

Private Overborrowing under Sovereign Risk

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What are the consequences of excessive private debt for sovereign risk?

Motivating facts: road to the Spanish debt crisis

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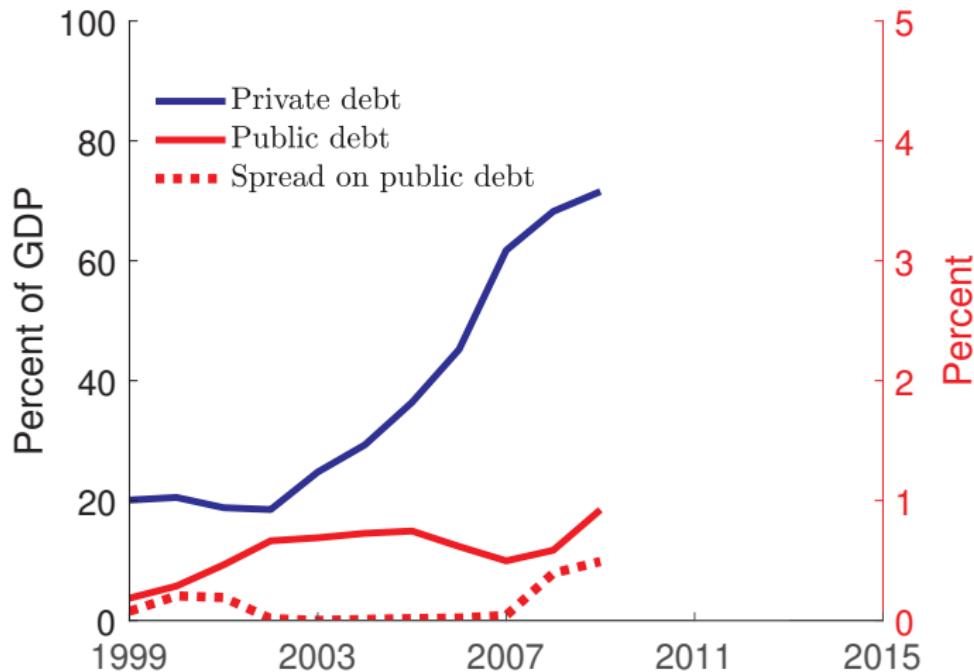
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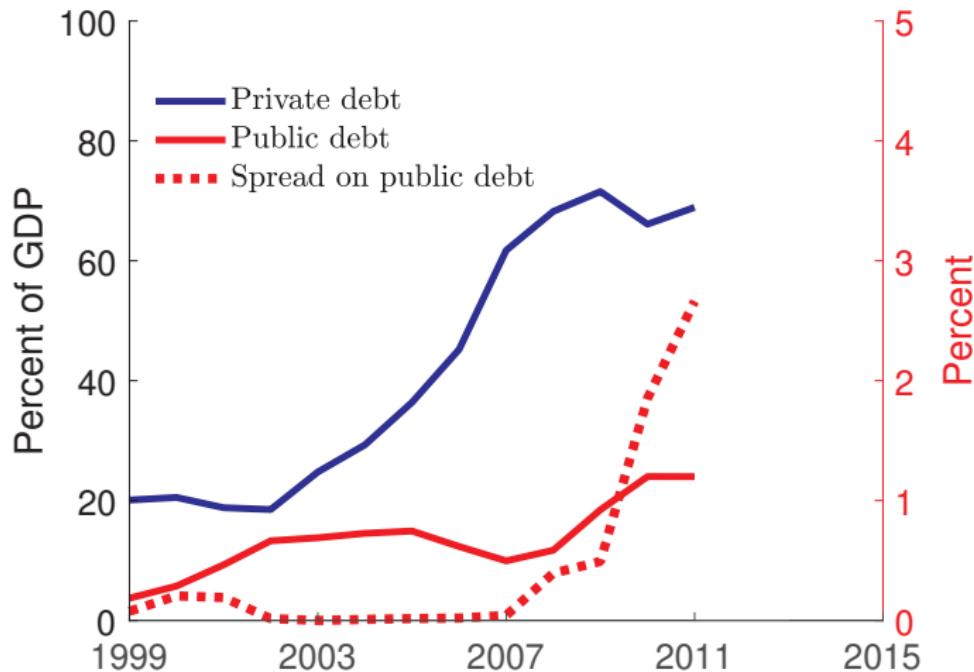
Motivation: The road to the 2012 Spanish debt crisis



Private and public net foreign liabilities and difference between Spanish and German 6-year T-bonds

Source: Bank of Spain, Eurostat, and Bloomberg

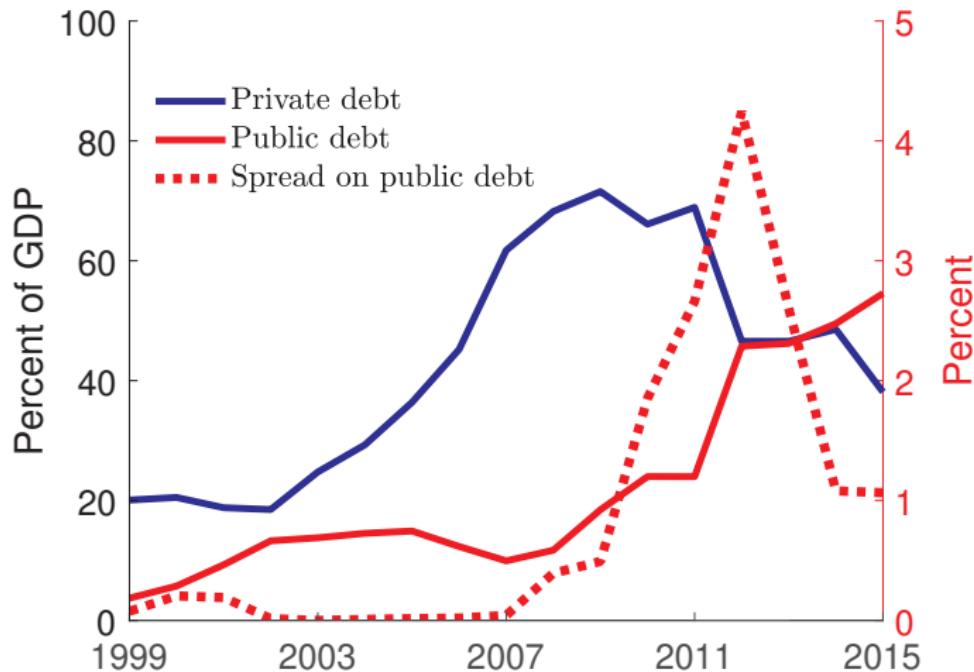
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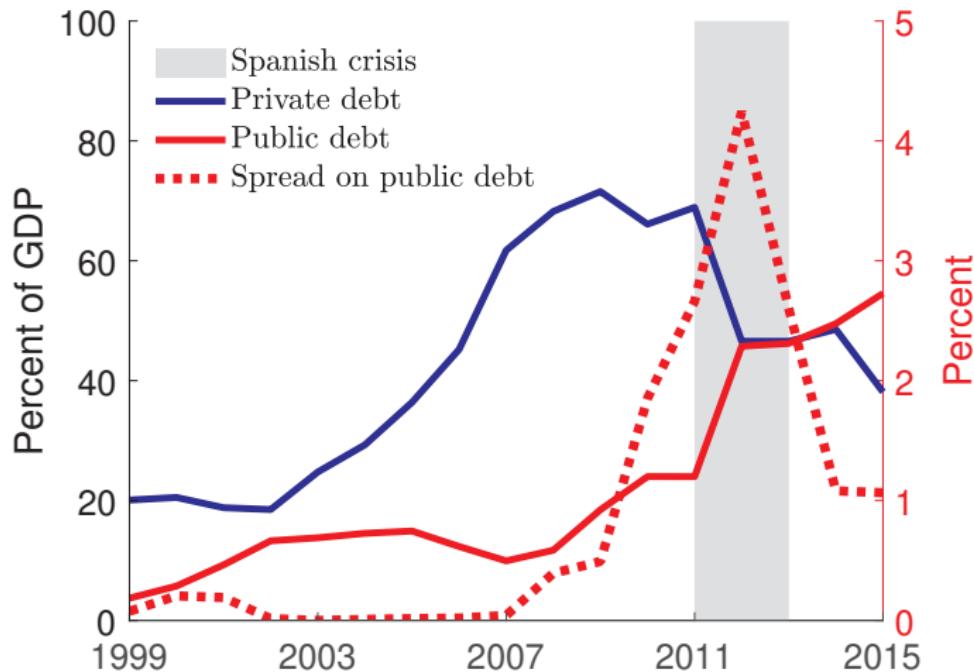


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In levels

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Total debt: Composition matters

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Policy implication: Sovereign risk tightens the optimal prudential tax by **.7 pp**

This paper: one model, two versions

Model with two types of international bonds

- Private debt, constrained by *market value of income* (**credit friction**)
- Public debt, strategically defaultable

Baseline version: Competitive households issue private debt. Government makes transfers and public debt decisions

Social planner (SP) version: Planner makes all aggregate borrowing decisions and faces the same constraints and also lacks commitment

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→ **Policy implementation:** Government sets taxes on private debt

Related literature

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- Sovereign default and private debt
 - Kaas, et al. (2020); Arellano, et al. (2020); Kim and Zhang (2011); Mendoza and Yue (2009); Jeske (2006); Wright (2006)
 - Inefficient international private debt during sovereign defaults
- Financial crisis and government's intervention
 - Jeanne and Korinek (2019); Davila and Korinek (2018); Bianchi (2016); Kester (2016); Chari and Kehoe (2016); in't Veld et al. (2014); in't Veld et al. (2012); Bianchi (2011)
 - Bailouts financed with external long term defaultable debt
- Doom loops between the sovereign and financial sectors
 - Farhi and Tirole (2018); Sosa-Padilla (2018); Boccola (2016) ; Brunnermeier et al. (2016); Perez (2015); Korinek (2012)
 - Quantify the channel from financial vulnerabilities to higher spreads

Model

Model: Small open endowment economy

Goods: Tradable (y^T) and nontradable (y^N). All bonds denominated in units of tradables

Real exchange rate (p^N): Relative price of nontradables

Shocks: Income (y^T) , financial (κ) , and private default (π).

Agents: Government, households, and risk-neutral foreign lenders

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Timing:

1. Shocks realize, summarized in $s = (y^T, \kappa, \pi)$
2. Government's default, borrowing, and tax decisions (*strategic*)
3. Households' consumption and borrowing decisions (*competitive*)
4. Lenders price both assets (*competitive*)
5. Markets clear

Public debt (L):

1. Perpetuity with exponential decay δ
2. Strategically defaultable
3. Issued at price Q

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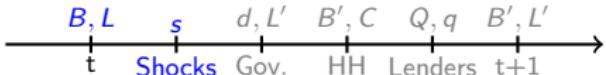
Private debt (B):

1. One period
2. Fraction π exogenously defaulted each period
3. Issued at price q
4. Constrained by current income:

$$q(\pi)B' \leq \kappa[y^T + p^N y^N]$$

Baseline: Government's problem

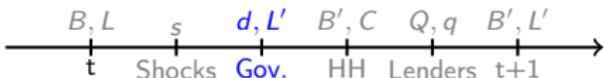
Government state space is $S = (s, B, L)$



Government takes households and lenders best responses as given

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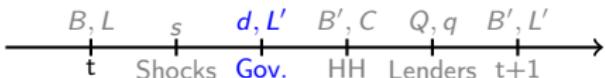
Government takes households and lenders best responses as given

Government chooses default d , and public debt L' to solve :

$$W(S) = \max_{d \in \{0,1\}} \underbrace{d}_{\text{Default}} \underbrace{W^D(S)}_{\text{Value in default}} + [1 - d] \underbrace{W^R(S)}_{\text{Repayment} \rightarrow \text{access to } L'}$$

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Period budget constraint that pins down transfers

$$\underbrace{T}_{\text{Transfers}} = (1 - d) \times \left[\underbrace{Q^{BR}}_{\text{Public debt price}} \times \underbrace{[L' - (1 - \delta)L]}_{\text{Public debt issuance}} - \underbrace{\delta L}_{\text{Repayment}} \right]$$

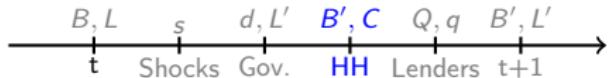
where Q^{BR} is the lenders best response to government policies

Repayment Problem

Default Problem

Baseline: Household's problem

Households face agg. state $S_G = (S, d, L')$

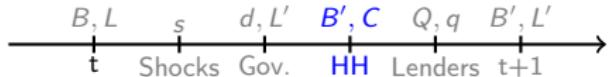


$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V(S'_G, b')]$$

subject to

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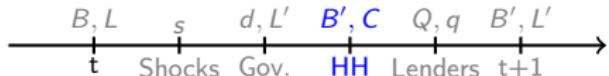
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$$\underbrace{c^T + p^N c^N}_{\text{Consumption}} + (1 - \pi) \underbrace{b}_{\text{Current private debt}} = \underbrace{y^T + p^N y^N}_{\text{Endowment}} + q(\pi) \underbrace{b'}_{\text{New private debt}} + \underbrace{T}_{\text{Transfers}}$$

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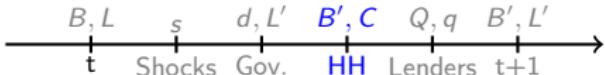
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$$\underbrace{q(\pi) b'}_{\text{Market value of new debt}} \leq \kappa \underbrace{(y^T + p^N y^N)}_{\text{Market value of current income}}$$

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$$\underbrace{q(\pi)b'}_{\text{Market value of new debt}} \leq \kappa \underbrace{(y^T + p^N y^N)}_{\text{Pecuniary aggregate demand externality}}$$

Taking aggregate prices, laws of motion and public policies as given

$$\underbrace{p^N}_{\text{Real exchange rate}} = p^N(S_G) ; \quad \underbrace{T = \mathcal{T}(S_G)}_{\text{Transfer function}} ; \quad S'_G = S'_G \left(\underbrace{s', \mathcal{B}'(S_G), L'}_{S'}, \underbrace{\mathbf{d}(S'), \mathcal{L}'(S')}_{\text{Next period policies}} \right) ;$$

Baseline: Definition of equilibrium

Decision rules and prices such that:

- (i) Government optimizes
- (ii) Households optimize
- (iii) Foreign lenders optimally price private and public debt ► Details
- (iv) Market of tradable and nontradable goods clear ► Details

Formal definition

Baseline: Equilibrium price of non tradables

Households' intratemporal optimality condition pins down p^N

The MRS between T and NT equals the relative price

$$p^N(S_G) = f\left(\frac{\mathcal{C}^T(S_G)}{y^N}\right)$$

Where $f(\cdot)$ is an increasing function

Functional form

Euler equation

Social planner (SP) problem

Agents: Planner and risk-neutral foreign lenders

Shocks: Same as baseline (y^T, κ, π)

Assets (B, L): Same structure, with credit constraint:

$$qB' \leq \kappa \left[y^T + f\left(\frac{c^T}{y^N}\right) y^N \right],$$

Planner chooses aggregates but takes pricing schedules as given

Foreign lenders optimally price both assets

Recursive planner problem

Euler equation planner

Why do baseline and socially planned allocations differ?

Individually optimal private borrowing decisions ignore two GE effects

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- *Government's response to a binding collateral constraint \rightarrow Bailouts*

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- Higher private debt \rightarrow \uparrow frequency binding credit constraint (financial crisis)
- Government's response to a binding collateral constraint \rightarrow Bailouts
- Bailouts \uparrow public debt and \uparrow the frequency of sovereign defaults
- Lenders internalize this \rightarrow \uparrow cost of public debt in baseline

Why bail out households when the credit constraint binds?

Bailouts increase consumption via **two** channels

- Bailouts \uparrow current consumption C^T [**1st channel**]
- Higher tradable consumption $\rightarrow \uparrow$ price of nontradables $f(\frac{C^T}{y^N})$
- Higher price of nontradables $\rightarrow \uparrow$ value of collateral $y^T + f(\frac{C^T}{y^N})y^N$
- Higher collateral $\rightarrow \uparrow$ borrowing limit $\rightarrow \uparrow$ borrowing b'
- Higher private borrowing $b' \uparrow$ consumption C^T [**2nd channel**]

Summary

Theoretical finding: Decentralization of the SP problem

Proposition I

The consumption, borrowing, and default rules of the social planner

coincide with the solutions of a decentralized equilibrium in the same

environment where the government can also set state contingent taxes

on private debt that satisfy:

$$1 - \tau = \frac{\beta \mathbb{E}_s \left[(1 - \pi') \left(u_T^{SP}(\mathcal{C}^{SP, T'}, y^{N'}) \right) \right] + \mu^{SP} q^{SP}}{q^{SP} u_T(\mathcal{C}^{SP, T}, y^N)}$$

Household problem with taxes

Quantitative analysis

Calibration

Calibrate **baseline** model to the pre-crisis years ([1999-2011](#)), assuming

- Discretized AR 1 process for income, private default, financial shock
- Default costs linear on the log of income
- Dynamic discrete choice method à la Dvorkin, Sanchez, Sapriza, and Yurdagul (2020) [► Details](#)

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Quantitative exercises: Use the calibrated model to

1. Measure overborrowing and its costs at the ergodic distribution
2. Simulate the 2012 Spanish debt crisis
3. Construct optimal macroprudential policies

Calibration

Results: Baseline vs. social planner at the ergodic

Average (in %)	Data	Baseline
Private debt	42	42
Public debt	14	15
Spread	.45	.45
Probability of a financial crisis	-	2.5
Probability of default	-	.46
Welfare gains (increase in c^T)	-	-

Welfare Calculations

Macropredutive policies at the ergodic

Results: Baseline vs. social planner at the ergodic

Average (in %)	Data	Baseline	Social planner
Private debt	42	42	37
Public debt	14	15	12
Spread	.45	.45	.034
Probability of a financial crisis	-	2.5	.10
Probability of default	-	.46	.030
Welfare gains (increase in c^T)	-	-	.41

Welfare Calculations

Macropredutive policies at the ergodic

Simulating the 2008-2015 Spanish crisis

Exercise I: Dynamics and counterfactual

Feed exogenous shocks from the data → compute models' dynamics

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Estimation method of κ_t : Particle filter on the baseline model ► Details

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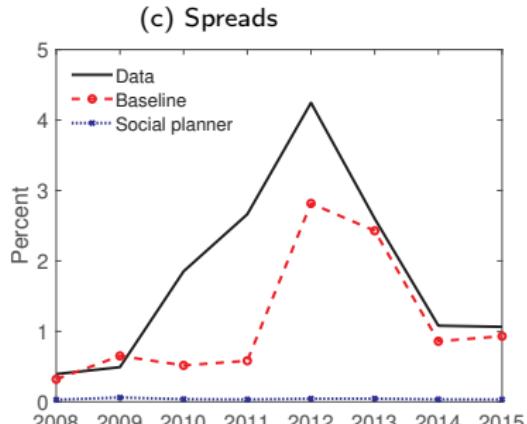
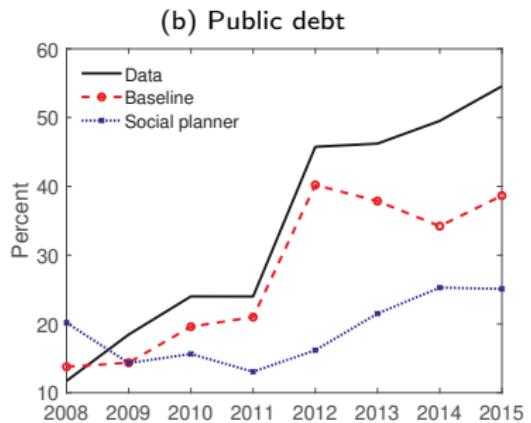
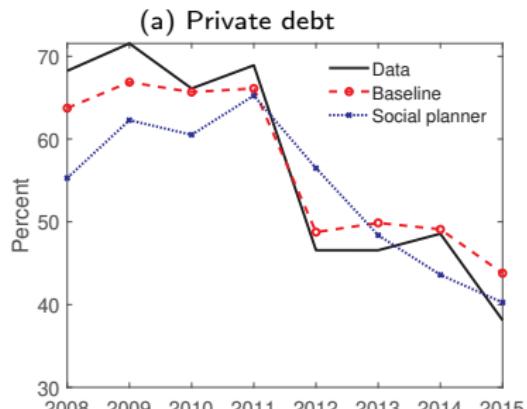
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Endogenous dynamics:

1. Public debt
2. Private debt
3. Interest rate spread on public debt

Exercise I: Endogenous private debt, public debt and spreads



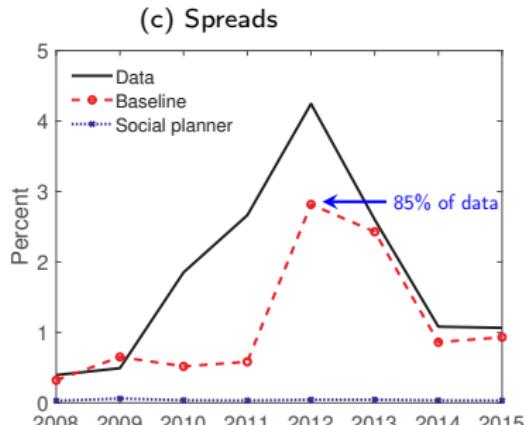
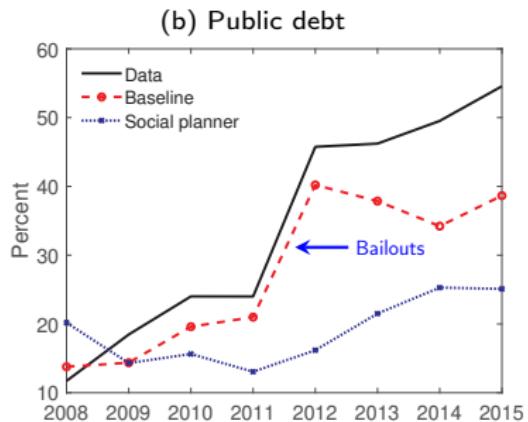
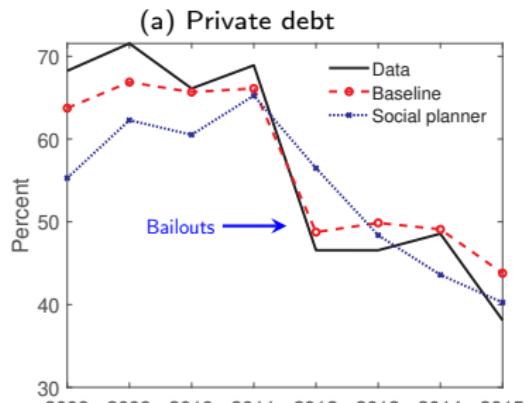
Income shock

Private default shock

Financial shock

Optimal macroprudential tax

Exercise I: Endogenous private debt, public debt and spreads



Income shock

Private default shock

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Optimal macroprudential tax

Exercise II

Consequences for macroprudential policy

Proposition I shows that a positive optimal tax on private debt can implement the planner's allocation

Consequences for macroprudential policy

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Bianchi (2011) [All Extensions](#)

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The difference with that model, nested here, is that I allow for financial crises that can lead to sovereign debt crises

Consequences for macroprudential policy

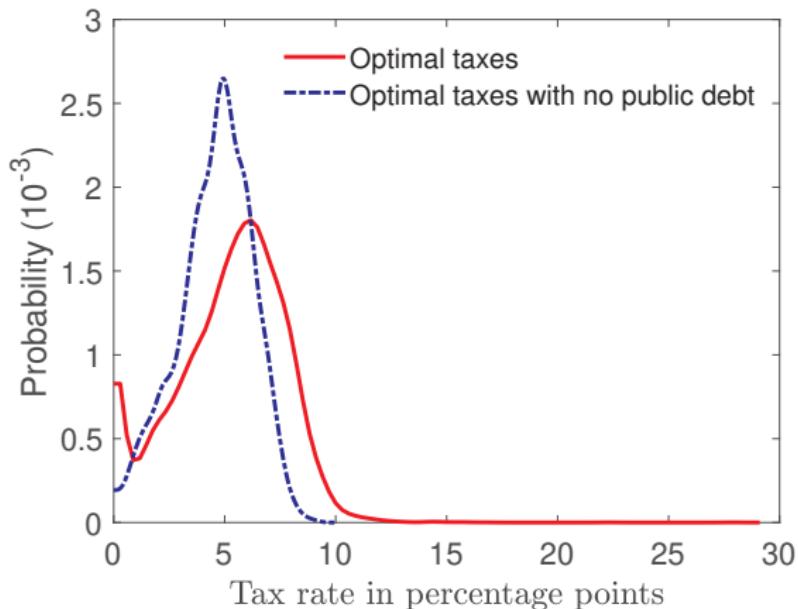
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The difference with that model, nested here, is that I allow for financial crises that can lead to sovereign debt crises

Policy implication : Higher taxes on private debt are needed in countries where sovereign risk is an issue

Policy implication : Higher taxes on private debt



Higher taxes than a model with no public debt

Household problem with taxes

The average tax rate increases from 4.6% to 5.3%

Simulations at the ergodic

Policy functions

Conclusion

Model of how financial crises lead to sovereign debt crises

Conclusion

Model of how financial crises lead to sovereign debt crises

1. Private borrowing above socially optimal levels
2. **Financial crisis:** Private debt is constrained. Contraction
3. Symmetric evolution of private and public debt
4. **Sovereign debt crisis:** Public interest spikes follow bailouts

Conclusion

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Measures the level of excessive private debt and its cost

- Private borrowing by 5.0% of GDP
- The probability of a financial crisis by 2.4 pp.

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Quantitatively consistent with the Spanish data

Measures the level of excessive private debt and its cost

- Private borrowing by 5.0% of GDP
- The probability of a financial crisis by 2.4 pp.

Policy recommendations:

- If sovereign risk is an issue, then stricter macroprudential policies (.7 p.p. higher)
- Optimal policy response combines preventive and remedial measures

Thank you

Appendix

Related literature

- Sovereign default and private debt

Kaas, et al. (2020); Arellano, et al. (2020); Kim and Zhang (2011); Mendoza and Yue (2009); Jeske (2006); Wright (2006)

- Financial crisis and government's intervention

Jeanne and Korinek (2019); Davila and Korinek (2018); Bianchi (2016); Kester (2016); Chari and Kehoe (2016); Bianchi (2011)

- Doom loops between the sovereign and financial sectors

Farhi and Tirole (2018); Sosa-Padilla (2018); Boccola (2016) ; Brunnermeier et al. (2016); Perez (2015); Korinek (2012)

Summary of the main mechanism

In the baseline relative to the socially planned economy

- Private debt is higher
- Credit constraint binds more often
- More frequent public debt-financed bailouts and sovereign defaults

Mechanism: Households overborrow → Bailouts → High spreads

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Calibrated parameters

Parameter	Value	Moments	Target (in %)	Model
Discount factor	$\beta = .92$	Av. Total Debt	56	56
Scale ϵ	$v = .020$	Std. Total Debt	4.8	5.0
Financial shock	$\bar{\kappa} = .45$	Av. Private Debt	42	42
$\kappa_{t+1} = (1 - \rho)\bar{\kappa} + \rho\kappa_t + \varepsilon_t^k$	$\sigma^\kappa = .020$	Std. Private Debt	7.1	5.8
Default Cost	$\phi_0 = .31$	Av. Spread	.45	.45
$\phi(y^T) = \phi_0 + \phi_1 \log(y^T)$	$\phi_1 = 1.9$	Std. Spread	.61	.61

Overborrowing in the policy functions

Parameters estimated outside of the model ↗ Back spreads

Exercise II: Effect of private debt on spreads

In Exercise I social planner avoids entirely the increase in spreads

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Planner achieves this by combining:

1. Lower private debt before crisis
2. Lower public debt during the crisis

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Estimate κ_t using the particle filter on the baseline model. Still endogenous:

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2. Interest rate spread on public debt

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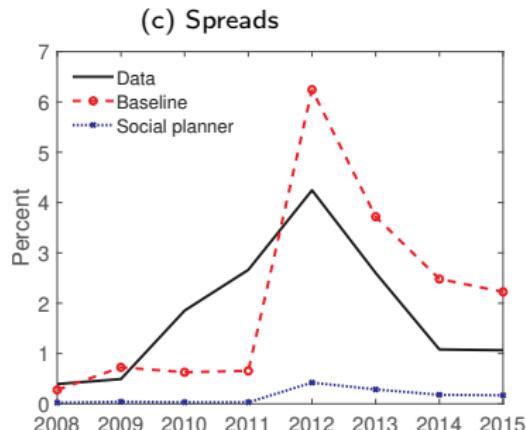
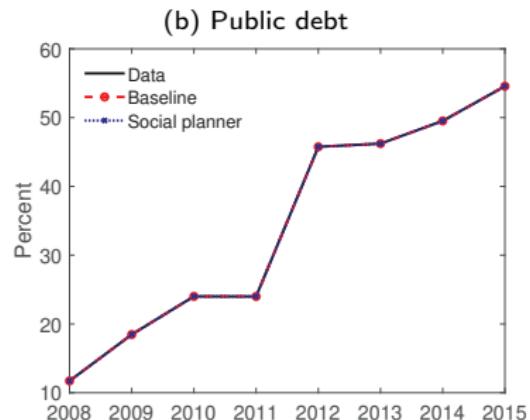
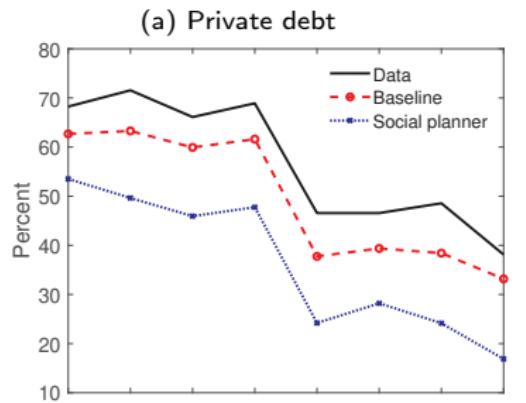
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Estimate κ_t using the particle filter on the baseline model. Still endogenous:

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Effect of private overborrowing: $spread^{SP} - spread^{data}$

Exercise II: Endogenous private debt and spreads



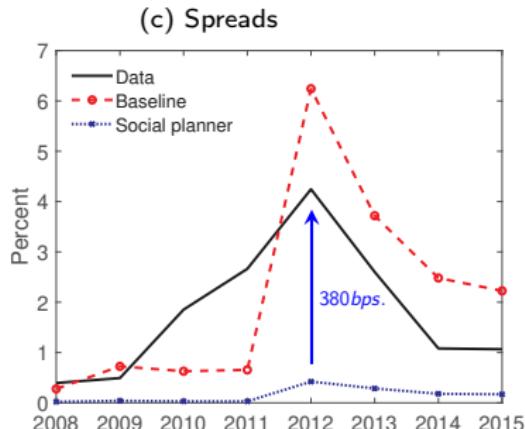
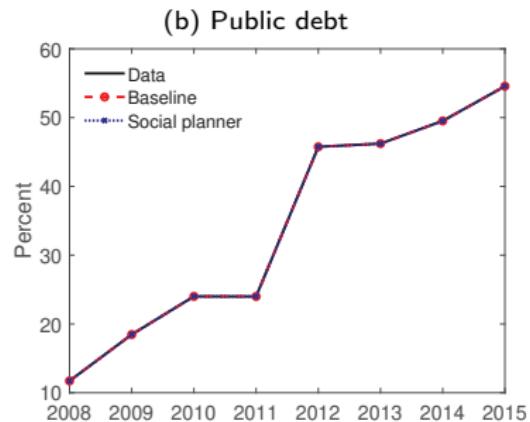
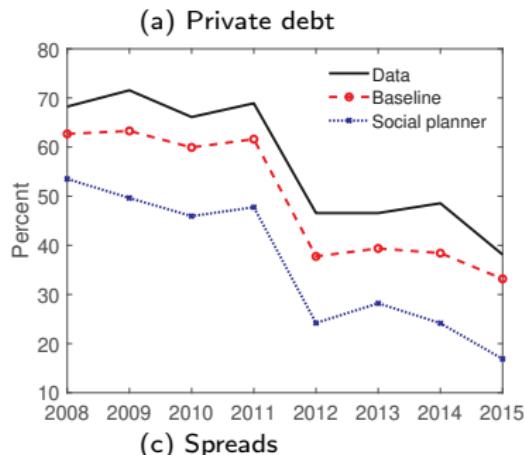
Income shock

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Exercise II: Endogenous private debt and spreads



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Extensions

A model with no public debt [Details](#)

A model with no private debt [Details](#)

A model with an exogenous credit constraint [Details](#)

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Households

Choose b_{t+1}, c_t^T, c_t^N to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(c(c^T, c^N)) = \frac{c(c^T, c^N)^{1-\sigma}}{1-\sigma}$,

$$c(c^T, c^N) = [\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}]^{-\frac{1}{\eta}} \quad \eta > -1, \quad \omega \in (0, 1),$$

subject to budget constraint

$$c_t^T + p_t^N c_t^N + b_t = qb_{t+1} + \textcolor{blue}{y_t^T} + p_t^N y^N + T_t,$$

and credit constraint

$$qb_{t+1} \leq \textcolor{blue}{\kappa_t} (y_t^T + p_t^N y^N)$$

Government

Picks debt L_{t+1}^j from a fixed grid $\{L^1, \dots L^J\}$ and makes default decision d_t to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(\mathcal{C}_t) + d_t (\epsilon_t^{J+1} - \phi(y_t^T)) + (1 - d_t) \epsilon_t^j],$$

subject to budget constraint

$$T_t = (1 - d_t) \left(Q_t [L_{t+1}^j - (1 - \delta)L_t^i] - \delta_t^i \right).$$

Foreign Lenders

Are risk neutral and lend to both the government and the households

$$Q_t = q\mathbb{E}_t \left[(1 - d_{t+1})(\delta + (1 - \delta)Q_{t+1}) \right]$$

Market clearing conditions

Market clearing for non-tradables

$$\mathcal{C}^N(s, L, B) = y^N$$

Resource constraint for tradables

$$\begin{aligned}\mathcal{C}^T(s, L, B) + (1 - \pi)B &= y^T + q(\pi)\mathcal{B}'(s, L, B) + \left\{ 1 - \mathbf{d}(s, L, B) \right\} \times \\ &\quad \left\{ Q(s, L, B) \left[\mathcal{L}'(s, L, B) - (1 - \delta)L \right] - \delta L \right\}.\end{aligned}$$

[Back](#)

Equilibrium relative price of nontradables

Household's CRRA on CES utility function:

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta} \right]^{-\frac{1-\sigma}{\eta}}$$

Solution to the intratemporal household optimization problem:

$$p^N = \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N} \right)^{\eta+1}$$

$$p^N(s, L, B) = \frac{1-\omega}{\omega} \left(\frac{\mathcal{C}^T(s, L, B)}{y^N} \right)^{\eta+1}$$

Functional forms: Utility

Household's CRRA on CES utility function:

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta} \right]^{-\frac{1-\sigma}{\eta}}$$

Elasticity of substitution between tradables and nontradables is $\frac{1}{1+\eta}$

Share of tradables in consumption is ω

Risk aversion coefficient is σ

[Back to equilibrium conditions](#)

Recursive Equilibrium

A Markov recursive competitive equilibrium is defined by,

1. value functions V , W , W^R , and W^D
2. policy functions for the private sector $\{\hat{b}, \hat{c}^T, \hat{c}^T\}$
3. pricing functions p^N and Q
4. perceived laws of motion $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N\}$
5. policy functions for the public sector $\{\mathcal{D}, \mathcal{L}'\}$

Such that,

Recursive Equilibrium II

1. Given $\{p^N, Q\}$, and $\{\mathcal{B}', \mathcal{L}'\}$, the private policy functions, $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ and V , solve the HH problem
2. Given Q , and $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N\}$, the public policy functions, $\{\mathcal{D}, \mathcal{L}'\}$ and W , W^R , and W^D , solve the government's problems
3. Rational expectations: $\{\mathcal{B}' = \hat{b}, \mathcal{C}^T = \hat{c}^T, \mathcal{C}^N = \hat{c}^N\}$
4. Given all policies Q satisfies the lenders problems
5. Markets clear: $y^N = \mathcal{C}^N$ and
$$\mathcal{C}^T + (1 - \pi)B = y^T + q\mathcal{B}' + (1 - \mathcal{D})\{Q[\mathcal{L}' - (1 - \delta)L] - \delta L\}$$

[Back](#)

Recursive government problem I

Discrete and bounded choices of public debt

$$L' \in \Lambda = \{L_1, \dots, L_J\}$$

Each period the government draws a taste vector shock ϵ

$$\epsilon = [\epsilon_1, \dots, \epsilon_J, \epsilon_{J+1}]$$

First J th elements are associated with a public debt level and the $J + 1$ st with the choice of default

ϵ follows a multivariate generalized extreme value with location $m = -v\gamma$, scale $v > 0$, shape $\xi = 0$

Where γ is the Euler constant

Recursive government problem II

Taking the household policy functions as given the government solves:

$$W(s, L_i, B, \epsilon) = \max_{d \in \{0,1\}} [1 - d] W^R(s, L_i, B, \epsilon) + d W^D(s, L_i, B, \epsilon_{J+1})$$

Where the value of default is:

$$\begin{aligned} W^d(s, L_i, B, \epsilon_{J+1}) &= u\left(\mathcal{C}^D(s, B)\right) - \phi(y^T) + \epsilon_{J+1} + \\ &\quad \beta \mathbb{E}_{s' | s} \mathbb{E}_{\epsilon'} \left[W(s', 0, \mathcal{B}'^D(s, B), \epsilon') \right] \end{aligned}$$

Subject to the resource constraints, $\mathcal{C}^N = y^N$ and :

$$\mathcal{C}^{T,D}(s, B) = y^T + q \mathcal{B}'^D(s, B) - B,$$

Recursive government problem III

And the value of repayment is:

$$W^c(s, L_i, B, \epsilon) = \max_{j \in \llbracket 1, J \rrbracket} u(\mathcal{C}(s, L_i, B)) + \epsilon_j +$$

$$\beta \mathbb{E}_{s' \mid s} \mathbb{E}_{\epsilon'} [W(s', L_j, \mathcal{B}'(s, L_i, B), \epsilon')]$$

Subject to the resource constraints, $\mathcal{C}^N = y^N$ and:

$$\mathcal{C}^T(s, , L_i, B) = y^T + q\mathcal{B}'(s, L_i, B) - B +$$

$$Q(s, L_i, \mathcal{B}') [L_j - (1 - \delta)L_i] - \delta L_i$$

Recursive government problem III

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$$\underbrace{Q(s, L_i, \mathcal{B}') [L_j - (1 - \delta)L_i] - \delta L_i}_{T(s, L_i, B')}$$

[Back to Government problem](#)

[Back to calibration](#)

Foreign lenders pricing in equilibrium

For all $j \in \llbracket 0, J \rrbracket$:

$$Q(s, L_j, B') = q \mathbb{E}_{s' \mid s} \mathbb{E}_{\epsilon'} \left[\left[1 - d(s', L_j, B', \epsilon') \right] \times \left[\delta + (1 - \delta) Q\left(s', L'(s', L_j, B', \epsilon'), B'(s, L_j, L', B')\right) \right] \right]$$

[Recursive equilibrium](#)

[Solution method](#)

[Back to Government problem](#)

Recursive problem of the government : properties

The Government expected values can be re-written:

$$W(s, L_i, B) = E_\epsilon[W(s, L_i, B, \epsilon)]$$

$$W(s, L_i, B) = E_\epsilon \left[\max \{ W^c(s, L_i, B, \epsilon); W^d(s, L_i, B, \epsilon_{J+1}) \} \right]$$

$$\begin{aligned} W(s, L_i, B) = E_\epsilon & \left[\max \left\{ \max_{j \in \llbracket 1, J \rrbracket} \{ u(\mathcal{C}_{i,j}) + \beta \mathbb{E}_{s' | s} W(s', L_j, \mathcal{B}'_{i,j}) + \epsilon_j \}; \right. \right. \\ & \left. \left. u(\mathcal{C}_{0,0}) - \phi(y^T) + \beta \mathbb{E}_{s' | s} W(s', 0, \mathcal{B}'_{0,0}) + \epsilon_{J+1} \right\} \right] \end{aligned}$$

Subject to the resource constraints:

$$\mathcal{C}^T(s, L_i, L_j, B) = y^T + q\mathcal{B}'(s, L_i, L_j, B) - B + T(s, L_i, L_j, \mathcal{B}'_{i,j})$$

$$\mathcal{C}^N(s, L_i, L_j, B) = y^N$$

Recursive problem of the government : properties II

We assume that the vector ϵ is i.i.d over time and follows a multivariate Generalized Extreme Value distribution with joint cumulative distribution function:

$$F(\mathbf{x}) = \exp \left[- \left(\sum_{j=1}^J \exp \left(- \frac{x_j - m}{\nu} \right) \right)^p - \exp \left(- \frac{x_{J+1} - m}{\nu} \right) \right]$$

It is useful to define:

$$\Upsilon_{i,j} = u(\mathcal{C}_{i,j}) + \beta \mathbb{E}_{s'|s} \mathbf{W}(s', L_j, \mathcal{B}'_{i,j})$$

$$\Upsilon_{i,def} = u(\mathcal{C}_{0,0}) - \phi(y^T) + \beta \mathbb{E}_{s'|s} \mathbf{W}(s', 0, \mathcal{B}'_{0,0})$$

Recursive problem of the government : properties III

The Government value can be re-written:

$$W(s, L_i, B) = \Upsilon_{i,def} + v \log \left[1 + \left(\sum_{k \in [1, J]} \exp \left(-\frac{\Upsilon_{i,def} - \Upsilon_{i,k}}{pv} \right) \right)^p \right]$$

There are also close form solutions for the probability of default:

$$d(s, L_i, B) = \mathbb{E}_\epsilon[d(s, L_i, B, \epsilon)]$$

$$d(s, L_i, B) = \frac{1}{1 + \left(\sum_{k \in [1, J]} \exp \left(-\frac{\Upsilon_{i,def} - \Upsilon_{i,k}}{pv} \right) \right)^p}$$

Recursive problem of the government : properties IV

The probability of issuance for each level of government debt conditional on repayment :

$$\mathbf{G}_{s,L_i,B}(L_j) = \mathbb{P}_\epsilon(L' = L_j | d(s, L_i, B, \epsilon) = 0)$$

$$\mathbf{G}_{s,L_i,B}(L_j) = \frac{1}{\sum_{k \in [1, J]} \exp\left(\frac{\gamma_{i,k} - \gamma_{i,j}}{pv}\right)}$$

The price of debt:

$$Q(s, L_j, B') = q \mathbb{E}_{s'|s} \left[[1 - \mathbf{d}(s', L_j, B')] [\delta + \right]$$

$$+ (1 - \delta) \sum_{k \in [1, J]} Q(s', L_k, , \mathcal{B}'(s', L_j, L_k, B')) \mathbf{G}_{s', L_j, B'}(L_k) \right]$$

Baseline: Foreign lenders problem

When lending to households, they maximize recursive profits:

$$\chi(s) = \max_q -qB' + \frac{B'}{1+r} \mathbb{E}_{s'|s} [1 - \pi']$$

When lending to the government they maximize recursive profits:

$$\begin{aligned} \Pi(s, L', B') = \max_Q -QL' + \frac{L'}{1+r} \mathbb{E}_{s'|s} & \left[\left\{ 1 - d' \right\} \times \right. \\ & \left. \left\{ \delta + (1 - \delta) Q(s', L'', B'') \right\} \right] \end{aligned}$$

Where,

$$B'' = \mathcal{B}'(s', L', B', d', L'')$$

$$L'' = \mathcal{L}'(s', L', B')$$

$$d' = \mathbf{d}'(s', L', B')$$

SP: Foreign lenders problem

When lending private bonds, they maximize recursive profits:

$$\chi(s) = \max_q -qB' + \frac{B'}{1+r} \mathbb{E}_{s'|s} [1 - \pi']$$

When lending to the public bonds they maximize recursive profits:

$$\Pi(s, L', B') = \max_Q -QL' + \frac{L'}{1+r} \mathbb{E}_{s'|s} \left[\left\{ 1 - d(s', L', B') \right\} \times \left\{ \delta + (1-\delta)Q(s', L'', B'') \right\} \right]$$

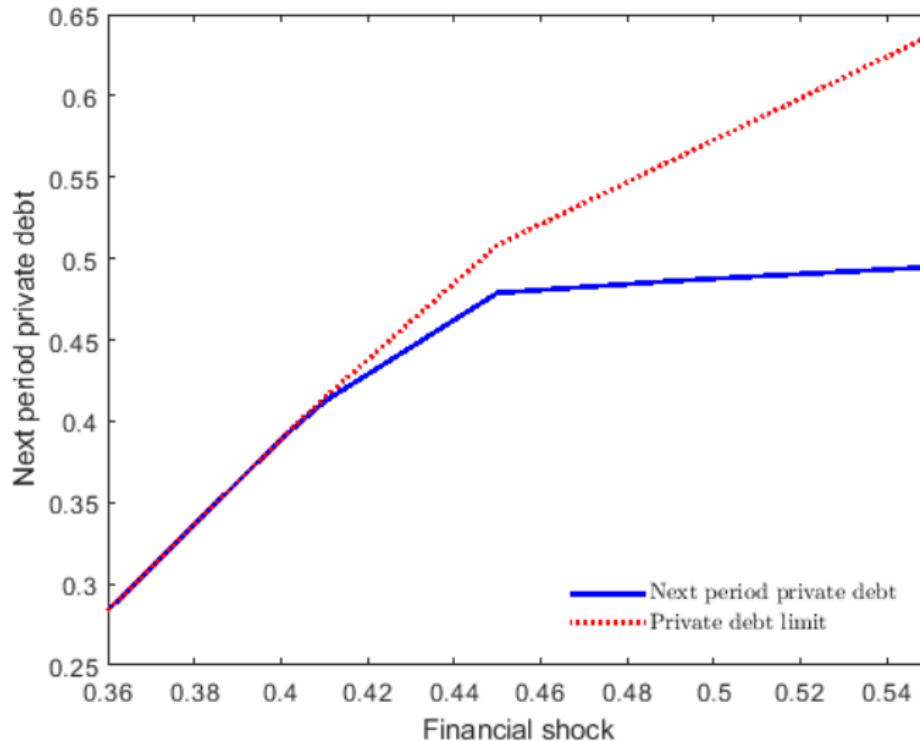
Where,

$$B'' = \mathcal{B}^{SP'}(s', L', B')$$

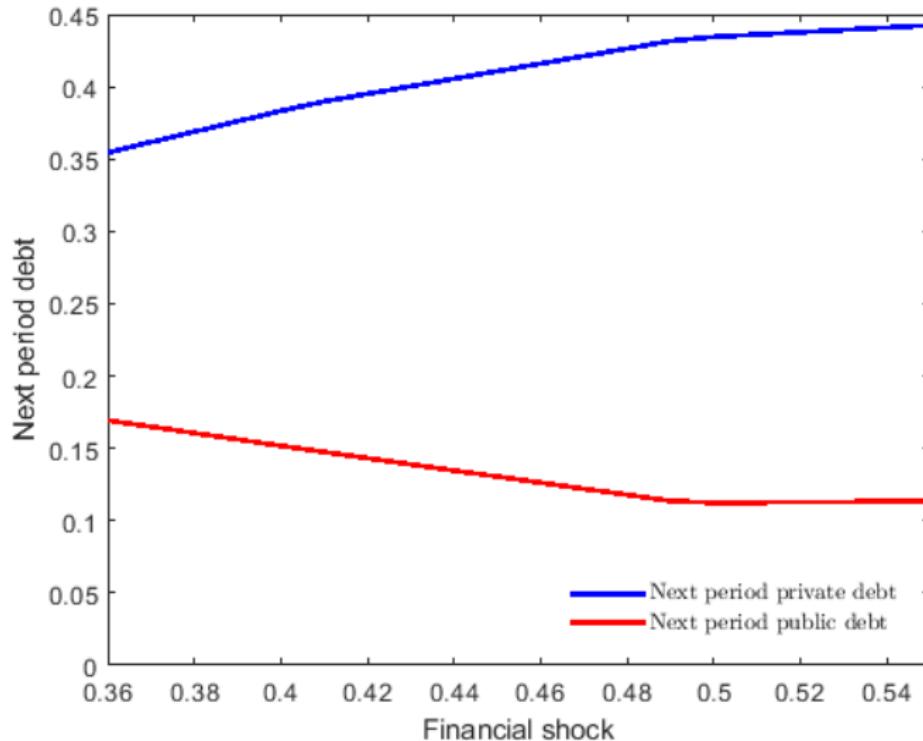
$$L'' = \mathcal{L}^{SP'}(s', L', B')$$

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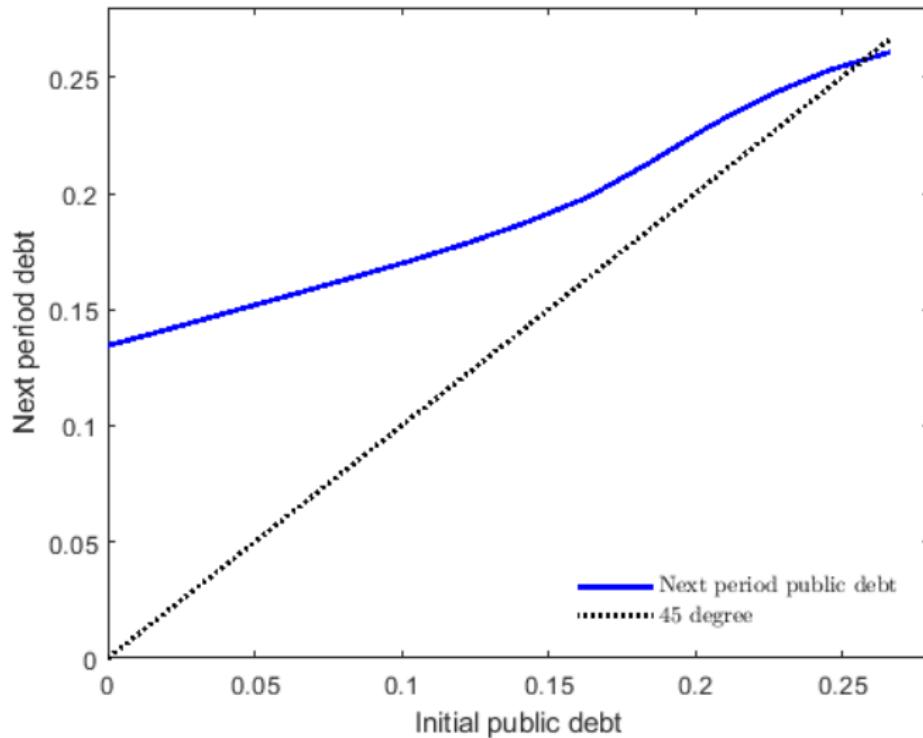
Private debt as function of the financial shock



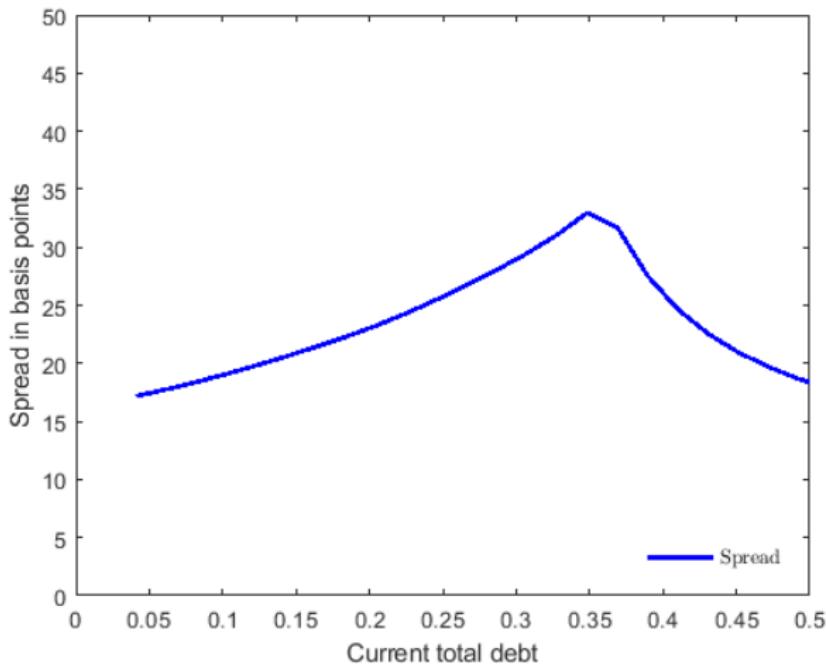
Public debt as function of the financial shock



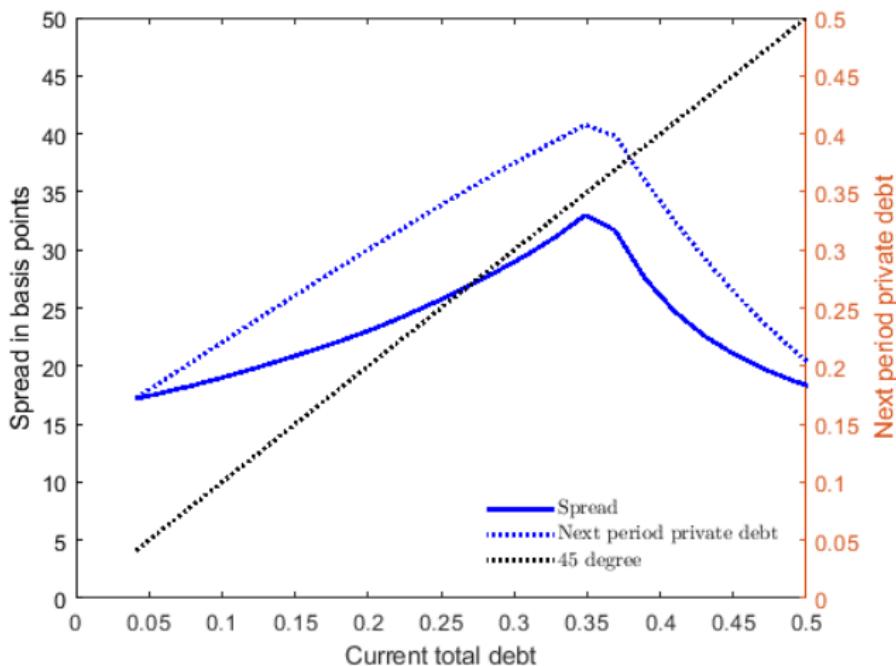
Next period public debt as a function of current private debt



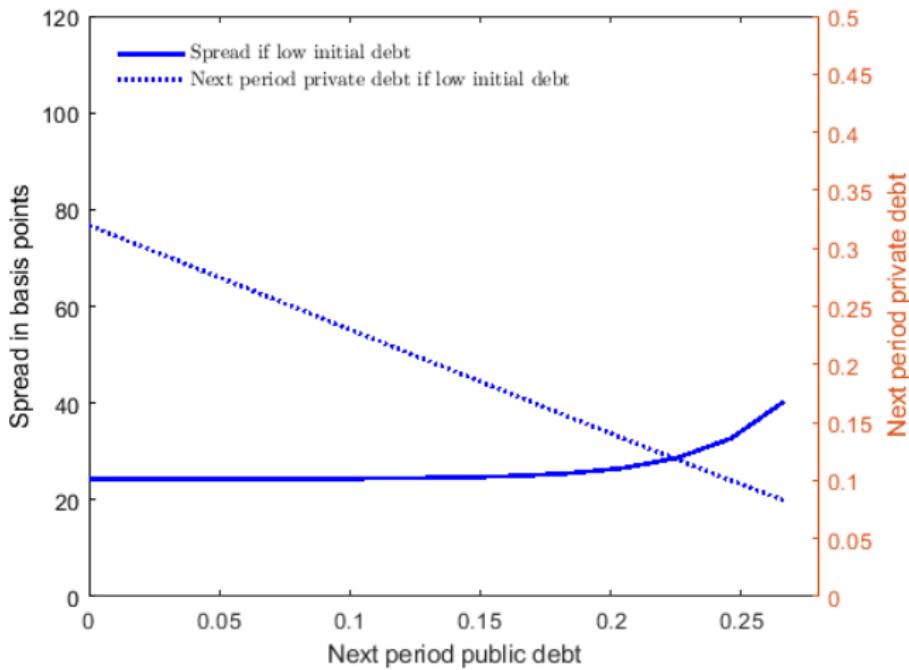
Spreads as a function of current private debt



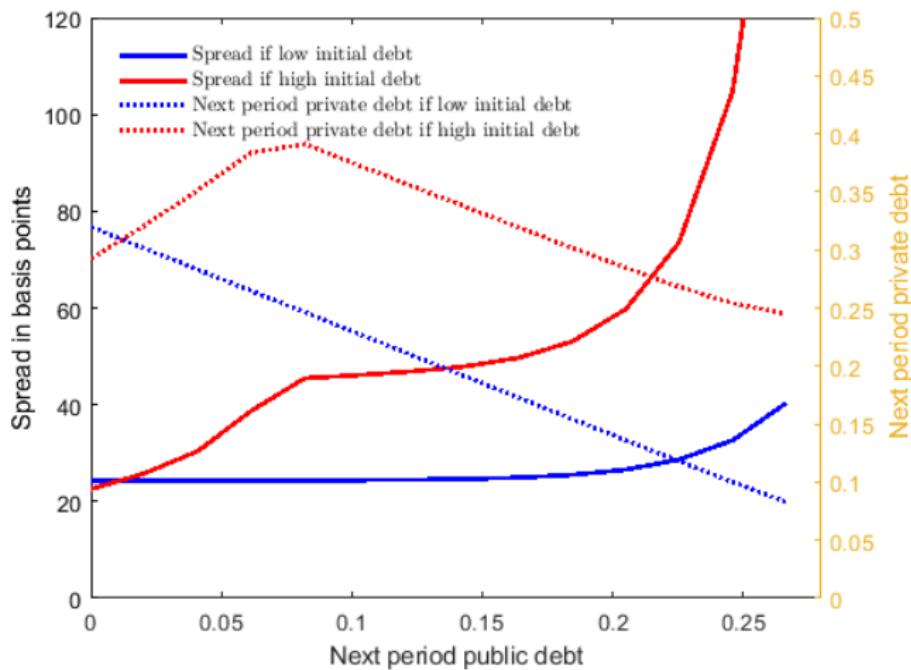
Spreads as a function of current private debt



Spreads as a function of next period public debt



Spreads as a function of next period public debt



Euler equations in the baseline

Intertemporal Euler equation for private bonds

$$qu_T(c) = \beta \mathbb{E}[(1 - \pi')u_T(c')] + \mu q$$

Where μ is the Lagrange multiplier associated with the credit constraint

[Back to equilibrium conditions](#)

Household problem with taxes

Households solve:

$$V^\tau(s, L, B, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V^\tau(S'_G, b')]$$

subject to

$$c^T + p^N c^N + (1 - \pi)b = y^T + p^N y^N + (1 - \tau(S_G))q(\pi)b' + T(S_G)$$

$$q(\pi)b' \leq \kappa[p^N y^N + y^T],$$

$$p^N = p^N(S_G),$$

$$B' = \mathcal{B}'(S_G),$$

$$L' = \mathcal{L}'(S_G)$$

[Government problem with taxes](#)
[Back to densities](#)

[Euler equations with taxes](#)

[Back to equivalence result](#)

Planner's Euler equation of private bonds

Intertemporal equation for the private bonds

$$(u_T(c_T^{SP}) + \mu\psi)(q^{SP} + Q_{B'}^{SP}(L' - (1 - \delta)L)) \\ = \beta \mathbb{E}[(1 - \pi')(u_T(c_{T'}^{SP}) + \mu^{SP'}\psi')] + q^{SP}\mu^{SP}$$

μ^{SP} is the Lagrange multiplier associated with the credit constraint

$$\psi = \kappa(1 + \eta) \frac{(1 - \omega)}{\omega} \left(\frac{c_T^{SP}}{y_N} \right)^\eta$$

Marginal effect of an extra unit of private debt on the credit constraint

$Q_{B'}^{SP}$ marginal effect of an extra unit of private debt on the price of L

Euler in the baseline model with taxes on b'

$$(1 - \tau)qu_T(c) = \beta\mathbb{E}[(1 - \pi')u_T(c')] + \mu q$$

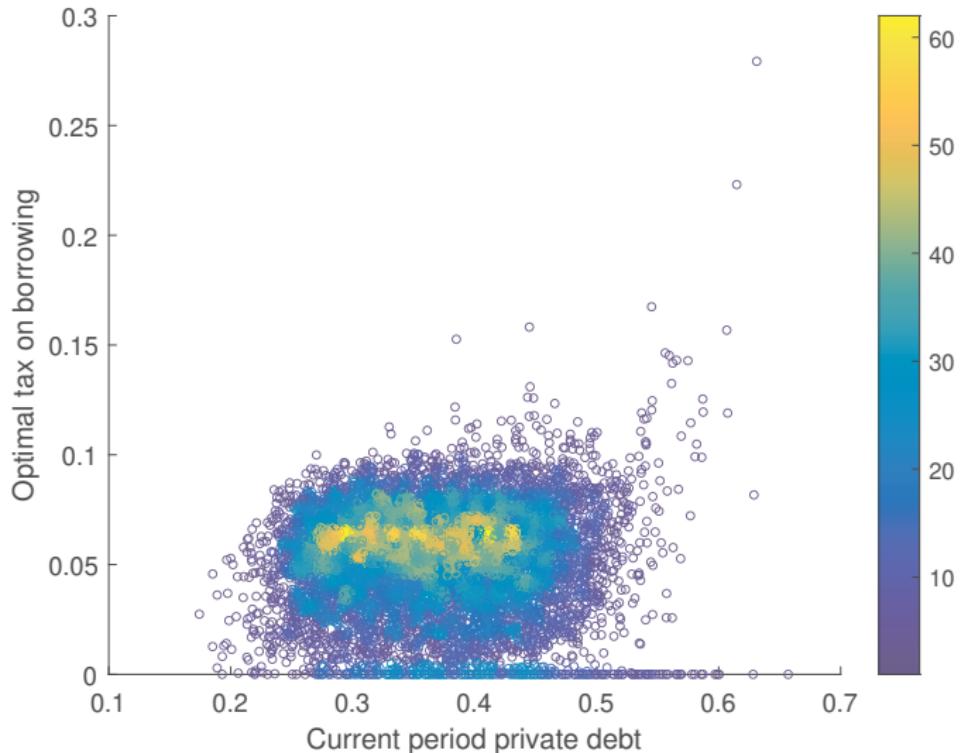
When the constraint does not bind:

$$(1 - \tau) = \frac{\beta\mathbb{E}[(1 - \pi')u_T(c')]}{qu_T(c)}$$

Otherwise

$$(1 - \tau) = \frac{\beta\mathbb{E}[(1 - \pi')u_T(c')]}{qu_T(c)} + \frac{\mu q}{qu_T(c)}$$

Optimal tax on borrowing



[Back to Ex. I](#)

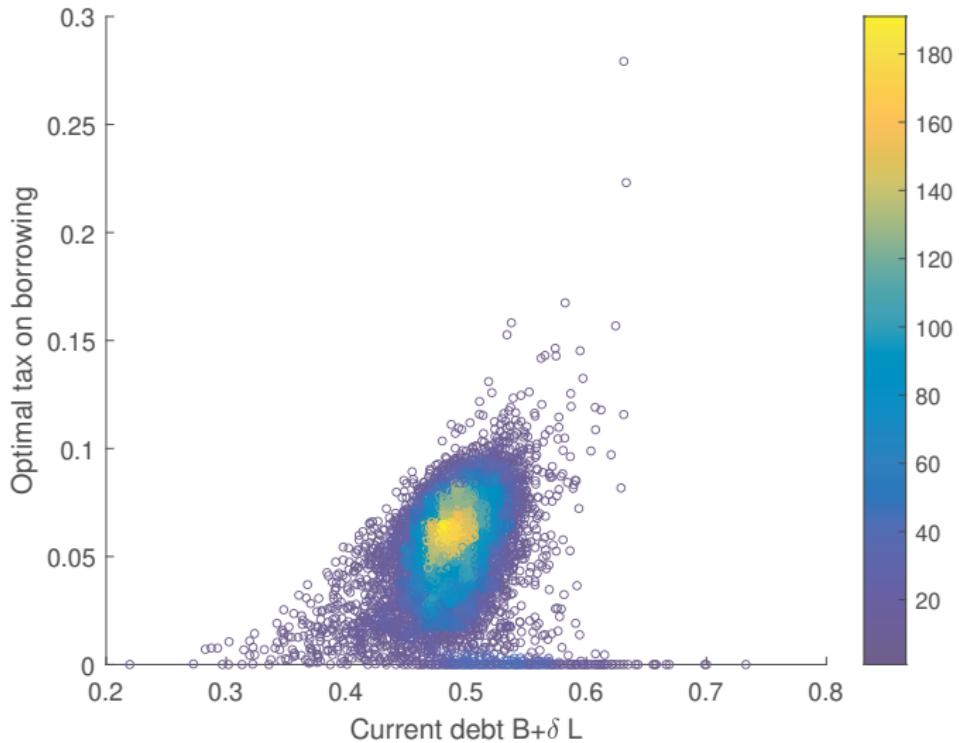
[Back to results](#)

[Back to densities](#)

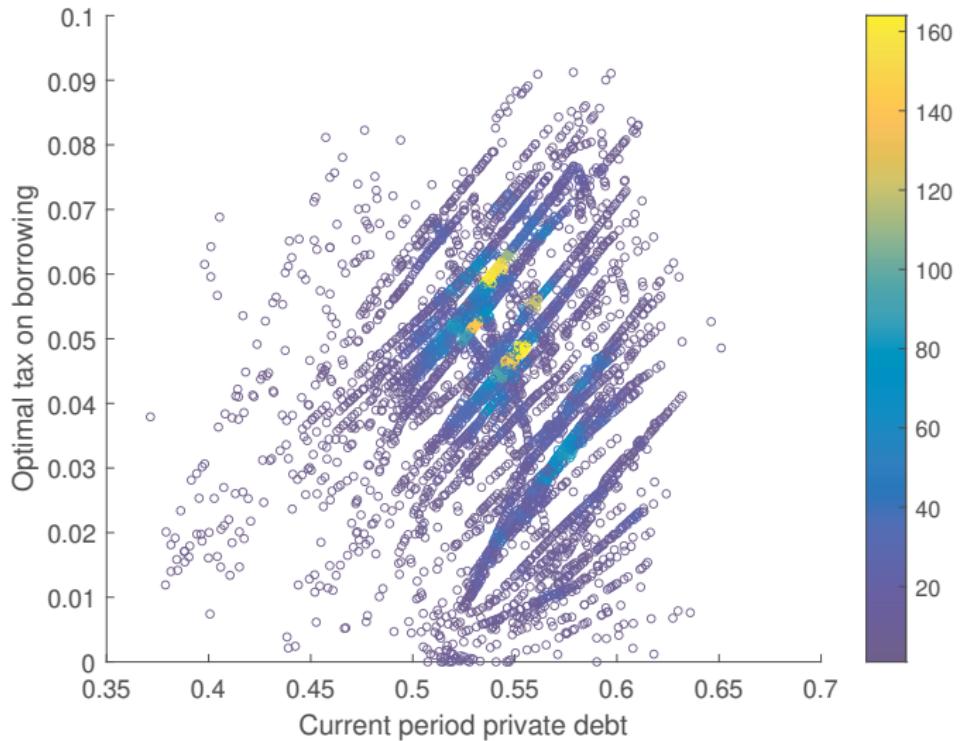
[With respect to total debt](#)

[With no \$L\$](#)

Optimal tax on borrowing relative to total debt



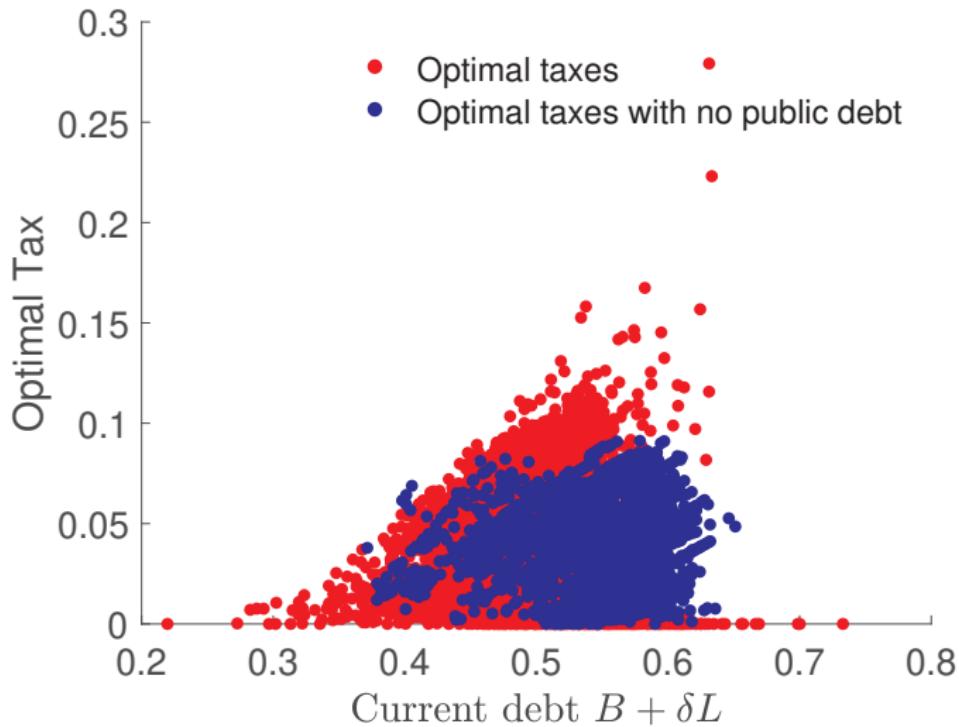
Taxes in a model with no public debt L



[Back](#)

[Compare models at the ergodic](#)

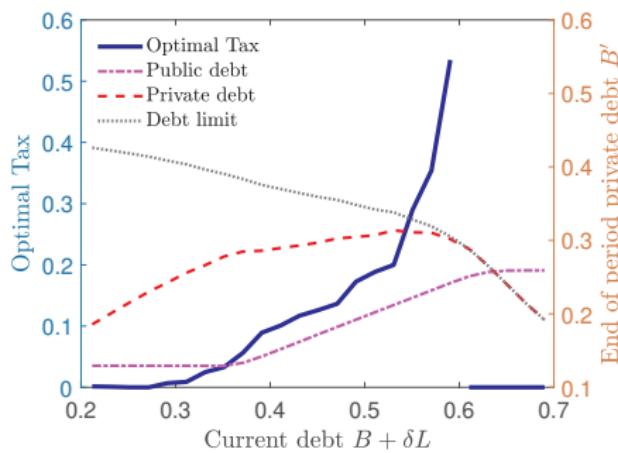
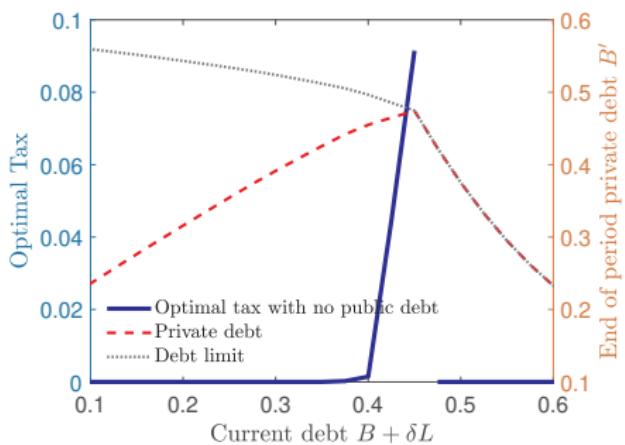
Taxes in models with and without L



[Back to no public debt](#)

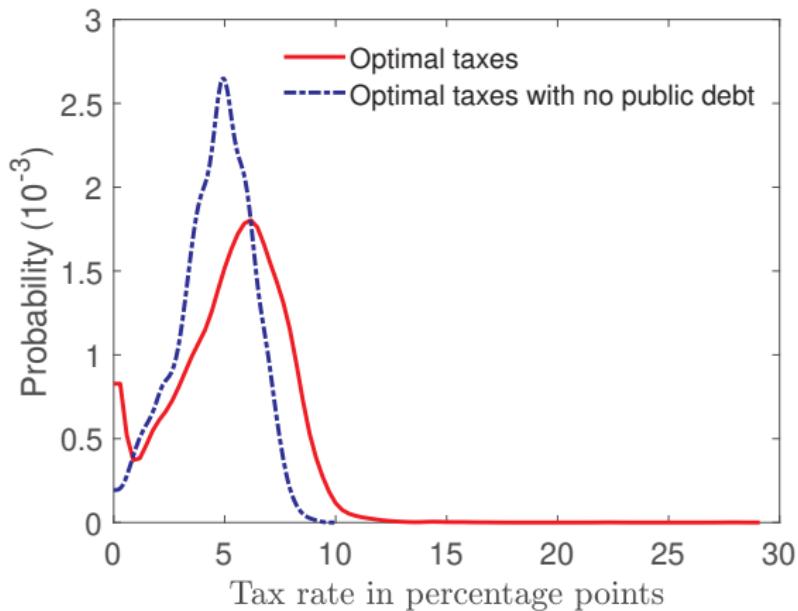
[Back to densities](#)

Optimal taxes as function of private debt



[Back](#)

Policy recommendation : Higher taxes on private borrowing



Higher taxes than a model with no public debt

Simulations at the ergodic

The average tax rate increases from **4.6%** to **5.3%**

Policy functions

Welfare calculations

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1 - \sigma}$$

Where:

$$C(c^T, c^N) = \left((c^T)^{-\eta} + (1 - \omega)(c^N)^{-\eta} \right)^{\frac{-1}{\eta}}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$\theta(s_0, L_0, B_0) = \left(\frac{W^{SP}(s_0, L_0, B_0) * (1 - \sigma) * (1 - \beta) + 1}{W(s_0, L_0, B_0) * (1 - \sigma) * (1 - \beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1$$

Alternative Welfare calculations

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1 - \sigma}$$

Where:

$$C(c^T, c^N) = \left((c^T)^{-\eta} + (1 - \omega)(c^N)^{-\eta} \right)^{\frac{-1}{\eta}}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$\theta(s_0, L_0, B_0, L'_0) = \left(\frac{V^{SP}(s_0, L_0, B_0, L'_0) * (1 - \sigma) * (1 - \beta) + 1}{V(s_0, L_0, B_0, L'_0) * (1 - \sigma) * (1 - \beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1$$

One Period Welfare gains

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1 - \sigma}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$\begin{aligned} W^{SP}(s, L, B) = & \left(1 - \hat{d}(s, L, B)\right) \left[(1 + \theta(s, L, B)) u(\hat{c}(s, L, B)) + \right. \\ & \left. + \beta \mathbb{E} V(s', \hat{B}', \hat{L}') \right] + \hat{d}(s, L, B) \left[(1 + \theta(s, L, B)) \times \right. \\ & \left. \times u(\hat{c}(s, 0, B)) + \beta \mathbb{E} V(s', \hat{B}', 0) \right] \end{aligned}$$

Models of sovereign debt

Average (in %)	Data	Baseline	SP	$\delta = .14$	$\delta = 1$
Private debt	42	42	37	-	-
Public debt	14	15	12	13	15
Spread	.45	.45	.034	.080	.28
Probability of a financial crisis	-	2.5	.10	-	-
Probability of default	-	.46	.030	.040	.35
Welfare gains	-	-	.41	2.4	2.7

Hatchondo
& Martinez
2009

Arellano
2008

For the all models welfare gains are calculated relative to baseline

Back

Models of private debt

Average (in %)	Data	Baseline	SP	Decentralized	Efficient
Private debt	42	42	37	44	43
Public debt	14	15	12	-	-
Spread	.45	.45	.034	-	-
Probability of a financial crisis	-	2.5	.10	6.4	1.8
Probability of default	-	.46	.030	-	-
Welfare gains (rel to Baseline)	-	-	.41	6.4	6.2
Welfare gains (rel to Bianchi DE)	-	-	-	-	.17

Bianchi
2011

Back

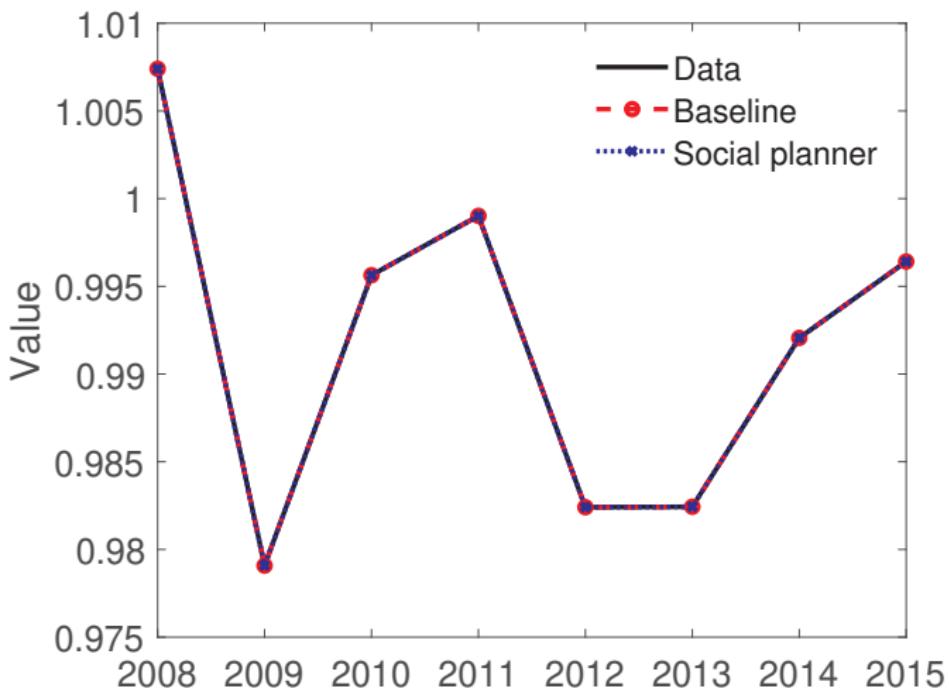
Model with an exogenous constraint

$$qB' \leq \kappa(y^N + y^T)$$

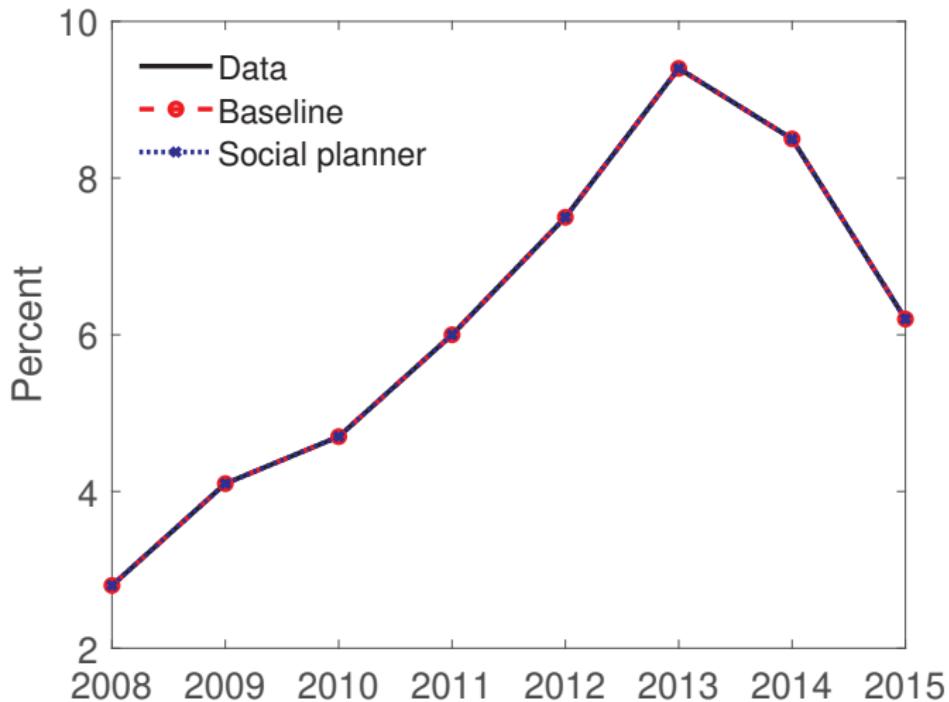
Average (in %)	Data	Baseline	SP	Decent.	Planner
Private debt	42	42	37	33	32
Public debt	14	15	12	12	12
Spread	.45	.45	.034	.038	.036
Probability of a financial crisis	-	2.5	.1	0	0
Probability of default	-	.46	.030	.030	.030
Welfare gains	-	-	.0041	-	-

Back

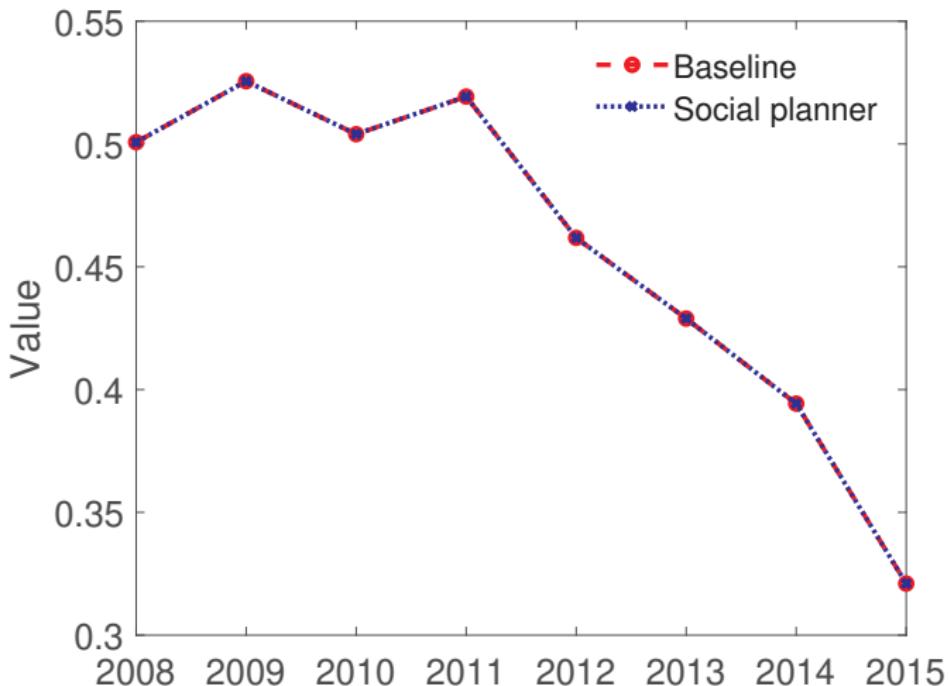
Exercise I: Exogenous dynamics of income



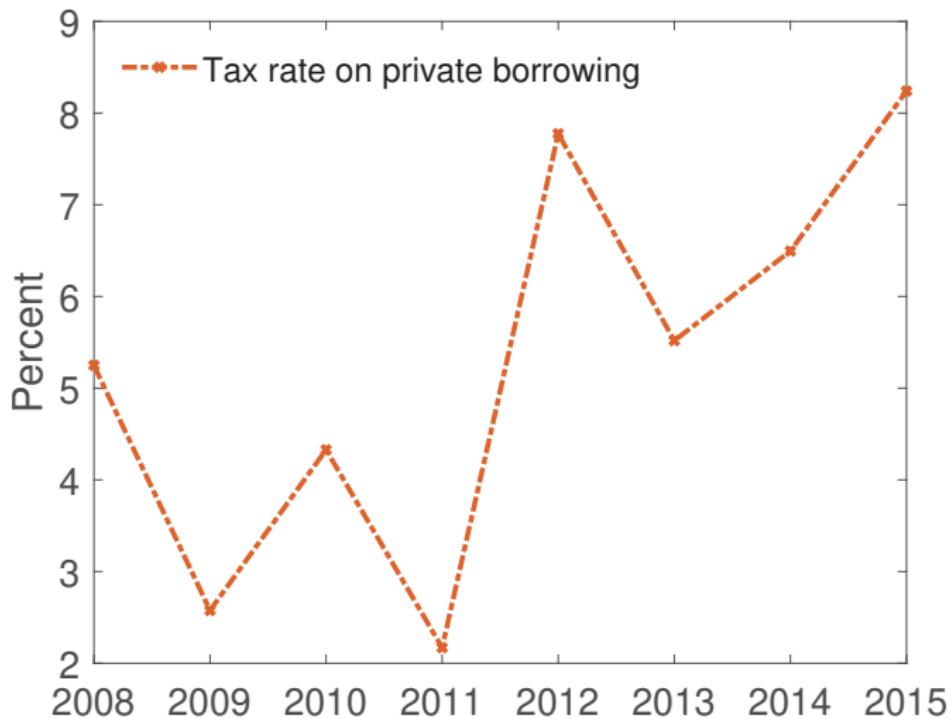
Exercise I: Exogenous dynamics of private default



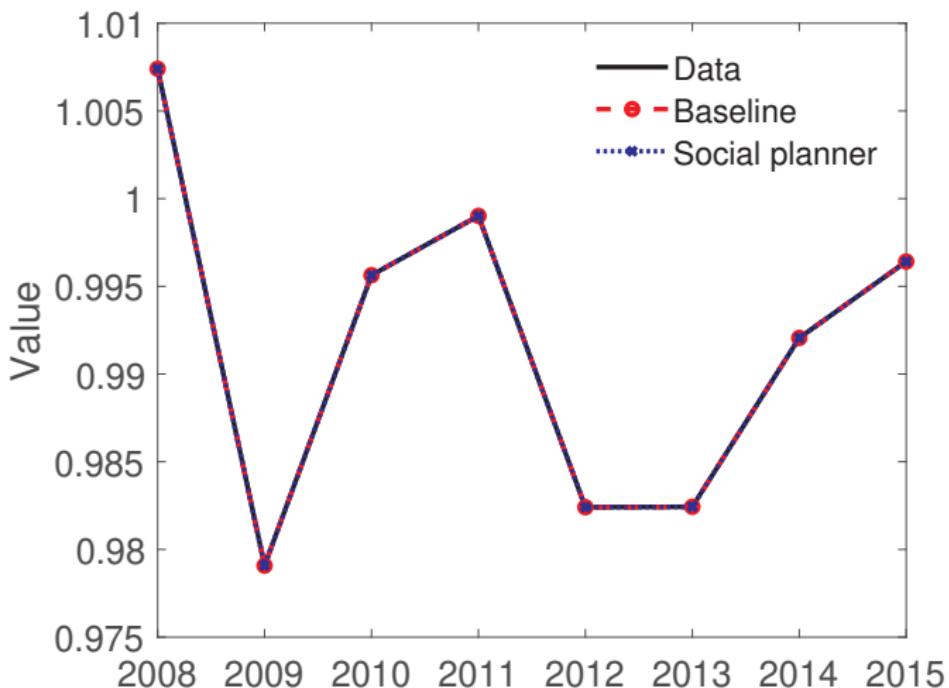
Exercise I: Exogenous financial shock



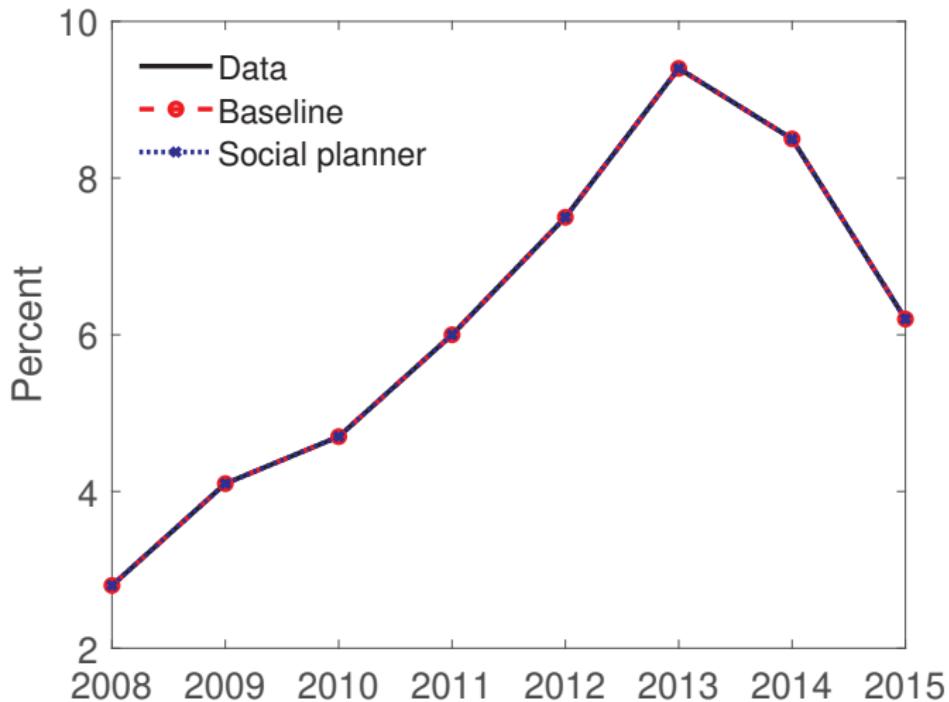
Exercise I: Optimal tax dynamics



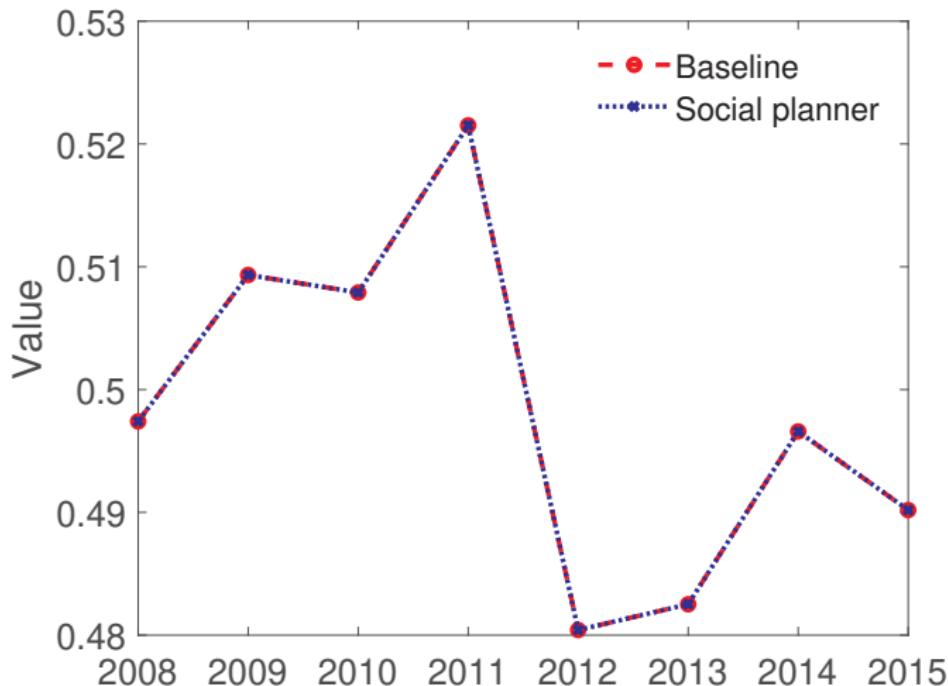
Exercise II: Exogenous dynamics of income



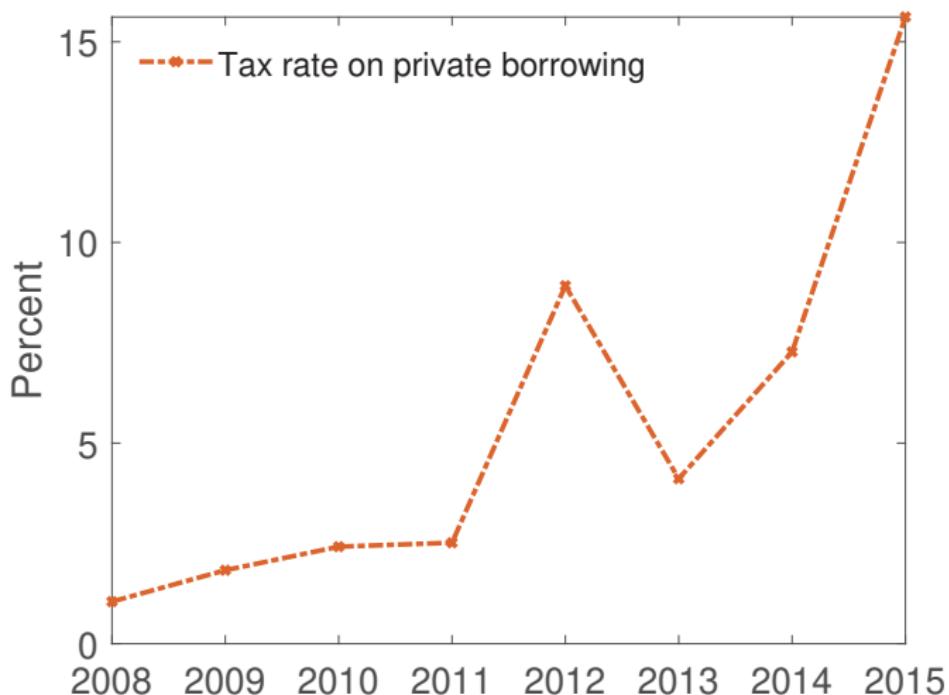
Exercise II: Exogenous dynamics of private default



Exercise II: Exogenous financial shock



Exercise II: Optimal tax dynamics



Why do households overborrow?

Households do not pick B' , thereby ignore **two** GE effects

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$$B' \uparrow \Rightarrow C^{T'} \downarrow \Rightarrow \mathbb{P} \text{ of } d' \uparrow \Rightarrow Q \uparrow$$

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2. Effect on the price of public debt

$$B' \uparrow \Rightarrow C^{T'} \downarrow \Rightarrow \mathbb{P} \text{ of } d' \uparrow \Rightarrow Q \uparrow$$

High private debt increases the cost of public debt

Planner incorporates these effects on its borrowing decision $\Rightarrow B' > B'^{SP}$

Why does the government respond with more public debt?

Private and **public** debt can be complements or substitutes

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When the credit constraint binds

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When the credit constraint binds

$$L' \uparrow \Rightarrow T \uparrow \Rightarrow C^T \uparrow \Rightarrow p^N(C^T) \uparrow \Rightarrow B' \uparrow$$

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$$L' \uparrow \Rightarrow T \uparrow \Rightarrow C^T \uparrow \Rightarrow p^N(C^T) \uparrow \Rightarrow B' \uparrow$$

Public and private debt are **complements**

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Public and private debt are **complements**

When the credit constraint binds does not bind

$$L' \uparrow \Rightarrow T \uparrow \Rightarrow B' \downarrow$$

Public and private debt are **substitutes**

Planner faces a bidding constraint **less** frequently $\Rightarrow L' > L'^{SP}$

Mechanism: Baseline relative to planner

Households **overborrow** because they don't internalize:

1. The probability of a binding credit constraint next period
2. The probability of default next period
3. Future issuances of public debt

The government responds by **issuing more public debt** because:

1. When the constraint doesn't bind, public and private debt are imperfect substitutes
2. When the constraint binds, public debt relaxes the constraint by depreciating the real exchange rate

This combination increases the frequency of **crises** and **spreads**

SP: Planner problem I

$$W^{SP}(s, L, B) = \max_{d \in \{0,1\}} [1 - d] W^{SP,R}(s, L, B) + d W^{SP,D}(s, L, B)$$

Where the value of default is:

$$W^{SP,D}(s, L, B) = \max_{B'} u(c) - \phi(y^T) + \beta \mathbb{E}_{s' | s} \left[W^{SP}(s', 0, B') \right]$$

Subject to the resource constraints :

$$c^T = y^T + q(\pi)B' - (1 - \pi)B,$$

$$q(\pi)B' \leq \kappa \left[\frac{(1 - \omega)}{\omega} \left(\frac{c^T}{c^N} \right)^{1+\eta} + y^T \right],$$
$$c^N = y^N$$

SP: Planner problem II

And the value of repayment is:

$$W^{SP,R}(s, L, B) = \max_{L' > 0, B'} u(c) + \beta \mathbb{E}_{s'|s} \left[W^{SP}(s', L, B') \right]$$

Subject to the resource constraints:

$$c^T = y^T + q(\pi)B' - (1 - \pi)B + Q(s, L', B') \left[L' - (1 - \delta)L \right] - \delta L$$

$$q(\pi)B' \leq \kappa \left[\frac{(1 - \omega)}{\omega} \left(\frac{c^T}{c^N} \right)^{1+\eta} + y^T \right],$$

Back

$$c^N = y^N$$

SP: Equilibrium pricing

With zero profits: :

$$q(s) = \frac{1}{1+r} \mathbb{E}_{s'|s} [1 - \pi']$$

With zero profits:

$$Q(s, L', B') = \frac{1}{1+r} \mathbb{E}_{s'|s} \left[\left\{ 1 - d^{SP}(s', L', B') \right\} \times \left\{ \delta + (1-\delta) Q(s', L'', B'') \right\} \right]$$

Where,

$$B'' = \mathcal{B}'^{SP}(s', L', B')$$

$$L'' = \mathcal{L}'^{SP}(s', L', B')$$

Government problem with taxes I

Taking the household policy functions as given the government solves:

$$W^\tau(s, L, B) = \max_{d \in \{0,1\}} [1 - d] W^{\tau,R}(s, L, B) + d W^{\tau,D}(s, L, B)$$

In default:

$$W^{\tau,D}(s, L, B) = \max_{\tau} u(\mathcal{C}_{L=0,\tau}^{T,N}) - \phi(y^T) + \beta \mathbb{E}_{s'|s} \left[W^\tau(s', 0, \mathcal{B}'_{L=0,\tau}) \right]$$

subject to

$$\mathcal{C}_{L=0,\tau}^N = y^N$$

$$\mathcal{C}_{L=0,\tau}^T = y^T + (1 - \tau)q(\pi)\mathcal{B}'_{L=0,\tau} - (1 - \pi)B + \underbrace{\tau q(\pi)\mathcal{B}'_{L=0,\tau}}_{T(s, 0, B, \tau)}$$

Government problem with taxes II

In repayment, the government solves:

$$W^R(s, L, B) = \max_{L' > 0, \tau} u(\mathcal{C}(s, L, B, \tau)) + \beta \mathbb{E}_{s'|s} \left[W(s', L, \mathcal{B}'(s, L, B, \tau)) \right]$$

Subject to:

$$\mathcal{C}^N = y^N$$

$$\begin{aligned} \mathcal{C}^T &= y^T + (1 - \tau)q(\pi)\mathcal{B}' - (1 - \pi)B + \\ &\underbrace{Q(s, L, B, \tau) \left[L' - (1 - \delta)L \right] - \delta L + \tau q(\pi)\mathcal{B}'}_{T(s, L, B, \tau)} \end{aligned}$$

Baseline: Recursive government problem, default

In default:

$$W^D(s, B) = u(\mathcal{C}^{BR}(s, \mathbf{0}, B; \mathbf{L}' = \mathbf{0}, \mathbf{d} = \mathbf{1})) - \phi(y^T) + \beta \mathbb{E}_{s' | s} \left[W(s', \mathbf{0}, \mathcal{B}'^{BR}(s, \mathbf{L}' = \mathbf{0}, \mathbf{d} = \mathbf{1})) \right]$$

Subject to

$$\mathcal{C}^{N, BR}(s, \mathbf{0}, B; \mathbf{L}' = \mathbf{0}, \mathbf{d} = \mathbf{1}) = y^N$$

$$\mathcal{C}^{T, BR}(s, \mathbf{0}, B; \mathbf{L}' = \mathbf{0}, \mathbf{d} = \mathbf{1}) = y^T + q(\pi) \mathcal{B}'^{BR}(s, \mathbf{0}, B; \mathbf{L}' = \mathbf{0}, \mathbf{d} = \mathbf{1}) - (1 - \pi) B$$

Baseline: Recursive government problem, repayment

In repayment, the government solves:

$$W^R(s, L, B) = \max_{L' > 0} u(\mathcal{C}(s, L, B; L', d)) + \beta \mathbb{E}_{s'|s} \left[W(s', L', \mathcal{B}'^{BR}(s, L, B; L', d)) \right]$$

Subject to

$$\mathcal{C}^{N, BR}(s, L, B; L', d) = y^N$$

$$\mathcal{C}^{T, BR}(s, L, B; L', d) = y^T + q(\pi) \mathcal{B}'^{BR}(s, L, B; L', d) - (1 - \pi) B$$

$$+ \underbrace{Q^{BR}(s, L', \mathcal{B}'^{BR}(s, L, B; L', d)) \left[L' - (1 - \delta)L \right] - \delta L}_{T(s, L, B; L', d)}$$

Baseline: Foreign lenders pricing in equilibrium

Price of private debt is:

$$q(\pi) = \frac{\mathbb{E}_{s'|s} [1 - \pi']} {1 + r}$$

Price of public debt is:

$$Q(s, L', B') = \frac{1}{1+r} \mathbb{E}_{s'|s} \left[\left\{ 1 - d' \right\} \times \left(\delta + (1-\delta) Q'(s', L'', B'') \right) \right]$$

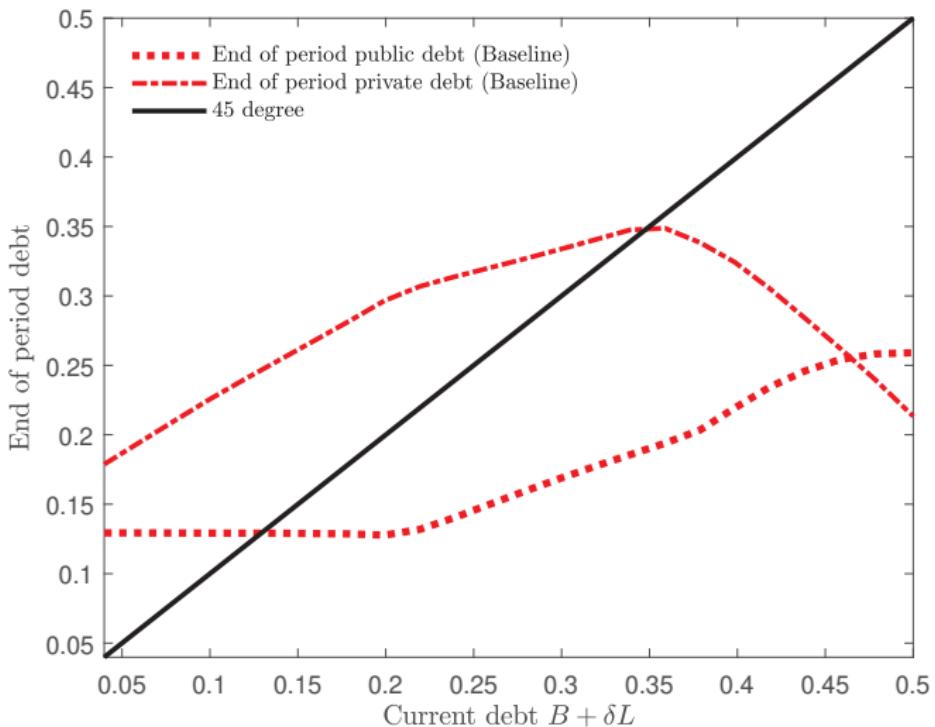
Where,

$$B'' = \mathcal{B}'(s', L', B', d', L'')$$

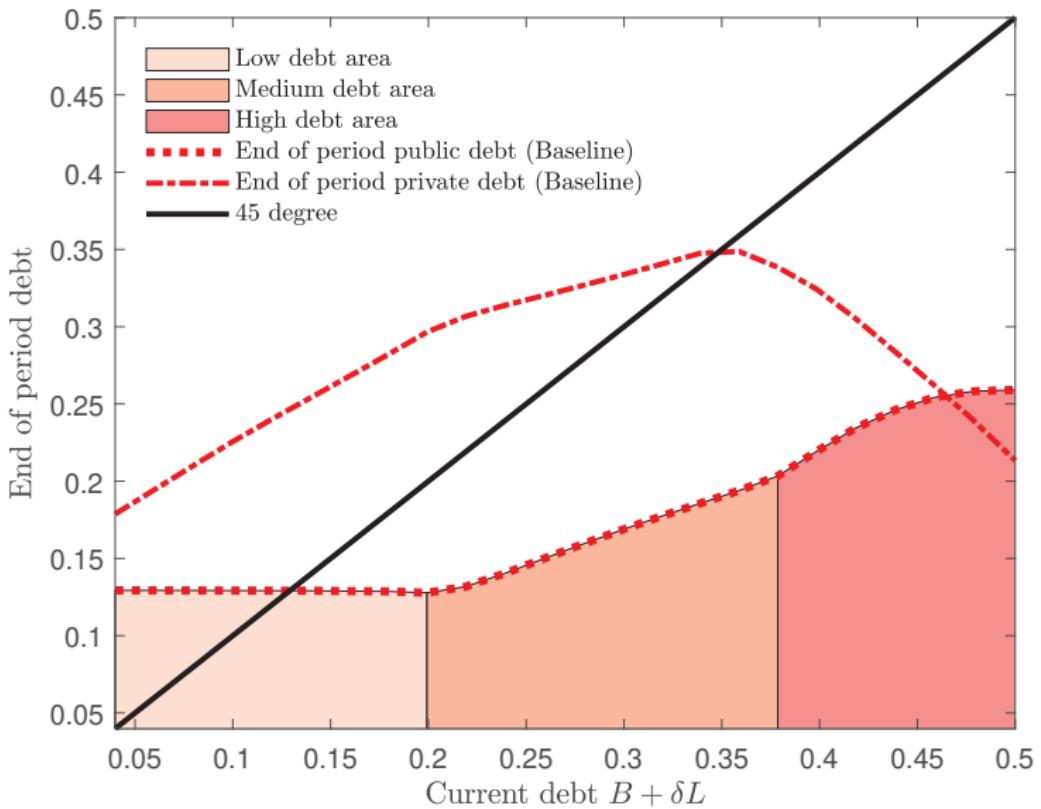
$$L'' = \mathcal{L}'(s', L', B')$$

$$d' = \mathbf{d}'(s', L', B')$$

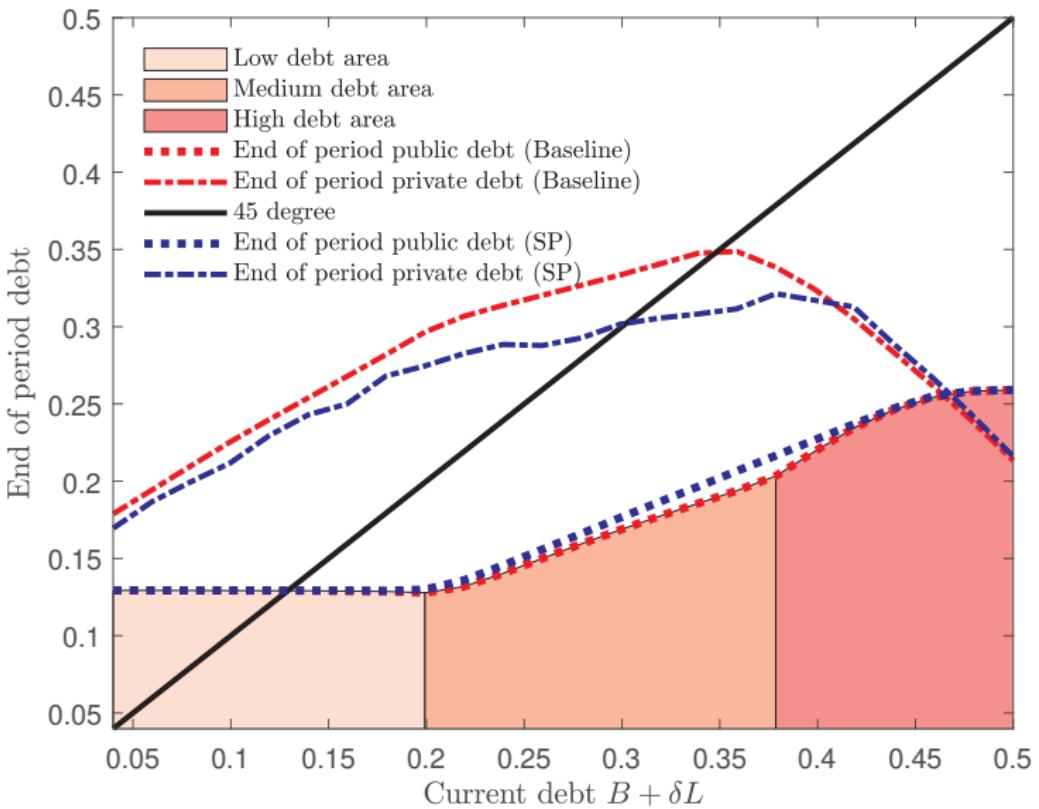
Public debt \mathcal{L}' as a function of current debt



Public debt \mathcal{L}' as a function of current debt



Public debt \mathcal{L}' as a function of current debt



Use particle filter to get κ_t

Write the problem in terms of state vectors $S_t = [B_{t+1}, L_{t+1}, y_t^T, \kappa_t, \pi_t]$.
And a vector of observable data

$$Y_t = [PrivateD_t, PublicD_t, Output_t, Nonperforming_t, Spread_t]$$

$$Y_t = g(S_t) + \eta_t \quad \text{with } \eta \sim N(0, \Sigma)$$

$$S_t = f(S_{t-1}, \epsilon_t) \quad \text{with } \epsilon_t \sim N(0, I)$$

Denote by $p(S_t | Y^t)$ the conditional distribution of the state vector given observations up to t . Approximate using $\{S_t^i, w_t^i\}_{i=1}^N$:

$$\frac{1}{N} \sum_{i=1}^N f(S_t^i) w_t^i \rightarrow \mathbb{E}[f(S_t) | Y^t]$$

Use particle filter algorithm to get w_t^i . Run counterfactual experiment with these shocks.

Simplifications

No measurement error in $\{y_t^T, \pi_t, L_{t+1}, \}$ (taken directly from the data).
Now $Y_t = [PrivateD_t, Spread_t]$

$$PrivateD_t = \frac{b_{t+1}}{p_t^N y_t^N + y_t^T} + \eta_t^{11} \quad \text{with } \eta \sim N(0, \sigma^\eta)$$

$$Spread_t = \frac{\delta - \delta Q_t}{Q_t} - r + \eta_t^{22} \quad \text{with } \eta \sim N(0, \sigma^\eta)$$

σ^η set to 1% sample variance of $[PrivateD_t, Spread_t]_{2008}^{2015}$

We use Kitagawa 1996 algorithm to calculate $p(Y_t | S_t)$

Algorithm for the particle filter (Kitagawa 1996)

1. Set $t = 1$. Set $w_i = 1/N$ for $i = 1 \dots N$. Draw $\{S_0^i\}_{i=0}^N$ from ergodic
2. For each draw compute $S_{t|t-1}^i$ using the model (For us $b'(S_{t-1}^i)$)
3. Construct particle weights as:

$$\tilde{w}_t^i = p(Y_t | S_{t|t-1}^i) w_{t-1}^i$$

4. Scale weights \tilde{w}_t^i so they add up to 1, now called w_t^i . Sample N values with replacement from $\{S_{t|t-1}^i, w_t^i\}$. Name the draws S_t^i .
Reset $w_t^i = 1/N$ for all i

Use the particle filter

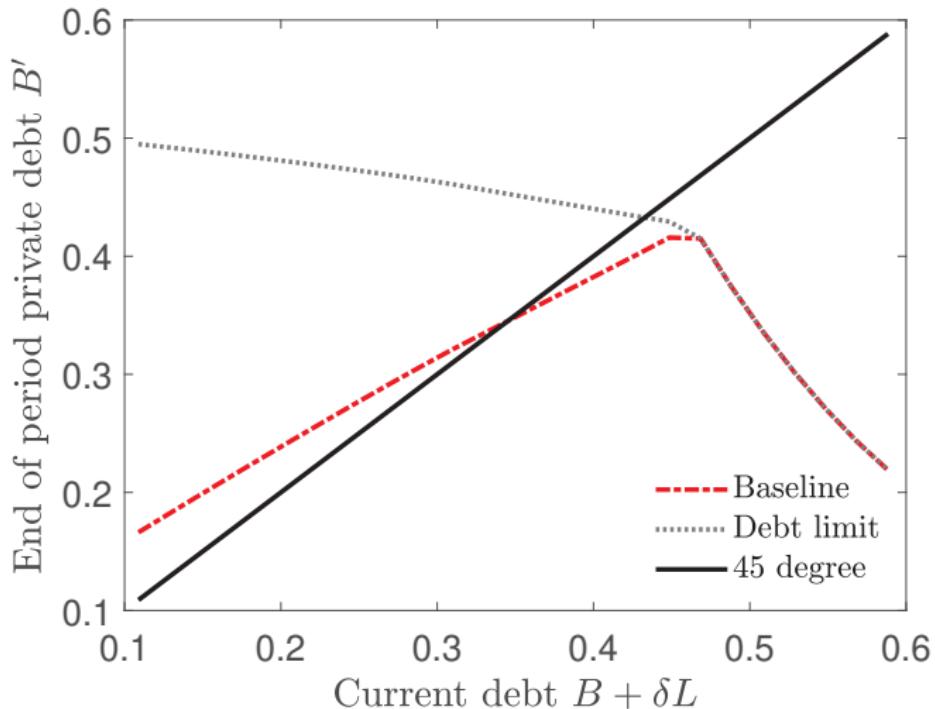
For each $t = 2008..2015$. We can compute the likely path of financial shocks

$$\hat{\kappa}_t^{filtered} = \sum_i^N w_t^i \kappa_t(S_t^i)$$

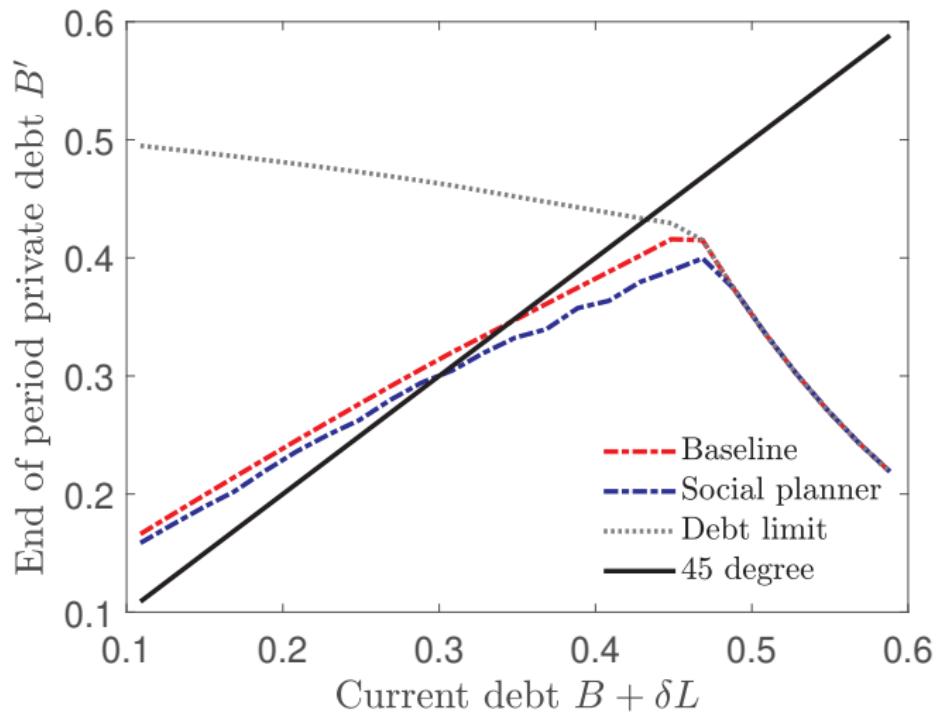
We use this series of financial shock to compute the dynamics

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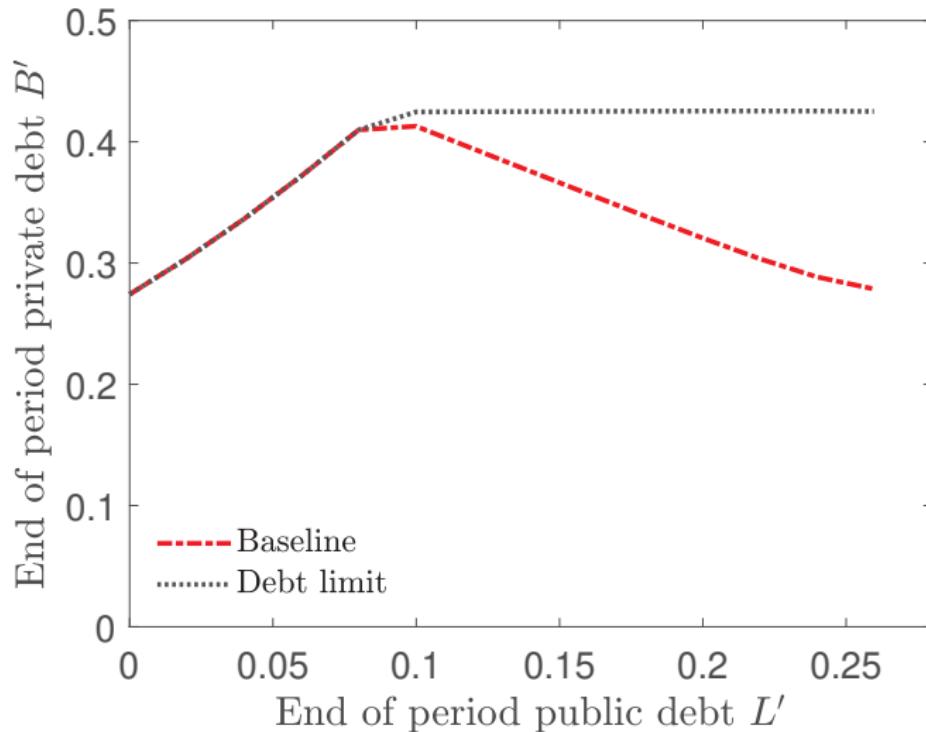
Private debt B' as a function of current debt



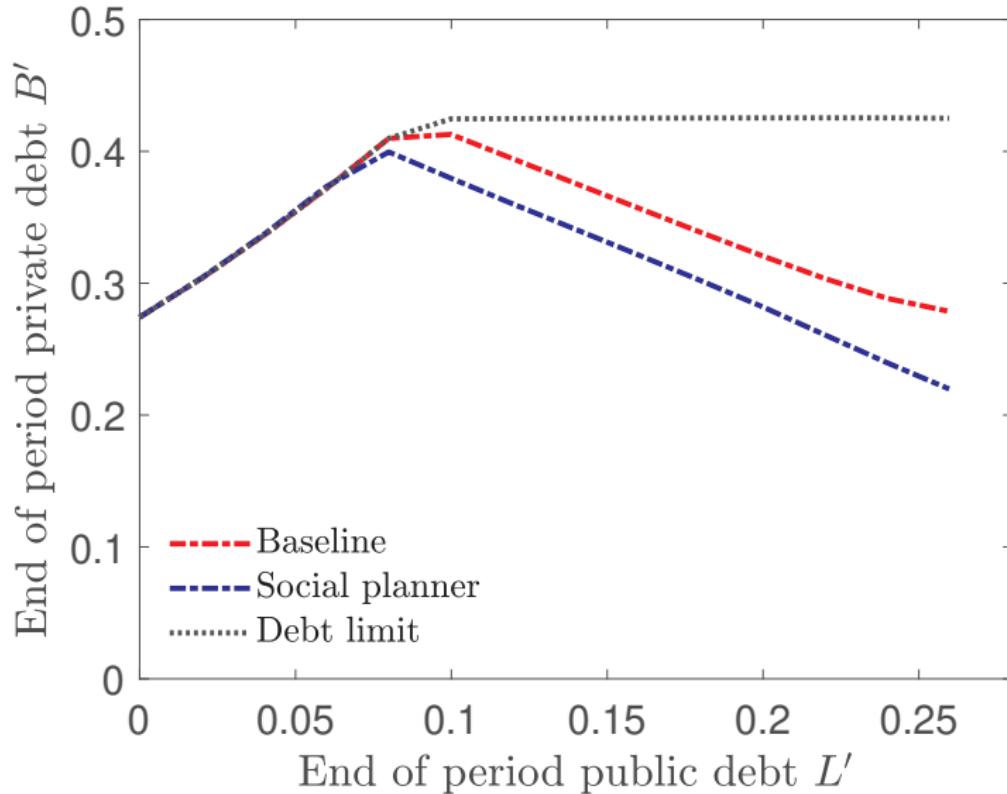
Private debt B' as a function of current debt



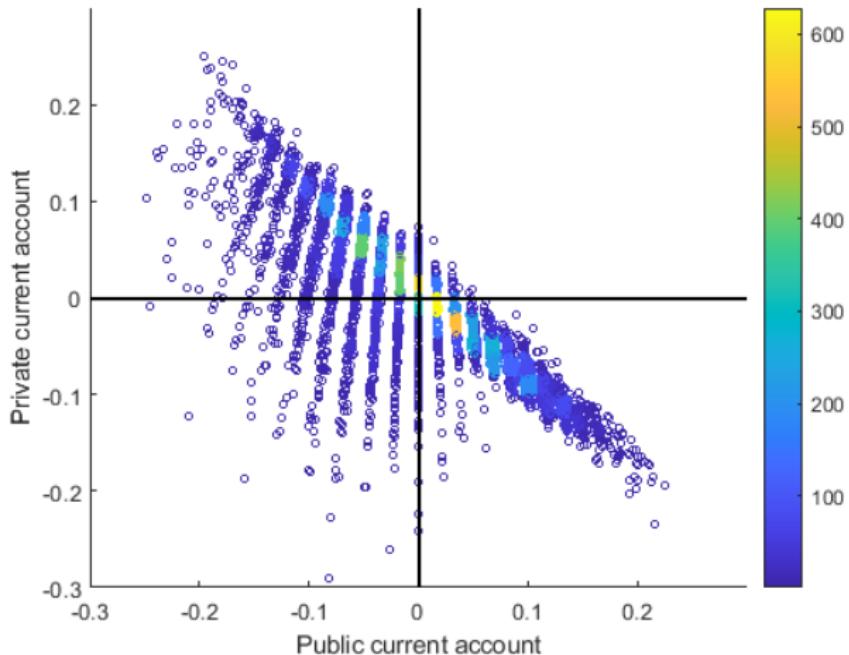
Private debt B' as a function of public debt L'



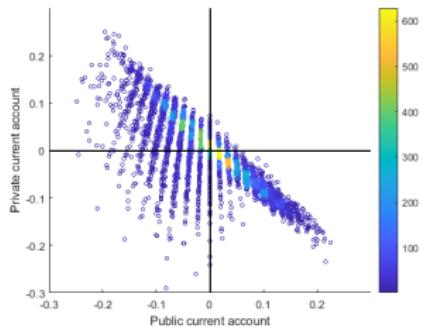
Private debt B' as a function of public debt L'



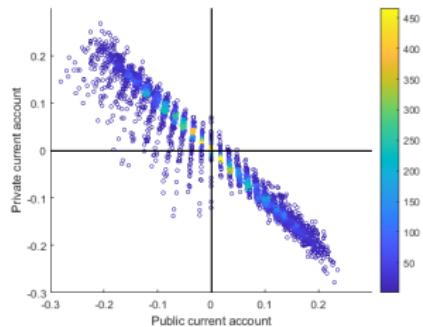
Current accounts: $B' - B$ and $L' - L$



Current accounts: $B' - B$ and $L' - L$



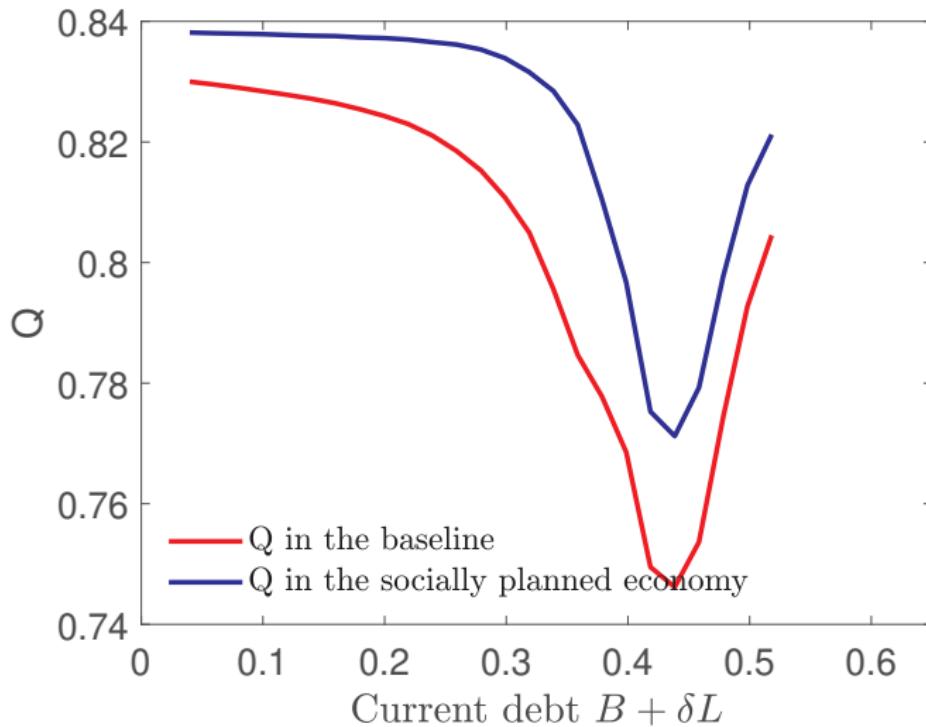
Baseline



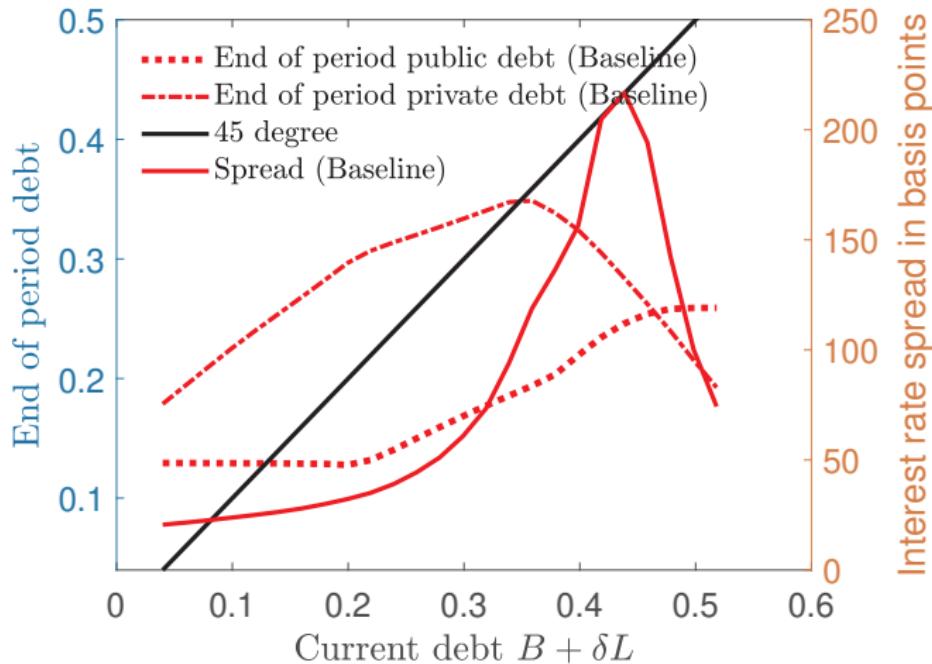
Social planner

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Interest rate spread schedule

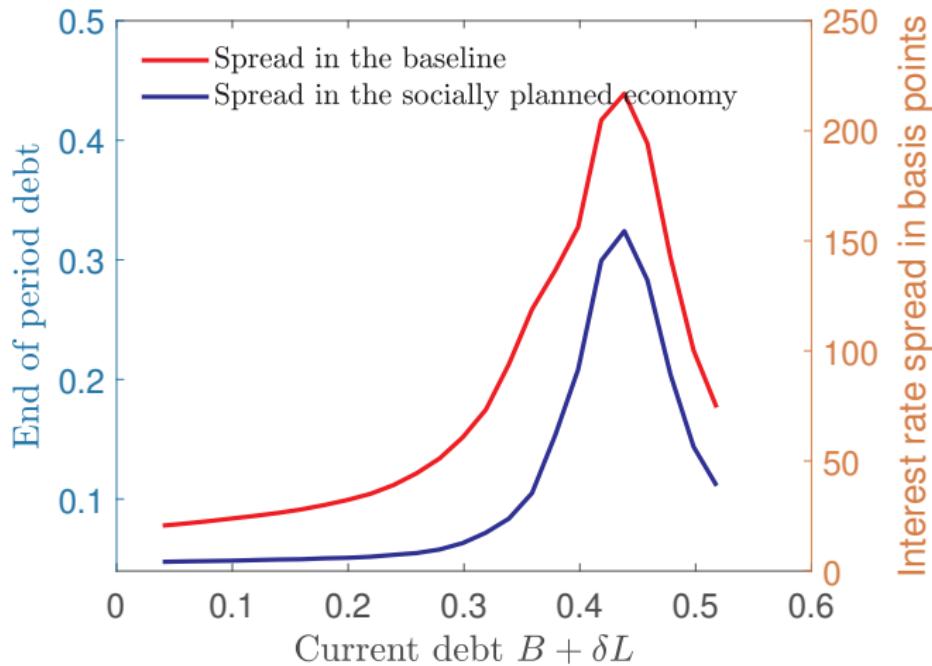


Interest rate spread on public debt



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Interest rate spread on public debt



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Parameters estimated outside of the model

Parameter	Value	Source/Transition
Risk aversion	$\sigma = 2.0$	Standard value
Elasticity of substitution	$1/(1 + \eta) = .83$	Standard value
Decay	$\delta = .14$	Avg. maturity 99-11 6 years
Risk free rate	$r = .027$	GER 1Year T-bill 99-11
Weight tradables in CES	$\omega = .39$	Tradable share 95-18
Income shock	$\rho = .75$	Estimated from
$\log(y_{t+1}^T) = \rho \log(y_t^T) + \varepsilon_t$	$\sigma^y = .010$	GDP 95-18
Private default shocks	$\bar{\pi} = .021$	Estimated from
$\log(\pi_{t+1}) - (1 - \rho^{p_i}) \log(\bar{\pi})$ $= \rho^\pi \log(\pi_t) + \varepsilon_t^\pi$	$\rho^\pi = .82$ $\sigma^\pi = .33$	Nonperforming loans 95-18

Baseline: Household's problem

Let $s = (y^T, \kappa, \pi)$. Households solve:

[Back to households problem in equilibrium](#)

$$V^{BR}(s, L, B, b, \underline{L}', \underline{d}') = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V^{BR}(s', L', B', b', \underline{L}'', \underline{d}'')]$$

subject to

$$\underbrace{c^T + p^N c^N}_{\text{Consumption}} + (1 - \pi) \underbrace{b}_{\text{Current private debt}} = \underbrace{y^T + p^N y^N}_{\text{Endowment}} + q(\pi) \underbrace{b'}_{\text{End-of-period private debt}} + \underbrace{T}_{\text{Transfers}}$$

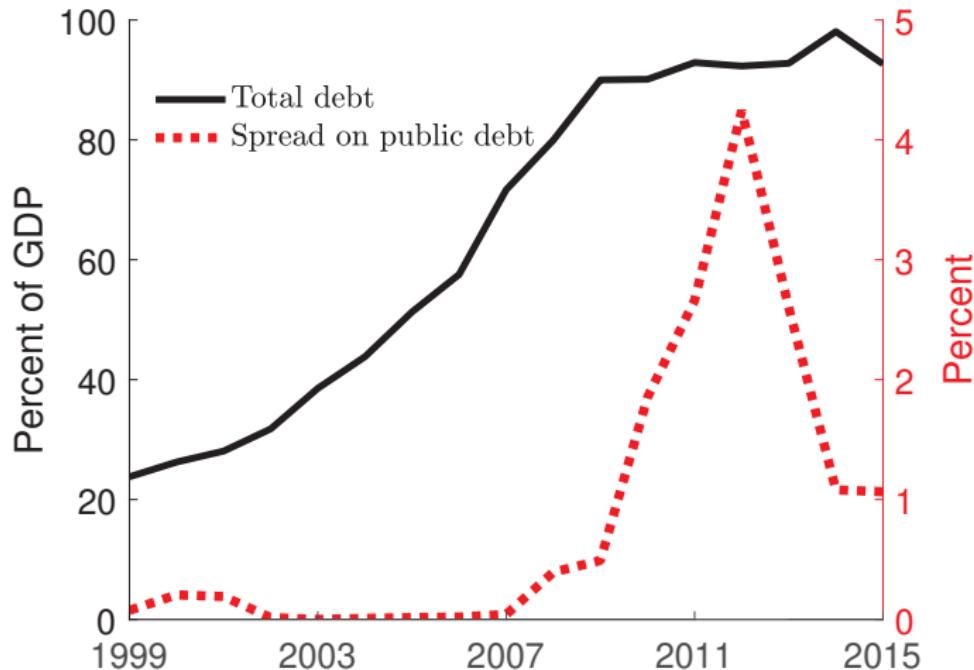
$$\underbrace{q(\pi)b'}_{\text{Market value of new debt}} \leq \kappa \underbrace{(y^T + p^N y^N)}_{\text{Market value of current income}}$$

Taking aggregate prices, laws of motion and public policies as given

$$\underline{d}' = \underbrace{d^{BR}(s, L, B, \underline{L}', \underline{d})}_{\text{Default}}$$

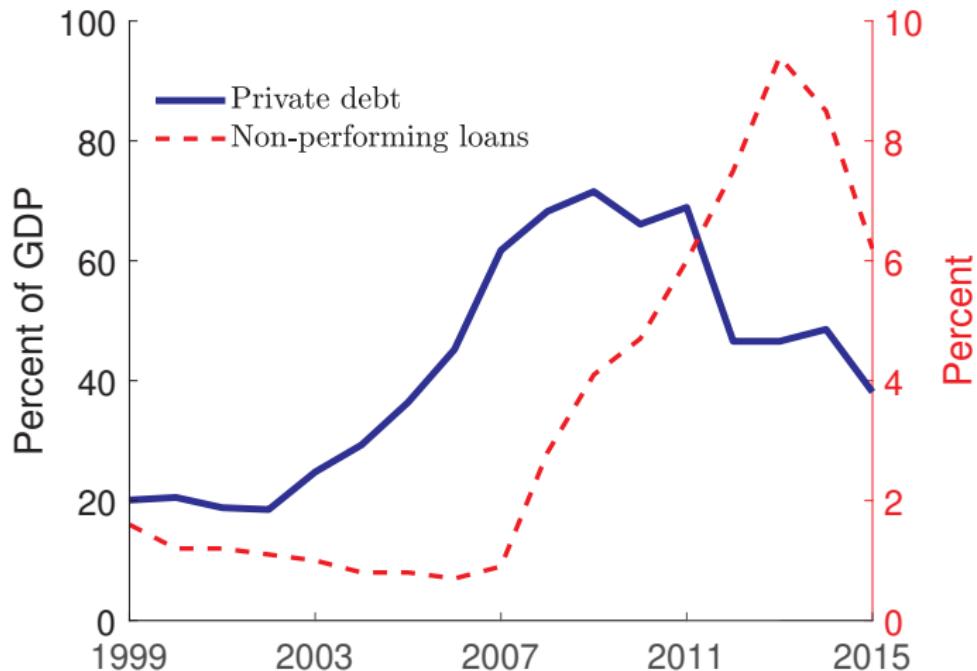
$$\underbrace{p^N}_{\text{Real exchange rate}} = p^{BR}(s, L, B, \underline{L}', \underline{d}'); \underbrace{B'}_{\text{Aggregate private debt}} = B^{BR'}(s, L, B, \underline{L}', \underline{d}'); \underbrace{\underline{L}''}_{\text{Aggregate public debt}} = \mathcal{L}^{BR'}(s, L, B, \underline{L}', \underline{d}); \underbrace{T}_{\text{Transfers}} = \mathcal{T}^{BR}(s, L, B, \underline{L}', \underline{d})$$

Total debt and spreads: Composition matters



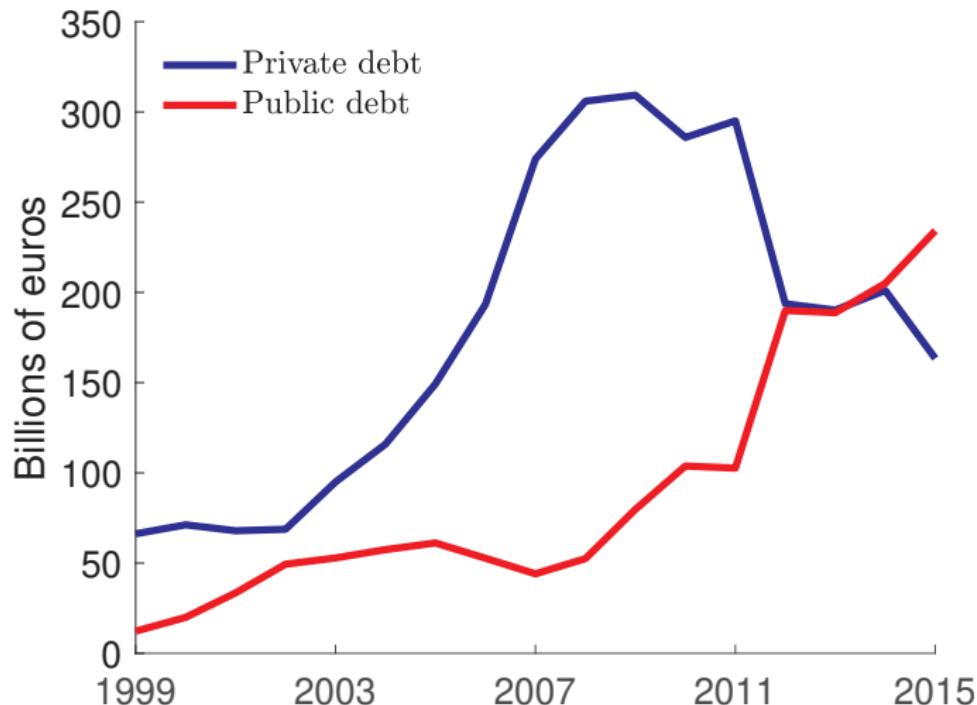
Net foreign liabilities and difference between Spanish and German 6-year T-bonds

Private origins of the crisis: Nonperforming loans



Private net foreign liabilities and non-performing loans as a percent of gross loans

Motivation: The road to the 2012 Spanish debt crisis



Private and public net foreign liabilities

Source: Bank of Spain, Eurostat, and Bloomberg

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Private origins of the crisis

Baseline: Equilibrium real exchange rate

The utility function is CRRA on the CES composite

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta} \right]^{-\frac{1-\sigma}{\eta}}$$

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Elasticity of substitution between tradables and nontradables is $\frac{1}{1+\eta}$

Share of tradables in consumption is ω

Risk aversion coefficient is σ

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Risk aversion coefficient is σ

Households' intratemporal optimality condition in equilibrium,

$$p^N(S_G) = \frac{1-\omega}{\omega} \left(\frac{\mathcal{C}^T(S_G)}{y^N} \right)^{\eta+1}$$

[Euler equation](#)

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Next Steps

1. Cross country evidence and regression analysis
2. Solve a simple rule for macroprudential policies under sovereign risk of the form:

<https://www.overleaf.com/project/60de01ed716f33baa1f39f8f>

$$\tau_t^b = \gamma_0 + \gamma_1 B_t + \gamma_2 L_t + \gamma_3 y_t + \gamma_4 \pi_t + \gamma_5 \kappa_t$$

Start from the OLS regression coefficients at the ergodic and search on a grid around those parameters