FEDERAL RESERVE BANK *of* NEW YORK

Investing in Capacity: Long-run Effects of Rational Inattention

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Motivation

- Amount of information available to market participants has grown over time
- Previous studies have shown the importance of information processing constraints for real decision, e.g. consumption, portfolio allocation...
- What about investing in information processing capacity?

This paper

- examine the case of perfectly elastic investment in information
- consider implications for the long-run behavior of consumption, portfolio allocation, beliefs
- compare to two standard set-ups:
  - “standard” learning (fixed, exogenous signal precision)
  - fixed information processing capacity

Results Preview

- different steady-states for the level of uncertainty, signal precision, information transmission rate

When agents are more uncertain in the short run:

- If agents invest in capacity, converge to a higher level of uncertainty and lower level of signal precision
- The mean reversion rate of the belief around the true realization is higher

When agents are more confident in the short run:

- If agents invest in capacity, converge to a lower level of uncertainty and higher level of signal precision
- The mean reversion rate of the belief around the true realization is higher

Economy structure

- Risky asset return:

$$d\eta_t = \theta_t + dz_{\eta t}$$

- Expected risky asset return:

$$d\theta_t = dz_{\theta t}$$

- External signal:

$$d\epsilon_t = \theta_t dt + n_t^{-\frac{1}{2}} dz_{\epsilon t}$$

Kalman-Bucy filter

Denote by $\mathcal{F}_t = \sigma\left\{\epsilon_s,\ \eta_s : s \leq t\right\}$ the time t information set of the representative agent in the economy. Then the agent's inference at time t of the expected risky asset return θ has a Gaussian distribution: $\theta|\mathcal{F}_t \sim N(\hat{\theta}_t, \gamma_t)$, with the inferred expected growth rate, $\hat{g}_t$, evolving according to:

$$d\hat{\theta}_t = \gamma_t d\nu_{\eta t} + \gamma_t \sqrt{n_t} d\nu_{\epsilon t},$$

and the conditional variance of the belief, γ_t, according to:

$$\frac{d\gamma_t}{dt} = 1 - \gamma_t^2 \left(1 + n_t\right).$$

Here, $d\nu_{\eta t}$ and $d\nu_{\epsilon t}$ are independent innovations of the standard Brownian motion under $\mathcal{F}_t$, given, respectively, by:

$$\begin{aligned} d\nu_{\eta t} &= d\eta_t - \hat{\theta}_t dt \\ d\nu_{\epsilon t} &= \sqrt{n_t}\left(d\epsilon_t - \hat{\theta}_t dt\right). \end{aligned}$$

Under partial information:

- Risky asset return:

$$d\eta_t = \hat{\theta}_t + d\nu_{\eta t}$$

$$d\theta_t = dz_{\theta t}$$

- External signal:

$$d\epsilon_t = \hat{\theta}_t dt + n_t^{-\frac{1}{2}} d\nu_{\epsilon t}$$

Optimization problem

The representative agent solves:

$$\max_{\{c_t, n_t, \omega_t\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} \log c_t dt\right]$$

s.t.

$$\begin{aligned} dW_t &= \omega_t W_t \left(d\eta_t - rdt\right) + rW_t dt - c_t dt - k(n_t) W_t dt \\ &= \omega_t W_t \left(\hat{\theta}_t - r\right) dt + rW_t dt - c_t dt - k(n_t) W_t dt + \omega_t W_t d\nu_{\eta t} \end{aligned}$$

- $k(n_t) = n_t^2/2$: cost of acquiring a signal of precision n_t

HJB equation

$$\rho J = \max_{\{c_t, n_t, \omega_t\}} \log c_t + J_W \left(\omega_t W_t \left(\hat{\theta}_t - r \right) + r W_t - c_t - k(n_t) W_t \right)$$
$$+ J_\gamma \left(1 - \gamma_t^2 (1 + n_t) \right) + \frac{1}{2} J_{WW} \omega_t^2 W_t^2 + \frac{1}{2} J_{\hat{\theta}\hat{\theta}} \gamma_t^2 (1 + n_t)$$
$$+ J_{W\hat{\theta}} \gamma_t \omega_t W_t + \phi_t n_t.$$

Guess:

$$J(W_t, \hat{\theta}_t, \gamma_t) = J_0 + \frac{1}{\rho} \log W_t + \frac{(\hat{\theta}_t - r)^2}{2\rho^2} + \Gamma(\gamma_t)$$

First order conditions

$$
\begin{aligned}
[c_t] : && c_t &= \rho W_t \\
[\omega_t] : && \omega_t &= \hat{\theta}_t - r \\
[n_t] : && n_t &= \rho \gamma_t^2 \left(\frac{1}{2\rho^2} - \Gamma' \right) + \phi_t
\end{aligned}
$$

$$
J_0 = \frac{\log \rho + 2(r-1)\rho - 1}{2\rho^2}
$$

$$
\rho \Gamma = - \frac{\left[\rho \gamma_t^2 \left(\frac{1}{2\rho^2} - \Gamma' \right) - 1 \right]^2}{2\rho} + \Gamma'
$$

Two benchmarks

1. Standard learning:

$$d\epsilon_t = \theta_t dt + \frac{1}{\sqrt{n}} dz_{\epsilon t}$$

2. Fixed information processing capacity:

$$\kappa = \frac{1}{2}\gamma_t(1 + n_t)$$

Standard learning

Optimization

$$\max_{\{c_t,\omega_t\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} \log c_t dt\right]$$

s.t.

$$\begin{aligned}
dW_t &= \omega_t W_t(\hat{\theta}_t - r)dt + rW_t dt - c_t dt + \omega_t W_t d\nu_{\eta t} \\
d\hat{\theta}_t &= \gamma_t d\nu_{\eta t} + \gamma_t \sqrt{n} d\nu_{\epsilon t} \\
\frac{d\gamma_t}{dt} &= 1 - \gamma_t^2(1+n)
\end{aligned}$$

Standard learning

HJB equation

$$\rho J^2 = \max_{c_t,\omega_t} \log c_t + J^2_W \left(\omega_t W_t(\hat{\theta}_t - r) + rW_t - c_t\right) + J^2_\gamma \left(1 - \gamma_t^2(1+n)\right) + \frac{1}{2} J^2_{WW}\omega_t^2 W_t^2 + \frac{1}{2} J^2_{\hat{\theta}\hat{\theta}}\gamma_t^2(1+n) + J^2_{W\hat{\theta}}\omega_t W_t$$

Value function

$$J^2(W_t, \hat{\theta}_t, \gamma_t) = \frac{1}{\rho}\log W_t + \frac{(\hat{\theta}_t - r)^2}{2\rho^2} + \frac{\rho\log\rho + r - \rho}{\rho^2} + \Gamma_2(\gamma)$$

$$\rho\Gamma_2 = \Gamma_2'(1 - \gamma_t^2(1+n)) + \frac{\gamma_t^2}{2\rho^2}(1+n)$$

Fixed capacity

Optimization

$$\max_{\{c_t,\omega_t\}} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t} \log c_t dt\right]$$

s.t.

$$dW_t = \omega_t W_t(\hat{\theta}_t - r)dt + rW_t dt - c_t dt - \frac{1}{2}\left(\frac{2\kappa - \gamma_t}{\gamma_t}\right)^2 dt + \omega_t W_t d\nu_{\eta t}$$

$$d\hat{\theta}_t = \gamma_t d\nu_{\eta t} + \gamma_t \sqrt{\frac{2\kappa - \gamma_t}{\gamma_t}} d\nu_{\epsilon t}$$

$$\frac{d\gamma_t}{dt} = 1 - 2\gamma_t \kappa$$

Fixed capacity

HJB equation

$$\rho J^3 = \max_{c_t,\omega_t} \log c_t + J^3_W \left(\omega_t W_t(\hat{\theta}_t - r) + rW_t - c_t - \frac{1}{2}\left(\frac{2\kappa - \gamma_t}{\gamma_t}\right)^2 \right)$$
$$+ J^3_\gamma (1 - 2\gamma_t\kappa) + \frac{1}{2} J^3_{WW}\omega_t^2 W_t^2 + J^3_{\hat{\theta}\hat{\theta}}\gamma_t\kappa + J^3_{W\hat{\theta}}\omega_t W_t$$

Value function

$$J^3(W_t, \hat{\theta}_t, \gamma_t) = \frac{1}{\rho}\log W_t + \frac{(\hat{\theta}_t - r)^2}{2\rho^2} + \frac{\rho\log\rho + r - \rho}{\rho^2} + \Gamma_3(\gamma)$$
$$\rho\Gamma_3 = -\frac{1}{2}\left(\frac{2\kappa - \gamma_t}{\gamma_t}\right)^2 + \Gamma_3'(1 - 2\kappa\gamma_t) + \frac{\gamma_t\kappa}{\rho^2}$$

Steady state

$$\frac{d\gamma_t}{dt} = 0$$

	γ_{ss}	n_{ss}
Learning	$\sqrt{\frac{1}{1+n}}$	n
Fixed capacity	$\frac{1}{2\kappa}$	$4\kappa^2 - 1$
Capacity Investment		$\frac{1}{\gamma_{ss}^2} - 1$

Learning: steady state

Fixed capacity steady state

Endogenous capacity steady state

Evolution of uncertainty

Evolution of signal precision

Infromation transmission rates

Mean reversion rates

Next steps

Irreversible investment in capacity

- So far, considered the case when cost is per unit of information
- What if the cost is per unit of additional capacity?

Multiple agents