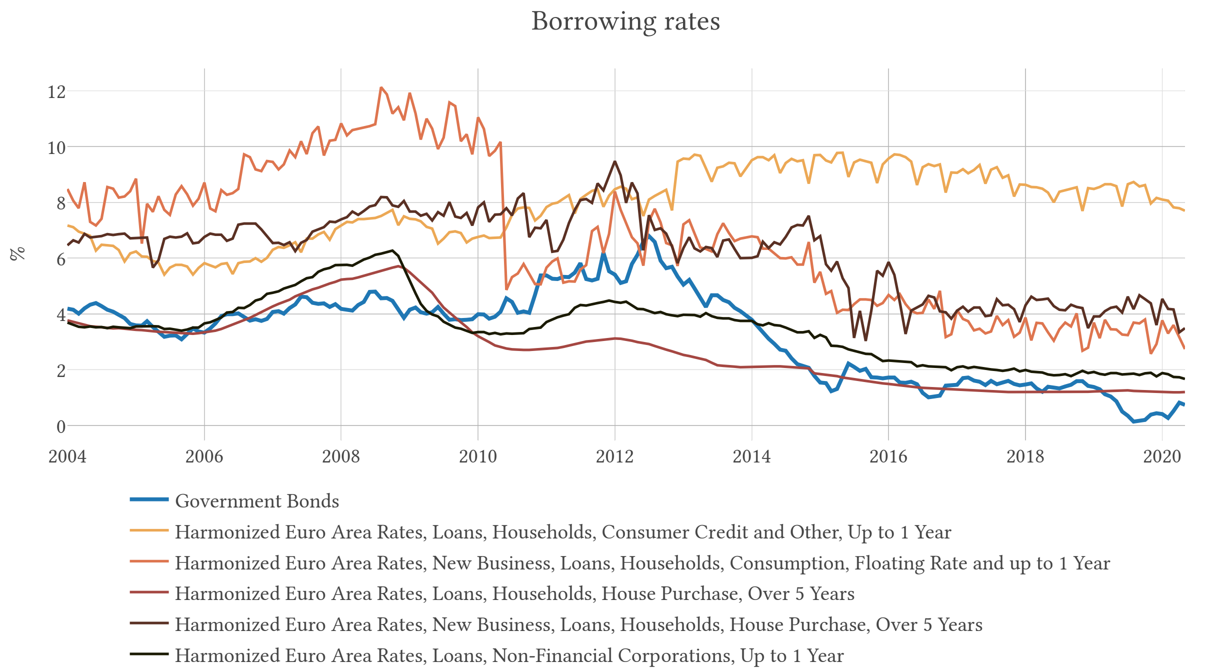



Figure 20: Spanish Firms' Self-reported Limits to Production

Source: Eurostat



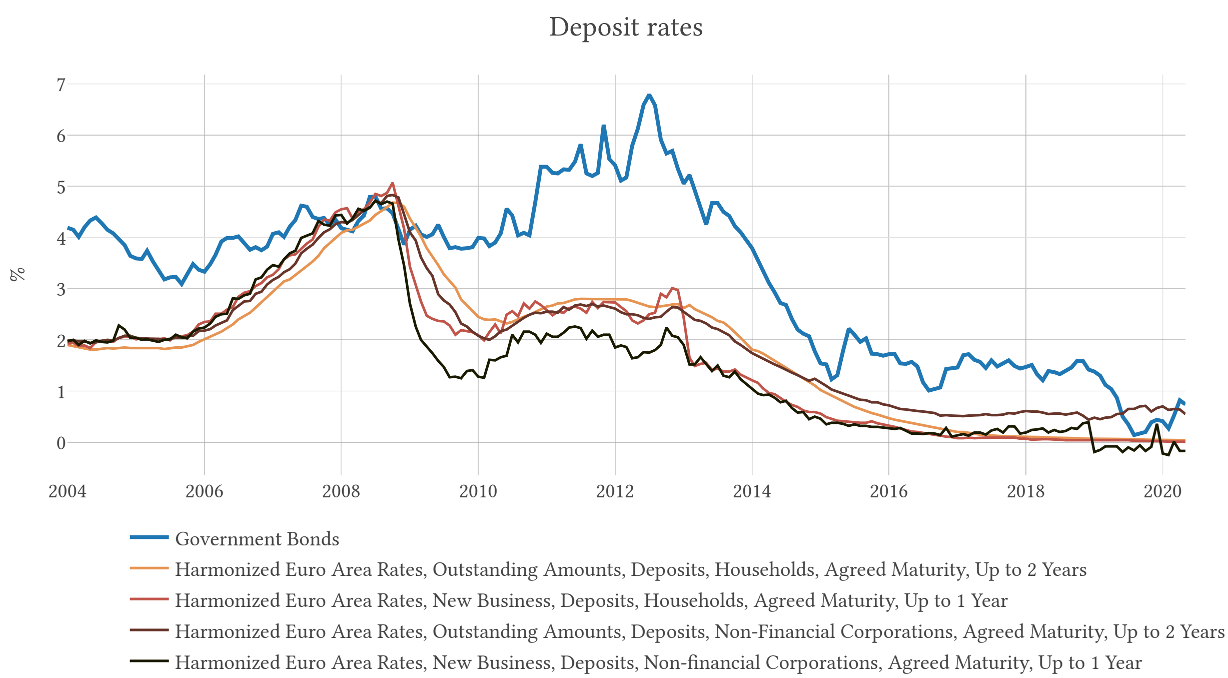




Figure 21: Interest rates in Spain

Source: IMF IFS

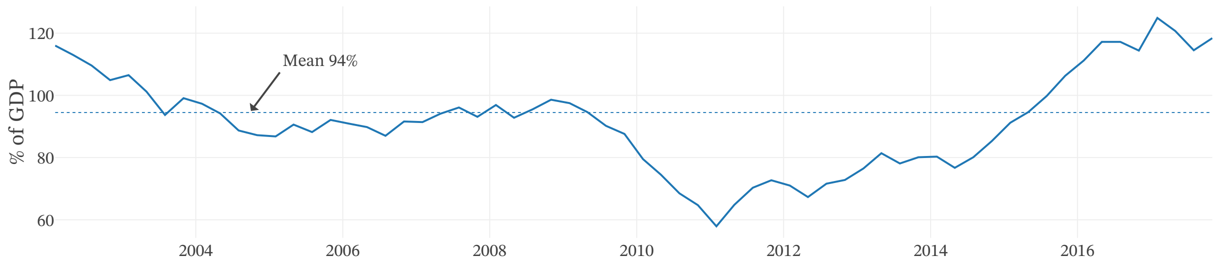



Figure 22: Net Worth of Spanish Households

Source: Eurostat

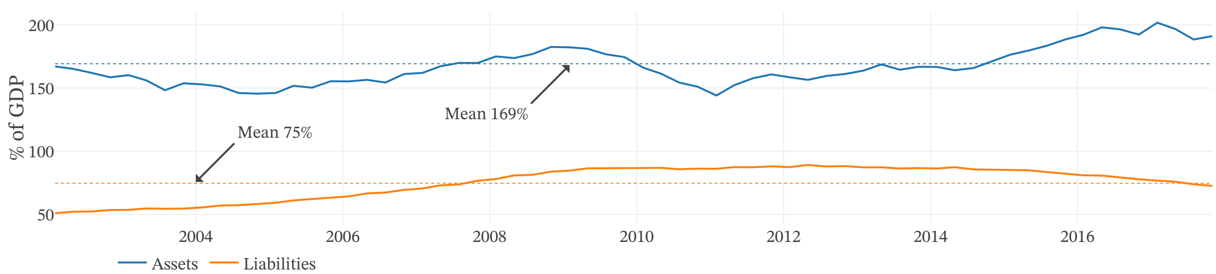



Figure 23: Net Worth of Spanish Households

Source: Eurostat

| | $\sigma(C)$ | $\sigma(Y)$ | $\sigma(C)/\sigma(Y)$ | $\sigma(C)/\sigma(Y)$ (AG) |
|---|---|---|---|---|
| Austria | 0.716 | 0.782 | 0.916 | 0.870 |
| Belgium | 0.556 | 0.795 | 0.700 | 0.810 |
| Denmark | 1.047 | 1.178 | 0.889 | 1.190 |
| Finland | 1.278 | 1.957 | 0.653 | 0.940 |
| France | 0.780 | 0.773 | 1.009 | – |
| Germany | 0.692 | 0.867 | 0.799 | – |
| Ireland | 3.140 | 3.680 | 0.853 | – |
| Italy | 1.165 | 0.978 | 1.191 | – |
| Netherlands | 1.726 | 1.244 | 1.388 | 1.070 |
| Portugal | 1.827 | 1.576 | 1.160 | 1.020 |
| Spain | 1.901 | 1.396 | 1.362 | 1.110 |

Series logged and HP-filtered with $\lambda = 1600$. Std deviations in %.

Table 6: The Cycle is the Trend

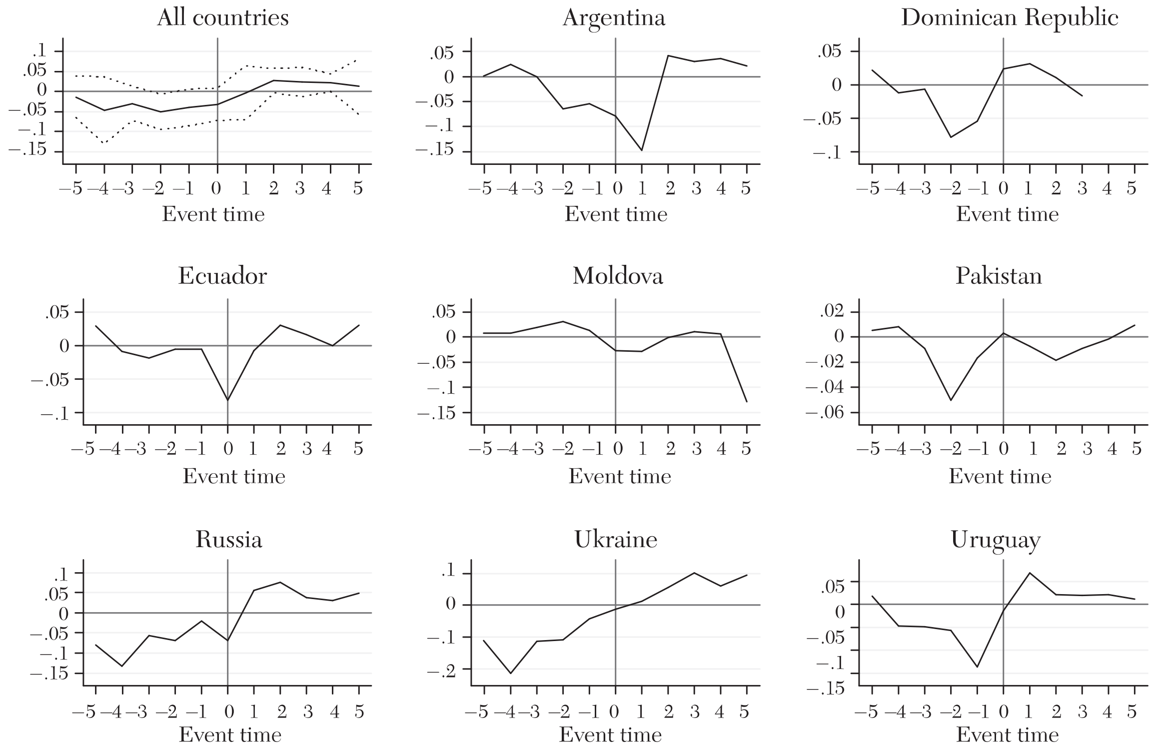



Figure 24: Defaults and output growth

Source: Panizza, Sturzenegger, and Zettelmeyer (2009)

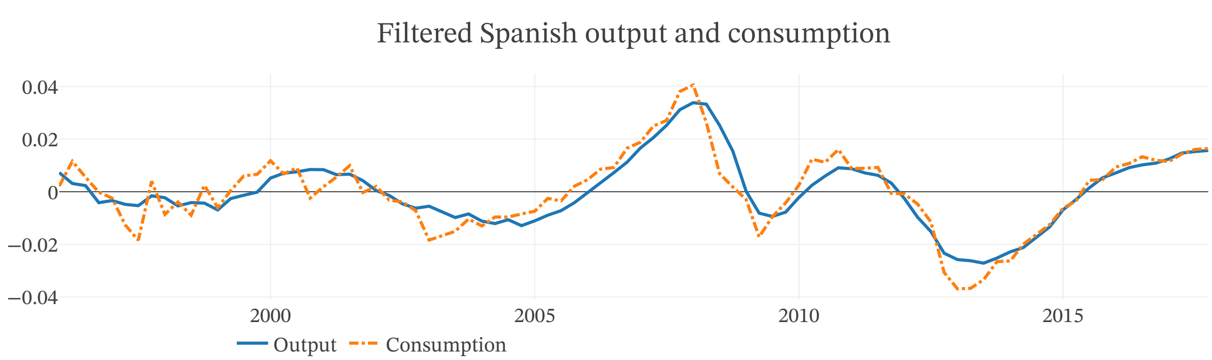



Figure 25: Spanish detrended output and consumption in the 2000s

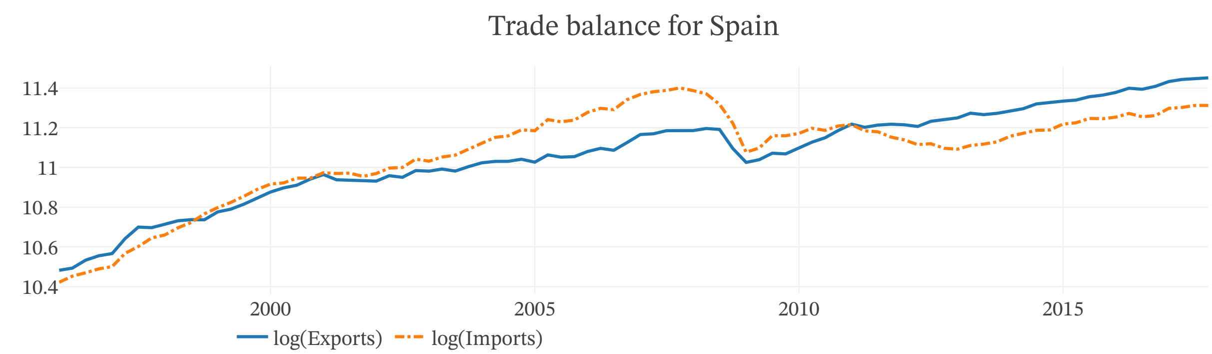



Figure 26: Spanish trade balance in the 2000s

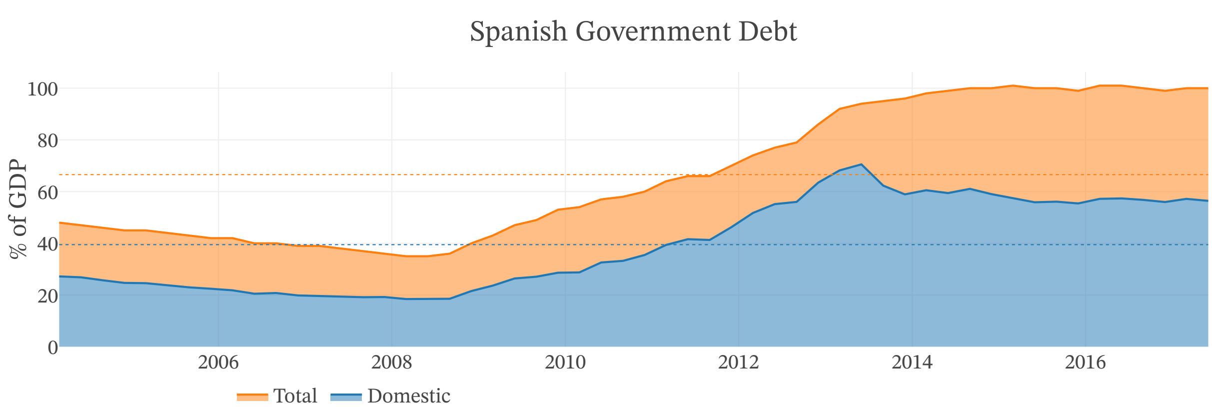



Figure 27: Composition of Spanish Debt

Dotted lines show average levels

## C. Appendix: Data sources

| Variable | Source | Coverage | Code |
|---|---|---|---|
| GDP | Eurostat | 1996Q1:2017Q4 | namq_10_gdp |
| HH Consumption | OECD | 1996Q1:2017Q4 | |
| Imports | Eurostat | 1996Q1:2017Q4 | namq_10_gdp |
| Exports | Eurostat | 1996Q1:2017Q4 | namq_10_gdp |
| Government Spending | Eurostat | 1996Q1:2017Q4 | namq_10_gdp |
| Government Debt | Eurostat | 1996Q1:2017Q4 | gov_10q_ggdebt |
| Interest Rate on Gov't Bonds | Eurostat | 1996Q1:2017Q4 | irt_lt_mcby_q |
| Unemployment | Eurostat | 1996Q1:2017Q4 | une_rt_q |
| Spanish HHs' wealth | Eurostat | 1998Q4:2018Q2 | nasq_10_f_bs |
| Spain's Debt Composition | Banco de España | 2004Q1:2017Q4 | |

Table 7: Data Sources