Credibility Dynamics and Disinflation Plans[*]

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Abstract

We study the optimal design of a disinflation plan by a planner who lacks commitment. Having announced a plan, the Central banker faces a tradeoff between surprise inflation and building reputation, defined as the private sector's belief that the Central bank is committed to the plan. Some plans are harder to sustain: the planner recognizes that paving out future grounds with temptation leads the way for a negative drift of reputation in equilibrium. Plans that successfully create low inflationary expectations balance promises of low inflation with dynamic incentives that make them credible. When announcing the disinflation plan, the planner takes into account these anticipated interactions. We find that, even in the zero reputation limit, a gradual disinflation is preferred despite the absence of inflation inertia in the private economy.

JEL Classification E52, C73

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two extreme assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In any of these extreme cases, the public understands that announcements do not bind the government in any way. Announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to policy design questions. Our notion of credibility is akin to the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model, the government could be rational and strategic, or one of many possible 'behavioral' types described by a policy that they stubbornly follow. The public is uninformed about the government's type and makes statistical inference about it after observing the government's announcements and actions. The inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government makes an announcement of its policy 'targets' and is then free to choose policy. However the private sector knows that, if the government is behavioral, it announced exactly what it will implement. As a consequence, the rational type has an incentive ex-post to stay close to any announced targets, which might earn it a reputation for being 'committed' to them. The incentive exists regardless of the level of reputation (as long as it is positive), although its strength depends on what targets were announced. In anticipation of these interactions, the rational type chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a monetary authority sets inflation subject to an expectations-augmented Phillips curve. The monetary authority dislikes inflation but faces a constant temptation to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard New Keynesian setup at the cashless limit for the private economy. To focus on incentives and the dynamics of reputation, we abstract from an IS curve and let the monetary authority control inflation directly.

A natural definition of the government's reputation is the private sector's belief that the government is indeed the behavioral type whose plan was announced. The credibility of a plan is a measure of closeness between expected inflation under the plan and what the plan actually calls for. We refer to the total, ex-ante probability of the behavioral types as the government's initial reputation. While credibility generally increases with reputation, the insights of the reputation literature mean in our case that credibility need not converge to zero as reputation vanishes.

A key assumption we introduce is that the government exerts imperfect control over current inflation, perhaps due to underlying shocks to money demand. This assumption distinguishes us in technical terms from the 1980s attempts referenced above, but it is also vital for our results. Imperfect control masks the government's actual choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. When Bayes' rule is used to update beliefs based on this imperfect signal, reputation does not jump to zero after any deviation, but rather moves continuously. This feature creates a tradeoff for the government: overshooting the target by more makes it expect to create a larger boom today, but also to lose reputation more quickly.

When designing policy, the planner takes into account its own expected future behavior. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Preserving reputation turns out to be a powerful disciplining force for the planner's future self. Crucially, the value of reputation depends on the plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, their remaining credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

We depart from the reputation literature by considering a special family of possible behavioral types, which correspond to the types of announcements that motivate us. While the reputation tradition typically considers behavioral types that repeat the same action in all periods, we allow for announcements of inflation targets that are constant as well as increasing or decreasing over time.

Our main result is that the planner chooses a policy under which inflation starts high and diminishes gradually (except maybe in the case when initial reputation is very high). Plans with a gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan while minimizing the costs of doing so. This leads to a slower pace of equilibrium reputational losses, which the planner values when designing the plan.

Seeing this policy, an outside observer might conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Rather, as it turns out, a plan that promises decreasing inflation is easier to keep.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only (Markov) equilibrium is a repetition of the static Nash with high inflation and output at the natural level. However, as is usual in the reputation literature, it turns out that our theory and our results do not depend on initial reputation being high. To make this point, we focus on the case of vanishingly small initial reputation. A discontinuity at zero reputation allows the credibility dynamics we emphasize have an impact on the government's plan even in the limit. While the solution of the model forces us to consider all levels of reputation, we view the limiting case as the most interesting and as a sensible refinement in the broader game played by the government and the private sector. Along the zero-reputation limit, optimal plans retain their gradualist property.

Discussion of the Literature We contribute to a long literature dealing with commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

We build on models such as Barro (1986), Backus and Driffill (1985), and more recently Sleet and Yeltekin (2007) and Dovis and Kirpalani (2019) who introduce reputation and behavioral types in models of monetary policy. The key departure from that literature is our assumption of imperfect control of inflation. With perfect control, all deviations by the government are detected by the private sector: on the equilibrium path, any deviation completely destroys the reputation. Imperfect control complicates the private sector's inference and enables the tradeoffs that shape our optimal plans.

A related literature looks at subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). However, this notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001) generally generates a large set of equilibria, which limits the theory's predictions.

There is also a large literature that makes use of imperfect control in the same way we do, along with uncertainty about the preferences of the planner. Examples include Phelan (2006), Cukierman and Meltzer (1986), Faust and Svensson (2001), among many others. We view our model with behavioral types as more directly suited to address the issue of announcements about future policy that motivates us.

Even though the announcements in our model do not constrain the actions of the rational government, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes our model from cheap talk models of monetary policy such as Turdaliev (2010).

Recent promising work by King, Lu, and Pastén (2008, 2016), or Lu (2013) shares some of our ingredients. They also consider a model with imperfect control of inflation and behavioral types, but in these papers the planner has commitment power. The tradeoff becomes that the planner wants to make it clear that it is the rational type but also deliver good outcomes. If the behavioral type resembles the Ramsey plan at any point in time, these objectives create a tension. Finally, in the rational limit that we are interested in, the tension between delivering the Ramsey plan and separating from a behavioral type dissipates. These plans then simply converge to the Ramsey outcome.

On the theoretical side, Faingold and Sannikov (2011) consider a similar model to ours, in the context of a monopolist selling to competitive buyers. They also make use of behavioral types and imperfect control. They find that in the continuous-time limit the equilibrium is unique and Markovian in reputation, which informs our strategy of looking for a Markovian equilibrium in our model. However, since they only consider static behavioral types, they cannot address the issue of gradualist policy and the incentives it generates, which are at the core of our argument.

Layout The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section 3. Section 4 lays out our main results. Finally, Section 5 concludes.

2. Model

We consider a government who dislikes inflation and deviations of output from a target according to a loss function

$$L_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(y^\star - y_t)^2 + \gamma \pi_t^2 \right] \right] \tag{1}$$

while a Phillips curve relates current output to current and expected inflation

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right] \tag{2}$$

We further assume that the government has imperfect control over inflation so that

$$\pi_t = g_t + \sigma \epsilon_t \tag{3}$$

where the government controls g_t at time t and $\epsilon \overset{iid}{\sim} \mathcal{N}(0,1)$.

2.1 *Reputation*

We introduce reputation by considering the possibility of behavioral types for the government. Each behavioral type is committed to a particular strategy. For convenience, we identify each possible behavioral type by the strategy it follows. Types are indexed by a set $\mathcal{C}$. For $c \in \mathcal{C}$, a government of behavioral type c is committed to an *inflation plan* $(a_t^c)_{t=0}^{\infty}$. An inflation plan consists of inflation announcements for each t.

We assume that the government is rational with probability z. A probability defined over $\mathcal{C}$ with density ν describes the distribution of possible behavioral types.

2.2 *Timing of play*

At time 0 an announcement $a = (a_t)_{t=0}^{\infty}$ of *inflation targets* takes place. The announcement includes targets a_t for all time periods into the future.

If the government happens to be behavioral of type $c \in \mathcal{C}$, it announces c for sure. However, the rational type of the government chooses an announcement r, possibly $r \in \mathcal{C}$. The government understands that announcing $r \notin \mathcal{C}$ reveals rationality. If $r \in \mathcal{C}$, then with probability 1 the government is either rational or behavioral of type r.

At time $t \geq 0$, the government sets inflation. If the government happens to be behavioral of type c, it sets $g_t = a_t^c$. The rational type may instead choose g_t strategically. Actual inflation is noisy so the private sector has to apply Bayes' rule to update beliefs about the government's type.

2.3 *Beliefs*

After the initial announcement, the private sector applies Bayes' rule to update beliefs about the government's type. By our discussion above, if an announcement $c \in \mathcal{C}$ has been made, the government can only be rational or of type c.

Furthermore, suppose that in equilibrium the rational type announces c with density $\mu(c)$. By Bayes' rule, the posterior probability of the government being behavioral of type c, the reputation with which it will start implementing its announcement, is

$$p_0(c; z, \mu) = \frac{z\nu(c)}{z\nu(c) + (1-z)\mu(c)} \tag{4}$$

At time t, the private sector's posterior of the government being of behavioral type c is formed by applying Bayes' rule to the private sector's information. Suppose that inflation π_t is realized at time t. If the government is behavioral of type c, then it must have chosen $g_t = a_t^c$ and the current shock must have been $\epsilon_t = \pi_t - a_t^c$, which happens with density $f_\epsilon(\pi_t - a_t^c)$. If on the other hand the government was rational, it chose $g_t = g_t^\star$, the rational type's strategy, which means that the shock must have been $\epsilon_t = \pi_t - g_t^\star$. Therefore, updating from a prior belief of p_t, we have that

$$p_{t+1} = \frac{p_t \cdot f_\epsilon(\pi_t - a_t)}{p_t \cdot f_\epsilon(\pi_t - a_t) + (1-p_t) \cdot f_\epsilon(\pi_t - g_t^\star)}$$

where $g_t^\star$ is the (conjectured) choice of inflation by the rational type of the central bank. In a rational-expectations equilibrium, this must also be the rational government's *actual* choice of inflation.

It is useful to rewrite this condition as

$$p_{t+1} = p_t + p_t(1-p_t)\frac{f_\epsilon(\pi_t - a_t) - f_\epsilon(\pi_t - g_t^\star)}{p_t f_\epsilon(\pi_t - a_t) + (1-p_t) f_\epsilon(\pi_t - g_t^\star)} \tag{5}$$

which makes it evident that reputation moves when (i) it started far away from 0 and 1 and (ii) when realized inflation is closer to either the target or the rational type's strategy. Only if a_t is far away from $g_t^\star$ can actual inflation fall closer to one than to the other. Some of this intuition is leveraged heavily later on.

2.4 *The set of behavioral types*

We parametrize the set $\mathcal{C}$ of possible behavioral types. We assume that behavioral type c's inflation plan is defined by three parameters (a_0, ω, a_∞) so that

$$a_t^c = (a_0 - a_\infty)\, e^{-\omega t} + a_\infty$$

This parametrization makes $\mathcal{C}$ finitely-dimensional but also allows us to write each plan recursively. For each $c \in \mathcal{C}$, $a_{t+1}^c = a_\infty + e^{-\omega}\left(a_t^c - a_\infty\right) = \phi_c(a_t)$. We also assume that both $a_0, a_\infty \in \mathcal{A} = [0, \pi^N]$.

Figure 1 illustrates some possible paths. Paths start at a_0 and converge towards a_∞ with a exponential decay rate of ω. The set $\mathcal{C}$ contains constant, decreasing, and increasing paths, which obtain by appropriately setting a_0 and a_∞.

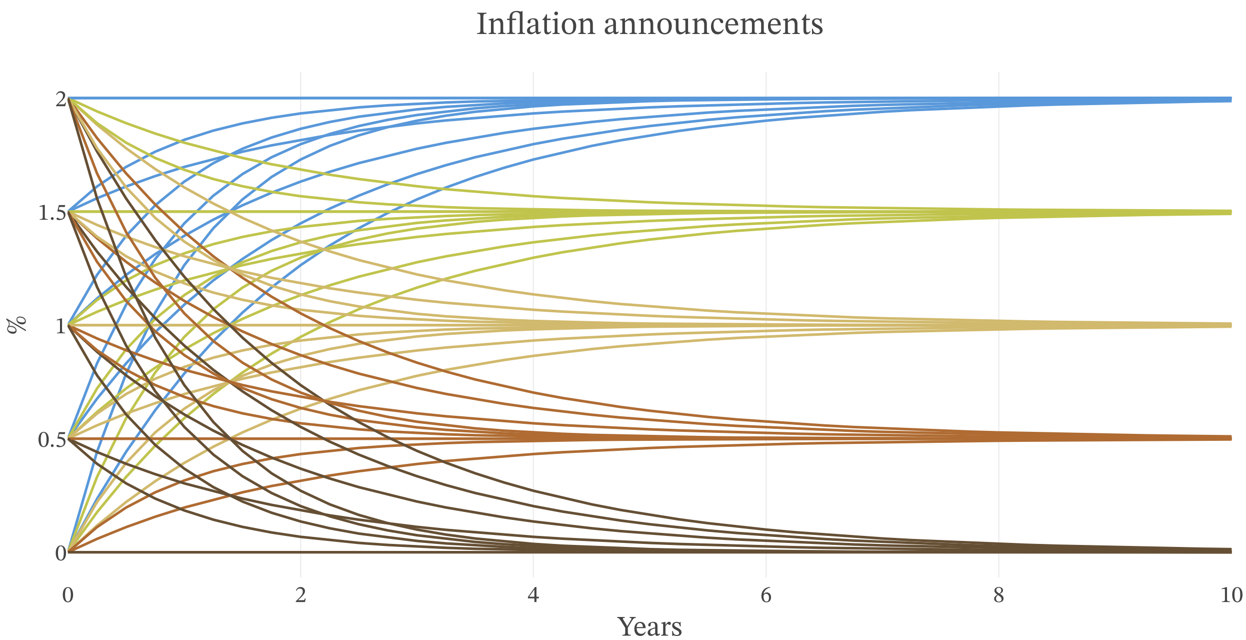


FIGURE 1: POSSIBLE BEHAVIORAL TYPES' ANNOUNCEMENTS

2.5 *Bellman equations after an announcement*

Given an announcement c, the problem of the rational type is to choose mean inflation g_t in period t to maximize (1) subject to (2), (3), and (5). The time-t government chooses taking as given its reputation p_t, its future strategy, and the private sector's expectations about the behavioral and rational types' choices.

At time t, the private sector expects the behavioral type to choose $g_t = a_t^c$. Let $g_t^\star$ denote the private sector's expectations of the rational type's choice. We focus on Markovian strategies with $g_t^\star = g^\star(p_t, a_t^c)$. This allows us to write the rational government's problem recursively as

$$\mathcal{L}^c(p, a) = \min_g \mathbb{E}\left[(y^\star - y)^2 + \gamma\pi^2 + \beta\mathcal{L}^c(p', \phi_c(a))\right] \tag{6}$$

$$\begin{aligned} \text{subject to } \pi &= g + \epsilon \\ \pi &= \kappa y + \beta\left[p'\phi_c(a) + (1-p')g^\star(p', \phi_c(a))\right] \\ p' &= p + p(1-p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p, a))}{pf_\epsilon(\pi - a) + (1-p)f_\epsilon(\pi - g^\star(p, a))} \end{aligned}$$

Problem (6) illustrates how the government best-responds to the public's beliefs $g^\star(p, a)$. In the Phillips curve, expected inflation is a weighted average between $\phi_c(a)$, the strategy of the behavioral type, and $g^\star(p', \phi_c(a))$, the conjectured choice of the rational type in next period's state. This gives the government a degree of freedom: it can influence expected inflation by affecting its reputation.

At low levels of p, the Phillips curve puts most of the weight on the expected choice of the rational type. Therefore, at low levels of p, the government affects inflationary expectations mostly through $g^\star(p', \phi_c(a))$. If future governments are expected to value their reputation and choose $g^\star(p, a)$ close to a when $p > 0$, the current government has an incentive not to let its reputation go to zero and, therefore, choose the current $g^\star$ close to a.

3. Expectations and Equilibrium

A solution to (6) describes the government's choices $g(p, a)$ as a function of the private sector's expectations $g^\star$. Our equilibrium definition makes it clear that rational expectations requires finding a fixed point of that function.

Definition Given an announcement $c \in \mathcal{C}$, a *continuation equilibrium* consists of a loss function $\mathcal{L}^c : [0,1] \times \mathcal{A} \to \mathbb{R}$ and a policy function $g^\star_c : [0,1] \times \mathcal{A} \to \mathbb{R}$ such that

1. The loss function $\mathcal{L}^c$ solves the government's Bellman equation (6) taking as given expectations $g^\star_c$
2. $g^\star_c$ is the policy function that corresponds to the solution of (6)

A useful property of continuation equilibria follows from close observation of problem (6): given the decay and asymptote parameters, it is equivalent to start the plan at a different initial announcement a or to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Observation Suppose $(\mathcal{L}, g^\star)$ is a continuation equilibrium for announcement $c = (a_0, a_\infty, \omega) \in \mathcal{C}$. Then for any b_0, the same pair $(\mathcal{L}, g^\star)$ is a continuation equilibrium for plan $c' = (b_0, a_\infty, \omega)$.

Lemma 1. *In any continuation equilibrium, the rational type's reputation is a supermartingale:*

$$\mathbb{E}\left[p_{t+1} \mid \textit{rational}, \mathcal{F}_t\right] \leq p_t$$

That is, the planner cannot design a policy that generates expected reputational gains.

See Appendix A.1 for a proof. Conditional on a rational government, either $g^\star(p, a) = a$, in which case $p' = p$ a.s., or $g^\star(p, a) \neq a$, in which case π is a signal centered away from a, which is revealing on average.

One reason why one might think a planner prefers a gradual disinflation is that gradualism allows the planner to promise easy things first, accumulate credibility, and then be able to promise more difficult things. Lemma 1 says that the planner cannot strategize in this way. It cannot design its plan in a way that makes it expect to increase reputation over time. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Definition Given an initial reputation z, an *equilibrium* is a distribution μ_z over $\mathcal{C}$ along with continuation equilibria $\{\mathcal{L}^c, g^\star_c\}_{c \in \mathcal{C}}$ and a posterior reputation $p_0 : \mathcal{C} \to [0,1]$ such that

1. Posterior reputation is set according to Bayes' rule (4), given the distribution μ_z.

2. The distribution of mimicked types μ_z minimizes the starting reputation-adjusted loss function

$$\mathcal{L}_r^\star(\mu_z, z) = \int_{\mathcal{C}} \mathcal{L}^c(p_0(c), a_0(c))d\mu_z(c)$$

taking as given the starting reputation function p_0.

Notice that, as a consequence of 2, in an equilibrium the planner is indifferent among plans in the support of μ_z and prefers them to plans outside the support

$$\begin{aligned} \mathcal{L}^c(p_0(c), a_0(c)) &= \mathcal{L}^{c'}(p_0(c'), a_0(c')) && \text{for } c, c' \in \text{supp}(\mu_z) \\ \mathcal{L}^c(p_0(c), a_0(c)) &\leq \mathcal{L}^{c'}(1, a_0(c')) && \text{for } c \in \text{supp}(\mu_z),\, c' \notin \text{supp}(\mu_z) \end{aligned}$$

where we highlight the fact that types that are not played start with full reputation: $p_0(c) = 1$ for $c \notin \text{supp}(\mu_z)$.

Definition An *equilibrium with vanishingly small reputation* is the limit of equilibria as $z \to 0$.

$$\mu^\star = \lim_{z\to 0} \mu_z$$

An important part of finding an equilibrium is determining which plans are played and which ones are not. A plan c can only be outside the support if its expected loss (at full reputation) is still greater than what the plans that are played deliver.

Definition For a given $p_0 \in [0, 1]$, a *K-equilibrium* is an announcement c and a continuation equilibrium $\{\mathcal{L}^c, g_c^\star\}$ such that c minimizes the loss function

$$c_{\mathrm{K}}^\star(p_0) = \arg\min_c \mathcal{L}^c(p_0, a_0(c))$$

Once more, in our application we are especially interested in $\lim_{p_0\to 0} c_{\mathrm{K}}^\star(p_0)$.

Finding a K-equilibrium is simple. For given plan c and reputation p_0, the value of starting plan c at reputation p_0, $\mathcal{L}^c(p_0, a_0(c))$ can be found applying our previous discussion. We then optimize over this function by choosing c in $\mathcal{C}$, keeping p_0 fixed.

To find an equilibrium, we proceed as follows: given $k \in \mathbb{R}$, we partition the space of plans according to whether

$$\mathcal{L}(1, c) \lessgtr k$$

Plans that have a loss greater than k are assigned probability zero of being played, $\mu(c) = 0$. For the remainder of plans, we find a probability $p_0(c)$ that delivers k by requiring

$$\mathcal{L}(p_0(c), c) = k$$

For plan c to start with a reputation of $p_0(c)$, the initial application of Bayes' rule (4) tells us that c must be played with a probability $\mu(c)$ such that

$$p_0(c) = \frac{z\nu(c)}{z\mu(c) + (1-z)\mu(c)}$$

Finally, the planner's strategy is required to be a probability distribution. Therefore, we pin down k by requiring that at the end of this process the non-negative function $\mu(c)$ integrates to 1 over the set of possible plans $\mathcal{C}$.

An alternative definition of equilibrium follows Kambe (1999) and does away with the initial inference by the private sector. Instead, it corresponds to the case where the government first announces a plan c and subsequently becomes committed to following it with some exogenous probability p_0, independent of what c is.

3.1 *Reputation-building incentives*

First-order conditions in the government's problem involve critically the marginal effect of inflation on output and on future reputation. Solving for output in the Phillips curve yields that output is affected by inflation according to

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa}\left[1 - \beta\frac{\partial p'}{\partial \pi}\left(\phi_c(a) - g^\star(p', \phi_c(a)) + (1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}\right)\right] \tag{7}$$

Inflation affects current output through three different channels, corresponding to three terms in equation (7). The first term, $\frac{1}{\kappa} \cdot 1$, describes the standard, direct effect of inflation on output.

The second term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(\phi_c(a) - g^\star(p', \phi_c(a)))$, describes an expectation-shifting effect by which more inflation reduces the posterior p' and therefore moves expectations of future inflation away from the target $\phi_c(a)$ and toward the expected choice of the rational type $g^\star(p', \phi_c(a))$.

Finally, the third effect is given by $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}$. It describes how more inflation today moves the expected choice of future rational governments through its effect on their reputation.

3.2 *Bounds on optimal actions*

Let π^N be the (mean) level of inflation in the Nash equilibrium of the stage game or, equivalently, when $p = 0$. First-order conditions of the government's choice in this case imply that

$$\pi^N = y^\star \frac{\kappa}{1 - \beta + \kappa^2\gamma}$$

Lemma 2. *In any continuation equilibrium, the rational type's choice of inflation is bounded above by the choice in the Nash equilibrium of the stage game:*

$$\forall c \in \mathcal{C}: \qquad g^\star_c(p, a) \leq \pi^N$$

See Appendix A.2 for a proof.

3.3 *Reputation and credibility*

Definition Given a plan c, its *remaining credibility* in state (p, a) is

$$C(p, a; c) = \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta C(p'_c(p,a), \phi_c(a))\right] \tag{8}$$
$$= (1-\beta)\frac{\pi^N - [pa + (1-p)g^\star_c(p,a)]}{\pi^N - a} + \beta\mathbb{E}\left[C(p'_c(p,a), \phi_c(a))\right]$$

where π^N is Nash inflation. The *credibility* of a plan in a K-equilibrium is then given by

$$C^K(c) = \lim_{p\to 0} C(p, a_0(c); c)$$

while in an equilibrium it is

$$C^\star(c) = \lim_{z\to 0}\int C(p_0(c), a_0(c); c)\, d\mu_z(c)$$

4. Analysis and Numerical Results

We solve the model numerically for different announcements $c \in \mathcal{C}$.

4.1 *Parametrization*

We parametrize our model following King, Lu, and Pastén (2016) and pick our preference and technology parameters $\gamma, \kappa, y^\star$ consistently with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices.

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	King, Lu, and Pastén (2016)
σ	1%	Std of control shock	King, Lu, and Pastén (2016)
κ	0.17	Slope of Phillips curve	King, Lu, and Pastén (2016)
$y^\star$	5%	Output target	King, Lu, and Pastén (2016)

Table 1: Benchmark calibration

4.2 *Continuation equilibrium after announcement* c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c. All plots have current reputation p in the x-axis. Darker lines correspond to lower current announcements a.

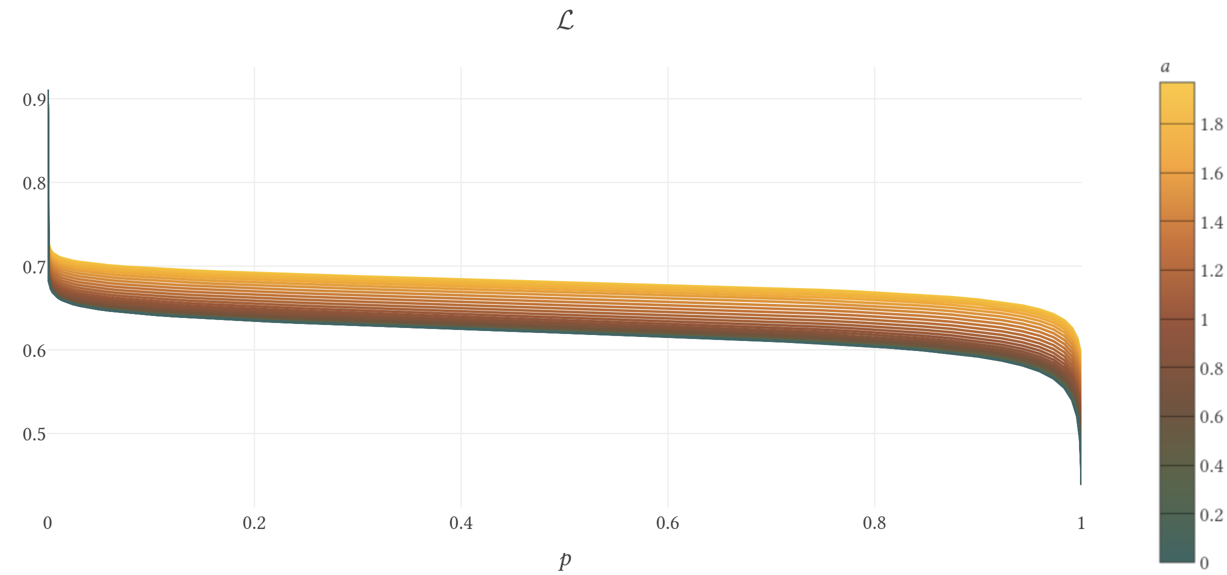


Figure 2: Loss function after announcement c

There are three observations to make. First, $\mathcal{L}$ is decreasing in p. More reputation generally decreases expected inflation, leading to more current output.

Second, the loss function has a convex-concave shape. Reputation has a large marginal value when it is low, as rational (continuation) governments place a high value on not being discovered. At high levels of reputation, however, governments start to prefer to gamble as the loss function turns concave. At high levels of reputation, the private sector is almost convinced that it is facing the behavioral type and updates beliefs by little. The government is then willing to set $g^{\star}$ far away from the current target a and trade off an expected loss of reputation for a boost conditional on a shock that makes observed inflation π closer to a.

Finally, at high levels of reputation, a lower current target a is unambiguously good. When p is high, lower a mostly means lower inflation expectations. However, when p is small a lower a also means a larger expected loss of reputation (as a more ambitious target fosters a larger deviation), which makes more modest targets preferable.

On the other hand, as reputation decreases, the gap between a-lines shrinks as two effects arise. The first effect is that with lower reputation the current announcement becomes less relevant as its weight in expected inflation decreases. The second effect concerns the government's choice and how close to a does it choose to set mean inflation. If at low reputation the government chooses to deviate more from lower announcements, this second force might make it prefer a higher announcement today.

Figure 3 shows deviations from the current target as a function of current reputation and target. It confirms that as reputation approaches zero governments tend to deviate more from their target. The figure also reveals a discontinuity at zero reputation, where the government reverts to Nash inflation regardless of announcements. When reputation is exactly zero, Bayes' rule prevents it from moving. This makes the government entirely disregard the plan. By the same logic, a planner that starts with zero reputation is indifferent across all plans, which it anticipates to yield the stage Nash payoff.

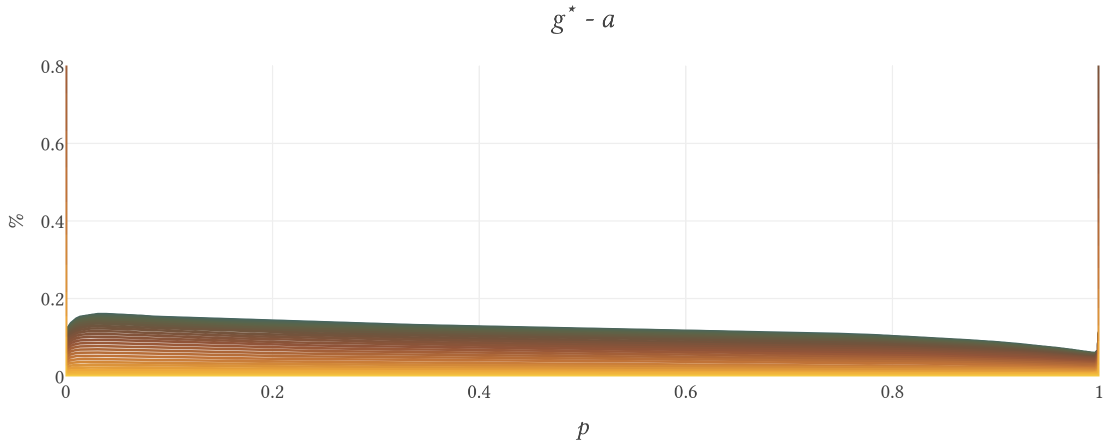


FIGURE 3: INFLATION DEVIATIONS

The effect of reputation p on the deviation $g^\star(p, a) - a$ is complicated and arises from the sum of many forces. On the one hand, a larger stock of reputation makes the planner more inclined to spending it. Moreover, a higher levels of reputation Bayes' rule implies that reputation is more difficult to lose, which increases incentives to gamble. But on the other hand, at higher reputation delivering on the announcement is less costly, especially when the current announcement is also high.

A higher current announcement a has a more clear effect on the deviation: the lower a, the further away from it will the rational type set inflation. The reason is simple: getting inflation close to target rewards the government in roughly the same way, but it is more costly to set inflation close to target when the target is lower.

Figure 4 shows average reputation p' as function of current reputation p and announcement a. First, $\mathbb{E}[p']$ is always below p, as predicted by Lemma 1. At the highest announcement we consider, which coincides with the Nash equilibrium of the stage game, by definition the government has no incentives to deviate so it chooses $g = a$ for all levels of p. As a result, reputation does not move. For all announcements lower than Nash inflation, reputation falls on average.

Second, lower announcements are associated with a larger expected reputation loss. Lower current announcements generate weaker incentives to deliver target inflation: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve more output.

Third, Bayes' rule forces p' to be close to p when p is close to either 0 or 1. However, the picture looks skewed to the right, which means that the government 'spends' more reputation when it has more of it. This is especially true at high levels of p, consistent with Figure 3. At low levels of reputation, the government expects to lose more reputation when its current target a is lower.

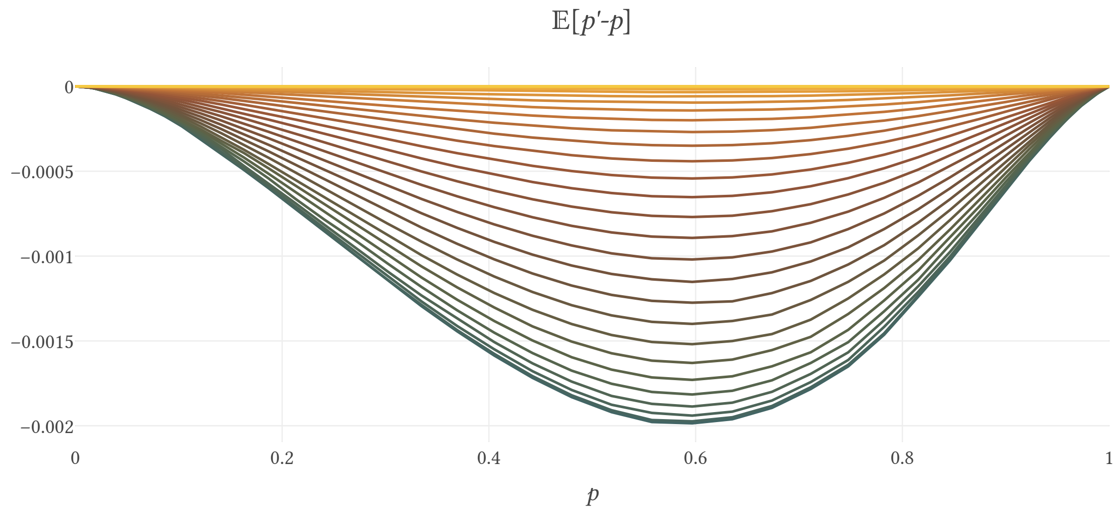


Figure 4: Expected reputation losses

4.3 *Equilibrium announcements*

Figure 5 shows the K-equilibrium as a function of p_0. The top panel shows the decay rate ω while the bottom panel shows the choice of initial inflation a_0 and asymptote χ.

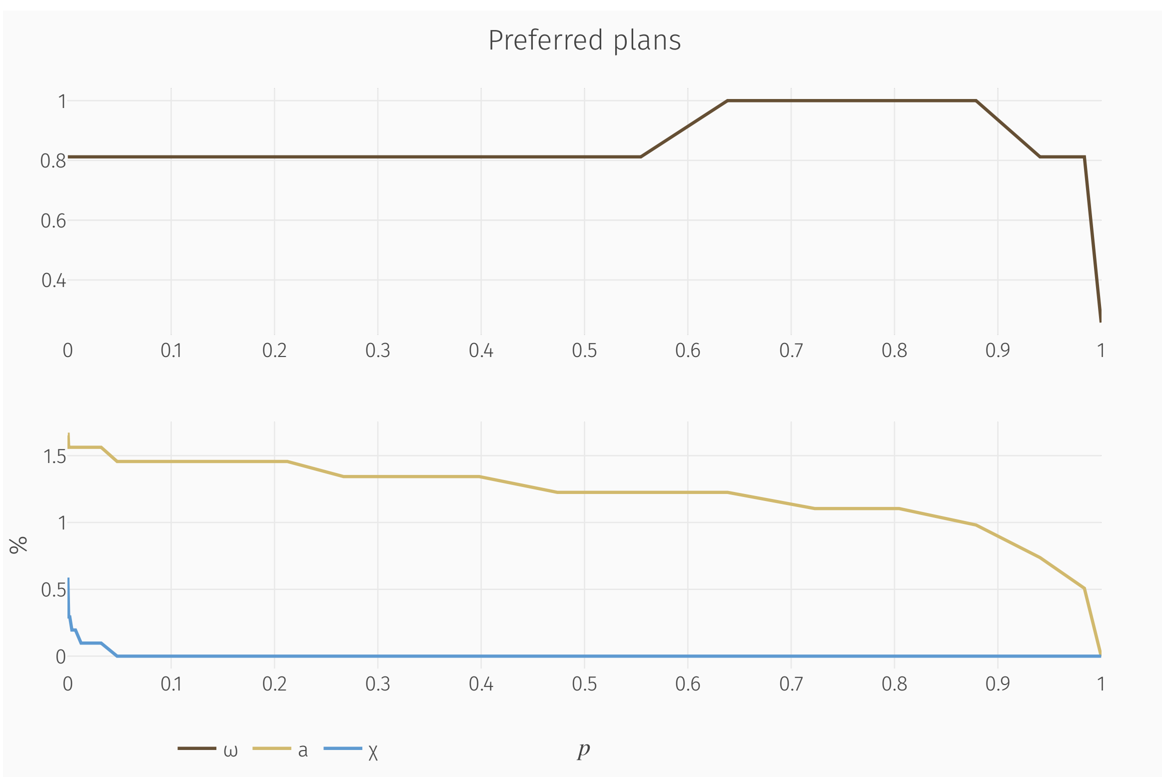


Figure 5: Preferred plans with different p_0

At $p_0 = 1$, any announcement is believed by the private sector, regardless of expectations about the behavior of the rational type. The planner sets expectations at their most advantageous level by promising zero inflation throughout. The rational type intends to break this promise, given that at full reputation the private sector never learns. As soon as the initial posterior p_0 is less than one, the planner starts caring about incentivizing future governments to behave, so as to conserve reputation. This leads the planner to prefer plans that have a higher initial inflation a_0. The planner also chooses plans that make inflation decrease over time by setting $a_0 > a_\infty$, meaning that the planner attempts a gradual disinflation. This property holds even as p_0 approaches zero.

Figure 6 shows the determination of the K-equilibrium when p_0 is small. For each decay ω and asymptote a_∞ we plot the minimized loss function $\min_{a_0} \mathcal{L}(p_0, (a_0, \omega, a_\infty))$. The x-axis moves ω while different curves plot different values of a_∞. Figure 10 in the Appendix shows the associated loss-minimizing choices of initial inflation a_0.

Some patterns are evident from Figure 6. The overall minimum is achieved at a point with both $\omega, a_\infty > 0$: the K-equilibrium has the initial planner promise a gradual disinflation that does not converge to the first best level of zero inflation.

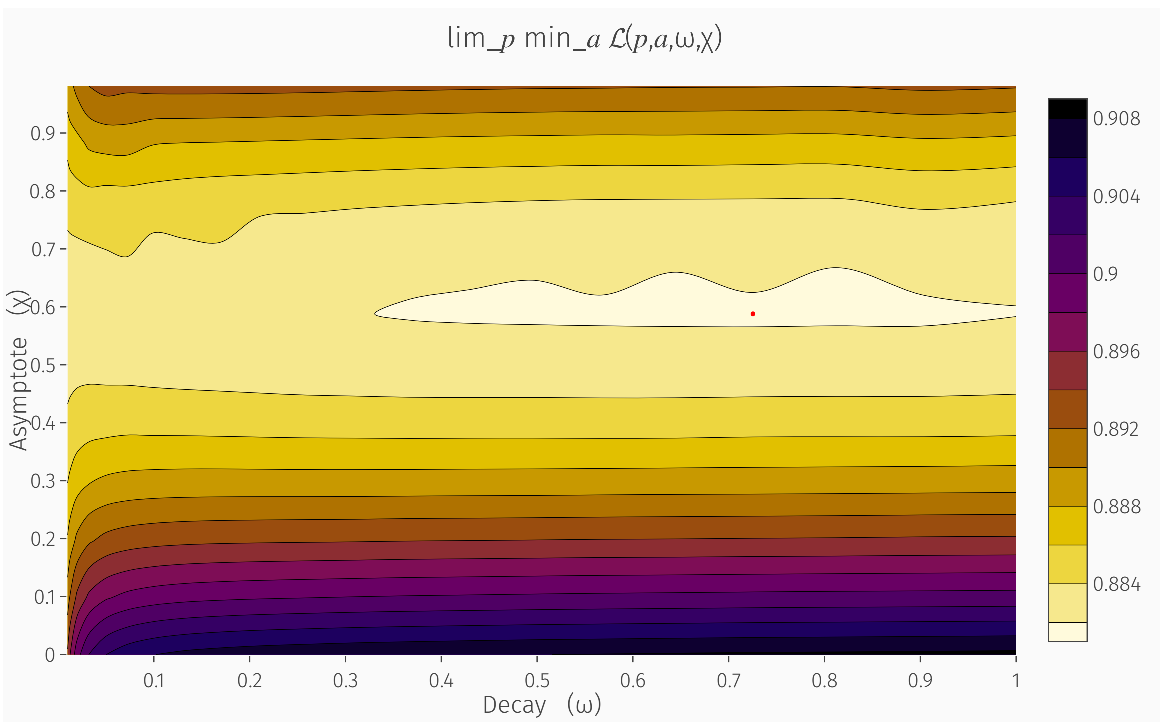


Figure 6: Loss function across announcements

When a_∞ is small, plans eventually imply very low levels of inflation which makes them more dificult to sustain: reputation is lost quickly when a approaches zero. This gives rise to unfavorable continuation values as the government is revealed to be rational and reverts back to the high-inflation stage Nash. For this reason, at low a_∞ the planner prefers to make the decay rate slow by choosing ω as low as possible. This way, the plan only promises very low inflation in the far future. When a_∞ is higher, the planner uses a decay rate that provides incentives even in the short run. These values of a_∞ turn out to be preferred.

Finally, a_∞ cannot grow too much either. As a_∞ approaches Nash inflation, the plan becomes arbitrarily easy to keep, but provides very small gains.

4.4 Credibility

Our setup distinguishes reputation p, the posterior that the government is the behavioral type that was announced, from credibility $C(p, a; c)$, the expected discounted deviations from plan c at reputation p and current announcement a, as defined in (8). Figure 7 plots the credibility of different plans at vanishingly small reputation, as a function of the decay rate ω and the asymptote χ, for the loss-minimizing initial inflation a_0 at those parameters.

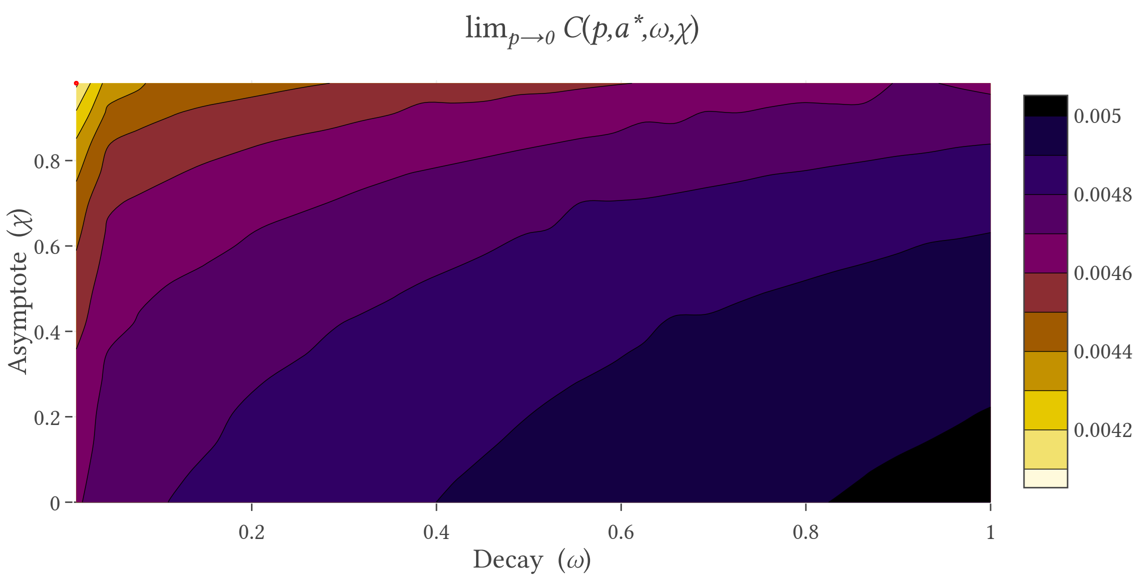


FIGURE 7: CREDIBILITY

The figure makes it clear that plans that promise a steeper descent of inflation (because of higher decay rate and asymptote) are the most credible.

4.5 Comparative statics

Figure 8 shows the average plan in the equilibrium, as function of the variance of the control shock σ. Broadly speaking, more noise in the control imply lower adherence to plans, as deviations would be less observable. This makes the planner choose less ambitious plans: as σ increases, the average plan has a higher asymptote χ, a slightly higher starting point a_0, and a slower rate of decay ω.

Figure 9 repeats the exercise varying the discount factor and the slope of the Phillips curve. It reveals some

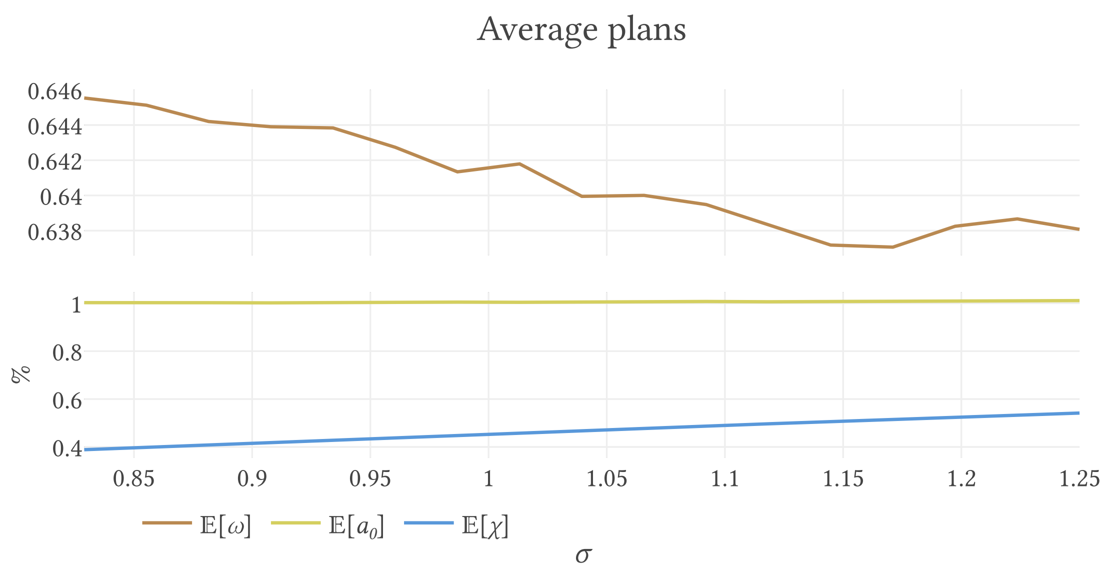


Figure 8: Average plans and the control shock variance

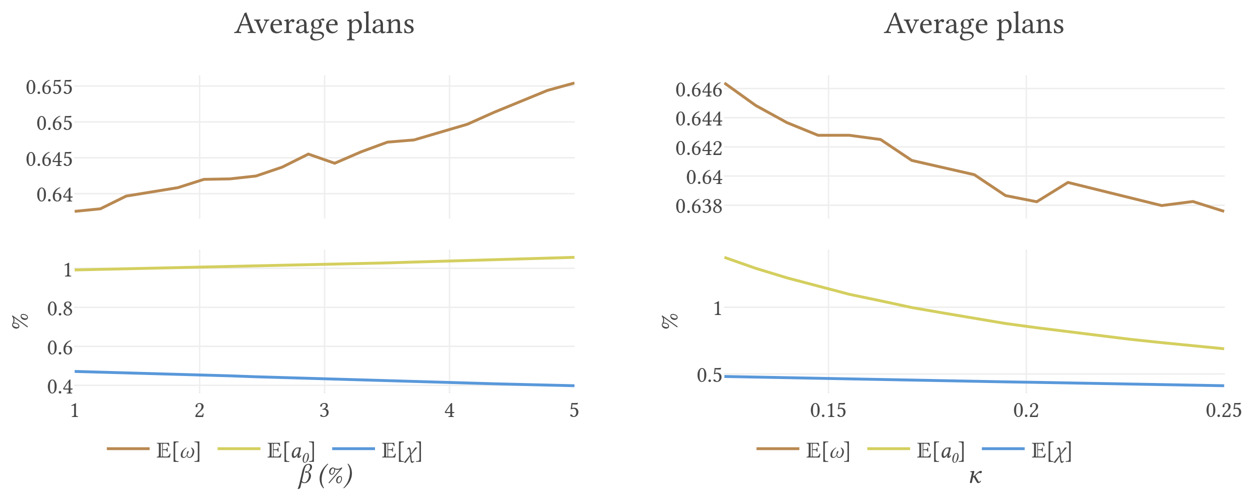
subtleties in the manipulation of the three parameters that describe our plans.

Figure 9a shows the average plan as a function of the discount rate (whose benchmark value is 2%). As the planner becomes more impatient, average plans start higher and converge to lower inflation, with a faster decay rate. When the planner is more impatient, it becomes understood that the inflation bias will be larger. For this reason, it tends to choose plans that are more 'resilient'. Increasing initial inflation makes the plan easier to keep, while decreasing asymptotic inflation makes it more costly to deviate early on. Having a steeper descent of inflation contributes to both objectives.

When we vary the slope of the Phillips curve, the planner chooses plans that have lower inflation throughout. Figure 9b shows that when the Phillips curve is steeper (meaning that the same increase in current inflation produces a smaller output boom), the planner chooses to announce lower inflation. Here the logic is that with a steep Phillips curve there are weaker incentives to create surprise inflation, which allows the planner to announce less inflation throughout.

5. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The central bank's optimal policy after a plan was announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the monetary authority's reputation becomes an important state variable in the optimal policy problem under discretion.



(a) Varying the discount factor (b) Varying the slope of the Phillips curve

Figure 9: Average plans

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations in which current announced inflation is higher now than in the future. In our model, gradualism is therefore an artifact of incentives and not the reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes to zero. We interpret this limit case as a sensible refinement of the original game between the monetary authority and the private sector.

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A. Proofs

A.1 Proof of Lemma 1

Proof. Start from the expression for Bayes' rule

$$p' = p + p(1-p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p,a))}{pf_\epsilon(\pi - a) + (1-p)f_\epsilon(\pi - g^\star(p,a))}$$

In the case of a rational government, we have $\pi = g^\star(p,a) + \epsilon$. Therefore, dropping the dependence of $g^\star$ on the state,

$$\mathbb{E}\left[\frac{p' - p}{p(1-p)}\right] = \int_{\mathbb{R}} \frac{f_\epsilon(g^\star + \epsilon - a) - f_\epsilon(\epsilon)}{pf_\epsilon(g^\star + \epsilon - a) + (1-p)f_\epsilon(\epsilon)} f_\epsilon(\epsilon) d\epsilon$$

TBW □

A.2 Proof of Lemma 2

TBW

B. Strategy details

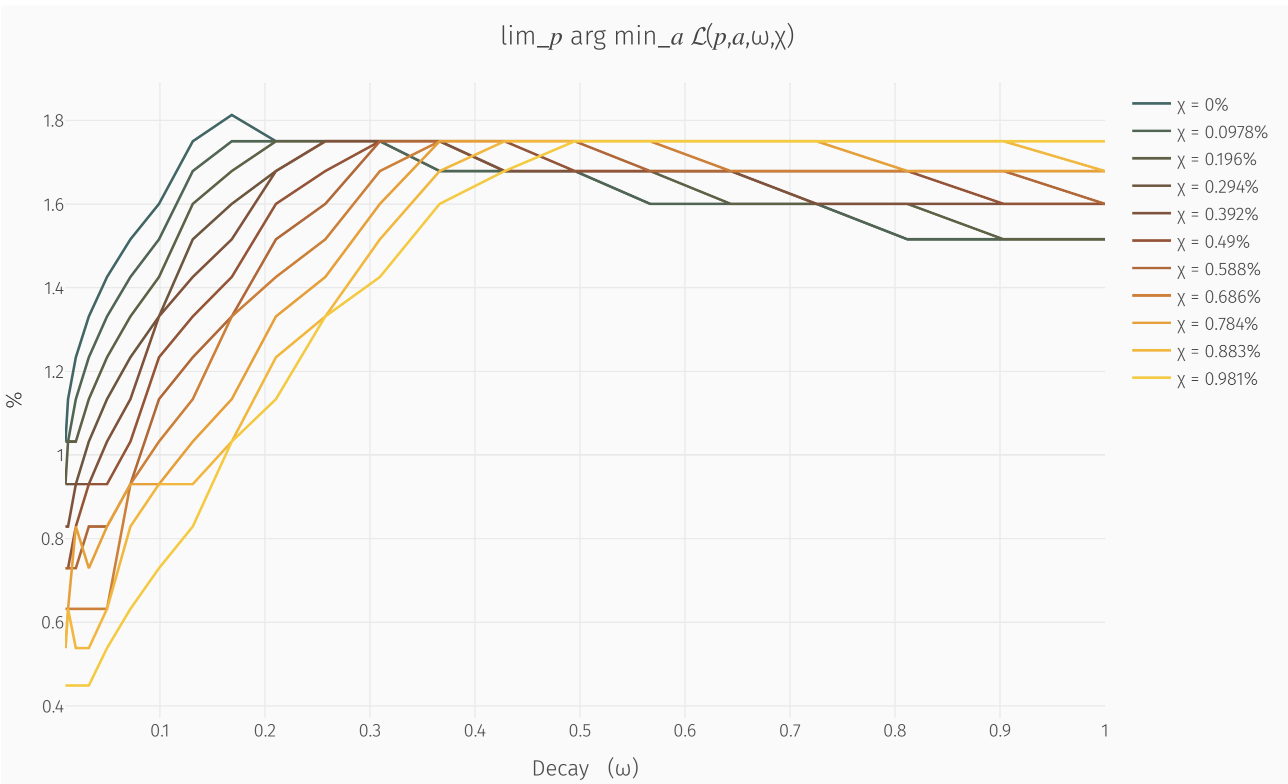

Figure 10: Initial inflation choice across announcements