# Overborrowing, Underborrowing, and Macroprudential Policy\*

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#### Abstract

In this paper, we revisit the scope for macroprudential policy in production economies with pecuniary externalities and collateral constraints. We study competitive equilibria and constrained-efficient equilibria and examine the extent to which the gap between the two depends on the production structure and the policy instruments available to the planner. We argue that macroprudential policy is desirable regardless of whether the competitive equilibrium features more or less borrowing than the constrained-efficient equilibrium. In our quantitative analysis, macroprudential taxes on borrowing turn out to be larger when the government has access to ex-post stabilization policies.

Keywords: Macroprudential policy, overborrowing, under-borrowing

JEL Classifications: E58, F31, F32, F34

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## 1 Introduction

After the 2008 global financial crisis, many countries upgraded their policy toolkit with macroprudential tools designed to prevent the buildup of excessive risks and vulnerabilities. A key theoretical foundation for the adoption of these policies is pecuniary externalities. In particular, a body of work has shown that collateral constraints linked to market prices may cause "overborrowing" and therefore call for macroprudential interventions to restrict external borrowing ahead of financial crises (e.g., Bianchi 2011). Other studies, however, have questioned the robustness of this prescription, arguing that, in the context of production economies, agents may instead "underborrow," and suggested that macroprudential policies may be either undesirable or unnecessary (e.g., Benigno, Chen, Otrok, Rebucci and Young 2013).<sup>1</sup>

In this paper, we revisit the foundations for macroprudential policies in a production economy. Using a canonical model with pecuniary externalities, we study constrained-efficient equilibria where we vary the planner's ability to control market allocations and examine the associated decentralization. We show that even in configurations in which the constrained-efficient equilibrium features higher levels of borrowing than under laissez-faire, macroprudential policy remains desirable. The logic is that macroprudential taxes are needed ex-ante to make agents internalize the full social costs of debt, regardless of whether the planner is able to alleviate the severity of crises ex-post. Furthermore, we also find that taxes turn out to be larger when the government has access to ex-post stabilization policies.

We analyze a two-sector small open economy model with production and imperfect financial markets, building on the framework developed by Mendoza (2002) and Bianchi (2011). In this economy, households face a credit constraint linked to the market price of non-tradable goods. Standard shocks to income can trigger sharp contractions in borrowing capacity through a feedback loop between consumption and the price of non-tradables. Individual households do not internalize these general equilibrium effects and the competitive equilibrium is constrained inefficient.

We first consider a scenario in which the planner can control the level of borrowing subject to the credit constraint, letting goods and labor markets clear competitively. We

<sup>&</sup>lt;sup>1</sup>Benigno et al. (2013) find that, in a production economy, a constrained planner borrows more than private agents in the competitive equilibrium and therefore argue that "...adopting only ex-ante interventions such as macro-prudential policies or capital controls may be costly in welfare terms. For example, a small macro-prudential tax on debt that lowers the probability of a crisis to zero is welfare-reducing in our model because it also lowers average consumption."

Echoing these findings, in a recent survey, Rebucci and Ma (2020) argue that: "...with a production margin that affects the collateral value in the borrowing constraint, capital controls alone cannot restore constrained efficiency and are generally suboptimal policy instruments."

define "macroprudential policy" as a policy featuring a positive tax on borrowing, and define "overborrowing" as a situation where the implementation of the planner's allocations requires a macroprudential policy. In this first scenario, the optimal tax on debt is strictly positive, as in Bianchi (2011)'s endowment economy model. We then consider a scenario in which the planner can also resort to ex-post stabilization policy by reallocating labor across sectors. In this case, we find that such interventions lead to an increase in borrowing ex-ante, a result that echoes Benigno et al. (2013), who consider an analogous planning problem but do not examine the question of decentralization. Our analysis demonstrates that a strictly positive tax on debt in normal times remains needed to decentralize the constrained-efficient allocations. That is, there is overborrowing and macroprudential policy is warranted regardless of whether the planner has an ex-post policy instrument or not.

In our quantitative analysis, we find that the optimal tax on borrowing is higher when the government has access to ex-post stabilization policy. This occurs despite borrowing exceeding its level in the laissez-faire economy. However, in circumstances where ex-post interventions are more distortionary, the ranking of taxes on borrowing may reverse. For instance, we find that very high curvature in production, which amplifies distortions from ex-post intervention, can make the tax on borrowing in the economy with ex-post policy become lower than in the economy with only ex-ante policy. Moreover, in this case, borrowing under ex-post policy falls below its laissez-faire level, highlighting that the level of borrowing alone is not informative about the desirability of macroprudential policy.

In addition to the work mentioned above, our paper is complementary to other studies on pecuniary externalities and inefficient borrowing. Dávila and Korinek (2018) find that externalities operating through redistributive effects under incomplete markets can lead to over- or under-borrowing as well as over- or under-investment. Drechsel and Kim (2022) show that with earning-based borrowing constraints, firms may underborrow as higher debt reduces equilibrium factor prices and thereby relaxes borrowing constraints.<sup>2</sup> Benigno et al. (2016) study Ramsey optimal policy in an endowment economy and find that when price support policies require distortionary taxes, the optimal policy also involves a tax on debt.<sup>3</sup> Schmitt-Grohé and Uribe (2021) find that the possibility of self-fulfilling financial crises leads to too little borrowing relative to the constrained planner's allocations. They also show how a sophisticated capital control policy that responds to off-equilibrium deviations can implement the good equilibrium during financial crises. Our results suggest that an ex-ante

<sup>&</sup>lt;sup>2</sup>In a similar vein, Bianchi and Mendoza (2010) point to an attenuation effect of overborrowing through the wage bill and working capital.

<sup>&</sup>lt;sup>3</sup>Benigno et al. (2023) study a richer set of policy instruments in a production economy and show how the unconstrained equilibrium can be implemented. Vargas and Parra-Polania (2021) highlight how fiscal constraints can impose limits on price support policies.

positive tax on borrowing remains optimal in normal times despite the level of borrowing in the competitive equilibrium being lower than the constrained efficient allocation.

Our paper is also related to a literature that has examined the interaction between ex-ante and ex-post policies in financial markets, particularly in the context of the Greenspan put. One strand of the literature has considered environments where the lack of government commitment to an ex-post stabilization policy introduces strategic complementarities in risk-staking decisions and leads to multiple equilibria (Diamond and Rajan, 2001; Acharya and Yorulmazer, 2007; Farhi and Tirole, 2012; Nosal and Ordoñez, 2016). In these settings, an ex-ante macroprudential policy implements the good equilibrium and eliminates the need for ex-post intervention in equilibrium. A key distinction between this strand of the literature and our work is that in our model ex-post policies remain desirable even when the government has commitment.

Another strand of the literature also examines environments where there is scope for both ex-ante and ex-post policies. Keister (2016) shows that prohibiting bailouts is undesirable because it leads to excessive liquidity hoarding. Bianchi (2016) shows how the moral hazard effects of bailouts can be mitigated by making them contingent on a systemic crisis. Jeanne and Korinek (2020) find that untargeted liquidity provision makes macroprudential policy less desirable. Bornstein and Lorenzoni (2018) present a model with an aggregate demand externality where countercyclical monetary policy leads to more borrowing and reduces the need for macroprudential policy. In contrast, in our quantitative analysis, we find that ex-ante and ex-post policies are complements. That is, the availability of ex-post policies raises the ex-ante macroprudential tax on borrowing.

#### 2 Model

We consider a dynamic small open economy model with production of tradable and non-tradable goods. The economy is populated by a continuum of identical households that borrow externally, subject to an occasionally binding borrowing constraint.

## 2.1 Households' problem

Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where  $\mathbb{E}_0$  is the expectation operator conditional on date 0 information;  $0 < \beta < 1$  is a discount factor;  $u(\cdot)$  is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada conditions. Consumption  $c_t$  is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution  $1/(\eta + 1)$  between tradable goods  $c_t^T$  and non-tradable goods  $c_t^N$ , given by

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1 - \omega) \left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \quad \text{with} \quad \eta > -1, \omega \in (0, 1).$$

In each period, households are endowed with a fixed number of hours  $\bar{h}$ , are perfectly mobile across sectors, and do not value leisure. They receive a competitive wage for their labor, as well as profits from firms in the tradable and non-tradable sectors.<sup>4</sup>

Households can borrow (or save) using a one-period non-state-contingent bond  $b_{t+1}$  denominated in units of tradables that pays a constant interest rate R exogenously determined in international capital markets.<sup>5</sup>

Their budget constraint, in units of tradables, is given by

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R} = w_t \bar{h} + \pi_t^T + \pi_t^N - b_t,$$
 (2)

where  $p_t^N$  is the price of non-tradables,  $w_t$  is the wage, and  $\pi_t^T$  and  $\pi_t^N$  are profits from tradable and non-tradable goods producing firms. In addition, following Mendoza (2002), Bianchi (2011), and Benigno et al. (2013), households face a credit constraint given by

$$\frac{b_{t+1}}{R} \le \kappa \left( w_t \bar{h} + \pi_t^T + \pi_t^N \right). \tag{3}$$

The constraint captures in a parsimonious way the empirical fact that income is critical in determining households' credit-market access (see, e.g., Jappelli, 1990; Lian and Ma, 2021). From a macro perspective, it has been shown to be important for accounting for the dynamics of capital flows in emerging markets.<sup>6</sup>

<sup>&</sup>lt;sup>4</sup>In a minor departure from Benigno et al. (2013), we do not consider leisure in the utility function. This assumption simplifies the analysis without affecting our theoretical results. For the quantitative analysis, it also helps avoid a counterfactual increase in employment during sudden stops events.

<sup>&</sup>lt;sup>5</sup>Assuming no foreign inflation, this condition is equivalent to denominating the bonds in foreign currency, capturing the liability dollarization phenomenon.

<sup>&</sup>lt;sup>6</sup>The credit constraint can be derived endogenously from a problem of limited enforcement under the assumption that if households default at the end of the period, they lose a fraction  $\kappa$  of the current income, and immediately regain access to credit markets (see Bianchi and Mendoza, 2018, for a derivation of a similar constraint). The ability to borrow could also depend on future income in addition to current income, but as emphasized by Ottonello et al. (2022), what is crucial for the analysis is that borrowing is at least partially leveraged on current income.

Households choose consumption and borrowing to maximize their utility (1), subject to their budget constraint (2) and credit constraint (3), taking prices as given. Their optimality conditions are given by

$$p_t^N = \frac{1 - \omega}{\omega} \left(\frac{c_t^T}{c_t^N}\right)^{\eta + 1},\tag{4}$$

$$\lambda_t = u_T(t),\tag{5}$$

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{6}$$

$$0 = \mu_t \left[ \kappa \left( w_t \bar{h} + \pi_t^T + \pi_t^N \right) - \frac{b_{t+1}}{R} \right], \tag{7}$$

where  $u_T(t)$  is shorthand notation for  $\frac{\partial u}{\partial c} \frac{\partial c}{\partial c^T}$  and  $\mu_t$  denotes the non-negative Lagrange multiplier on the borrowing constraint. Condition (4) is a static optimality condition equating the marginal rate of substitution between tradable and non-tradable goods to their relative price. Condition (5) equates the marginal utility of tradable consumption to the shadow value of current wealth. Condition (6) is the household's Euler equation for debt, where  $\mu_t \geq 0$  is the multiplier on the credit constraint (3). Finally, condition (7) is the household's complementary slackness condition. When  $\mu_t > 0$ , the credit constraint binds, and the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit and repaying next period.

## 2.2 Firms' problem

The tradable and non-tradable goods are produced by competitive firms that maximize profits and respectively solve

$$\max_{h_t^T} z_t^T (h_t^T)^\alpha - w_t h_t^T, \tag{8}$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^\alpha - w_t h_t^N, \tag{9}$$

where  $z_t^T$  is a stochastic productivity shock, while  $z^N$  and  $\alpha$  are constant parameters.

The optimality conditions of the firms producing tradable and non-tradable goods are respectively given by

$$w_t = z_t^T \alpha \left( h_t^T \right)^{\alpha - 1}, \tag{10}$$

$$w_t = p_t^N z^N \alpha \left( h_t^N \right)^{\alpha - 1}. \tag{11}$$

Combining (10) and (11), we obtain

$$1 = \frac{z^N (h_t^N)^{\alpha - 1}}{z_t^T (h_t^T)^{\alpha - 1}} p_t^N.$$
 (12)

This expression implies that increases in the relative price of non-tradables are associated with a shift in employed hours toward the non-tradable sector.

#### 2.3 Competitive equilibrium

In equilibrium, the market for non-tradable goods must clear domestically and total labor demand by firms must equal  $\bar{h}$ . That is, we must have

$$c_t^N = z^N (h_t^N)^{\alpha}, \tag{13}$$

$$\bar{h} = h_t^N + h_t^T. (14)$$

We can now define a competitive equilibrium:

**Definition 1.** The competitive equilibrium is given by a sequence of prices  $\{p_t^N, w_t\}_{t=0}^{\infty}$  and allocations  $\{c_t^N, c_t^T, h_t^N, h_t^T, b_{t+1}\}_{t=0}^{\infty}$  such that:

- i) Households optimize. That is, (2)-(7) are satisfied.
- ii) Firms optimize. That is, (10) and (11) are satisfied.
- iii) Markets clear. That is, (13) and (14) hold.

# 3 Normative Analysis

We now analyze the normative properties of this economy. We consider two cases. First, we look at the case of a planner that directly controls the level of debt, but lets markets for labor and goods clear competitively. Second, we consider a case in which the planner can also choose the allocation of labor across sectors in addition to choosing the level of debt, and lets only the goods market clear competitively.

## 3.1 Ex-ante macroprudential intervention

We first assume that the planner directly controls only the level of debt, while the labor and goods markets clear competitively. While the planner cannot directly choose the labor allocations, it internalizes how its borrowing choices affect employment across sectors. In recursive form, the planner's problem can be written as

$$V(b, z^{T}) = \max_{c^{T}, c^{N}, h^{T}, h^{N}, b'} u(c^{T}, c^{N}) + \beta \mathbb{E}V(b', z^{T'})$$
(15)

subject to

$$c^{T} - \frac{b'}{R} = z^{T} \left( h^{T} \right)^{\alpha} - b, \tag{15a}$$

$$c^{N} = z^{N} \left( h^{N} \right)^{\alpha}, \tag{15b}$$

$$\bar{h} = h^T + h^N, \tag{15c}$$

$$\frac{b'}{R} \le \kappa \left[ z^T \left( h^T \right)^{\alpha} + \frac{1 - \omega}{\omega} \left( \frac{c^T}{c^N} \right)^{\eta + 1} z^N \left( h^N \right)^{\alpha} \right], \tag{15d}$$

$$\frac{z^T}{z^N} \left(\frac{h^T}{h^N}\right)^{\alpha - 1} = \frac{1 - \omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta + 1},\tag{15e}$$

where (15a) is the resource constraint for tradable goods, (15b) is the resource constraint for non-tradable goods, (15c) is the time constraint for labor, (15d) is the credit constraint, and (15e) is an implementability constraint associated to firms' and households' optimal intratemporal choices. When both the labor and goods markets clear competitively, the marginal rate of substitution is equal to the marginal rate of transformation between goods.

Returning to sequential notation, the planner's Euler equation for debt is given by

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{16}$$

its credit constraint is given by

$$-\frac{b_{t+1}}{R} + \kappa \left[ z_t^T (h_t^T)^{\alpha} + \frac{1-\omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{\eta+1} z^N (h_t^N)^{\alpha} \right] \ge 0, \tag{17}$$

and its complementary slackness condition is

$$\left\{ -\frac{b_{t+1}}{R} + \kappa \left[ z_t^T (h_t^T)^{\alpha} + \frac{1-\omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{\eta+1} z^N (h_t^N)^{\alpha} \right] \right\} \mu_t = 0.$$
 (18)

Meanwhile, its remaining optimality conditions for  $c_t^T$ ,  $c_t^N$ ,  $h_t^T$  and  $h_t^N$  are given by

$$\lambda_t = u_T(t) + \left(\mu_t \kappa + \frac{\nu_t}{c_t^N}\right) (1+\eta) \frac{p_t^N c_t^N}{c_t^T},\tag{19}$$

$$\delta_t = u_N(t) - \left(\mu_t \kappa + \frac{\nu_t}{c_t^N}\right) (1+\eta) p_t^N, \tag{20}$$

$$\chi_t = (\lambda_t + \mu_t \kappa) z_t^T \alpha \left( h_t^T \right)^{\alpha - 1} - \nu_t \left( \alpha - 1 \right) p_t^N \frac{1}{h_t^T}, \tag{21}$$

$$\chi_t = \left(\delta_t + \mu_t \kappa p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha - 1} + \nu_t \left(\alpha - 1\right) p_t^N \frac{1}{h_t^N},\tag{22}$$

where  $\lambda_t$ ,  $\delta_t$ ,  $\chi_t$ ,  $\mu_t$  and  $\nu_t$  respectively denote the multipliers on constraints (15a), (15b), (15c), (15d) and (15e). Equations (19) and (20) equate the marginal utility benefits of tradable and non-tradable consumption to their marginal utility costs while (21) and (22) are the analogous conditions for employment in the tradable and non-tradable goods sectors. Crucially, these conditions incorporate how changes in allocations alter the tightness of the collateral constraint, as captured by the terms featuring the multiplier  $\mu_t$ .

Combining (15c) and (19)-(22) to eliminate the multipliers  $\delta_t$ ,  $\chi_t$  and  $\nu_t$  leads to the following expression

$$\lambda_t = u_T(t) + \mu_t \widetilde{\Psi}_t, \tag{23}$$

where  $\widetilde{\Psi}_t = \Psi_t \Upsilon_t$ , with

$$\begin{split} \Psi_t &\equiv (1+\eta)\kappa(p_t^N c_t^N)/c_t^T \\ \Upsilon_t &\equiv \frac{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{h_t^T}}{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{h_t^T} + (1+\eta)\left[c_t^T/(c_t^T + p_t^N c_t^N)\right]^{-1}}. \end{split}$$

Comparing (5) with (23) reveals that, for given allocations, the social planner has a higher shadow value of wealth than private agents whenever the collateral constraint binds. The wedge arises from the product of the planner's shadow value of the collateral constraint,  $\mu_t$ , and a collateral elasticity term,  $\widetilde{\Psi}_t$ , which measures how the value of collateral responds to changes in aggregate tradable consumption. It is useful to decompose the collateral elasticity into two components:  $\Psi_t$  and  $\Upsilon_t$ . The term  $\Psi_t$  corresponds to the elasticity term that appears in Bianchi (2011)'s endowment economy model. In our production economy, however, the planner internalizes how wealth affects the allocation of employment across the two sectors, which in turn influences the relative price of non-tradables. Specifically, higher tradable consumption raises the relative price of non-tradables, inducing a reallocation of labor toward

the non-tradable sector. This additional effect partly offsets the response of the relative price, and it is captured by the term  $\Upsilon_t$ .

We obtain the following proposition:

**Proposition 1** (Decentralization with ex-ante intervention). Consider the allocations that solve the optimal policy problem (15). These allocations can be decentralized as a competitive equilibrium with a state contingent tax on debt rebated with lump sum transfers. In states in which the borrowing constraint is not binding, the tax on debt is given by:

$$\tau_t = \frac{\mathbb{E}_t \mu_{t+1} \widetilde{\Psi}_{t+1}}{\mathbb{E}_t u_T(t+1)} \ge 0. \tag{24}$$

*Proof.* See Appendix A.1.  $\blacksquare$ 

Thus, the desirability of a strictly positive tax on borrowing in Bianchi (2011) extends to the case with production. Failing to internalize that additional tradable consumption would support the equilibrium price of non-tradable goods and relax credit constraints, private households borrow too much relative to what is socially optimal.<sup>7</sup>

#### 3.2 Ex-ante and ex-post interventions

In the preceding section, we examined an economy where the planner only had the ability to control the debt level. We now allow the planner to also control the allocation of labor across sectors. In this case, the planner's problem is given by

$$V(b, z^{T}) = \max_{c^{T}, c^{N}, h^{T}, h^{T}, b'} u(c^{T}, c^{N}) + \beta \mathbb{E}V(b', z^{T'})$$
(25)

subject to

$$c^{T} - \frac{b'}{R} = z^{T} \left( h^{T} \right)^{\alpha} - b, \tag{25a}$$

$$c^{N} = z^{N} \left( h^{N} \right)^{\alpha}, \tag{25b}$$

$$\bar{h} = h^T + h^N, \tag{25c}$$

$$\frac{b'}{R} \le \kappa \left[ z^T \left( h^T \right)^{\alpha} + \frac{1 - \omega}{\omega} \left( \frac{c^T}{c^N} \right)^{\eta + 1} z^N \left( h^N \right)^{\alpha} \right]. \tag{25d}$$

<sup>&</sup>lt;sup>7</sup>When the borrowing constraint is binding, one can show that the tax on borrowing that implements the optimal policy is zero if  $\mu_{t+1}\tilde{\Psi}_{t+1} + \mu_t (1 - \Psi_t) \geq 0$ .

The difference with problem (15) is that the implementability constraint (15e) associated with the intratemporal allocation of labor across sectors can now be dropped. The planner can now choose to clear the labor market at a marginal rate of transformation between goods that can be different from the marginal rate of substitution between goods.

Again using the same sequential notation for convenience, we get that the planner's Euler equation for debt, its credit constraint and its complementary slackness condition are still given by (16), (17) and (18), while its remaining optimality conditions for  $c_t^T$ ,  $c_t^N$ ,  $h_t^T$  and  $h_t^N$  are now given by

$$\lambda_t = u_T(t) + \mu_t \Psi_t, \tag{26}$$

$$\delta_t = u_N(t) - \mu_t \kappa (1 + \eta) p_t^N, \tag{27}$$

$$\chi_t = (\lambda_t + \mu_t \kappa) z_t^T \alpha \left( h_t^T \right)^{\alpha - 1}, \tag{28}$$

$$\chi_t = \left(\delta_t + \mu_t \kappa p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha - 1}, \tag{29}$$

where  $\lambda_t$ ,  $\delta_t$ ,  $\chi_t$  and  $\mu_t$  again respectively denote the multipliers on constraints (25a), (25b), (25c) and (25d).

Condition (26) equates the marginal benefit of tradable consumption to its marginal cost. The key difference from condition (23) analyzed above is that the planner now optimally chooses the allocation of labor. Unlike in the economy without ex-post intervention, higher tradable consumption does not induce a labor reallocation toward non-tradables, which would otherwise offset the increase in their relative price. Given an allocation, this implies that collateral elasticity is higher in the economy with ex-post policy. Moreover, this elasticity coincides with its counterpart in the endowment economy (cf. Bianchi 2011, equation 12).

We obtain the following proposition.

**Proposition 2** (Decentralization with ex-ante and ex-post intervention). Consider the allocations that solve the optimal policy problem (25). These allocations can be decentralized as a competitive equilibrium with a state contingent tax on debt and a payroll tax on non-tradable labor, both rebated lump sum. In states in which the borrowing constraint is not binding, the tax on debt is given by

$$\tau_t = \frac{\mathbb{E}_t \mu_{t+1} \Psi_{t+1}}{\mathbb{E}_t u_T(t+1)} \ge 0. \tag{30}$$

The payroll tax on non-tradable labor is given by

$$\tau_t^N = \mu_t \kappa c_t^N (1+\eta) p_t^N \frac{\left(c_t^T\right)^{-1} + \left(c_t^N\right)^{-1} \frac{z^N \left(h_t^N\right)^{\alpha-1}}{z_t^T \left(h_t^T\right)^{\alpha-1}}}{u_T(t) + \mu_t \kappa} \ge 0.$$
 (31)

*Proof.* See Appendix A.2.  $\blacksquare$ 

The proposition shows that a macroprudential tax on debt is still warranted in this economy with ex-post government interventions. Irrespective of whether the planner has access to an ex-post stabilization policy or not, the optimal policy thus features a macroprudential tax on debt. The reason is that regardless of the planner's ability to distort the allocation of labor ex-post, private households always undervalue wealth in states of nature in which the credit constraint binds.

In addition, the proposition also shows that implementation requires a payroll tax on non-tradable labor. This tax is positive whenever the credit constraint binds for the planner (i.e., whenever  $\mu_t > 0$ ). The logic is that in such cases, the planner optimally redirects production away from the non-tradable sector so as to support the price of non-tradable goods and relax the credit constraint at the margin. In Appendix B.1, we extend the environment with working capital constraints and show that an equivalent decentralization involves a subsidy on credit in the tradable sector.

Underborrowing and macroprudential policy. These results clarify a common misconception in the literature, which posits that the case for macroprudential debt taxes is weaker (or absent) in production economies and/or when ex-post stabilization policies are available. This interpretation stems from the fact that in the presence of production and an ex-post policy margin, the optimal policy intervention often results—as we will see below—in higher borrowing than in the laissez-faire economy, a phenomenon referred to by Benigno et al. (2013) as "underborrowing."

Benigno et al. (2013), however, do not formally consider the decentralization of their planner problem. Our results of Propositions 1 and 2 clarify that even when the planner borrows more than under laissez-faire, restrictions or taxes on borrowing are desirable.

<sup>&</sup>lt;sup>8</sup>Benigno et al. (2013) find that a 1% non-state contingent tax on debt is welfare reducing in their model. Proposition 2 indicates that the optimal state-contingent tax is necessarily non-negative. As we show in Appendix B.4, however, a small non-state contingent tax does lead to welfare gains. The intuition is that even though the optimal tax is state contingent and zero in a subset of the state space, there is a first-order gain from correcting the externality and a second-order loss from distorting consumption when the optimal tax should be zero.

That is, regardless of whether ex-post policies are available or whether there is production, macroprudential policy is desirable. Intuitively, what is key for the macroprudential policy motive is that in the competitive equilibrium households face a private shadow cost of debt that is lower than the social one, or equivalently, a lower shadow value of wealth compared to the social one. This result holds irrespective of whether the laissez-faire economy exhibits more or less borrowing relative to the planner's allocations.

# 4 Quantitative Analysis

We next proceed to study quantitatively how the set of instruments available to the planner matters for the levels of borrowing and taxes.<sup>10</sup>

#### 4.1 Calibration

Our model calibration largely follows Bianchi (2011)'s calibration to Argentina. A period in the model represents a year. A first set of parameters is taken from the literature or estimated in the data outside of the model. The preference parameters for risk aversion and the elasticity of substitution are set to  $\sigma = 2$ ,  $1/(1+\eta) = 0.83$ . The value for the interest rate is set to 4%. The labor share  $\alpha = 0.67$  coincides with the share of aggregate income that goes to labor as a percentage of GDP in household surveys (see Garcia-Verdú, 2005), a standard value in quantitative sudden stop models with production (see Benigno et al., 2013).

A second set of parameters is calibrated so that the unregulated competitive equilibrium matches a set of moments from the data (following the same targets as in Bianchi, 2011). The relative preference for tradables is set to  $\omega = 0.307$  to match the share of tradable GDP in the data. The discount factor  $\beta = 0.91$  is set to match the average net foreign asset position to GDP ratio. The collateral constraint parameter is set to  $\kappa = 0.32$  to match the observed frequency of sudden stops.<sup>11</sup>

<sup>&</sup>lt;sup>9</sup>We note that for a given level of beginning of period debt level, the shadow value of wealth is higher in the competitive equilibrium than in the economy with ex-ante and ex-post policy tools, as in (Benigno et al., 2013, see Figure 6). This happens because the household's level of consumption is higher in the economy with ex-ante and ex-post policy tools than under laissez-faire. However, what is key for our result on the tax on debt being positive is that the social shadow value of wealth is higher than the private one when evaluating the shadow values at the *same allocations*.

<sup>&</sup>lt;sup>10</sup>We solve the model globally, following Bianchi (2011). We use a grid of 500 points for debt and 20 different values of the exogenous shocks. The laissez-faire economy and the economy with an ex-ante intervention are solved using time iteration. The economy with ex-ante and ex-post intervention is solved using value function iteration.

<sup>&</sup>lt;sup>11</sup>These parameters are equal to those used in Bianchi (2011).

To estimate the stochastic process for productivity in the tradable goods sector, we follow Arce et al. (2019). First, we construct a tradable GDP index for Argentina from the World Development Indicators from 1965 to 2007. We assume a first-order autoregressive process for the cyclical component of this series:  $\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y$  with  $\varepsilon_t^y \sim N(0, \sigma_y)$ . We estimate values of  $\rho^y = 0.5$  and  $\sigma_y = 0.0502$ . We then set the persistence parameter of the productivity shocks in the tradable sector to the persistence of tradable GDP and calibrate the volatility of the productivity shocks in the tradable sector to ensure that the standard deviation of the simulated tradable output at the ergodic distribution of the unregulated economy coincides with the one in the data. This method yields values of  $\rho^z = 0.5$  and  $\sigma_z = 0.0535$ . A summary of parameter values is provided in Table 1.

Table 1: Parameter Values

|                            | Value                             | Source/Targets                          |  |
|----------------------------|-----------------------------------|---|--|
| Interest Rate              | r = 0.04                          | Standard value                          |  |
| Risk Aversion              | $\sigma = 2$                      | Standard value                          |  |
| Elasticity of Substitution | $1/(1+\eta) = 0.83$               | Standard value                          |  |
| Weight on Tradables in CES | $\omega = 0.32$                   | Share of tradable consumption           |  |
| Discount Factor            | $\beta = 0.91$                    | Net foreign asset position              |  |
| Credit coefficient         | $\kappa = 0.32$                   | Frequency of sudden stops               |  |
| Labor share in production  | $\alpha = 0.67$                   | Share of labor income in GDP            |  |
| Stochastic structure $z^T$ | $\rho^z = 0.5, \sigma_z = 0.0535$ | Persistence and volatility of tradables |  |

## 4.2 Ex-ante intervention only

Figure 1 plots the policy functions for debt issuance and for employment in the tradable sector as a function of the initial level of assets in the case in which the planner has access only to an ex-ante instrument. Policy functions from the laissez-faire economy are also plotted for comparison. As in Bianchi (2011) and most of the literature on overborrowing, we find that the planner chooses a lower level of borrowing when the credit constraint is not binding (panel a), relative to its level in the laissez-faire economy. We also find that the effect of this lower debt on relative prices leads to a higher level of labor allocated in the tradable sector when the constraint is not binding (panel b). However, for high enough initial levels of debt, the credit constraint binds, and the allocations of the regulated and laissez-faire economies coincide.

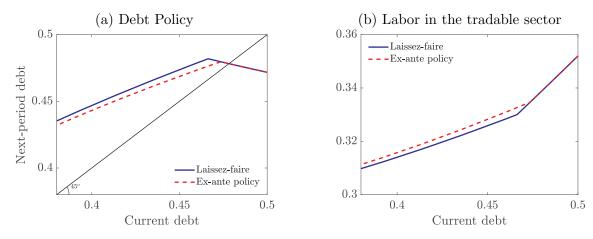


Figure 1: Policy functions: ex-ante intervention

*Note:* In both panels, the tradable productivity shock is set at the mean value.

#### 4.3 Ex-ante and ex-post interventions

Figure 2 plots the policy functions for debt issuance and employment in the tradable sector as a function of the initial level of assets in the case in which the planner has access to both ex-ante and ex-post policies. Policy functions from the laissez-faire economy are once again plotted as a benchmark. This time, the level of borrowing in the regulated economy is always more than that in the laissez-faire economy (panel a), a phenomenon often referred to as underborrowing in the literature (Benigno et al., 2013). While the policy function for debt still exhibits a kink due to the credit constraint, the level of borrowing is always increasing in the current debt level.

The policy function for labor allocated to the tradable sector is helpful for understanding this feature (panel b). When the current level of debt is high enough to lead to a binding credit constraint, the planner seeks to relax the constraint by allocating more labor to the tradable sector. This in turn increases the collateral value and allows the economy to sustain a higher level of debt. Moreover, given that a binding collateral constraint can be alleviated in this way, the incentive to accumulate precautionary savings when the credit constraint is not binding is weaker than in the laissez-faire economy. As a result, the level of borrowing is also higher. This in turn appreciates the real exchange rate, increases wages, and reduces the share of labor employed in the tradable sector relative to their levels in the laissez-faire economy.

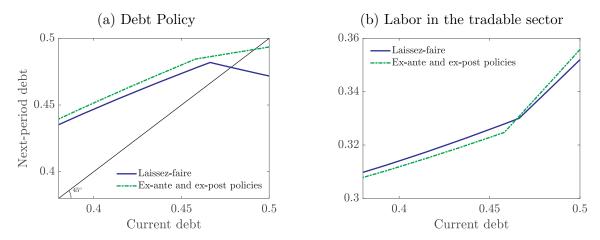


Figure 2: Policy functions: ex-ante and ex-post intervention

Note: In both panels, the tradable productivity shock is set at the mean value.

#### 4.4 Policy Comparison: Ex-ante vs. Ex-ante and Ex-post

Figure 3 compares the economy with only ex-ante intervention and the economy with both ex-ante and ex-post interventions. Panel (a) shows the ergodic distribution of debt for the two regulated economies as well as for the laissez-faire economy (blue). The economy with only ex-ante policies exhibits the least amount of debt, followed by the laissez-faire economy, and then the economy with ex-ante and ex-post policies.

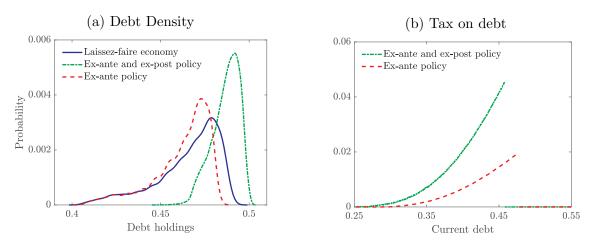


Figure 3: Distribution of debt at the ergodic and optimal taxes on debt *Note:* In panel (b), the tradable productivity shock is set at the mean value.

Panel (b) shows the optimal tax on debt that implements the planner's allocations with only ex-ante policies (red) and with ex-ante and ex-post policies (green). In both cases, macroprudential policy is active only when the credit constraint is not binding. Interestingly,

taxes on debt are higher in the economy with ex-ante and ex-post policies for any initial level of debt where the credit constraint is not binding. The finding that the optimal macroprudential tax on debt is higher in the economy where the government also has access to ex-post policy tools may seem surprising. One might have expected that the ability to intervene ex-post, by relaxing a binding credit constraint, would reduce the need for ex-ante intervention. In what follows, we unpack the forces underlying this result.

To unpack this result, we first conduct a one-period deviation exercise around the optimal allocation. Specifically, we consider a situation where households face no tax in period t but anticipate that from t+1 onward, allocations will be regulated by a government with either only an ex-ante tool (panel a) or both ex-ante and ex-post tools (panel b).

Figure 4 presents the results. In both panels, the dotted line depicts households' policy function for debt at time t, while the dashed line represents the benchmark policy function in the permanently regulated economy.<sup>12</sup> By placing households in an unregulated environment for a single period (t), this experiment captures their incentive to deviate from the planner's choice, allowing us to shed light on the corrective tax on borrowing required to decentralize the planner's allocations.

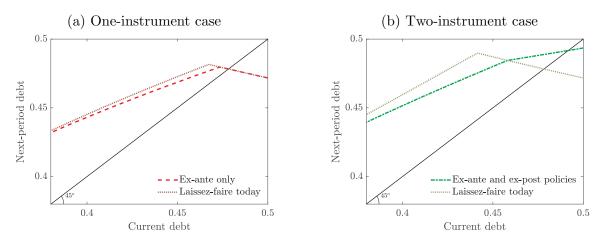


Figure 4: Policy functions for one-period deviations.

*Note:* The tradable productivity shock is set at the mean value. In the experiment, denoted by "Laissez-faire today" on the Figure, there are no taxes at period t but households anticipate that from t+1 onwards the allocations will be regulated by a government that has access to: an ex-ante instrument only (panel a), or ex-ante and ex-post instruments (panel b).

The experiment reveals a contrast between the ex-ante-only case (panel a) and the case with both ex-ante and ex-post instruments (panel b). In both panels, the gap between

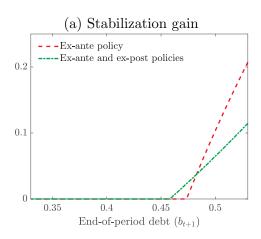
The permanently regulated economy features only ex-ante instruments in panel (a) and both ex-ante and ex-post instruments in panel (b).

households' desired borrowing (dotted line) and borrowing under permanent regulation (dashed line) illustrates how the anticipation of ex-post policy (or lack thereof) affects the current borrowing choice. The larger gap in panel (b) suggests that the planner requires a higher macroprudential tax to curb borrowing when an ex-post instrument is available. Intuitively, anticipating ex-post stabilization weakens households' precautionary savings motive, amplifying the need for ex-ante intervention.

Stabilization gains and collateral elasticity. To shed further light on why the taxes are higher with ex-post policy, it is useful to go back to the optimal tax expressions in Proposition 1 and 2. There are two key terms in those expressions: a stabilization-gain term reflecting the value to the planner of relaxing the collateral constraint  $(\mu_{t+1})$ , and a collateral elasticity term reflecting the sensitivity of the collateral value to a reduction in borrowing  $(\Psi_{t+1}\Upsilon_{t+1})$  in the one instrument and  $\Psi_{t+1}$  in the one instrument case). On the one hand, the stabilization gain term captures the idea that the greater the benefit of relaxing households' credit constraints ex-post, the higher the optimal tax. On the other hand, the collateral elasticity term reflects how effectively taxes on debt support future collateral values—a higher elasticity calls for a higher tax on debt.

Figure 5 plots these two terms as functions of debt  $(b_{t+1})$ , assuming the productivity shock is at its mean value. Panel (a) shows that at high debt levels where the credit constraint binds, the stabilization gain is larger in the ex-ante-only case than in the case with both instruments. This is because, with ex-post policy, the planner can relax the credit constraint by shifting labor to the tradable sector ex-post. As a result, the need for ex-ante stabilization declines, justifying lower debt taxes in the ex-ante-only economy.

Panel (b) presents the collateral elasticity term for the two cases. In the case with the ex-ante instrument only, any rise in the future price of non-tradable goods brought about by prudential policy necessarily triggers an ex-post reallocation of labor toward the non-tradable sector, which mutes some of the support for the collateral value. This leads to lower collateral elasticity. With ex-post tools, this effect is absent because the planner is not bound by market allocation of labor, leading to a higher collateral elasticity, and ultimately to higher taxes on debt.



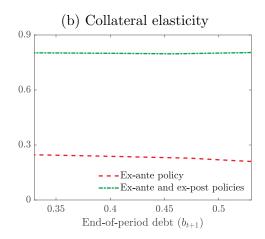


Figure 5: Stabilization gains and collateral elasticity

Note: The productivity shock is set at its mean value. Panel (a) shows the ratio of the multiplier of the credit constraint to the marginal utility of tradable consumption as a function of the starting level of debt. Panel (b) shows the collateral elasticity term. For the economy with only an ex-ante instrument, this corresponds to  $\Psi_{t+1}\Upsilon_{t+1}$  as defined in equation (23), while for the economy with ex-ante and ex-post instruments, this corresponds to  $\Psi_{t+1}$  as defined in equation (26).

#### 4.5 Curvature in production and ex-post effectiveness

In our baseline calibration, the optimal ex-ante tax on debt is higher when the planner has access to an ex-post policy instrument. Moreover, borrowing in this economy exceeds both the laissez-faire level and the level observed in the economy with only one instrument.

In this section, we argue that while the availability of ex-post policy always raises borrowing relative to the economy with only one instrument, there also exist configurations where the planner with an ex-post instrument imposes a lower tax on debt than the planner with only an ex-ante instrument. Specifically, we show that with higher curvature in production, the ranking of taxes on debt reverses.<sup>13</sup> The curvature in production effectively determines the effectiveness of ex-post policies in the model. A lower  $\alpha$  increases the cost of labor reallocation between the tradable and non-tradable sectors, as the marginal product of labor declines more sharply with increases in labor input.

Panel (a) of Figure 6 shows the policy function for debt when  $\alpha = 0.1$  (compared to 0.67 in our baseline calibration).<sup>14</sup> With access to both ex-ante and ex-post policies, the planner now borrows less than in the laissez-faire economy during periods when the credit

<sup>&</sup>lt;sup>13</sup>In Appendix B.3, we show that changes in parameters other than  $\alpha$  have only a modest impact on the relative differences in taxes on debt across the two economies.

<sup>&</sup>lt;sup>14</sup>We recalibrate the volatility of the tradable productivity shock to match the volatility of tradable output in the data. This yields  $\sigma_z = 0.0514$ .

constraint does not bind. As mentioned above, this highlights that as ex-post policies become less effective, the planner relies more on reducing debt to mitigate crisis vulnerability.

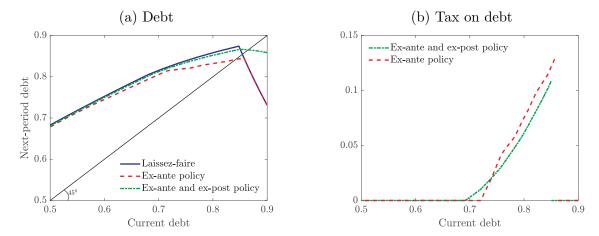


Figure 6: Policy functions under limited ex-post policy effectiveness

Note: Policy functions when the curvature in production parameter  $\alpha$  is set to 0.1. Tradable productivity shock is fixed at two standard deviations below its mean value. Panel (a) depicts the policy function of end-of-period debt as a function of initial debt. Panel (b) shows the macroprudential tax on debt that implements the allocations of the economy with ex-ante only tools and the economy with ex-ante and ex-post tools.

Panel (b) of Figure 6 shows the optimal tax on debt that implements the planner's allocation in both cases. In contrast to the baseline calibration, the optimal tax on debt is now lower with two instruments than with one for most initial debt levels. <sup>15</sup> As shown in Figure B-4 in the appendix, with high concavity in production, differences in the collateral elasticity narrow, while the stabilization term becomes more pronounced.

The key takeaway is that while access to ex-post policies generally induces higher borrowing, debt taxes may be either lower or higher than in an economy with only ex-ante policies. In other words, the level of borrowing is not informative about the scope for macroprudential policies.

<sup>&</sup>lt;sup>15</sup>For intermediate debt values, the tax is zero under the ex-ante policy because borrowing is lower, and there is a zero probability of a binding constraint next period, while the economy with two instruments borrows more.

## 5 Conclusions

Collateral constraints linked to market prices generate a pecuniary externality that leads to inefficient private borrowing. In this paper, we revisit the question of whether this externality leads to overborrowing and the extent to which this depends on the production structure.

Our main findings are threefold. First, configurations where the unregulated competitive equilibrium features less borrowing than the constrained-efficient equilibrium emerge when the planner has access to ex-post policy instruments. Second, regardless of whether the unregulated competitive equilibrium features more or less borrowing, optimal taxes on borrowing are always positive, implying that macroprudential policy is always desirable. Finally, macroprudential taxes on borrowing may be higher when the government has access to ex-post stabilization policies.

There are several promising avenues for future research. In our paper, we have focused on ex-post interventions that alter the real exchange rate and through this channel, relax households' credit constraints. We have shown that these interventions may take the form of labor taxes in the non-tradable sector, which induce a reallocation of employment towards tradables. Moreover, we have shown that subsidies on credit lending towards tradables can achieve the same outcomes. The implications of our analysis, however, may extend to other forms of interventions, in particular those in financial markets aimed at credit easing and stabilizing asset prices. For instance, a substantial body of research has examined the role of asset purchases in economies with financially constrained intermediaries (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Kiyotaki and Moore, 2019; Amador and Bianchi, 2024a). To the extent that these economies exhibit overborrowing in the laissez-faire equilibrium (see Amador and Bianchi, 2024b), our analysis suggests that credit easing may need to be complemented by stricter macroprudential regulation. Further exploring these interactions presents an interesting direction for future research.

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## A Proofs

#### A.1 Proof of Proposition 1

Denote by  $\tau_t$  the tax charged on debt issued at time t, the Euler equation for bonds in the regulated decentralized equilibrium (6) becomes

$$u_T(t) = \beta R(1 + \tau_t) \mathbb{E}_t u_T(t+1) + \mu_t. \tag{A.1}$$

Combining the optimality conditions (16) and (23) from the planning problem (15)-(15e) yields

$$u_T(t) = \beta R \mathbb{E}_t \left[ u_T(t+1) + \mu_{t+1} \widetilde{\Psi}_{t+1} \right] + \mu_t \left( 1 - \widetilde{\Psi}_t \right). \tag{A.2}$$

The optimal policy with ex-ante macroprudential intervention consists of stochastic sequences  $\{c_t^T, c_t^N, b_{t+1}, h_t^T, h_t^N, p_t^N, w_t, \mu_t\}_{t\geq 0}$  such that the following conditions hold: (4), (10), (11), (13), (14), (16), (18), (23), (A.2), and  $\mu_t \geq 0$ .

Meanwhile, the decentralized equilibrium with taxes on debt consists of of stochastic sequences  $\{c_t^T, c_t^N, b_{t+1}, h_t^T, h_t^N, p_t^N, w_t, \mu_t, \tau_t, T_t\}_{t\geq 0}$  such that the following conditions hold: (4), (7), (10), (11),

(13), (14), (A.1), 
$$T_t = b_t R \tau_{t-1}$$
 and  $\mu_t \ge 0$ .

Setting the tax to  $\tau_t = \mathbb{E}_t \mu_{t+1}^{\star} \widetilde{\Psi}_{t+1}^{\star} / \mathbb{E}_t u_T^{\star}(t+1)$  yields that the conditions characterizing the decentralized equilibrium with a tax on debt are identical to those characterizing the optimal policy outcome with ex-ante macroprudential intervention. Finally, the result that  $\tau_t \geq 0$  is immediate from the fact that  $\widetilde{\Psi}_t \geq 0$ .

## A.2 Proof of Proposition 2

With the payroll tax on non-tradable labor, firms maximize  $\max_{h_t^N} p_t^N z^N (h_t^N)^{\alpha} - w_t (1 + \tau_t) h^N$  and optimality implies

$$w_t(1+\tau_t^N) = p_t^N z^N \alpha \left(h_t^N\right)^{\alpha-1}.$$
 (B.1)

The proof for the tax on debt follows the same steps as Proposition 1, this time with

$$u_T(t) = \beta R \mathbb{E}_t \left[ u_T(t+1) + \mu_{t+1} \Psi_{t+1} \right] + \mu_t \left( 1 - \Psi_t \right)$$
(B.2)

replacing (A.2), in addition to condition (26) replacing condition (23).

Regarding the payroll tax on non-tradable labor, we can obtain (31) by combining (26)-(28) and (29), together with (B.1).

## **B** Additional Results

#### **B.1** Interventions in Credit Markets

Suppose that firms are subject to a working capital constraint. At the beginning of the period, firms must borrow intra-period loans to pay workers. At the end of the period firms collect the revenue and repay the loans. We assume that the government can provide credit subsidies to firms and that the profits of the financial intermediaries are reverted to households.

The problem of firms is

$$\max_{h_t^T} z_t^T (h_t^T)^{\alpha} - w_t h_t^T (1 + R(1 - s^T)),$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^{\alpha} - w_t h_t^N (1 + R(1 - s^N)),$$

where  $s^T, s^N$  are the subsidies for the two types of firms.

Optimality implies

$$w_t(1 + R(1 - s^T)) = z_t^T \alpha (h_t^T)^{\alpha - 1},$$
  

$$w_t(1 + R(1 - s^N)) = p_t^N z^N \alpha (h_t^N)^{\alpha - 1}.$$

Similar to Proposition 2, the optimal policy problem (25) can be decentralized with a pair of subsidies on credit. In particular, we obtain that by setting  $s_t^N = 0$  to zero and

$$s_{t}^{T} = \mu_{t}(1+R) \frac{\kappa c_{t}^{N} (1+\eta) p_{t}^{N} \left( \left( c_{t}^{T} \right)^{-1} + \left( c_{t}^{N} \right)^{-1} \frac{z^{N} \left( h_{t}^{N} \right)^{\alpha-1}}{z_{t}^{T} \left( h_{t}^{T} \right)^{\alpha-1}} \right)}{R(u_{T}(t) + \mu_{t} \kappa) + \mu_{t} \kappa c_{t}^{N} (1+\eta) p_{t}^{N} \left( \left( c_{t}^{T} \right)^{-1} + \left( c_{t}^{N} \right)^{-1} \frac{z^{N} \left( h_{t}^{N} \right)^{\alpha-1}}{z_{t}^{T} \left( h_{t}^{T} \right)^{\alpha-1}} \right)} \ge 0$$

this policy can also achieve the same outcomes as the payroll tax on labor.

#### B.2 Economy with high curvature in production

In this section we show all the policy functions and distributions computed in the baseline model for an economy with high curvature in production ( $\alpha = 0.1$ ). All policy functions are plotted keeping the tradable productivity shock fixed at two standard deviations below its mean.

Figure B-1 depicts the policy functions of debt (panel a) and tradable sector labor (panel b) in this economy, and is thus analogous to Figure 1 and 2 for the baseline calibration. Figure B-2 depicts the density distribution of debt at the ergodic distribution (panel a) and the optimal tax on debt (panel b) in the economy with low  $\alpha$ . It is the counterpart of Figure 3 in the main body of the paper. Next, Figure B-3 depicts the results of the experiment described in Section 4.4 in the economy with low  $\alpha$ . Once again, we compare the optimal borrowing choices in this experiment with the one in the economies regulated with one or two instruments. Finally, B-4 depicts the decomposition of the optimal tax on debt in this economy, following the steps described in the construction of Figure 5 in the main text.

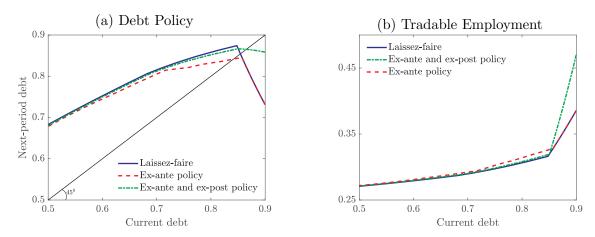


Figure B-1: Policy functions

*Note:* In both panels, the tradable productivity shock is set two standard deviations below the mean.

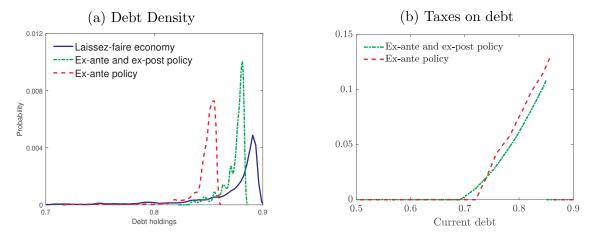


Figure B-2: Ergodic Distribution of debt and optimal taxes on debt

Note: In panel (b), the tradable productivity shock is set two standard deviations below the mean.

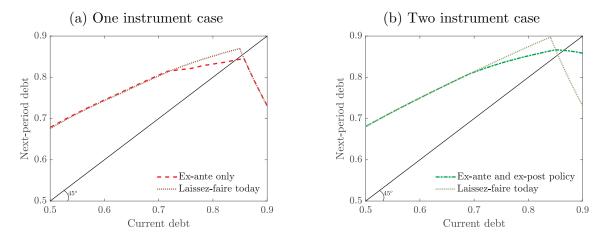
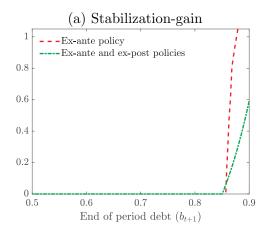


Figure B-3: One-period deviations

*Note:* The tradable productivity shock is set two standard deviations below the mean. In the experiment, there are no taxes at period t but households anticipate that from t+1 onward the allocations will be regulated by a government that has access to: an ex-ante instrument only (panel (a)) or ex-ante and ex-post instruments (panel (b)).



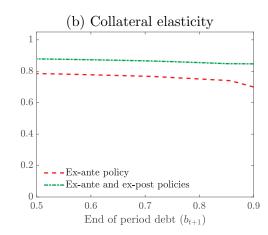


Figure B-4: Stabilization gains and collateral elasticity with high curvature in production.

Note: The tradable productivity shock is set two standard deviations below the mean. Panel (a) shows the ratio of the multiplier of the credit constraint to the marginal utility of tradable consumption as a function of the starting level of debt. Panel (b) shows the collateral elasticity term as a function of the initial level of debt. For the economy with only an ex-ante instrument, this corresponds to  $\Psi_{t+1}\Upsilon_{t+1}$  as defined in equation (23), while for the economy with ex-ante and ex-post instruments, this corresponds to  $\Psi_{t+1}$  as defined in equation (26).

#### B.3 Sensitivity of the results

To numerically assess the robustness of the results discussed in the previous section, we present solutions to our model for variations of the fundamental parameters  $(\eta, \kappa, \omega, \sigma)$  relative to our baseline values. We consider deviations of  $\pm 20\%$  around the baseline values of the parameters.

Table B1 shows the debt-to-output ratio at the ergodic distribution for the laissez-faire and socially planned economies. In all specifications, the same ordering identified in the baseline is preserved: The laissez-faire economy accumulates an average level of debt between the average levels of debt of the planner with only ex-ante instruments and the planner with both ex-ante and ex-post instruments.

Figure B-5 shows the policy functions for the optimal level of taxes on debt for the alternative calibrations and the baseline. Once again, in all specifications, we find the same result as in Figure 3, namely that the optimal tax on debt that implements the planner's allocations with only ex-ante policies is lower than in the economy with both ex-ante and ex-post policies.

Table B1: Debt to GDP

| Moments (in %)               | One-instrument | Laissez-Faire | Two-instrument |
|------------------------------|----------------|---------------|----------------|
| Baseline                     | 32.3           | 32.6          | 33.1           |
| Elasticity $1/(1+\eta)$      |                |               |                |
| Low                          | 32.4           | 32.6          | 33.1           |
| High                         | 32.3           | 32.5          | 33.1           |
| Collateral $\kappa$          |                |               |                |
| Low                          | 26.0           | 26.2          | 26.4           |
| High                         | 38.6           | 38.9          | 39.7           |
| Weight on tradables $\omega$ |                |               |                |
| Low                          | 32.2           | 32.5          | 33.2           |
| High                         | 32.4           | 32.6          | 33.0           |
| Risk aversion $\sigma$       |                |               |                |
| Low                          | 32.5           | 32.7          | 33.2           |
| High                         | 32.2           | 32.4          | 33.0           |

Note: We compute the average level of debt to output at the ergodic distribution for different parametrizations of the model. Low represents a value 20% below the baseline and high represents a value 20% above baseline.

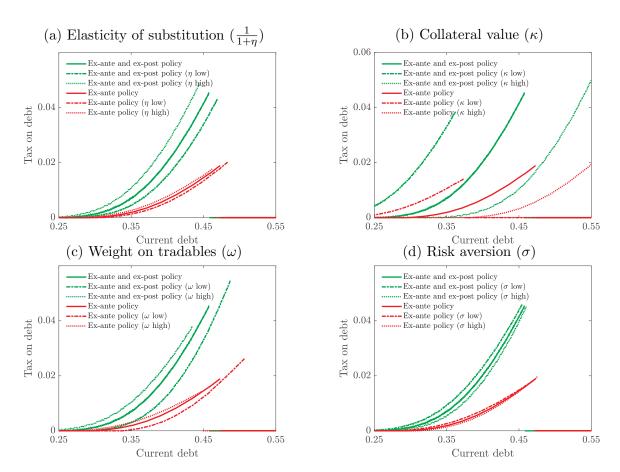


Figure B-5: Optimal taxes on debt under alternative calibrations *Note:* In all panels, the tradable productivity shock is set at the mean value.

## B.4 Constant tax on debt

Figure B-6 presents the unconditional welfare gains from a non-state contingent tax on debt. As the figure shows, there are welfare gains for constant taxes lower than 1.6%.

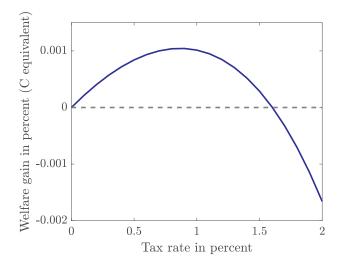


Figure B-6: Welfare gains in consumption equivalent terms from switching from a laissez-faire economy to an economy with a constant tax on debt on debt