Credibility Dynamics and Disinflation Plans*

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Abstract

We study the optimal design of a disinflation plan by a planner who lacks commitment. Having announced a plan, the Central banker faces a tradeoff between surprise inflation and building reputation, defined as the private sector's belief that the Central bank is committed to the plan. Some plans are harder to sustain: the planner recognizes that paving out future grounds with temptation leads the way for a negative drift of reputation in equilibrium. Plans that create low inflationary expectations balance promises of low inflation with dynamic incentives that make them credible. When announcing the disinflation plan, the planner takes into account these anticipated interactions. We find that a gradual disinflation is preferred despite the absence of inflation inertia in the private economy.

JEL Classification: E52, C73

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two extreme assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. While such promises typically do not actually constrain future choices, they are sometimes able to shift expectations. Moreover, they are made with the purpose of shifting expectations.

In this paper we develop a rational-expectations theory of government credibility and apply it to policy design questions. Our notion of credibility is akin to the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model, the government could be rational or one of many

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possible 'behavioral', 'obstinate' types. These behavioral types are described by a policy that they follow stubbornly. The government's type is unobserved by the private sector, who instead must form beliefs about it. This inference is nontrivial because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government announces its policy 'targets' and is then free to choose policy. If the government happened to be behavioral, it announces exactly what it will then implement. This creates an incentive ex-post for the rational type to stay close to any announced targets and to earn a reputation for being committed to the targets. This structure creates an incentive ex-ante for the rational type to think very hard about which targets to announce.

A natural definition of the government's reputation is the private sector's belief that the government is *not* rational. We refer to the total, ex-ante probability of the behavioral types as initial reputation. Clearly, if initial reputation is zero we recover the case of full discretion. On the other hand, we feel that the model would lose interest if it were to rely on a very high initial reputation. This is why we choose to focus on the case when initial reputation is arbitrarily small. While the solution of the model requires us to think about all levels of reputation, we view this limiting case as a sensible refinement in the game between the government and the private sector.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a monetary autority sets inflation subject to an expectations-augmented Phillips curve. The monetary authority dislikes inflation but faces the constant temptation to engineer surprise inflation which would deliver output close to potential. We introduce these features by considering the standard New Keynesian setup for the private economy, where we also eliminate the IS curve by letting the government directly choose inflation.

A key assumption we introduce is that the government exerts imperfect control over current inflation. This assumption masks the government's actual choice of policy: the private sector understands that realized inflation is an imperfect signal of intended inflation and applies Bayes' rule to infer the government's type. As a consequence, reputation becomes a continuous variable. This feature creates a tradeoff for the government. Missing its target by more makes it expect to create a larger boom today at the expense of a larger expected loss of reputation.

When designing policy, the planner takes into account its own expected future behavior. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Keeping their reputation turns out to be a powerful disciplining force for future governments. Crucially, the value of reputation depends on the plan in place. Plans differ in the outcomes they intend to deliver and in how closely are following governments expected to follow them. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

We obtain two main results. One is that the planner chooses a policy where inflation starts high and diminishes gradually, except maybe when initial reputation is very high. Seeing this policy, an outside observer might conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Rather, as it turns out, a plan that promises decreasing inflation is easier to keep. A decreasing path for inflation boosts the gains from increasing reputation, which leads future governments to be more inclined to respect the plan.

Our second result concerns the limit as initial reputation becomes arbitrarily small. When reputation is zero, the only (Markov) equilibrium is a repetition of the static Nash with high inflation and output at the natural level. We find a discontinuity at zero reputation: as initial reputation vanishes, the optimal plan converges to

a plan that is not a repetition of the stage game Nash equilibrium. Moreover, along the zero-reputation limit, optimal plans retain their gradualist property.

Discussion of the Literature We contribute to a long literature dealing with commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

We build on models such as Barro (1986), Backus and Driffill (1985), and more recently Sleet and Yeltekin (2007) who introduce reputation and behavioral types in models of monetary policy. The key departure from that literature is our introduction of imperfect control. With perfect control, any deviation by the government is detected by the private sector: on the equilibrium path, reputation can only stay where it is or become zero. Imperfect control complicates the private sector's inference and enables the tradeoffs that shape our optimal plans.

A related literature looks at subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). However, this notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001) generally generates a large set of equilibria, which limits the theory's predictions.

There is also a large literature that makes use of imperfect control in the same way we do, along with uncertainty about the preferences of the planner. Examples include Phelan (2006), Cukierman and Meltzer (1986), Faust and Svensson (2001), among many others. We view our model with behavioral types as more directly suited to address the issue of announcements about future policy that motivates us.

Some recent attempts by King, Lu, and Pastén (2008, 2016), or Lu (2013) share some of our ingredients. They consider the same model with imperfect control of inflation and behavioral types that we do, except that in these papers the planner has commitment power. The tradeoff becomes that the planner wants to make it clear that it is the rational type but also deliver good outcomes. If the behavioral type resembles the Ramsey plan, these objectives create a tension. Finally, in the rational limit that we are interested in, the tension between delivering the Ramsey plan and separating from a behavioral type dissipates. These plans then simply converge to the Ramsey outcome.

On the theoretical side, Faingold and Sannikov (2011) consider a similar model to ours, in the context of a monopolist selling to competitive buyers. They also make use of behavioral types and imperfect control. They find that in the continuous-time limit the equilibrium is unique and Markovian in reputation, which informs our strategy of looking for a Markovian equilibrium in our model. However, since they only consider static behavioral types, they cannot address the issue of gradualist policy and the incentives it generates, which are at the core of our argument.

Layout The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section 3. Section 4 lays out our main results. Finally, Section 5 concludes.

2. Model

We consider a government who dislikes inflation and deviations of output from a target according to a loss function

$$L_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(y^\star - y_t)^2 + \gamma \pi_t^2 \right] \right] \tag{1}$$

while a Phillips curve relates current output to current and expected inflation

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] \tag{2}$$

We further assume that the government has imperfect control over inflation so that

$$\pi_t = g_t + \sigma \epsilon_t \tag{3}$$

where the government controls g_t at time t and $\epsilon \overset{iid}{\sim} \mathcal{N}(0, 1)$.

2.1 Reputation

We introduce reputation by considering the possibility of behavioral types for the government. Each behavioral type is committed to a particular strategy. For convenience, we identify each possible behavioral type by the strategy it follows. Types are indexed by a set $\mathcal{C}$. For $c \in \mathcal{C}$, a government of behavioral type c is committed to an *inflation plan* $(a_t^c)_{t=0}^{\infty}$. An inflation plan consists of inflation announcements for each t.

We assume that the government is rational with probability z. A probability defined over $\mathcal{C}$ with density ν describes the distribution of possible behavioral types.

2.2 Timing of play

At time 0 an announcement $a = (a_t)_{t=0}^{\infty}$ of *inflation targets* takes place. The announcement includes targets a_t for all time periods into the future.

If the government happens to be behavioral of type $c \in \mathcal{C}$, it announces c for sure. However, the rational type of the government chooses an announcement r, possibly $r \in \mathcal{C}$. The government understands that announcing $r \notin \mathcal{C}$ reveals rationality. If $r \in \mathcal{C}$, then the probability that the government is rational or behavioral of type r equals 1.

At time $t \geq 0$, the government sets inflation. If the government happens to be behavioral of type c, it sets $g_t = a_t^c$. The rational type may instead choose g_t strategically. Actual inflation is noisy so the private sector has to apply Bayes' rule to update beliefs about the government's type.

2.3 Beliefs

After the initial announcement, the private sector applies Bayes' rule to update beliefs about the government's type. By our discussion above, if an announcement $c \in \mathcal{C}$ has been made, the government can only be rational or of type c.

Furthermore, suppose that in equilibrium the rational type announces c with density $\mu(c)$. Bayes' rule states that initial reputation (for being behavioral of type c) is the ratio of the probability of the government actually being of type c (who announces c with probability 1) over the total probability of announcing c (which also includes announcements by the rational type). That is,

$$p_0(c; z, \mu) = \frac{z\nu(c)}{z\nu(c) + (1 - z)\mu(c)} \tag{4}$$

At time t, the private sector's posterior of the government being of behavioral type c is formed by applying Bayes' rule to the private sector's information. Suppose that inflation π_t is realized at time t. If the government is behavioral of type c, then it must have chosen $g_t = a_t^c$ and the current shock must have been $\epsilon_t = \pi_t - a_t^c$, which happens with density $f_\epsilon(\pi_t - a_t^c)$. If on the other hand the government was rational, it chose $g_t = g_t^\star$, the

rational type's strategy, which means that the shock must have been $\epsilon_t = \pi_t - g_t^\star$. Therefore, updating from a prior belief of p_t, we have that

$$p_{t+1} = \frac{p_t \cdot f_\epsilon(\pi_t - a_t)}{p_t \cdot f_\epsilon(\pi_t - a_t) + (1 - p_t) \cdot f_\epsilon(\pi_t - g_t^\star)}$$

where $g_t^\star$ is the (conjectured) choice of inflation by the rational type of the central bank. In a rational-expectations equilibrium, this must be the rational government's *actual* choice of inflation.

It is useful to rewrite this condition as

$$p_{t+1} = p_t + p_t(1 - p_t)\frac{f_\epsilon(\pi_t - a_t) - f_\epsilon(\pi_t - g_t^\star)}{p_t f_\epsilon(\pi_t - a_t) + (1 - p_t) f_\epsilon(\pi_t - g_t^\star)} \tag{5}$$

which makes it evident that reputation moves when (i) initial reputation is far away from 0 and 1 and (ii) when realized inflation is closer to either the target or the rational type's strategy. Only if a_t is far away from $g_t^\star$ can actual inflation fall closer to one than to the other. Some of this intuition is leveraged heavily later on.

2.4 *The set of behavioral types*

We parametrize the set $\mathcal{C}$ of allowed behavioral types. We assume that behavioral type c's inflation plan is defined by three parameters (a_0, ω, a_∞) so that

$$a_t^c = (a_0 - a_\infty)\, e^{-\omega t} + a_\infty$$

This parametrization makes $\mathcal{C}$ finitely-dimensional but also allows us to write each plan recursively. For each $c \in \mathcal{C}$, $a_{t+1}^c = a_\infty + e^{-\omega}\,(a_t^c - a_\infty) = \phi_c(a_t)$.

Figure 1 illustrates some possible paths. Paths start at a_0 and converge towards a_∞ with a exponential decay rate of ω. The set $\mathcal{C}$ contains constant, decreasing, and increasing paths, which obtain by appropriately setting a_0 and a_∞.

2.5 *Bellman equations after an announcement*

Given an announcement c, the problem of the rational type is to choose mean inflation g_t in period t to maximize (1) subject to (2), (3), and (5). The time-t government chooses taking as given its reputation p_t, its future strategy, and the private sector's expectations about the behavioral and rational types' choices.

At time t, the private sector expects the behavioral type to choose $g_t = a_t^c$. Let $g_t^\star$ denote the private sector's expectations of the rational type's choice. We focus on Markovian strategies with $g_t^\star = g^\star(p_t, a_t^c)$. This allows us to write the rational government's problem recursively as

$$\begin{aligned}
\mathcal{L}^c(p, a) = \min_g\ & \mathbb{E}\left[(y^\star - y)^2 + \gamma\pi^2 + \beta\mathcal{L}^c(p', \phi_c(a))\right] \\
\text{subject to } & \pi = g + \epsilon \\
& \pi = \kappa y + \beta\left[p'\phi_c(a) + (1 - p')g^\star(p', \phi_c(a))\right] \\
& p' = p + p(1 - p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p, a))}{p f_\epsilon(\pi - a) + (1 - p) f_\epsilon(\pi - g^\star(p, a))}
\end{aligned} \tag{6}$$

Problem (6) makes it clear that the government best-responds to the private sector current beliefs $g^\star(p, a)$. In the Phillips curve, expected inflation is a weighted average between $\phi_c(a)$, the strategy of the behavioral type, and $g^\star(p', \phi_c(a))$, the conjectured choice of the rational type in next period's state. This gives the government a degree of freedom: it can influence expected inflation by affecting its reputation.

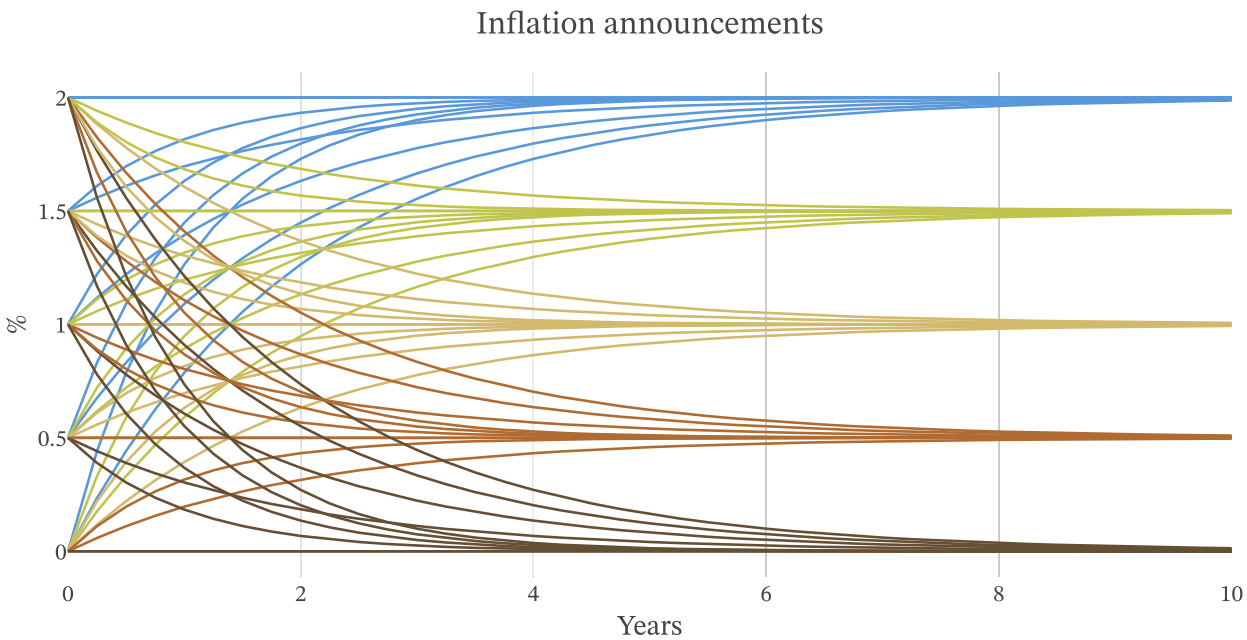

Figure 1: Possible behavioral types' announcements

At low levels of p, the Phillips curve puts most of the weight on the expected choice of the rational type. Therefore, at low levels of p, the government affects inflationary expectations mostly through $g^\star(p', \phi_c(a))$. If future governments are expected to value their reputation and choose $g^\star(p, a)$ close to a when $p > 0$, the current government has an incentive not to let its reputation go to zero and, therefore, choose the current $g^\star$ close to a.

3. Expectations and Equilibrium

A solution to (6) describes the government's choices $g(p, a)$ as a function of the private sector's expectations $g^\star$. Our equilibrium definition makes it clear that rational expectations requires finding a fixed point of that function.

Definition Given an announcement $c \in \mathcal{C}$, a *continuation equilibrium* consists of a loss function $\mathcal{L}^c : [0, 1] \times \mathcal{A} \to \mathbb{R}$ and a policy function $g_c^\star : [0, 1] \times \mathcal{A} \to \mathbb{R}$ such that

1. The loss function $\mathcal{L}^c$ solves the government's Bellman equation (6) taking as given expectations $g_c^\star$
2. $g_c^\star$ is the policy function that corresponds to the solution of (6)

A useful property of continuation equilibria follows from close observation of problem (6): given the decay and asymptote parameters, it is equivalent to start the plan at a different initial announcement a and to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Observation Suppose $(\mathcal{L}, g^\star)$ is a continuation equilibrium for announcement $c = (a_0, a_\infty, \omega) \in \mathcal{C}$. Then for any b_0, the same pair $(\mathcal{L}, g^\star)$ is a continuation equilibrium for plan $c' = (b_0, a_\infty, \omega)$.

Lemma 1. *In any continuation equilibrium, the rational type's reputation is a supermartingale:*

$$\mathbb{E}\left[p_{t+1} \mid \textit{rational}, \mathcal{F}_t\right] \leq p_t$$

That is, the planner cannot design a policy that generates expected reputational gains.

See Appendix A.1 for a proof. The idea is that, conditional on a rational government, either $g^\star(p, a) = a$, in which case $p' = p$ a.s., or $g^\star(p, a) \neq a$, in which case π is a signal centered away from a, which is revealing on average.

One reason why one might think a planner prefers a gradual disinflation is that gradualism allows the planner to promise easy things first, accumulate credibility, and then be able to promise more difficult things. Lemma 1 says that the planner cannot strategize in this way. It cannot design its plan in a way that makes it expect to increase reputation over time. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Definition Given an initial reputation z, an *equilibrium* is a distribution μ_z over $\mathcal{C}$ along with continuation equilibria $\{\mathcal{L}^c, g_c^\star\}_{c\in\mathcal{C}}$ and initial reputation $p_0 : \mathcal{C} \to [0, 1]$ such that

1. Initial reputation is set according to Bayes' rule (4), given the distribution μ_z.
2. The distribution of mimicked types μ_z minimizes the initial reputation-adjusted loss function

$$\mathcal{L}_r^\star(\mu_z, z) = \int_{\mathcal{C}} \mathcal{L}^c(p_0(c), a_0(c))d\mu_z(c)$$

   taking as given the initial reputation function p_0.

Notice that, as a consequence of 2, in an equilibrium the planner is indifferent among plans in the support of μ_z and prefers them to plans outside the support

$$\begin{aligned}
\mathcal{L}^c(p_0(c), a_0(c)) &= \mathcal{L}^{c'}(p_0(c'), a_0(c')) && \text{for } c, c' \in \operatorname{supp}(\mu_z) \\
\mathcal{L}^c(p_0(c), a_0(c)) &\leq \mathcal{L}^{c'}(1, a_0(c')) && \text{for } c \in \operatorname{supp}(\mu_z),\ c' \notin \operatorname{supp}(\mu_z)
\end{aligned}$$

where we highlight the fact that types that are not played start with full reputation: $p_0(c) = 1$ for $c \notin \operatorname{supp}(\mu_z)$.

Definition An *equilibrium with vanishingly small reputation* is the limit of equilibria as $z \to 0$.

$$\mu^\star = \lim_{z\to 0} \mu_z$$

An alternative definition of equilibrium follows Kambe (1999) and does away with the initial inference by the private sector. Instead, it corresponds to the case where the government first announces a plan c and subsequently becomes committed to following it with some exogenous probability p_0, independent of what c is.

Definition Given p_0, a *K-equilibrium* is an announcement c and a continuation equilibrium $\{\mathcal{L}^c, g_c^\star\}$ such that the mimicked type c minimizes the initial reputation-adjusted loss function

$$c_{\mathrm{K}}^\star(p_0) = \arg\min_c \mathcal{L}^c(p_0, a_0(c))$$

Once more, in our application we are especially interested in $\lim_{p_0\to 0} c_{\mathrm{K}}^\star(p_0)$. For the remainder of the paper in this version we will consider K-equilibria.

3.1 Reputation-building incentives

First-order conditions in the government's problem involve critically the marginal effect of inflation on output and on future reputation. Solving for output in the Phillips curve yields that output is affected by inflation according to

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa}\left[1 - \beta\frac{\partial p'}{\partial \pi}\left(\phi_c(a) - g^\star(p', \phi_c(a)) + (1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}\right)\right] \tag{7}$$

Inflation affects current output through three different channels, corresponding to three terms in equation (7). The first term, $\frac{1}{\kappa} \cdot 1$, describes the standard, direct effect of inflation on output.

The second term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(\phi_c(a) - g^\star(p', \phi_c(a)))$, describes an expectation-shifting effect by which more inflation reduces the posterior p' and therefore moves expectations of future inflation away from the target $\phi_c(a)$ and toward the expected choice of the rational type $g^\star(p', \phi_c(a))$.

Finally, the third effect is given by $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}$. It describes how more inflation today moves the expected choice of future rational governments through its effect on their reputation.

3.2 Bounds on optimal actions

Let π^N be the (mean) level of inflation in the Nash equilibrium of the stage game or, equivalently, when $p = 0$. First-order conditions of the government's choice in this case imply that

$$\pi^N = y^\star \frac{\kappa}{1 - \beta + \kappa^2\gamma}$$

Lemma 2. *In any continuation equilibrium, the rational type's choice of inflation is bounded above by the choice in the Nash equilibrium of the stage game:*

$$\forall c \in \mathcal{C}: \qquad g_c^\star(p, a) \leq \pi^N$$

See Appendix A.2 for a proof.

3.3 Reputation and credibility

Definition Given a plan c, its *remaining credibility* in state (p, a) is

$$\begin{aligned} C(p, a; c) &= \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta C(p'_c(p, a), \phi_c(a))\right] \\ &= (1-\beta)\frac{\pi^N - [pa + (1-p)g_c^\star(p, a)]}{\pi^N - a} + \beta\mathbb{E}\left[C(p'_c(p, a), \phi_c(a))\right] \end{aligned} \tag{8}$$

where π^N is Nash inflation. The *credibility* of a plan in a K-equilibrium is then given by

$$C^\star(c) = \lim_{p \to 0} C(p, a_0(c); c)$$

4. Analysis and Numerical Results

We solve the model numerically for different announcements $c \in \mathcal{C}$.

4.1 *Parametrization*

We parametrize our model following King, Lu, and Pastén (2016) and pick our preference and technology parameters $\gamma, \kappa, y^\star$ consistently with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices.

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	King, Lu, and Pastén (2016)
σ	1%	Std of control shock	King, Lu, and Pastén (2016)
κ	0.017	Slope of Phillips curve	King, Lu, and Pastén (2016)
$y^\star$	5%	Output target	King, Lu, and Pastén (2016)

Table 1: Benchmark calibration

4.2 *Continuation equilibrium after announcement* c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c. All plots have current reputation p in the x-axis. Darker lines correspond to lower current announcements a.

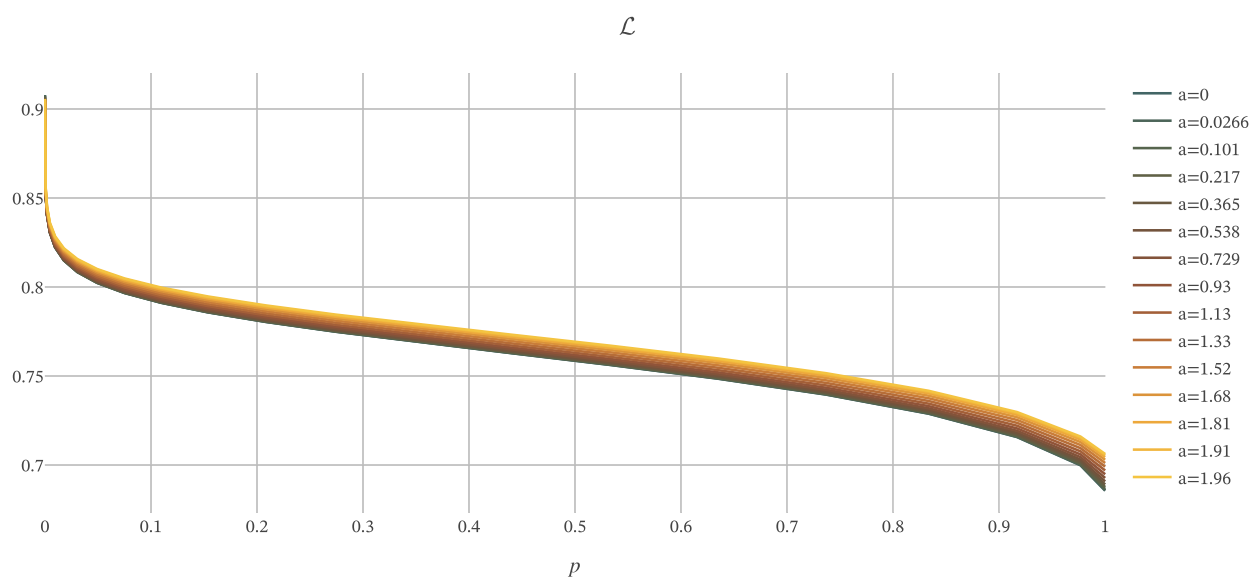

Figure 2: Loss function after announcement c

There are three observations to make. First, $\mathcal{L}$ is decreasing in p. More reputation generally decreases expected inflation, leading to more current output.

Second, the loss function has a convex-concave shape. Reputation has a large marginal value when it is low, as rational (continuation) governments place a high value on not being discovered. At high levels of reputation, however, governments start to prefer to gamble as the loss function turns concave. At high levels of reputation, the private sector is almost convinced that it is facing the behavioral type and updates beliefs

by little. The government is then willing to set $g^\star$ far away from the current target a and trade off an expected loss of reputation for a boost conditional on a shock that makes observed inflation π closer to a.

Finally, at high levels of reputation, a lower current target a is unambiguously good. When p is high, lower a mostly means lower inflation expectations. However, when p is small a lower a also means a larger expected loss of reputation (as a more ambitious target fosters a larger deviation), which makes more modest targets preferable.

On the other hand, as reputation decreases, the gap between a-lines shrinks as two effects arise. The first effect is that with lower reputation the current announcement becomes less relevant as its weight in expected inflation decreases. The second effect concerns the government's choice and how close to a does it choose to set mean inflation. If at low reputation the government chooses to deviate more from lower announcements, this second force might make it prefer a higher announcement today.

Figure 3 shows deviations from the current target as a function of current reputation and target. It confirms that as reputation approaches zero governments tend to deviate more from their target. The figure also reveals a discontinuity at zero reputation, where the government reverts to Nash inflation regardless of announcements. When reputation is exactly zero, Bayes' rule prevents it from moving. This makes the government entirely disregard the plan. By the same logic, a planner that starts with zero reputation is indifferent across all plans, which it anticipates to yield the stage Nash payoff.

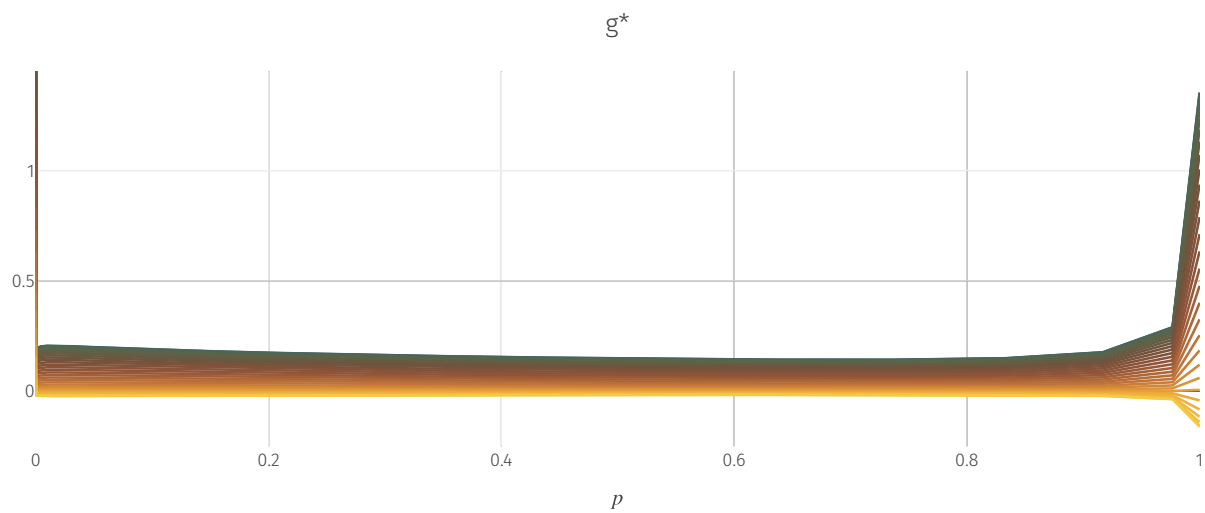

Figure 3: Inflation deviations

The effect of reputation p on the deviation $g^\star(p, a) - a$ is complicated and arises from the sum of many forces. On the one hand, a larger stock of reputation makes the planner more inclined to spending it. Moreover, a higher levels of reputation Bayes' rule implies that reputation is more difficult to lose, which increases incentives to gamble. But on the other hand, at higher reputation delivering on the announcement is less costly, especially when the current announcement is also high.

A higher current announcement a has a more clear effect on the deviation: the lower a, the further away from it will the rational type set inflation. The reason is simple: getting inflation close to target rewards the government in roughly the same way, but it is more costly to set inflation close to target when the target is lower.

Figure 4 shows average reputation p' as function of current reputation p and announcement a. First, $\mathbb{E}[p']$ is always below p, as predicted by Lemma 1. At the highest announcement we consider, which coincides with

the Nash equilibrium of the stage game, by definition the government has no incentives to deviate so it chooses $g = a$ for all levels of p. As a result, reputation does not move. For all announcements lower than Nash inflation, reputation falls on average.

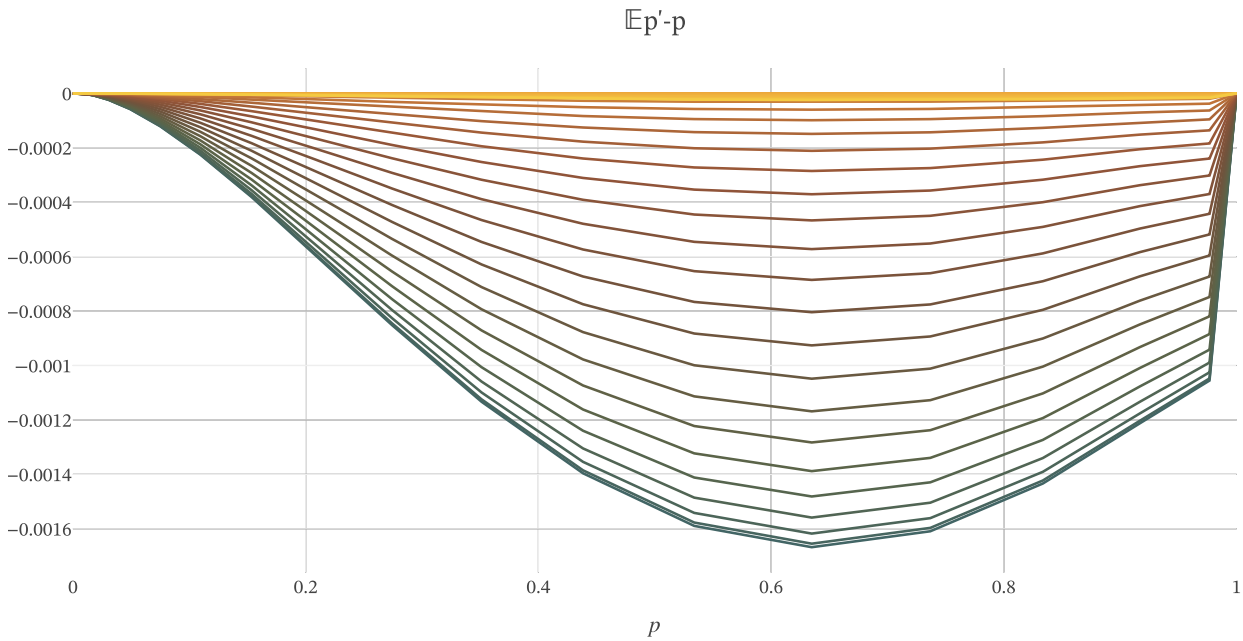

Figure 4: Expected reputation losses

Second, lower announcements are associated with a larger expected reputation loss. Lower current announcements generate weaker incentives to deliver target inflation: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve more output.

Third, Bayes' rule forces p' to be close to p when p is close to either 0 or 1. However, the picture looks skewed to the right, which means that the government 'spends' more reputation when it has more of it. This is especially true at high levels of p, consistent with Figure 3. At low levels of reputation, the government expects to lose more reputation when its current target a is lower.

4.3 *Equilibrium announcements*

Figure 5 shows the K-equilibrium as a function of p_0. The top panel shows the decay rate ω while the bottom panel shows the choice of initial inflation a_0 and asymptote χ.

At $p_0 = 1$, any announcement is believed by the private sector, regardless of expectations about the behavior of the rational type. The planner sets expectations at their most advantageous level by promising zero inflation throughout. The rational type intends to break this promise, given that at full reputation the private sector never learns. As soon as initial reputation p_0 is less than one, the planner starts caring about incentivizing future governments to behave, so as to conserve reputation. This leads the planner to prefer plans that have a higher initial inflation a_0. The planner also chooses plans that make inflation decrease over time by setting $a_0 > a_\infty$, meaning that the planner attempts a gradual disinflation. This property holds even as p_0 approaches zero.

Figure 6 shows the determination of the K-equilibrium when p_0 is small. For each decay ω and asymptote a_∞ we plot the minimized loss function $\min_{a_0} \mathcal{L}(p_0, (a_0, \omega, a_\infty))$. The x-axis moves ω while different curves

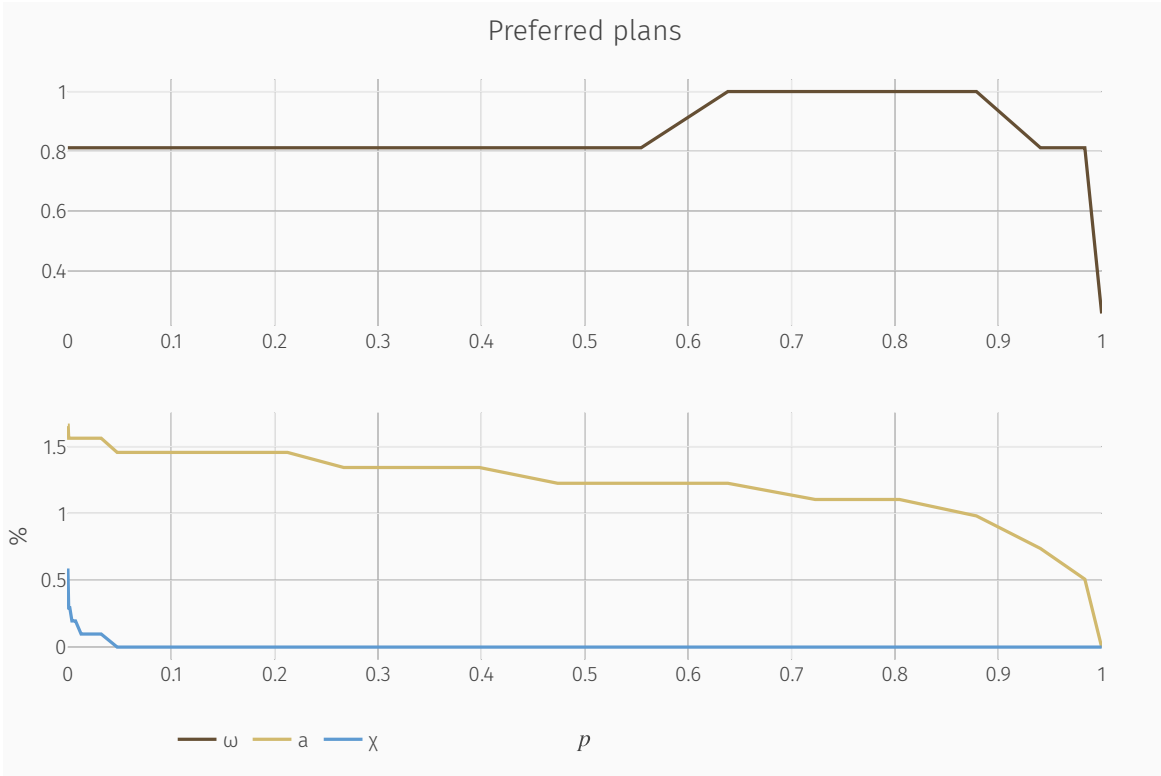

Figure 5: Preferred plans with different p_0

plot different values of a_∞. Figure 7 shows the associated loss-minimizing choices of initial inflation a_0.

Some patterns are evident from Figure 6. The overall minimum is achieved at a point with both ω, $a_\infty > 0$: the K-equilibrium has the initial planner promise a gradual disinflation that does not converge to the first best level of zero inflation.

When a_∞ is small, plans eventually imply very low levels of inflation which makes them more dificult to sustain: reputation is lost quickly when a approaches zero. This gives rise to unfavorable continuation values as the government is revealed to be rational and reverts back to the high-inflation stage Nash. For this reason, at low a_∞ the planner prefers to make the decay rate slow by choosing ω as low as possible. This way, the plan only promises very low inflation in the far future. When a_∞ is higher, the planner uses a decay rate that provides incentives even in the short run. These values of a_∞ turn out to be preferred.

Finally, a_∞ cannot grow too much either. As a_∞ approaches Nash inflation, the plan becomes arbitrarily easy to keep, but provides very small gains.

4.4 *Simulations*

Figure 8 shows simulations of the K-equilibrium type.

5. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The central bank's optimal policy after a plan was announced trades off

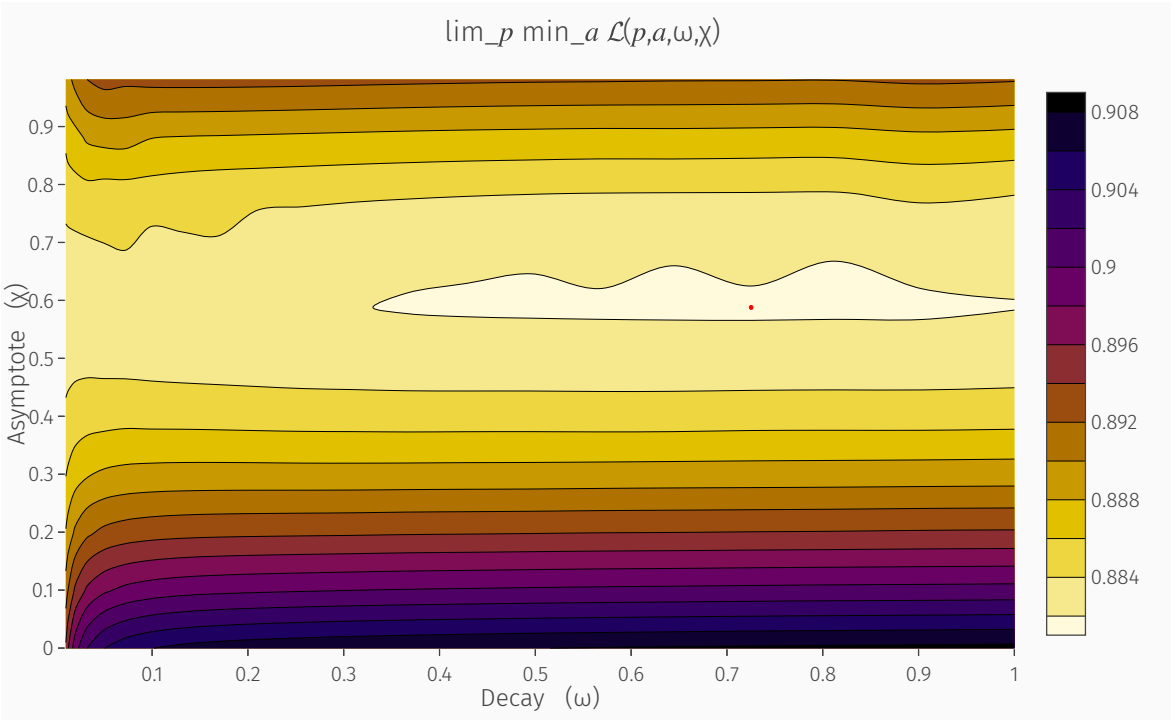

Figure 6: Loss function across announcements

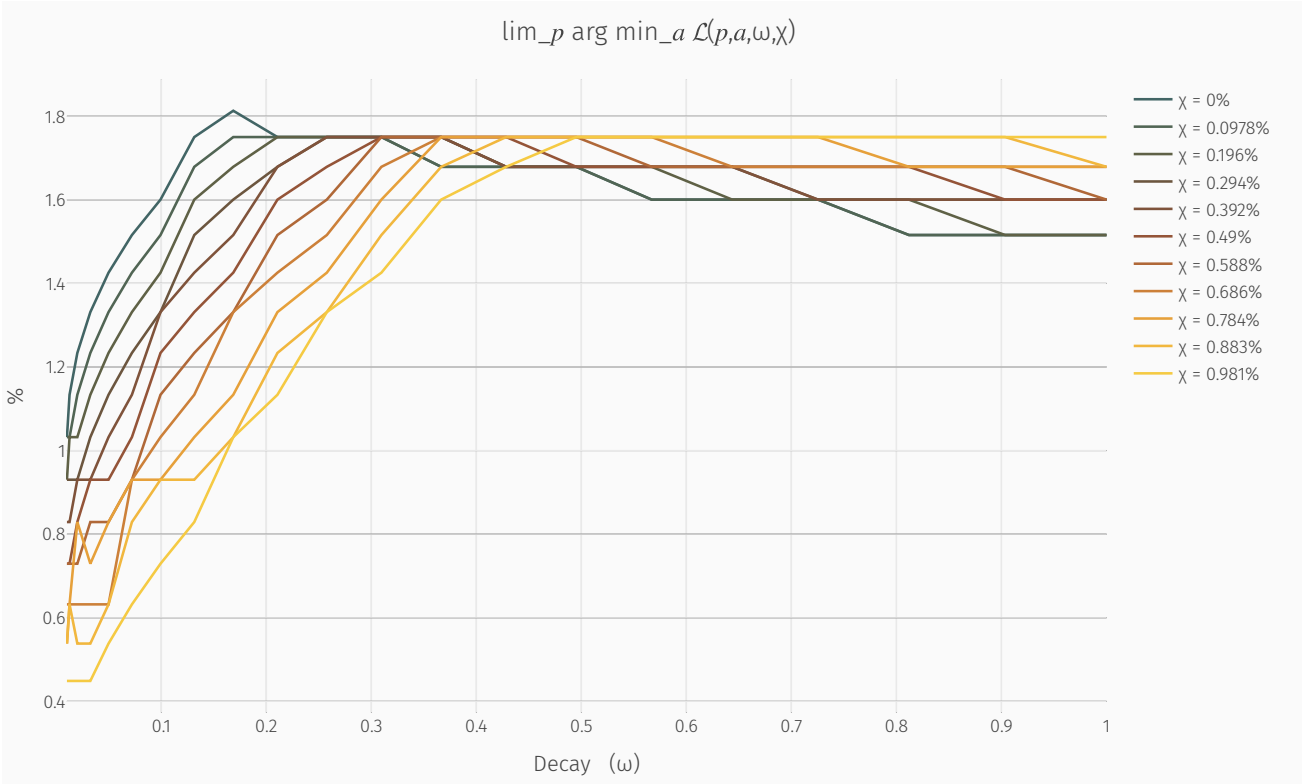

Figure 7: Initial inflation choice across announcements

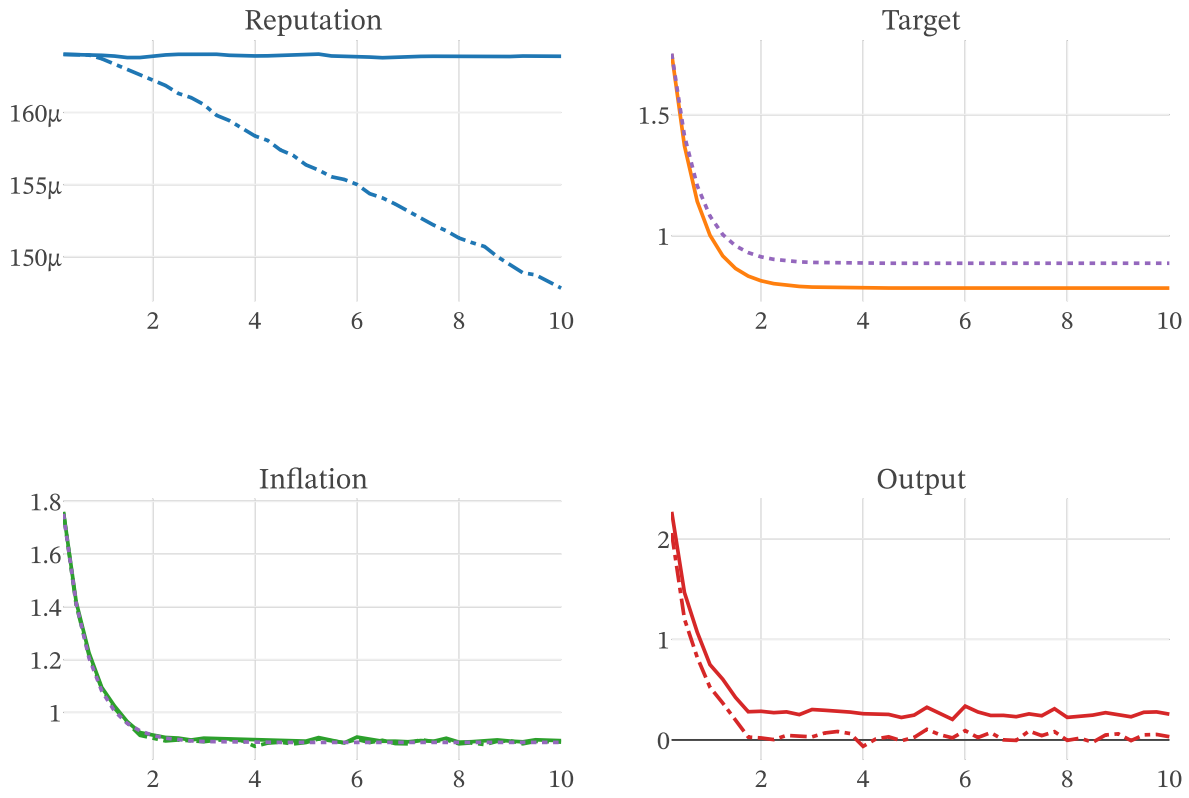

Figure 8: Simulation paths

the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the monetary authority's reputation becomes an important state variable in the optimal policy problem under discretion.

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations in which current announced inflation is higher now than in the future. In our model, gradualism is therefore an artifact of incentives and not the reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes to zero. We interpret this limit case as a sensible refinement of the original game between the monetary authority and the private sector.

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A. Proofs

A.1 Proof of Lemma 1

Proof. Start from the expression for Bayes' rule

$$p' = p + p(1-p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p,a))}{pf_\epsilon(\pi - a) + (1-p)f_\epsilon(\pi - g^\star(p,a))}$$

In the case of a rational government, we have $\pi = g^\star(p,a) + \epsilon$. Therefore, dropping the dependence of $g^\star$ on the state,

$$\mathbb{E}\left[\frac{p' - p}{p(1-p)}\right] = \int_{\mathbb{R}} \frac{f_\epsilon(g^\star + \epsilon - a) - f_\epsilon(\epsilon)}{pf_\epsilon(g^\star + \epsilon - a) + (1-p)f_\epsilon(\epsilon)} f_\epsilon(\epsilon)\, d\epsilon$$

TBW □

A.2 Proof of Lemma 2

TBW