

Aggregate Investment and Stock Returns*

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Abstract

We study the relation between returns on the aggregate stock market and aggregate real investment. While it is well known that the aggregate investment rate is negatively correlated with subsequent excess stock market returns, we find that it is also positively correlated with future stock market volatility. Thus, conditionally on past aggregate investment, the mean-variance tradeoff in aggregate stock returns is negative. We interpret these patterns within a general equilibrium production economy. In our model, investment is determined endogenously in response to two types of shocks: shocks to productivity and preference shocks affecting discount rates. Preference shocks affect expected stock returns, the aggregate investment rate, and stock return volatility in equilibrium, helping the model reproduce the empirical relations between these variables. Thus, our results emphasize that the time-varying price of aggregate risk plays an important role in shaping the aggregate investment dynamics.

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1 Introduction

In this paper we explore the relation between aggregate real investment and stock market volatility, an important aspect of the broader relation between financial markets and the real economy. In a recent influential paper, Bloom (2009) analyzes the impact of large transient volatility shocks on aggregate investment. We focus on a different aspect of the joint dynamics of volatility and investment, relating aggregate investment to persistent changes in return volatility. We document a new empirical pattern: high aggregate investment rate forecasts persistently high subsequent market volatility. It is well known (e.g., Abel (1983), Caballero (1991)) that the sign of the investment-volatility relation depends on the structure of the economic environment. To help narrow down the range of possible structural explanations for the observed positive correlation between aggregate real investment and expected future stock market volatility, we rely on the additional empirical patterns in the joint dynamics of stock returns, investment, and output.

An important feature of the stock return-investment dynamics is negative correlation between aggregate investment rate and subsequent excess stock market returns, studied in Cochrane (1991).¹

In our model, time-varying discount rates generate both the negative relation between investment and future excess returns and the positive relation between aggregate investment and future stock market volatility. The first relation is well understood. It is consistent with the basic partial-equilibrium intuition that, *ceteris paribus*, an exogenous decline in discount rates should increase the net present value of potential investment projects, and thus should raise the aggregate investment rate.

¹The relation between investment and subsequent excess stock market returns has also been studied in the cross-section of firms, e.g., Titman, Wei, and Xie (2004), Chen, Novy-Marx, and Zhang (2010), Kogan and Papanikolaou (2010).

To see the intuition behind the second relation, consider the classic Gordon model for stock valuation. The price of the stock is proportional to the expected future dividend, and inversely proportional to the difference between the expected growth rate of dividends, and the discount rate, both assumed constant:

$$P_0 = \frac{E_0[D_1]}{r - g}, \quad (1)$$

where time is discrete, P_0 is the stock price at time 0, $E_0[D_1]$ is the expected time-1 dividend, r denotes the cost of capital, and g denotes the expected dividend growth rate. Assume, furthermore, that dividend growth is homoscedastic, so heteroscedastic stock return volatility is not generated mechanically by a similar pattern in cash flows. Consider a comparative-statics experiment: holding the expected future dividends fixed, reduce the discount rate by a small amount. This has an effect of increasing the stock price at time 0, which is a well-known effect of time-varying discount rates on the volatility in stock returns. Note that the magnitude of the impact of a discount rate change on the stock price depends on the initial difference between the discount rate and the expected growth rate: if $r - g$ is relatively low, the same change in the discount rate has larger impact on the stock price than it would at higher levels of $r - g$. This simple observation prompts a conjecture: if discount rates experience homoscedastic shocks, an exogenous decline in discount rates should give rise to higher future return volatility. Since a decline in discount rates also naturally leads to an increase in the aggregate investment rate, we thus conclude that time-varying discount rates may give rise to a positive correlation between real investment and future stock market volatility.

The above conjecture is based on ad hoc arguments ignoring the general equilibrium considerations and liberally using comparative statics in lieu of rigorous dynamic analysis.

We formalize these arguments using a general-equilibrium production economy model. The economy in our model is affected by two types of shocks: productivity shocks and preference shocks. Our framework is very similar to canonical real business cycle models in its treatment of production. The only deviation from the standard setting is in our assumption that the representative household is subject to preference shocks. Effectively, preference shocks generate exogenous variation in risk aversion of the representative household, and with it variation in the market prices of risk. We calibrate our model to match the key unconditional moments of consumption growth and financial asset returns. We then verify that our model generates the same qualitative predictive relations as we document empirically and comes close in replicating the magnitude of the observed effects.

Our analysis further supports the idea that accounting for the time-varying price of risk in financial markets is important for understanding the dynamics of real economic activity. Modern asset pricing literature has emphasized the significance of time-varying price of risk, or return predictability, for understanding the key properties of asset return behavior, such as excess volatility of asset returns and high equity premium (e.g., Campbell and Cochrane (1999), Cochrane (1999)). Our paper adds to this body of work by arguing that time-varying price of risk may also be the cause of persistent changes in return volatility that we document. Thus, we tie together the core asset pricing results on return predictability and the growing literature on the connections between real economic activity and time-varying uncertainty (e.g., Bernanke (1983), Leahy and Whited (1996), Bloom, Bond, and Reenen (2007), Bloom (2009)). As shown in Bloom (2009), stock market volatility is a key indicator of economic uncertainty. Our analysis in this paper offers an economic interpretation of the empirical relations between market volatility and real investment.

The rest of the paper is organized as follows. Section 2 describes the data and empirical

results. Section 3 presents the theoretical model. Section 4 presents calibration results and robustness checks. Section 5 concludes.

2 Empirical Results

2.1 Data and procedures

Our sample starts 1947Q1 and ends 2009Q4 for a total of 252 quarters.² We use lowercase letters for logs of all variables throughout this section and the rest of the paper.

As a measure of aggregate stock returns, we use returns on the CRSP value weighted portfolio, available from Kenneth French’s website. We construct quarterly returns, r_t ,³ from the daily returns. To construct excess returns, we subtract the three-month T-bill rate, r_t^f , available in Kenneth French’s website.

We also use quarterly data on realized volatility. Specifically, we construct a quarterly series vol_t , defined as the log of the standard deviation of daily returns within quarter t .⁴ We find that our results are robust to Winsorizing the volatility series or using alternative measures of realized volatility, such as absolute values of quarterly returns.

Quarterly data for the macroeconomic variables is from U.S. National Income and Product Accounts (NIPA) available directly from the Bureau of Economic Analysis (BEA).⁵ For GDP, Y_t , we use real⁶ business gross domestic product (Table 1.3.6, line 2). For investment,

²We find that excluding the 2008-2009 period from our sample has no effect on the qualitative results, and has only minor effect on the point estimates. Thus, our conclusions are robust to excluding the financial crises period. We also find that our conclusions are unchanged if we exclude the immediate post-war period of 1947-1952.

³ r_t is cumulative log return on the value-weighted portfolio of NYSE, NASDAQ, and AMEX stocks.

⁴For example, in order to obtain the standard deviation of the market return for 1952Q1, we use the standard deviation of the time series of daily market returns from January 1, 1952 to March 31, 1952

⁵In 2003, the BEA undertook comprehensive revisions of all NIPA data series. Our data incorporates these revisions.

⁶In billions of chained (2005) dollars.

I_t , we use real nonresidential fixed private domestic investment (we multiply the real index in Table 1.5.3, line 28, by the nominal investment in 2005 —the base year for the real index— in Table 1.5.5, line 28 and then we divide by 100). Quarterly capital stock values, K_t , are interpolated from annual values using the quarterly investment series I_t . The annual series for real capital stock is private nonresidential fixed assets (we multiply the real index in NIPA’s fixed assets table 4.2, line 1, by the nominal capital in 2005 —the base year for the real index— in table 4.1, line 2 and then we divide by 100), with year-end 1946 as the starting point. For each quarter, a fraction of the annual capital increment is added to the current end-of-year stock, with the fraction given as the year’s investment to date, divided by total investment for that year. Unlike the standard inventory-based method of constructing capital stock (e.g., Cochrane (1991)), the above method does not rely on any particular model of capital accumulation.

2.2 Empirical findings

Summary statistics

We use two variables in predictive regressions. The first is the natural log of the investment rate, $i_t - k_t = \ln(I_t/K_t)$, where investment rate is measured as the ratio of the quarterly GPDI to the end-of-quarter capital stock (our timing convention is analogous to the one used in defining trailing dividend yield). The second variable, $y_t - k_t = \ln(Y_t/K_t)$, is the log of the ratio of quarterly output to the end-of-quarter capital stock. We view this variable as a proxy for average profitability in the economy.⁷

We start by summarizing the key moments of investment, profitability, and financial asset returns in our sample.

⁷Our interpretation is justified as long as the output share of capital is approximately constant. Alternatively, one may simply view the two variables as jointly approximating the state vector in the economy.

[Table 1]

In addition to the first two moments of the key variables, we estimate their autocorrelation. We find that profitability is highly persistent in our sample, with an eight-quarter autocorrelation coefficient of 0.56. Aggregate investment rate shows less persistence, with autocorrelations of 0.48 and below beyond the eight-quarter horizon. Consistent with commonly reported results, consumption and output growth rates exhibit very little autocorrelation. Investment growth is also close to being uncorrelated over time. Stock returns are virtually uncorrelated over time, but stock return volatility is persistent. Autocorrelations of volatility decline at a relatively slow rate, starting at 0.58 at a one-quarter horizon and declining to 0.34 and 0.10 at four and eight-quarter horizons respectively. This pattern of decline suggests that market volatility possesses more persistence than what could be generated by a simple first-order autoregressive specification. Our results in Table 4 reinforce this observation.

Predictability of excess stock returns and return volatility

We first analyze predictability of stock returns. Tables 2 and 3 report predictive regressions of single-quarter and multi-quarter excess stock returns on lagged values of investment rate and profitability. Our regressions extend the results in Cochrane (1999) to our longer sample and to a more general specification. We run two predictive regressions:

$$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h} \quad (2)$$

and

$$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h} \quad (3)$$

Before running these regressions, we de-trend all right-hand-side variables. We do this so that low-frequency movements in the variables under consideration do not drive our results, since we cannot evaluate statistical significance of such effects in our sample. De-trending has little effect on the predictive regressions for returns, but is potentially important for predictive regressions of return volatility below, since return volatility exhibits some low-frequency persistence. As a robustness check, we perform the same regression on the original series, and find qualitatively similar results.

In the second specification, we include profitability as a second predictive variable in addition to the investment rate. The predictive relation between the investment rate and future excess stock returns indicates time-variation in expected stock returns. According to conventional intuition, the aggregate investment rate is negatively affected by the discount rates on future projects because higher discount rates imply lower net present value of cash flows produced by new investments. Therefore, as long as discount rates on cash flows from new investments and on those produced by existing assets are not too different, it is natural that the investment rate is negatively correlated with future excess stock returns. This argument can be refined by observing that investment decisions are affected by profitability of new investment projects in addition to their discount rates. Persistence of aggregate profitability (see Table 1) suggests that lagged aggregate profitability may be a useful predictor of future profitability of new investments, as long as profitability of new projects is not too different from profitability of existing physical assets. Therefore, aggregate profitability is a potentially useful control in predictive regression of excess stock returns on the lagged aggregate investment rate.

It is worth noting at this point that, while our regressions are inspired by the common intuition, our interpretation of the empirical results relies on a fully specified general equi-

librium model that we develop in the following sections. In the model, some of the vague statements used in the previous intuitive argument are not necessary. All of the relevant variables are derived endogenously in equilibrium, and relations between them can be quantified. For instance, discount rates on cash flows from new investments are equal to those on cash flows from existing assets, and thus the aggregate investment rate is a useful predictor of future excess stock returns. In anticipation of the formal equilibrium analysis below, we note that in our model economy there are two structural shocks: shocks to productivity and preference shocks. Discount rates, as well as profitability, are affected by both shocks in equilibrium. Without arguing further that investment rate and profitability are each primarily affected by a single structural shock, the two-shock structure of the model implies that the two-variable predictive regression emerges as a natural agnostic linear approximation to the model's equilibrium relation between the conditional moments of financial asset returns and equilibrium state variables.

[Table 2]

As in Cochrane (1999), we find that the investment rate predicts future excess stock returns negatively. Adding lagged profitability as a control does not seem to affect the results substantially, and coefficients on lagged profitability are not statistically significant.

In Table 5, we report analogous predictive regressions for a cumulative sum of excess stock returns over multiple quarters. These regressions highlight the joint significance of predictability in excess returns at multiple horizons. The explanatory power of these regressions is quite low, but there is evidence of predictability in returns at horizons up to 16 quarters.

[Table 3]

We next summarize the results on predictability of stock return volatility in Tables 4

and 5. In Table 4, we report the results of a univariate predictive regression of volatility on lagged investment rate,

$$\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h} \quad (4)$$

and of multivariate regressions, adding lagged profitability and lagged realized volatility to the forecasting equation:

$$\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1} \quad (5)$$

and

$$\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3\text{vol}_{t-1} + \varepsilon_{t,t+h+1} \quad (6)$$

As in Tables 3, we use both the investment rate and profitability in predictive regressions.

[Table 4]

The investment rate predicts future return volatility with a positive sign at all horizons up to 16 quarters. This pattern is stronger and has higher statistical significance in a two-variable regression, which also includes aggregate profitability.

In the third panel of Table 4, we add lagged realized volatility to the regression. We do this for two reasons. First, as shown in Bloom (2009), market volatility has a negative impact on future investment at short horizons. Our model is not flexible enough to capture this pattern, but such a relation may affect our empirical findings. In particular, at relatively short horizons, it may weaken the positive relation between investment rate and future market volatility produced by our current specification. We find that adding lagged volatility leaves the results qualitatively unchanged, while significance and point estimates of coefficients on investment rate increase at short horizons.

Second, realized volatility is a noisy proxy for the true conditional volatility, and it is

useful to see how well realized volatility can predict its own future values compared to the two macro-economic state variables we use in our regressions. Looking at the first panel of Table 4, we find that investment rate and profitability jointly explain 22 percent of variation in realized return volatility at an eight-quarter horizon, while lagged realized volatility adds only 2 of explanatory power when added to the regression. percent of its own future variation. The pattern of R^2 's is consistent with this observation. Adding lagged volatility to the forecasting regression significantly boosts its explanatory power at short horizons, for instance, raising the R^2 from 0.22 to 0.40 at a one-quarter horizon, but has negligible effect at horizons longer than six quarters. This indicates that there are sources of short-horizon predictability in return volatility not captured by our predictive variables.

[Table 5]

In Table 5, we report analogous predictive regressions for a cumulative sum of realized intra-quarter volatility over multiple quarters. These regressions highlight the joint significance of predictability in volatility at multiple horizons. We find that the investment rate is a highly statistically significant predictor of future stock return volatility at horizons up to 16 quarters. Profitability enters negatively in a multi-variate forecasting regression, and is marginally statistically significant. Lagged realized volatility enters positively, and is highly statistically significant. The pattern of coefficients and t-statistics on lagged volatility is consistent with its predictive ability being relatively short-lived.

3 The Model

3.1 Formulation

Technology

We assume that there exists a competitive representative firm. This firm uses capital and labor to produce a single consumption good. We denote the capital stock by K_t , the input of labor by L_t , and the flow of output by Y_t . We assume the standard Cobb-Douglas production function

$$Y_t = e^{x_t} K_t^\alpha L_t^{1-\alpha}, \quad (7)$$

where the productivity shock x_t follows

$$dx_t = \left(\mu - \frac{\sigma_X^2}{2} \right) dt + \sigma_X dW_t.$$

We assume that capital depreciates at the constant rate δ and can be replenished through investment. Denoting the investment rate by i_t ,

$$dK_t = (i_t - \delta) K_t dt. \quad (8)$$

We assume that new capital can be created from the consumption good subject to convex adjustment costs, so that the flow cost of creating new capital at the investment rate i_t is given by

$$I_t = \frac{a}{\lambda} i_t^\lambda K_t, \quad (9)$$

where $\lambda > 1$. Thus, the marginal cost of capital creation, measured in units of the consumption good, is positively related to the investment rate.

Households

We model households as a representative consumer. The representative agent owns the representative firm and supplies labor competitively in the labor market. We assume that the representative household is endowed with a constant flow of labor, normalized to one, which it supplies inelastically.

We describe preferences of the representative household by a time-separable iso-elastic utility function subject to preference shocks. In particular, the representative household evaluates consumption streams $\{C_t\}$ according to

$$E_0 \left[\int_0^\infty e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right].$$

We assume that the preference shock ξ_t evolves according to

$$d\xi_t = (b_t - \frac{\sigma_t^2}{2})dt + \sigma_t dW_t, \quad (10)$$

$$d\sigma_t = -\theta\sigma_t dt + v dW_{\sigma t}, \quad W_{\sigma t} = \rho W_t + \sqrt{1-\rho^2} W'_t. \quad (11)$$

In our specification, stochastic σ_t implies that the representative household has a state-dependent marginal utility with respect to consumption. Specifically, state-dependence is driven by the same shocks as the productivity process. This specification can be viewed as a reduced-form description of time-varying aversion to risk or time-varying beliefs.⁸⁹ The

⁸⁹If we set $b_t = 0$ in the definition of ξ_t , our preference specification is isomorphic to a model of a household with the same isoelastic preferences but distorted beliefs. In particular, under the distorted beliefs, the Brownian motion (W_t) that drives the productivity process acquires a drift σ_t . Thus, the representative household exhibits the time-varying degree of optimism or pessimism, and perceives the productivity process as

$$dx_t = \left(\mu - \frac{\sigma_X^2}{2} + \sigma_t \right) dt + \sigma_X d\widetilde{W}_t, \quad (12)$$

where \widetilde{W}_t is a Brownian motion under the subjective distorted beliefs of the representative household. Clearly, such a distortion in beliefs affects the equilibrium discount rates.

⁹⁰Our reduced-form description may reflect a variety of economic phenomena. The most immediate connection is with the models emphasizing habit formation, e.g., Campbell and Cochrane (1999). Gallmeyer,

exact interpretation is not critical for our analysis. Our interpretation of the empirical patterns is based on the time-varying discount rates, and does not hinge on the exact source of discount rate variation. The process b_t plays the role of time-varying subjective rate of time preferences. This process does not affect the qualitative implications of our model and is introduced for purely technical reasons, helping stabilize the risk-free interest rate in equilibrium.

Financial markets and asset prices

We assume that there exists a complete set of zero-net-supply state-contingent claims, prices of which are summarized by the state-price density process $\pi > 0$ and that the time- t price of any long-lived asset with cash flow X is given by the bubble-free pricing equation

$$E_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} X_s \right] dt.$$

We denote the equilibrium short-term risk-free rate by r_t .

In addition to the state-contingent claims, we assume that the representative household is endowed with a single stock share, which is a claim on the dividends of the representative firm. Thus, the representative firm is all equity financed.¹⁰ The dividends are equal to output net of investment costs and labor costs. Denoting the wages paid by the representative firm by w , the aggregate dividend flow rate is

$$D_t = Y_t - \frac{a}{\lambda} i_t^\lambda K_t - w_t L_t. \quad (13)$$

Hollifield, and Zin (2005) use a similar description of preferences to capture time-varying risk aversion. Time-varying discount rates also arise naturally as a result of the dynamic wealth re-distribution across a heterogeneous population of market participants, e.g., Chan and Kogan (2002), Gârleanu and Panageas (2007), Guvenen (2009).

¹⁰The assumption of equity financing for the representative firm is without loss of generality, since the assumptions of the Modigliani-Miller theorem hold in our setting, and therefore the choice of capital structure does not affect equilibrium policies.

3.2 Equilibrium

We adopt the standard definition of competitive equilibrium. In equilibrium, the representative household and the representative firm take prices of state-contingent claims and the wage rate as given. The representative household maximizes its expected utility, while the representative firm maximizes its market value. All markets clear.

Definition 1 *The competitive equilibrium is described by a collection of stochastic processes π^* , w_t^* , L^* , C^* , Y^* , K^* , i^* , and D^* , such that*

1. Y^* , K^* , L^* , and i^* satisfy the technological constraints (7) and (8).
2. C^* and L^* maximize the representative household's objective, taking the state-price density, dividends, and wages as given,

$$\max_{\{C, L\}} E_0 \left[\int_0^\infty e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right],$$

subject to

$$E_0 \left[\int_0^\infty \frac{\pi_t^*}{\pi_0^*} (C_t - D_t^* - w_t^* L_t^*) dt \right] = 0;$$

3. i^* , L^* , and D_t^* maximize the representative firm's value, taking the state-price density as given,

$$\max_{\{i, L, D\}} E_0 \left[\int_0^\infty \frac{\pi_t^*}{\pi_0^*} D_t dt \right],$$

subject to (7), (8), and (13).

4. Labor market clears,

$$L_t^* = 1,$$

and consumption market clears,

$$C_t^* = D_t^* + w_t L_t^*.$$

3.3 Solution

Since financial markets in our model are frictionless and there are no externalities, equilibrium consumption and investment policies can be determined by solving the central planner's problem. The central planner maximizes the expected utility of the representative household

$$E_0 \left[\int_0^\infty e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]. \quad (14)$$

subject to the aggregate resource constraint

$$C_t + I_t = Y_t \quad (15)$$

and to (8), (9), (10), (11).

Equilibrium prices can be recovered from the central planner's solution for equilibrium quantities using individual optimality conditions:

$$\pi_t = e_t^{-\beta t + \xi_t} (C_t^*)^{-\gamma}, \quad (16)$$

and

$$w_t^* = (1 - \alpha) Y_t^*. \quad (17)$$

We solve for equilibrium numerically using finite-difference approximations. We first solve the dynamic program of the central planner and determine equilibrium consumption and investment policies, and the state-price density. We then compute the price of the aggregate stock market as the expected value of future dividends discounted with the equilibrium

state-price density. We derive the risk-free interest rate as the negative of the drift of the state-price density.

4 Calibration and Simulation Results

4.1 Parameter calibration

[Table 8]

The starting point of our calibration is the canonical real business-cycle model. Indeed, if our model had no preference shocks, $v = 0$, $\theta = 0$, $\rho = 0$, we could pick parameters that are standard in the literature, simplifying calibration. We find that if we set the parameters not relating to preference shocks in that manner, we can successfully match all the unconditional moments in Table 1 that do not relate to asset prices. We then pick the values of v , θ , and ρ to reproduce the empirically observed moments for asset prices and the conditional moments of Tables 2-5. This strategy is possible because most of the moments relating to quantities are decoupled from preference shock parameters. For example, the steady-state level of the investment rate is

$$E[i_t - k_t] = \frac{\mu - \sigma_X^2/2}{(1 - \alpha)} + \delta, \quad (18)$$

while the mean and volatility of output growth are given by

$$E[dy_t] = \frac{\mu - \sigma_X^2/2}{(1 - \alpha)} \quad (19)$$

$$\text{st.dev.}(dy_t) = \sigma_X, \quad (20)$$

which do not depend on any preference parameters.

We set the values for the model's production technology to $\alpha = 0.33$, $\mu = 0.015$, $\sigma_X = 0.03$, $a = 10$ and $\lambda = 5$, which are similar to those found in the literature. For the standard

preference parameters, we pick a discount parameter of $\beta = 0.02$ and a reasonable coefficient of relative risk aversion $\gamma = 10$.

We pick an auto-regressive coefficient for the preference shock of $\theta = 0.4$, which represents a half-life of about 7 quarters. Tables 4 and 5 show that this is exactly the horizon for which investment has its strongest predictive force. While this persistence parameter helps control the timing of predictability, we set the volatility of preference shocks to $\nu = 0.3$ to match the magnitude of predictability of stock volatility and returns. Finally, we set the correlation between shocks to productivity and preference shocks to $\rho = -0.9$. The negative correlation implies that times of low productivity coincide with times of high volatility.

To understand the mechanics of the model and gain further intuition into how different parameters affect our results, we analyze how macroeconomic variables and asset prices respond to preference shocks.

[Figure 1]

Figure 1 shows the steady-state probability distribution function of the two state variables, the preference shock σ and profitability $y_t - k_t$. Since $\rho < 0$, they are negatively correlated. Figure 2(a) shows how the investment rate and consumption (normalized by capital) behave as a function of the state variables. Investment is decreasing in the preference shock: a positive shock to marginal utility will, *ceteris paribus*, reduce investment and increase consumption. On the other hand, investment is increasing in profitability, since the latter is simply a capital-adjusted measure of productivity: $y_t - k_t = x_t + (\alpha - 1)k_t$.

Figure 3 shows the impulse-response functions of the key variables to a positive preference shock. Because the capital stock cannot change instantaneously but productivity x_t decreases when a positive preference shock hits the economy (because $\rho < 0$), profitability decreases after the shock. The representative agent's optimal response to a drop in productivity is to

reduce the investment rate. In addition, because the preference shock raises marginal utility, the representative agent has an added incentive to invest less and consume more. Both effects lead to a sharp decrease in investment. As the preference shock reverts to its mean, investment slowly returns to its steady-state levels, with the speed of adjustment controlled mainly by the convexity of adjustment costs.

Figure (4) shows the impulse response function for asset prices. Even though we have a general equilibrium model and cash flows are not constant, the partial equilibrium intuition of the Gordon formula given in the introduction still holds. In this case, an increase in the cost of capital is associated with persistently high expected stock returns and low volatility. Tables 9-12 confirm the result by replicating our empirical regressions using 2,500 sample paths generated by the model and then computing the averages of regression coefficients, t -statistics, and R^2 across the simulated samples replications.

[Table 9]

[Table 10]

[Table 11]

[Table 12]

Simulation results suggest that our equilibrium model captures, at least qualitatively, the key empirical patterns: the negative predictive relation between the investment rate and future excess stock returns, and the positive relation between the investment rate and future return volatility. Comparing the empirical numbers to simulation results, we identify two areas for future improvements.

First, excess stock returns in the model are much more predictable than in the data, as indicated by the high values of R^2 's in Tables 11, 12. This may be partly because too much of stock return volatility in the model is driven by the preference shocks. In addition,

since we know that high explanatory power for future excess stock returns can be obtained using financial valuation ratios (e.g., Campbell and Cochrane (1999)), it appears that the aggregate investment rate is a more precise proxy for the preference shocks in the model than it is in the data.

Second, the hump-shaped pattern of the regression coefficients of stock return volatility on the aggregate investment rate is an interesting feature of the data that is not captured by the model. In our model, even though the effects on returns and volatility are persistent, the largest responses occur contemporaneously with the arrival of shocks. Our model therefore matches the observed empirical patterns at the frequencies of 4 to 6 quarters and onwards, and over-estimates the effects of investment on volatility in the short run. A potential resolution of this discrepancy is to introduce transient components, either in the volatility of productivity or in productivity itself. Indeed, Bloom (2009) considers exactly these type of shocks and obtains a negative relation between investment and volatility of returns at short horizons.

5 Conclusion

In this paper we establish a new empirical fact: the aggregate investment rate is strongly positively correlated with future stock market volatility. Together with the well-known negative relationship between the investment rate and subsequent excess stock returns, this implies that, conditionally on the aggregate investment rate, stock market exhibits a negative mean-variance tradeoff. We interpret these empirical patterns using a general-equilibrium production economy model. In our model, the qualitative empirical correlation patterns among the aggregate investment and productivity on one hand, and the conditional moments of the stock market returns on the other hand, arise because of time-varying discount rates.

Thus, our paper emphasizes the importance of time-varying discount rates for understanding not only the behavior of financial markets, but also for interpreting the dynamics of the key macroeconomic variables.

We are working on extending our model to incorporate the negative short-term correlation between stock-return volatility and subsequent real investment. Together with the results obtained in this paper, the extended model should further clarify the respective roles played by the technology and preference shocks in shaping the observed joint dynamics of aggregate investment and financial asset returns.

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6 Appendix: Solution and Numerical Procedure

6.1 Central's Plannner's Problem

The value function $J(\xi, \sigma, x, K)$ of the central planner's problem satisfies the following HJB equation:

$$\begin{aligned} \max_i \left[e^\xi \frac{[e^x K^\alpha - \frac{a}{\lambda} i^\lambda K]^{1-\gamma}}{1-\gamma} + iK J_K \right] - \delta K J_K - \theta \sigma J_\sigma + (b - \frac{\sigma^2}{2}) J_\xi + \\ (\mu - \frac{\sigma_X^2}{2}) J_x + \frac{v^2}{2} J_{\sigma\sigma} + \frac{\sigma^2}{2} J_{\xi\xi} + \frac{\sigma_X^2}{2} J_{xx} + \rho \sigma_X v J_{x\sigma} + \rho \sigma v J_{\xi\sigma} + \sigma \sigma_X J_{x\xi} = \beta J. \end{aligned} \quad (21)$$

We look for solutions of the form

$$J(\xi, \sigma, x, K) = \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} V(\sigma, z), \quad (22)$$

where

$$z \equiv x + (\alpha - 1)k.$$

Substituting (22) back into (21) and using

$$\begin{aligned} J_\xi &= J_{\xi\xi} = \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} V, \\ K J_K &= \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} [(1-\gamma)V + (\alpha-1)V_z], \\ J_x &= J_{x\xi} = \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} V_z, \\ J_{xx} &= \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} V_{zz}, \\ J_{x\sigma} &= \frac{e^{\xi+(1-\gamma)k}}{1-\gamma} V_{z\sigma}, \end{aligned}$$

we obtain

$$(e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} + \mathcal{L}V = 0 \quad (23)$$

where Dynkin operator \mathcal{L} has the following form

$$\begin{aligned} \mathcal{L} = & (\rho v - \theta) \sigma \frac{\partial}{\partial \sigma} + \left[\mu + \sigma \sigma_X - \frac{\sigma_X^2}{2} - (1 - \alpha)(i^* - \delta) \right] \frac{\partial}{\partial z} + \\ & + \frac{v^2}{2} \frac{\partial^2}{\partial \sigma^2} + \frac{\sigma_X^2}{2} \frac{\partial^2}{\partial z^2} + \rho \sigma_X v \frac{\partial^2}{\partial \sigma \partial z} - [\beta - b - (1 - \gamma)(i^* - \delta)]. \end{aligned} \quad (24)$$

The optimal investment rate, i^* , satisfies

$$a(i^*)^{\lambda-1} = \left[V - \frac{1-\alpha}{1-\gamma} V_z \right] \left[e^z - \frac{a}{\lambda} (i^*)^\lambda \right]^\gamma. \quad (25)$$

6.2 Stock Price

The stock price is given by the discounted sum of dividends:

$$\begin{aligned} S_t &= e^{-\xi_t} \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t) + \xi_s} \left(\frac{C_t^*}{C_s^*} \right)^\gamma D_s ds \right] = e^{-\xi_t} (C_t^*)^\gamma \alpha (1 - \gamma) J(\xi_t, \sigma_t, x_t, K_t) - \\ & - (1 - \alpha) e^{-\xi_t} (C_t^*)^\gamma \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t) + \xi_s + (1-\gamma)k_s} \left(\frac{C_s^*}{K_s} \right)^{-\gamma} \frac{a}{\lambda} (i_s^*)^\lambda ds \right] = \\ & = K_t \left(\frac{C_t^*}{K_t} \right)^\gamma [\alpha V(\sigma_t, z_t) - (1 - \alpha) \Phi(\sigma_t, z_t)], \end{aligned} \quad (26)$$

where the Feynman-Katz forumal implies that $\Phi(\sigma_t, z_t)$ satisfies the following PDE

$$\frac{\frac{a}{\lambda} (i^*)^\lambda}{\left(e^z - \frac{a}{\lambda} (i^*)^\lambda \right)^\gamma} + \mathcal{L} \Phi = 0. \quad (27)$$

6.3 Steady state distribution

The joint steady state distribution of (σ, z) , $p_\infty(z, \sigma)$, satisfies the following Kolmogorov backward equation

$$\begin{aligned} - \left[\theta + (1 - \alpha) \frac{\partial i^*}{\partial z} \right] p_\infty(z, \sigma) &= -\theta \sigma \frac{\partial p_\infty(z, \sigma)}{\partial \sigma} + \\ &+ \left[\mu - \frac{\sigma_X^2}{2} - (1 - \alpha) (i^* - \delta) \right] \frac{\partial p_\infty(z, \sigma)}{\partial z} + \\ &+ \frac{v^2}{2} \frac{\partial^2 p_\infty(z, \sigma)}{\partial \sigma^2} + \frac{\sigma_X^2}{2} \frac{\partial^2 p_\infty(z, \sigma)}{\partial z^2} + \rho \sigma_X v \frac{\partial^2 p_\infty(z, \sigma)}{\partial \sigma \partial z}. \end{aligned} \quad (28)$$

This equation is solved so that the probability density function integrates to one:

$$\iint_{-\infty}^{\infty} p_\infty(z, \sigma) dz d\sigma = 1.$$

6.4 Numerical Procedure

We discretize the HJB equation on the (σ, z) using the following approximations:

$$V_z = \frac{V(z + \Delta_z, \sigma) - V(z, \sigma)}{\Delta_z}, \text{ if } \sigma \geq 0 \quad (29)$$

$$V_z = \frac{V(z, \sigma) - V(z - \Delta_z, \sigma)}{\Delta_z}, \text{ if } \sigma < 0 \quad (30)$$

$$V_\sigma = \frac{V(z, \sigma + \Delta_\sigma) - V(z, \sigma)}{\Delta_\sigma}, \text{ if } (\rho v - \theta) \sigma \geq 0 \quad (31)$$

$$V_\sigma = \frac{V(z, \sigma) - V(z, \sigma - \Delta_\sigma)}{\Delta_\sigma}, \text{ if } (\rho v - \theta) \sigma < 0$$

Since $(\mu + (1 - \alpha) \delta) \geq 0$, $(\sigma_X^2/2 + (1 - \alpha) i) \geq 0$, we use the one-sided approximation for the second derivatives:

$$V_{\sigma\sigma} = \frac{V(z, \sigma + \Delta_\sigma) + V(z, \sigma - \Delta_\sigma) - 2V(z, \sigma)}{\Delta_\sigma^2}$$

$$V_{zz} = \frac{V(z + \Delta_z, \sigma) + V(z - \Delta_z, \sigma) - 2V(z, \sigma)}{\Delta_z^2}.$$

For the cross-partials, we use

$$V_{\sigma z} = \frac{2V(z, \sigma) + V(z + \Delta_z, \sigma + \Delta_\sigma) + V(z - \Delta_z, \sigma - \Delta_\sigma)}{2\Delta_z \Delta_\sigma} - \left[\frac{V(z + \Delta_z, \sigma) + V(z - \Delta_z, \sigma) + V(z, \sigma + \Delta_\sigma) + V(z, \sigma - \Delta_\sigma)}{2\Delta_z \Delta_\sigma} \right], \quad (32)$$

if $\rho \geq 0$ and

$$V_{\sigma z} = - \left[\frac{2V(z, \sigma) + V(z + \Delta_z, \sigma - \Delta_\sigma) + V(z - \Delta_z, \sigma + \Delta_\sigma)}{2\Delta_z \Delta_\sigma} \right] + \quad (33)$$

$$+ \left[\frac{V(z + \Delta_z, \sigma) + V(z - \Delta_z, \sigma) + V(z, \sigma + \Delta_\sigma) + V(z, \sigma - \Delta_\sigma)}{2\Delta_z \Delta_\sigma} \right] \quad (34)$$

if $\rho < 0$. We define the transition probabilities between the discretized points to be:

$$\begin{aligned} p((z, \sigma) \rightarrow (z, \sigma + \Delta_\sigma) | i) &= \frac{1}{Q^\Delta} \left[[(\rho v - \theta) \sigma]^+ \Delta_\sigma \Delta_z^2 + \frac{v^2}{2} \Delta_z^2 - \frac{\sigma_X v |\rho|}{2} \Delta_\sigma \Delta_z \right] \\ p((z, \sigma) \rightarrow (z, \sigma - \Delta_\sigma) | i) &= \frac{1}{Q^\Delta} \left[[(\rho v - \theta) \sigma]^- \Delta_\sigma \Delta_z^2 + \frac{v^2}{2} \Delta_z^2 - \frac{\sigma_X v |\rho|}{2} \Delta_\sigma \Delta_z \right] \\ p((z, \sigma) \rightarrow (z + \Delta_z, \sigma) | i) &= \frac{1}{Q^\Delta} \left[(\mu + (1 - \alpha)\delta + [\sigma]^+ \sigma_X) \Delta_\sigma^2 \Delta_z + \frac{\sigma_X^2}{2} \Delta_\sigma^2 - \frac{\sigma_X v |\rho|}{2} \Delta_\sigma \Delta_z \right] \\ p((z, \sigma) \rightarrow (z - \Delta_z, \sigma) | i) &= \frac{1}{Q^\Delta} \left[\left[\frac{\sigma_X^2}{2} + (1 - \alpha)i^* + [\sigma]^- \sigma_X \right] \Delta_\sigma^2 \Delta_z + \frac{\sigma_X^2}{2} \Delta_\sigma^2 - \frac{\sigma_X v |\rho|}{2} \Delta_\sigma \Delta_z \right] \\ p((z, \sigma) \rightarrow (z + \Delta_z, \sigma + \Delta_\sigma) | i) &= p((z, \sigma) \rightarrow (z - \Delta_z, \sigma - \Delta_\sigma) | i) = \frac{1}{Q^\Delta} \left[\frac{\sigma_X v [\rho]^+}{2} \Delta_\sigma \Delta_z \right] \\ p((z, \sigma) \rightarrow (z - \Delta_z, \sigma + \Delta_\sigma) | i) &= p((z, \sigma) \rightarrow (z + \Delta_z, \sigma - \Delta_\sigma) | i) = \frac{1}{Q^\Delta} \left[\frac{\sigma_X v [\rho]^-}{2} \Delta_\sigma \Delta_z \right] \end{aligned}$$

where

$$\begin{aligned} Q^\Delta((z, \sigma), i) &= |(\rho v - \theta) \sigma| \Delta_\sigma \Delta_z^2 + \left[\mu + \sigma_X |\sigma| + \frac{\sigma_X^2}{2} + (1 - \alpha)(i^* + \delta) \right] \Delta_\sigma^2 \Delta_z + \\ &\quad + v^2 \Delta_z^2 + \sigma_X^2 \Delta_\sigma^2 - \sigma_X v |\rho| \Delta_\sigma \Delta_z \end{aligned}$$

and

$$\tilde{\beta} = \beta - b - (1 - \gamma)(i^* - \delta)$$

The transition probabilities are positive if

$$\frac{v\Delta_z}{|\rho|\sigma_X} > \Delta_\sigma > \frac{v|\rho|\Delta_z}{\sigma_X}.$$

Plugging in the approximations into the HJB equation and using the definitions above, we get:

$$V(z, \sigma) \left[1 + \frac{\Delta_\sigma^2 \Delta_z^2}{Q^\Delta} \tilde{\beta} \right] = (e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} \frac{\Delta_\sigma^2 \Delta_z^2}{Q^\Delta} + \sum_{s \in S} V(s) p((z, \sigma) \rightarrow s)$$

where S is the set of nearest neighbors of (z, σ) . The discretized Bellman equation is:

$$V(z, \sigma) = \min_i \left\{ (e^z - \frac{a}{\lambda} (i^*)^\lambda)^{1-\gamma} \frac{\Delta t}{1 + \tilde{\beta} \Delta t} + \frac{1}{1 + \tilde{\beta} \Delta t} E_t [V(z', \sigma') | (z, \sigma)] \right\}$$

with

$$\Delta t = \frac{\Delta_\sigma^2 \Delta_z^2}{Q^\Delta}$$

and

$$\tilde{\beta} = [\beta - b - (1 - \gamma)(i^* - \delta)].$$

We solve the Bellman equation using policy iteration.

6.5 Steady state distribution

After solving the HJB equation, the steady state is given by the eigenvector of the transpose of the transition probability matrix associated with the eigenvalue of 1, and then normalized so that it integrates to unity.

Table 1: Summary Statistics

This table reports the summary statistics of data and model output. I_t is the real nonresidential fixed private domestic investment, Y_t is the real business gross domestic product, K_t is the real capital stock, Δy_t is the growth of Y_t , Δi_t is the growth of I_t , r_t is the market return (value-weighted CRSP index), r_t^f is the risk-free rate (3-month T-Bill yield), and vol_t is the time series for the natural log of the realized market volatility. Lowercase letters represent natural logarithms. The sample is quarterly and spans the period from 1947 to 2009. Model is simulated 2,000 times using parameters reported in Table 8 and the averages across simulations are reported.

	Data	Model
$E[i_t - k_t]$	-1.3965	-2.6562
$\text{std}[i_t - k_t]$	0.1365	0.0798
$E[y_t - k_t]$	-0.4871	-0.7379
$\text{std}[y_t - k_t]$	0.0677	0.0758
$E[r_t - r_t^f]$	0.0145	0.0123
$\text{std}[r_t - r_t^f]$	0.1106	0.06831
$E[\text{vol}_t]$	-2.1753	-2.9239
$\text{std}[\text{vol}_t]$	0.4255	0.1736
$E[r_t^f]$	0.0101	0.0061
$\text{std}[r_t^f]$	0.0088	0.0078
$E[\Delta y_t]$	0.0083	0.0054
$\text{std}[\Delta y_t]$	0.0124	0.0150
$E[\Delta i_t]$	0.0099	0.0052
$\text{std}[\Delta i_t]$	0.0276	0.0398

Table 2: Predictability of Excess Stock Returns: Single Period

This table reports the results of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill, $r_{t+h} - r_{f,t+h}$, on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Regressions are performed for values of the lag h between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted R^2 in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 1 for the definition of all relevant variables.

Variables	Horizon h (in quarters)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.50	-0.42	-0.28	-0.22	-0.15	-0.16	-0.12	-0.17	-0.17	-0.14	-0.16	-0.13	-0.11	-0.10	-0.13	-0.20
t -stat	(-2.42)	(-1.94)	(-1.27)	(-1.02)	(-0.68)	(-0.72)	(-0.54)	(-0.78)	(-0.76)	(-0.62)	(-0.69)	(-0.56)	(-0.47)	(-0.44)	(-0.61)	(-0.96)
R^2	[0.02]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.39	-0.23	-0.07	-0.09	-0.02	-0.11	-0.02	-0.07	-0.01	-0.01	-0.00	0.01	0.05	0.02	0.01	-0.04
t -stat	(-1.55)	(-0.93)	(-0.28)	(-0.37)	(-0.07)	(-0.41)	(-0.06)	(-0.25)	(-0.05)	(-0.05)	(-0.02)	(0.04)	(0.22)	(0.09)	(0.04)	(-0.18)
a_2	-0.42	-0.71	-0.79	-0.49	-0.50	-0.18	-0.39	-0.40	-0.60	-0.50	-0.60	-0.53	-0.62	-0.47	-0.55	-0.64
t -stat	(-0.74)	(-1.32)	(-1.28)	(-0.78)	(-0.79)	(-0.30)	(-0.65)	(-0.67)	(-1.05)	(-0.89)	(-1.10)	(-0.98)	(-1.20)	(-0.93)	(-1.08)	(-1.24)
R^2	[0.03]	[0.02]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.01]

Table 3: Predictability of Excess Stock Returns: Multiple Periods

This table reports the results of multi-period regressions of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Multi-period excess returns are defined as a sum of single-period excess returns, $\sum_{s=1}^h (r_{t+s} - r_{f,t+s})$. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones, Neuberger, and Nolte (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted R^2 in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 1 for the definition of all relevant variables.

Variables	Horizon h (in quarters)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_1	-0.50	-0.92	-1.17	-1.35	-1.51	-1.66	-1.75	-1.89	-1.99	-2.06	-2.14	-2.21	-2.26	-2.30	-2.39	-2.53
t -stat	(-2.42)	(-2.19)	(-1.87)	(-1.64)	(-1.47)	(-1.38)	(-1.29)	(-1.24)	(-1.19)	(-1.12)	(-1.07)	(-1.02)	(-0.96)	(-0.92)	(-0.89)	(-0.90)
R^2	[0.02]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.04]	[0.05]
$\sum_{s=1}^h (r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.39	-0.62	-0.66	-0.72	-0.77	-0.87	-0.86	-0.89	-0.86	-0.82	-0.78	-0.74	-0.66	-0.62	-0.60	-0.63
t -stat	(-1.55)	(-1.25)	(-0.91)	(-0.75)	(-0.64)	(-0.61)	(-0.52)	(-0.48)	(-0.41)	(-0.36)	(-0.31)	(-0.27)	(-0.23)	(-0.20)	(-0.18)	(-0.19)
a_2	-0.42	-1.13	-1.90	-2.35	-2.79	-2.98	-3.36	-3.75	-4.29	-4.72	-5.22	-5.66	-6.16	-6.55	-7.00	-7.51
t -stat	(-0.74)	(-1.04)	(-1.14)	(-1.04)	(-0.98)	(-0.87)	(-0.85)	(-0.84)	(-0.87)	(-0.88)	(-0.91)	(-0.92)	(-0.95)	(-0.96)	(-0.98)	(-1.01)
R^2	[0.03]	[0.04]	[0.05]	[0.05]	[0.06]	[0.06]	[0.06]	[0.06]	[0.07]	[0.07]	[0.08]	[0.08]	[0.08]	[0.09]	[0.09]	[0.10]

Table 4: Predictability of Volatility of Excess Stock Returns: Single Period

This table reports the results of regressions of log volatility of returns on CRSP value-weighted index, vol_{t+h} , on the log of investment rate, $i_t - k_t$, the log of profitability, $y_t - k_t$, and lagged log volatility vol_{t-1} . Volatility is calculated using intra-quarter daily returns and has a time trend removed. Regressions are performed for values of the lag h between 1 and 16. All series have been de-trended. The table reports OLS estimates of the regressors and t -statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags. Adjusted R^2 is shown in square brackets. The sample is quarterly and spans the period 1947 to 2009. See the caption to Table 1 for the definition of all relevant variables.

Variables	Horizon h (in quarters)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Univariate regression: $\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	1.12	1.26	1.33	1.50	1.57	1.65	1.62	1.60	1.54	1.45	1.33	1.15	1.02	0.83	0.67	0.53
t -stat	(3.84)	(4.42)	(4.53)	(5.14)	(5.52)	(5.95)	(5.91)	(5.75)	(5.51)	(4.83)	(4.14)	(3.28)	(2.67)	(2.04)	(1.58)	(1.23)
R^2	[0.10]	[0.13]	[0.15]	[0.18]	[0.20]	[0.23]	[0.22]	[0.21]	[0.20]	[0.17]	[0.14]	[0.11]	[0.08]	[0.06]	[0.04]	[0.02]
Multivariate regression I: $\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1}$																
a_1	2.02	2.08	2.07	2.19	2.16	2.17	1.97	1.81	1.61	1.47	1.28	1.08	0.92	0.67	0.48	0.35
t -stat	(6.05)	(6.02)	(5.76)	(6.22)	(6.37)	(6.76)	(6.17)	(5.45)	(4.87)	(4.13)	(3.33)	(2.57)	(2.00)	(1.41)	(1.03)	(0.75)
a_2	-3.38	-3.10	-2.78	-2.63	-2.21	-1.94	-1.30	-0.80	-0.27	-0.09	0.19	0.27	0.40	0.64	0.71	0.70
t -stat	(-3.96)	(-3.67)	(-3.40)	(-3.27)	(-2.84)	(-2.52)	(-1.75)	(-1.08)	(-0.36)	(-0.11)	(0.24)	(0.33)	(0.50)	(0.81)	(0.92)	(0.88)
R^2	[0.22]	[0.23]	[0.23]	[0.26]	[0.26]	[0.26]	[0.24]	[0.22]	[0.20]	[0.17]	[0.14]	[0.11]	[0.09]	[0.06]	[0.04]	[0.03]
Multivariate regression II: $\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3\text{vol}_{t-1} + \varepsilon_{t,t+h+1}$																
a_1	1.09	1.47	1.69	1.98	1.99	2.07	2.09	2.12	1.93	1.90	1.64	1.44	1.36	1.11	0.77	0.72
t -stat	(5.01)	(5.08)	(5.15)	(6.00)	(5.82)	(6.26)	(5.65)	(5.38)	(5.43)	(5.53)	(3.90)	(3.41)	(2.86)	(2.25)	(1.66)	(1.52)
a_2	-1.55	-1.91	-2.04	-2.23	-1.88	-1.76	-1.53	-1.41	-0.89	-0.92	-0.50	-0.42	-0.42	-0.17	0.19	0.04
t -stat	(-2.96)	(-2.74)	(-2.65)	(-2.77)	(-2.38)	(-2.25)	(-1.93)	(-1.75)	(-1.12)	(-1.26)	(-0.64)	(-0.55)	(-0.51)	(-0.20)	(0.25)	(0.04)
a_3	0.48	0.31	0.19	0.11	0.09	0.05	-0.06	-0.16	-0.17	-0.23	-0.19	-0.19	-0.23	-0.23	-0.15	-0.19
t -stat	(8.80)	(4.30)	(2.68)	(1.58)	(1.37)	(0.80)	(-0.78)	(-2.02)	(-2.24)	(-3.05)	(-2.07)	(-2.19)	(-2.53)	(-2.31)	(-1.64)	(-1.82)
R^2	[0.40]	[0.31]	[0.26]	[0.26]	[0.26]	[0.27]	[0.24]	[0.24]	[0.22]	[0.21]	[0.17]	[0.13]	[0.12]	[0.10]	[0.06]	[0.05]

Table 5: Predictability of Volatility of Excess Stock Returns: Multiple Periods

This table reports the results of multi-period regressions of log volatility of returns on CRSP value-weighted index, vol_{t+h} , on the log of investment rate $i_t - k_t$, the log of profitability $y_t - k_t$, and lagged log volatility vol_{t-1} . Volatility is calculated using intra-quarter daily excess returns and has a time trend removed. Multi-period log volatility defined as a sum of single-period log volatility, $\sum_{s=1}^h \text{vol}_{t+s}$. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted R^2 in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 1 for the definition of all relevant variables.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Horizon h (in quarters)																
Univariate regression: $\sum_{s=1}^h \text{vol}_{t+s} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	1.12 (3.84) [0.10]	2.39 (4.20) [0.15]	3.75 (4.42) [0.18]	5.29 (4.74) [0.23]	6.85 (5.04) [0.27]	8.44 (5.32) [0.30]	9.95 (5.55) [0.33]	11.44 (5.75) [0.36]	12.86 (5.93) [0.38]	14.18 (6.04) [0.40]	15.38 (6.08) [0.42]	16.43 (6.05) [0.43]	17.36 (5.91) [0.44]	18.14 (5.66) [0.44]	18.77 (5.33) [0.43]	19.27 (4.97) [0.43]
Multivariate regression I: $\sum_{s=1}^h \text{vol}_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$																
a_1	2.02 (6.05) -3.38	4.11 (6.11) -6.47	6.20 (6.09) -9.24	8.43 (6.24) -11.84	10.55 (6.38) -13.96	12.64 (6.59) -15.85	14.51 (6.71) -17.17	16.22 (6.76) -18.01	17.72 (6.79) -18.41	19.08 (6.80) -18.64	20.26 (6.73) -18.67	21.27 (6.60) -18.61	22.15 (6.38) -18.50	22.78 (6.00) -18.02	23.24 (5.57) -17.48	23.57 (5.15) -16.99
a_2	-3.96 (-3.82) [0.22]	-3.71 (-3.71) [0.28]	-3.71 (-3.71) [0.32]	-3.62 (-3.62) [0.37]	-3.49 (-3.49) [0.41]	-3.35 (-3.35) [0.44]	-3.17 (-3.17) [0.46]	-2.95 (-2.95) [0.47]	-2.72 (-2.72) [0.48]	-2.50 (-2.50) [0.49]	-2.31 (-2.31) [0.50]	-2.13 (-2.13) [0.50]	-1.98 (-1.98) [0.50]	-1.83 (-1.83) [0.49]	-1.69 (-1.69) [0.48]	-1.57 (-1.57) [0.47]
Multivariate regression II: $\sum_{s=1}^h \text{vol}_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3 \text{vol}_{t-1} + \varepsilon_{t,t+h}$																
a_1	1.09 (5.01) -1.55	2.57 (5.97) -3.46	4.26 (6.14) -5.50	6.28 (6.40) -7.77	8.28 (6.43) -9.67	10.36 (6.47) -11.49	12.33 (6.38) -13.03	14.42 (6.26) -14.56	16.29 (6.21) -15.64	18.12 (6.22) -16.77	19.55 (6.12) -17.31	20.87 (5.99) -17.85	22.04 (5.79) -18.29	23.05 (5.47) -18.53	23.79 (5.17) -18.50	24.45 (4.88) -18.57
a_2	-2.96 (-3.34) [0.48]	-3.33 (-3.33) [0.80]	-3.33 (-3.33) [0.99]	-3.33 (-3.33) [1.09]	-3.19 (-3.19) [1.16]	-3.06 (-3.06) [1.18]	-2.84 (-2.84) [1.13]	-2.63 (-2.63) [0.94]	-2.42 (-2.42) [0.75]	-2.27 (-2.27) [0.50]	-2.11 (-2.11) [0.37]	-1.98 (-1.98) [0.21]	-1.86 (-1.86) [0.06]	-1.73 (-1.73) [0.14]	-1.62 (-1.62) [0.29]	-1.53 (-1.53) [0.45]
a_3	0.48 (8.80) [0.40]	0.80 (8.23) [0.44]	0.99 (7.09) [0.45]	1.09 (5.95) [0.46]	1.16 (5.38) [0.48]	1.18 (4.67) [0.49]	1.13 (3.73) [0.50]	0.94 (2.60) [0.49]	0.75 (1.77) [0.50]	0.50 (1.03) [0.50]	0.37 (0.71) [0.50]	0.21 (0.36) [0.50]	0.06 (0.09) [0.50]	-0.14 (-0.22) [0.49]	-0.29 (-0.41) [0.48]	-0.45 (-0.62) [0.47]
R^2	[0.40]	[0.44]	[0.45]	[0.46]	[0.48]	[0.49]	[0.50]	[0.49]	[0.50]	[0.50]	[0.50]	[0.50]	[0.50]	[0.49]	[0.48]	[0.47]

Table 6: Predictability of S&P 500 Earnings Growth Volatility

This table reports results of regressions of the absolute value of demeaned log growth rate of real one-period S&P 500 earnings, $|e_{t+h} - e_{t+h-1}|$, on the log of investment rate, $i_t - k_t$, and the log of profitability, $y_t - k_t$. Regressions are performed for values of the lag h between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, Newey and West (1987) corrected t -statistics (in parentheses), and adjusted R^2 in square brackets. The sample is quarterly and spans the period from 1962 to 2007. See the caption to Table 1 for the definition of all other relevant variables.

	Horizon h (in quarters)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Univariate regression: $ e_{t+h} - e_{t+h-1} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.119	-0.094	-0.072	-0.049	-0.023	-0.015	-0.004	0.001	0.003	0.016	0.029	0.035	0.039	0.054	0.057	0.057
t -stat	(-4.588)	(-3.182)	(-1.910)	(-1.089)	(-0.467)	(-0.302)	(-0.079)	(0.015)	(0.065)	(0.318)	(0.526)	(0.571)	(0.624)	(0.808)	(0.954)	(1.081)
R^2	[0.094]	[0.058]	[0.034]	[0.016]	[0.004]	[0.001]	[0.000]	[0.000]	[0.000]	[0.002]	[0.006]	[0.008]	[0.010]	[0.019]	[0.022]	[0.021]
Multivariate regression: $ e_{t+h} - e_{t+h-1} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.111	-0.075	-0.047	-0.017	0.013	0.019	0.031	0.036	0.037	0.053	0.069	0.076	0.081	0.099	0.101	0.097
t -stat	(-3.696)	(-2.162)	(-1.030)	(-0.320)	(0.227)	(0.339)	(0.562)	(0.666)	(0.680)	(0.974)	(1.150)	(1.133)	(1.156)	(1.306)	(1.500)	(1.795)
a_2	-0.018	-0.042	-0.058	-0.074	-0.085	-0.080	-0.081	-0.083	-0.081	-0.088	-0.093	-0.097	-0.099	-0.107	-0.104	-0.095
t -stat	(-0.552)	(-1.181)	(-1.390)	(-1.696)	(-1.804)	(-1.632)	(-1.519)	(-1.449)	(-1.327)	(-1.431)	(-1.460)	(-1.448)	(-1.433)	(-1.444)	(-1.390)	(-1.316)
R^2	[0.090]	[0.057]	[0.037]	[0.024]	[0.016]	[0.011]	[0.010]	[0.011]	[0.010]	[0.014]	[0.020]	[0.024]	[0.027]	[0.040]	[0.041]	[0.036]

Table 7: Predictability of Dividend Growth Volatility

This table reports results of regressions of the absolute value of demeaned log growth rate of real one-period dividends of the CRSP value-weighted index, $|d_{t+h} - d_{t+h-1}|$, on the log of investment rate, $i_t - k_t$, and the log of profitability, $y_t - k_t$. Regressions are performed for values of the lag h between 1 and 16. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted R^2 in square brackets. The sample is quarterly and spans the period from 1947 to 2009. See the caption to Table 1 for the definition of all other relevant variables.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	Horizon h (in quarters)															
	Univariate regression: $ d_{t+h} - d_{t+h-1} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$															
a_1	0.01	-0.02	-0.03	-0.05	-0.02	-0.01	-0.00	-0.04	-0.01	0.03	0.02	-0.04	0.01	0.04	-0.00	-0.04
t -stat	(0.26)	(-0.51)	(-0.66)	(-1.17)	(-0.56)	(-0.28)	(-0.06)	(-0.86)	(-0.20)	(0.58)	(0.47)	(-0.72)	(0.31)	(0.73)	(-0.02)	(-0.60)
R^2	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	Multivariate regression: $ d_{t+h} - d_{t+h-1} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$															
a_1	-0.00	-0.02	-0.02	-0.04	-0.02	0.02	0.03	-0.03	-0.00	0.06	0.06	-0.02	0.03	0.07	0.02	-0.03
t -stat	(-0.04)	(-0.38)	(-0.24)	(-0.58)	(-0.42)	(0.27)	(0.36)	(-0.41)	(-0.05)	(0.92)	(0.72)	(-0.26)	(0.50)	(0.94)	(0.29)	(-0.32)
a_2	0.05	0.00	-0.04	-0.04	0.00	-0.11	-0.11	-0.04	-0.02	-0.12	-0.13	-0.06	-0.08	-0.11	-0.08	-0.04
t -stat	(0.45)	(0.04)	(-0.32)	(-0.29)	(0.03)	(-0.88)	(-0.84)	(-0.29)	(-0.17)	(-1.13)	(-0.98)	(-0.49)	(-0.61)	(-0.93)	(-0.73)	(-0.26)
R^2	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Table 8: Calibration Parameters

This table reports parameters used to calibrate the model.

Preferences:		
Risk Aversion	γ	10
Intertemporal Discount Parameter	β	0.02
Preference Shock ξ_t : Mean Reversion Rate	θ	0.4
Preference Shock ξ_t : Volatility	v	0.3
Subjective Rate of Time Preferences: $b_t = B^0 + B^1 \sigma_t^2$	B^0	0.10
	B^1	0.075
Technology:		
Capital Elasticity	α	0.33
Productivity Shock: Mean	μ	0.015
Productivity Shock: Variance	σ_X	0.03
Depreciation Rate	δ	0.05
Adjustment Costs Scale Parameter	a	10
Adjustment Costs Elasticity to Investment Rate	λ	5
Correlation Between dW_t and $dW_{\sigma t}$	ρ	-0.9

Table 9: Predictability of Volatility of One Period Excess Stock Returns: Model Simulation

This table reports the results of regressions of log volatility of returns on CRSP value-weighted index, vol_{t+h} , on the log of investment rate, $i_t - k_t$, the log of profitability, $y_t - k_t$, and lagged log volatility vol_{t-1} . Volatility is calculated using intra-quarter daily returns and has a time trend removed. Regressions are performed for values of the lag h between 1 and 16. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 8 and the averages across simulations are reported. Volatility series has been adjusted by removing the time trend. The table reports OLS estimates of the regressors and Newey and West (1987) corrected t -statistics (in parentheses). Adjusted R^2 is shown in square brackets.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Horizon h (in quarters)																
Univariate regression: $\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	0.837	0.714	0.599	0.450	0.429	0.394	0.282	0.287	0.224	0.180	0.162	0.095	0.107	0.068	0.080	-0.012
t -stat	(6.655)	(5.580)	(4.231)	(2.727)	(2.694)	(2.153)	(1.538)	(1.520)	(1.135)	(0.872)	(0.827)	(0.484)	(0.522)	(0.323)	(0.374)	(-0.068)
R^2	[0.121]	[0.088]	[0.062]	[0.035]	[0.031]	[0.026]	[0.014]	[0.014]	[0.009]	[0.006]	[0.005]	[0.002]	[0.002]	[0.001]	[0.001]	[0.000]
Multivariate regression I: $\text{vol}_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1}$																
a_1	0.829	0.718	0.610	0.469	0.451	0.420	0.310	0.309	0.249	0.206	0.191	0.130	0.138	0.100	0.114	0.026
t -stat	(6.166)	(5.293)	(4.044)	(2.676)	(2.663)	(2.133)	(1.618)	(1.686)	(1.339)	(1.081)	(1.093)	(0.719)	(0.741)	(0.498)	(0.553)	(0.146)
a_2	0.053	-0.023	-0.069	-0.113	-0.123	-0.131	-0.142	-0.110	-0.127	-0.136	-0.147	-0.170	-0.155	-0.166	-0.180	-0.202
t -stat	(0.377)	(-0.148)	(-0.422)	(-0.646)	(-0.659)	(-0.676)	(-0.711)	(-0.530)	(-0.613)	(-0.653)	(-0.714)	(-0.874)	(-0.793)	(-0.855)	(-0.916)	(-1.109)
R^2	[0.117]	[0.084]	[0.059]	[0.033]	[0.030]	[0.025]	[0.013]	[0.012]	[0.007]	[0.005]	[0.004]	[0.003]	[0.002]	[0.001]	[0.003]	[0.003]

Table 10: Predictability of Volatility of Multi-Period Excess Stock Returns: Model Simulations

This table reports the model results of multi-period regressions of log volatility of returns on CRSP value-weighted index, vol_{t+h} , on the log of investment rate $i_t - k_t$, the log of profitability $y_t - k_t$, and lagged log volatility vol_{t-1} . Volatility is calculated using intra-quarter daily excess returns and has a time trend removed. Multi-period log volatility defined as a sum of single-period log volatility, $\sum_{s=1}^h \text{vol}_{t+s}$. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 8 and the averages across simulations are reported. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 6 lags, and adjusted R^2 in square brackets.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Horizon h (in quarters)																
Univariate regression: $\sum_{s=1}^h \text{vol}_{t+s} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	0.837	1.552	2.152	2.608	3.040	3.404	3.669	3.953	4.188	4.394	4.531	4.601	4.734	4.850	4.977	4.962
t -stat	(6.655)	(6.674)	(6.233)	(5.424)	(4.980)	(4.528)	(4.179)	(3.996)	(3.763)	(3.500)	(3.262)	(2.983)	(2.792)	(2.605)	(2.466)	(2.324)
R^2	[0.121]	[0.182]	[0.215]	[0.211]	[0.208]	[0.203]	[0.194]	[0.188]	[0.183]	[0.177]	[0.167]	[0.155]	[0.151]	[0.145]	[0.139]	[0.127]
Multivariate regression I: $\sum_{s=1}^h \text{vol}_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3 \text{vol}_{t-1} + \varepsilon_{t,t+h}$																
a_1	0.829	1.549	2.159	2.636	3.093	3.482	3.757	4.042	4.281	4.491	4.646	4.736	4.876	5.002	5.142	5.156
t -stat	(6.166)	(6.213)	(5.809)	(5.066)	(4.671)	(4.264)	(3.982)	(3.884)	(3.753)	(3.565)	(3.384)	(3.124)	(2.931)	(2.731)	(2.581)	(2.426)
a_2	0.053	0.025	-0.046	-0.165	-0.293	-0.397	-0.442	-0.448	-0.476	-0.504	-0.582	-0.670	-0.715	-0.784	-0.871	-1.024
t -stat	(0.377)	(0.087)	(-0.108)	(-0.280)	(-0.386)	(-0.429)	(-0.410)	(-0.361)	(-0.337)	(-0.316)	(-0.326)	(-0.341)	(-0.333)	(-0.337)	(-0.347)	(-0.383)
R^2	[0.117]	[0.179]	[0.212]	[0.209]	[0.207]	[0.203]	[0.193]	[0.187]	[0.182]	[0.176]	[0.166]	[0.155]	[0.150]	[0.145]	[0.140]	[0.129]

Table 11: Predictability of Single Period Excess Stock Returns: Model Simulations

This table reports the model results of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill, $r_{t+h} - r_{f,t+h}$, on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Regressions are performed for values of the lag h between 1 and 16. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 8 and the averages across simulations are reported. Standard errors are adjusted for conditional heteroscedasticity and autocorrelation. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted R^2 in square brackets.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.256	-0.239	-0.207	-0.178	-0.148	-0.114	-0.130	-0.101	-0.101	-0.083	-0.067	-0.050	-0.022	-0.017	-0.033	-0.002
t -stat	(-7.407)	(-8.196)	(-6.355)	(-5.854)	(-5.078)	(-3.857)	(-3.746)	(-3.128)	(-2.874)	(-2.166)	(-1.914)	(-1.384)	(-0.605)	(-0.464)	(-0.845)	(-0.045)
R^2	[0.172]	[0.152]	[0.113]	[0.083]	[0.057]	[0.033]	[0.044]	[0.027]	[0.027]	[0.018]	[0.012]	[0.007]	[0.001]	[0.001]	[0.003]	[0.000]
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.264	-0.250	-0.219	-0.194	-0.168	-0.137	-0.155	-0.129	-0.127	-0.110	-0.095	-0.079	-0.051	-0.046	-0.062	-0.031
t -stat	(-8.251)	(-9.217)	(-7.213)	(-6.683)	(-6.010)	(-5.536)	(-5.872)	(-5.049)	(-5.141)	(-4.238)	(-4.164)	(-2.880)	(-2.062)	(-1.574)	(-2.132)	(-1.118)
a_2	0.055	0.071	0.081	0.096	0.113	0.118	0.129	0.140	0.136	0.140	0.144	0.145	0.146	0.148	0.150	0.154
t -stat	(2.257)	(3.129)	(3.313)	(3.992)	(4.786)	(4.959)	(5.903)	(6.182)	(5.680)	(5.384)	(5.541)	(5.475)	(5.521)	(5.401)	(5.144)	(5.825)
R^2	[0.178]	[0.166]	[0.131]	[0.109]	[0.094]	[0.073]	[0.092]	[0.082]	[0.080]	[0.073]	[0.071]	[0.066]	[0.061]	[0.064]	[0.068]	[0.067]

Table 12: Predictability of Multi-Period Excess Stock Returns: Model Simulations

This table reports the model results of multi-period regressions of log returns on CRSP value-weighted index in excess of a log gross return on a 3-month Treasury Bill on the log of investment rate $i_t - k_t$ and the log of profitability $y_t - k_t$. Multi-period excess returns are defined as a sum of single-period excess returns, $\sum_{s=1}^h (r_{t+s} - r_{f,t+s})$. The model is simulated 2,000 times using 252 quarters starting from steady state and the parameters reported in Table 8 and the averages across simulations are reported. Standard errors are adjusted for the conditional heteroscedasticity and autocorrelation. We treat overlapping observations using the inference method of Britten-Jones et al. (2010) together with Newey and West (1987) corrections. For each regression, the table reports OLS estimates of the regressors, t -statistics in parentheses obtained using the correction of Newey and West (1987) with 3 lags, and adjusted R^2 in square brackets.

Variables	Horizon h (in quarters)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\sum_{s=1}^h (r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.256	-0.495	-0.700	-0.874	-1.014	-1.115	-1.242	-1.342	-1.446	-1.535	-1.595	-1.638	-1.666	-1.700	-1.746	-1.747
t -stat	(-7.40)	(-9.43)	(-10.48)	(-11.21)	(-11.28)	(-10.37)	(-9.39)	(-8.48)	(-7.62)	(-6.88)	(-6.29)	(-5.83)	(-5.40)	(-5.10)	(-4.83)	(-4.47)
R^2	[0.17]	[0.37]	[0.49]	[0.53]	[0.56]	[0.55]	[0.56]	[0.55]	[0.55]	[0.53]	[0.50]	[0.47]	[0.44]	[0.41]	[0.40]	[0.36]
$\sum_{s=1}^h (r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$																
a_1	-0.264	-0.514	-0.732	-0.926	-1.087	-1.216	-1.368	-1.495	-1.621	-1.731	-1.823	-1.898	-1.950	-2.000	-2.066	-2.095
t -stat	(-8.25)	(-11.0)	(-12.9)	(-15.3)	(-17.4)	(-18.6)	(-19.6)	(-19.2)	(-17.9)	(-17.2)	(-16.8)	(-16.2)	(-15.8)	(-15.6)	(-15.1)	(-14.8)
a_2	0.055	0.126	0.205	0.301	0.408	0.519	0.636	0.767	0.889	1.016	1.154	1.292	1.426	1.555	1.691	1.837
t -stat	(2.257)	(3.220)	(4.099)	(5.249)	(6.659)	(7.889)	(9.133)	(9.740)	(9.985)	(9.729)	(9.408)	(9.082)	(8.824)	(8.636)	(8.420)	(8.324)
R^2	[0.178]	[0.400]	[0.540]	[0.604]	[0.670]	[0.694]	[0.732]	[0.761]	[0.784]	[0.795]	[0.797]	[0.800]	[0.789]	[0.790]	[0.795]	[0.792]

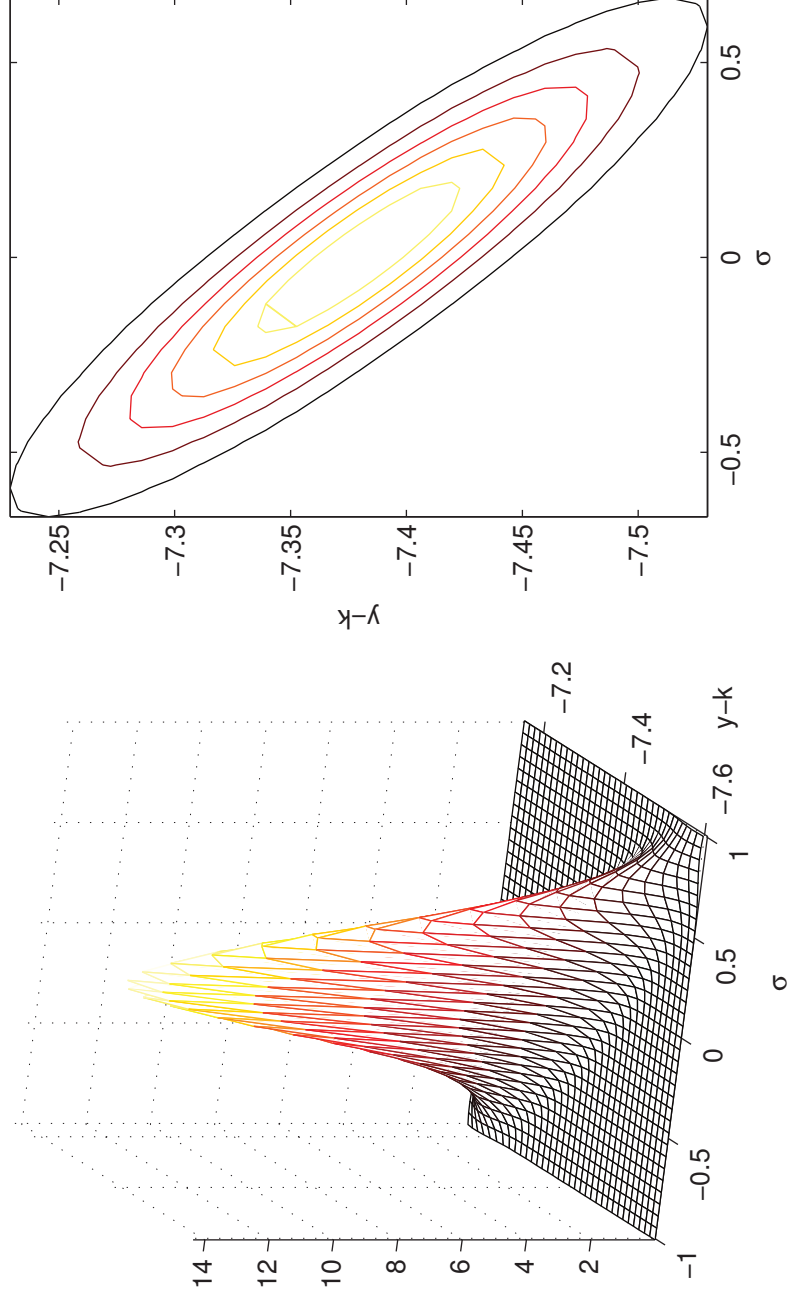


Figure 1: Steady-state distribution of the state variables σ and $y-k$

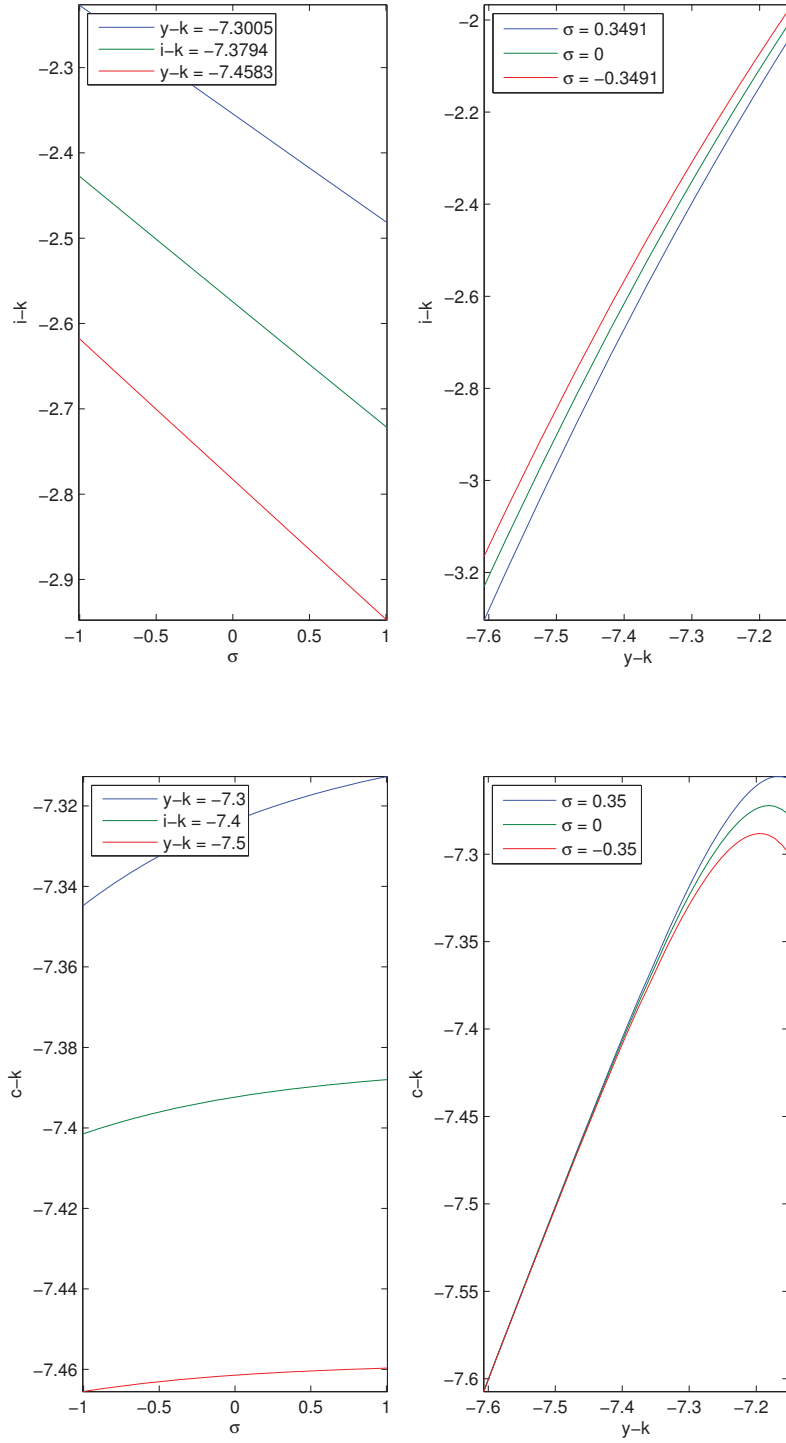


Figure 2: Investment and consumption as a function of the state variables σ and $y - k$

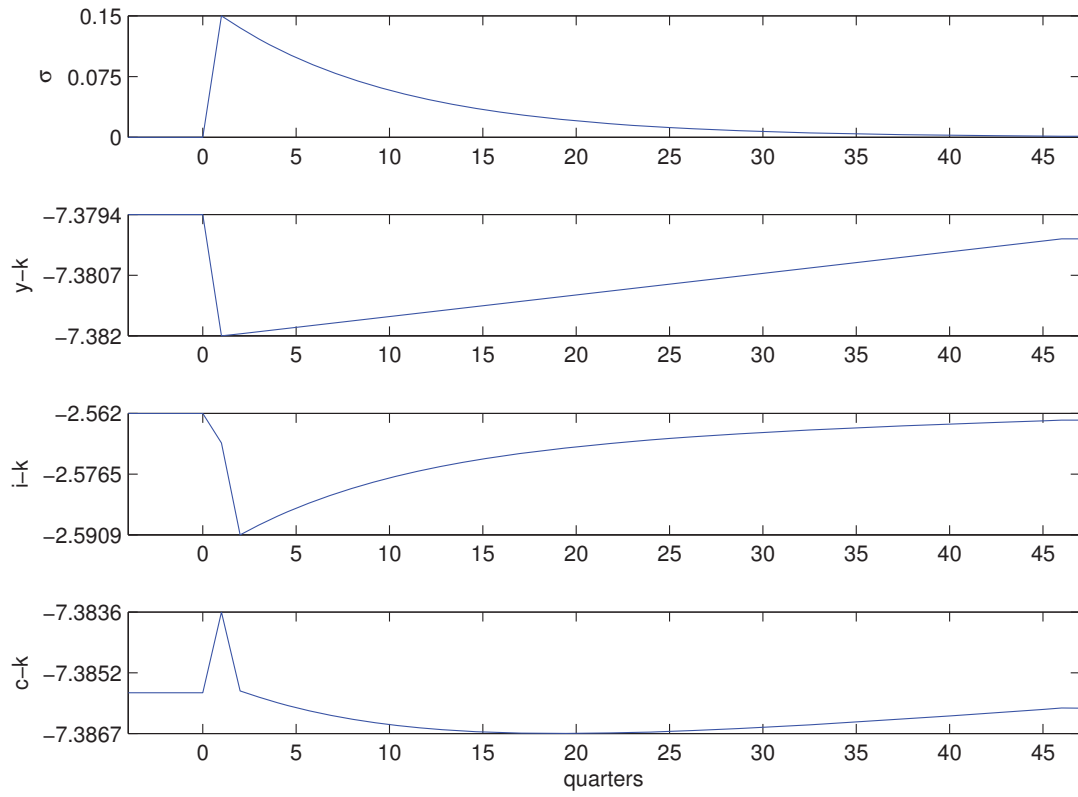


Figure 3: Model's response of the investment rate, profitability and consumption after a preference shock

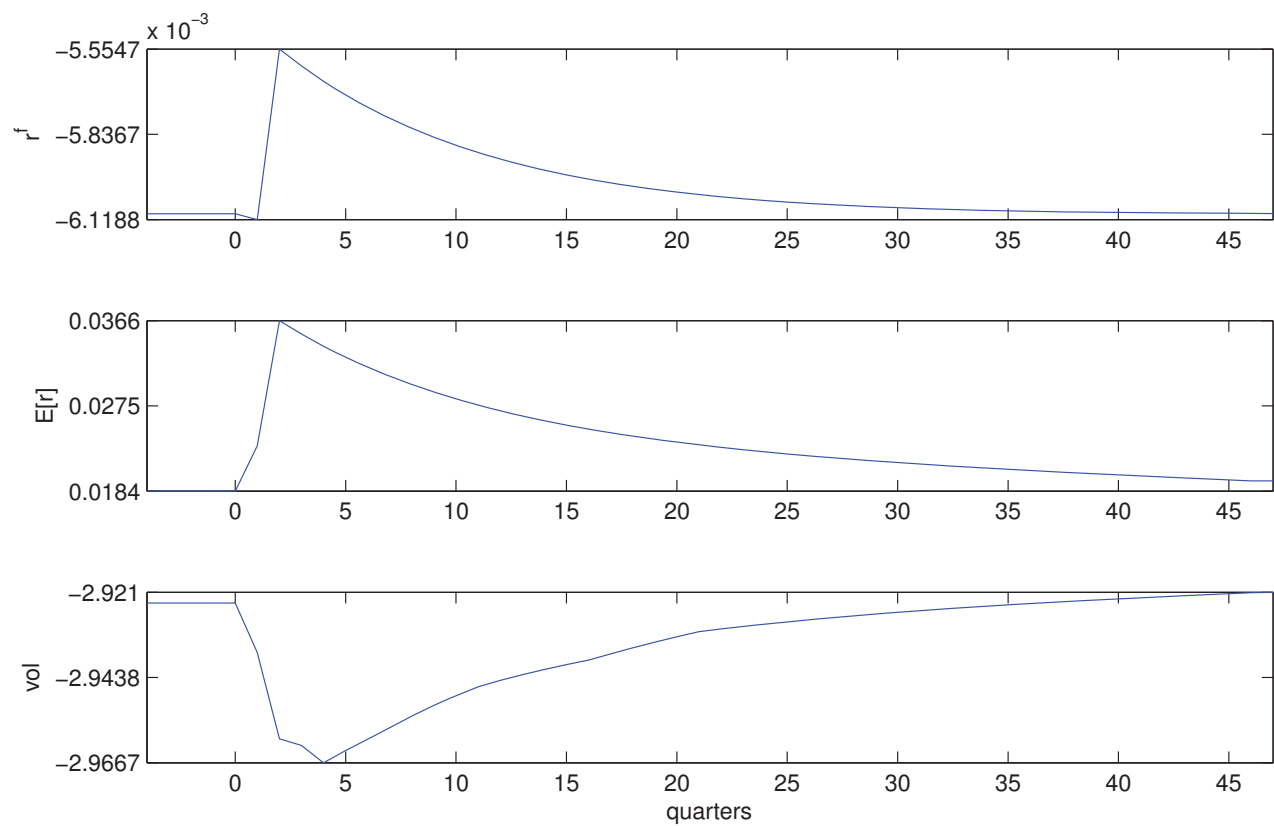


Figure 4: Model's response of the risk-free rate, expected stock returns and volatility of stock returns after a preference shock