Tutorial on exact inference in the Undirected PGM

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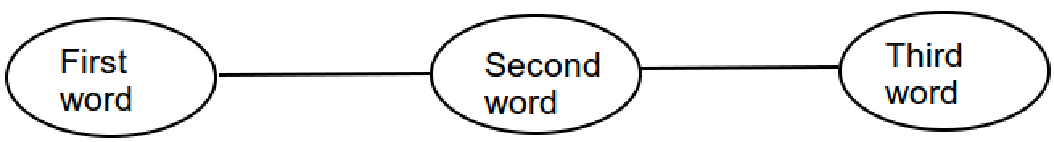
This tutorial will help you to understand basic algorithms for exact inference in the Probabilistic Graphical Models (PGM) better by implementing them.

1 Theory

Please, read carefully the attached file with a theory from the Daphne Koller book before starting this tutorial.

2 Exercises

Let's consider very simple Markov Model for the words in the sentence :



Each word in the sentence can have one of 3 values, corresponding to its part of speech : 'noun', 'verb' or 'other'. The factor function for the class of the next word, given a previous word will be the same for the both pairs (first-second and second-third) and is given in the table below :

ψ	Noun	Verb	Other
Noun	1	2	3
Verb	10	1	3
Other	3	5	2

Columns stays for the first word and raws - for the next one. For compactness, I will omit 'word' in the following. So $\psi(Prev = Noun, Next = Verb) = 10$ and $\psi(Prev = Verb, Next = Other) = 5$
In order to make an inference on the undirected graphical model, we need to obtain Clique Tree first. Try to do it yourself and then look the answer in the next page.

FIGURE 1 – Clique tree for the exercise 1

Now you are ready to do exact inference on this model. You should always remember that you are working with tables, but not matrices. So the operation you will be doing are not operations over the matrices, but rather over the table. The meaning of each value is more important that usual conventions for the matrix-vector operations.

The assignment requires using pen and paper and is the following :

1. Perform calibration using Sum-Product Message Passing algorithm (it is given in the D. Koller book on the page 357, while $\delta_{1->2}$ is defined at the page 352)
   (a) Pass a message from the left clique to the right $\delta_{1->2}$: calculate β_2
   (b) Pass a message from the right clique to the left $\delta_{2->1}$: calculate β_1
   (c) Check if the resulting beliefs β_i are calibrated (if $\mu_{1,2} = \mu_{2,1}$)

   Note : you should not include $\delta_{1->2}$, while passing message $\delta_{2->1}$ back, in order not to count it twice.
2. Calculate the marginal distribution over the third word
   (a) Marginalize the belief $\beta(Second, Third)$ over the second word
   (b) Normalize resulting distribution

   Note : You will get the partition function as a side product of normalization
3. Make the following queries :
   (a) You know that the first word is noun. What does it tell you about the third word ? Find condition distribution $P(Third|First = Noun)$
   Hint : The formula for unnormalized distribution from the page 369 :

$$\widetilde{P}(T = N|F = N) = \sum_S \frac{\beta_1(F = N, S) * \beta_2(S, T = N)}{\mu_{1,2}(S)}$$

   where F,S,T means words : 'First, Second, Third' and N means 'Noun'

(b) Now you also know that the second word is verb. Update your probability that the 3rd word is "other".

Note : Do you really need to use belief in order to answer to the last query ?