Chen-Mangasarian Smoothing Function

```
ClearAll["Global`*"]
```

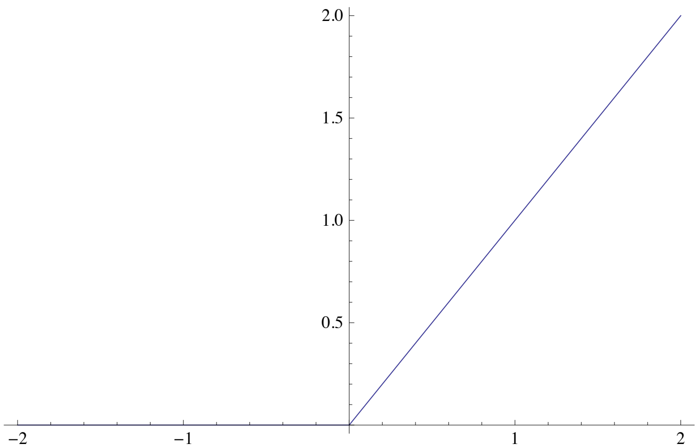
Suppose you want to use the following function

In[2]:=
```
xplus[x_] = Max[0, x]
```

Out[2]=
```
Max[0, x]
```

In[3]:=
```
Plot[xplus[x], {x, -2, 2}]
```

Out[3]=



However, you don’t like the kink at x=0. Consider the following functional form, where a and x0 are parameters: x0 centers the function, and a is a steepness paramter.

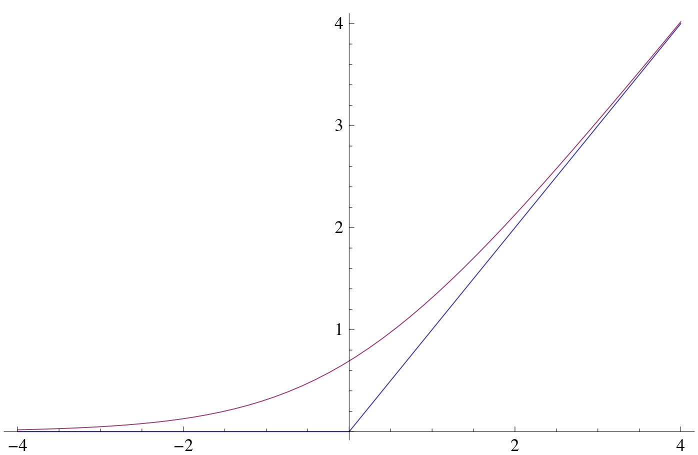
In[4]:=
```
xcmf[x_, a_, x0_] := (x - x0 + Log[1 + Exp[-a (x - x0)]] / a)
```

Consider the following example.

In[25]:=
```
a = 1;
xcm[x_] = xcmf[x, a, 0]
Plot[{xplus[x], xcm[x]}, {x, -4, 4}]
```

Out[26]= $x + \mathrm{Log}[1 + e^{-x}]$

Out[27]=

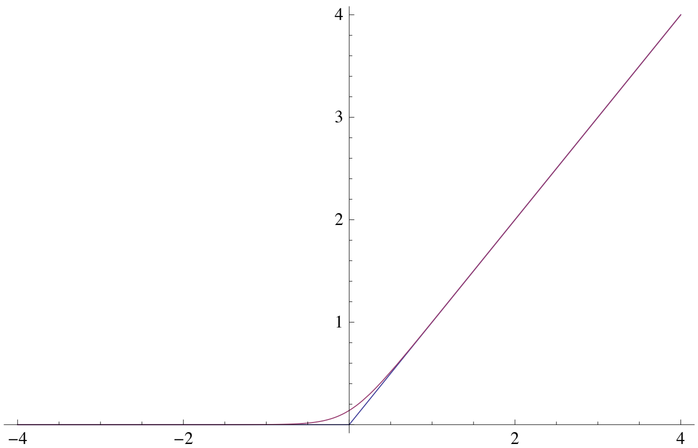


Let’s increase the steepness paramter.

In[28]:=
```
a = 5;
xcm[x_] = xcmf[x, a, 0]
Plot[{xplus[x], xcm[x]}, {x, -4, 4}]
```

Out[29]= $x + \frac{1}{5} \text{Log}\left[1 + e^{-5 x}\right]$

Out[30]=



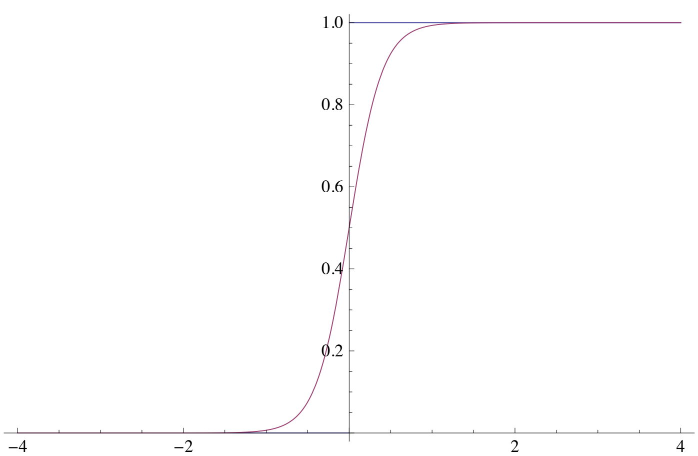
As we increase a, the Chen-Mangasarian function, xcm[x], converges to xplus[x].

Notice that the derivative is well-behaved:

In[31]:=
```
a = 5;
xcm[x_] = xcmf[x, a, 0]
Plot[{xplus'[x], xcm'[x]}, {x, -4, 4}]
```

Out[32]= $x + \frac{1}{5} \text{Log}\left[1 + e^{-5 x}\right]$

Out[33]=

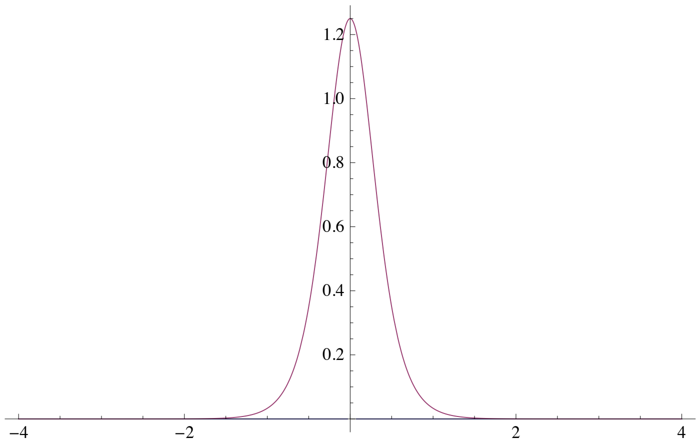


Even the second derivative is well-behaved:

In[37]:=
```
a = 5;
xcm[x_] = xcmf[x, a, 0]
Plot[{xplus''[x], xcm''[x]}, {x, -4, 4}, PlotRange → All]
```

Out[38]= $x + \frac{1}{5} \mathrm{Log}\left[1 + e^{-5 x}\right]$

Out[39]=



This last property is important. Other smoothers, such as hyperbolic functions, have substantially higher curvature near the kink.