$$\frac{\partial}{\partial t}\left(\log \lambda(t)\right) = \rho - r(t)$$

$$\log \lambda(t) = \log \lambda(0) + \int_0^t [\rho - r(s)]ds$$

$$\int_0^T d\log(\lambda(t)) = \int_0^T [\rho - r(t)]dt$$

$$\log \lambda(T) - \log \lambda(0) = \int_0^T [\rho - r(t)]dt$$

USE (2) :

$$\lambda(t) = u'(c^*) = \lambda(0)\exp\left(-\int_0^t [r(s) - \rho]ds\right)$$

(3) TRANSVERSALITY :

$$\lim_{t\to\infty} e^{-(\rho-m)} a(t)\lambda(t) = 0$$

(FINITE HORIZON : $a(T) = 0$)

↳ EITHER NEED $a(t) \geq$ LOWER BOUND

OR

↳ ADMISSIBLE CONTROLS / STATES ARE FINITE FOR ALL t

COMBINE (2) AND (3). THEN,

NO PONZI HOLDS WITH EQUALITY

COMBINE $u'(c(t)) = \lambda(t)$ AND

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -(r(t) - \rho)$$

$$\frac{\partial}{\partial t} u'(c(t)) = u''(c(t))\,\dot{c}(t) = \dot{\lambda}(t)$$

$$\Rightarrow u''(c(t))\,\dot{c}(t) = -(r(t)-\rho)\,\lambda(t)$$

$$= -(r(t)-\rho)\,u'(c(t))$$

$$\frac{\dot{c}(t)}{c(t)} = -\underbrace{\frac{u'(c(t))}{c(t)\,u''(c(t))}}_{\substack{\text{ELASTICITY OF} \\ \text{INTERTEMPORAL} \\ \text{SUBSTITUTION} \\ := [\theta(t)]^{-1}}} (r(t) - \rho)$$

ELASTICITY B/W t, s:

$$\theta(t,s) := \frac{\partial\left(\log\left[u'(c(t))/u'(c(s))\right]\right)}{\partial\left(\log\left[c(t)/c(s)\right]\right)}$$

$$\lim_{s\to t} \theta(t,s) = \theta(t)$$

USE FIRM OPTIMALITY:

$$f'(k) = R(t)$$

$$r(t) = f'(k) - \delta$$

ZERO PROFITS:

$$w(t) = f(k) - f'(k)k$$

USE MARKET CLEARING:

$$a(t) = k(t)$$

(SINCE BONDS IN ZERO NET SUPPLY)

BUDGET CONSTRAINT:

$$\dot{a}(t) = a(t)(r(t) - n) + w(t) - c(t)$$

$$\dot{k}(t) = k(t)(f'(k) - \delta - n) + f(k) - f'(k)k(t) - c(t)$$

$$\boxed{\dot{k}(t) = f(k(t)) - c(t) - (\delta + n)k(t)}$$

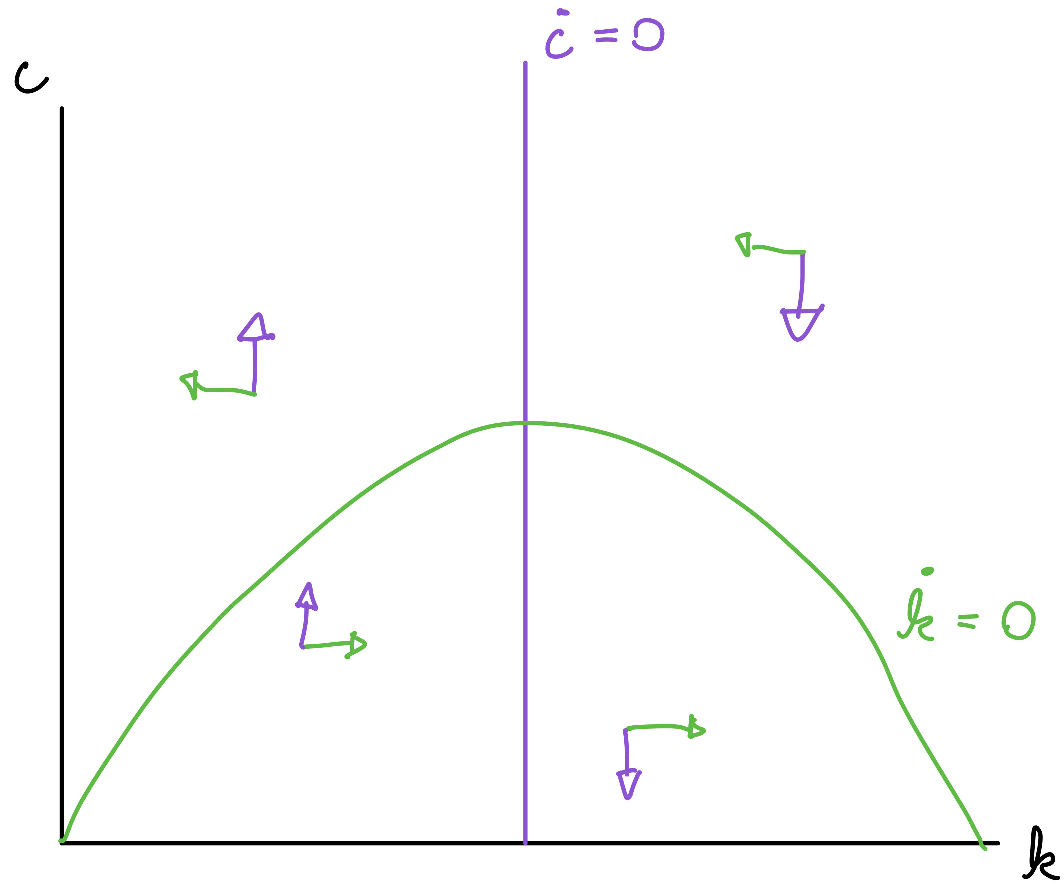
$\boxed{k(0) = k_0}$ + $\boxed{\text{TRANSVERSALITY}}$

$$\frac{\dot{c}(t)}{c(t)} = -\frac{u'(c(t))}{c(t)\,u''(c(t))}(r(t) - \rho)$$

$$\boxed{\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta(t)}\left(f'(k(t)) - \delta - \rho\right)}$$

$\dot{c} = 0 \Rightarrow f'(k) = \delta + \rho$

PHASE DIAGRAM



$$\dot{k} = 0 \Rightarrow 0 = f(k) - c - (\delta + n)k$$

$$\Rightarrow c = f(k) - (\delta + n)k$$