Private Overborrowing under Sovereign Risk

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What are the consequences of excessive private debt for sovereign risk?

Motivating facts: road to the Spanish debt crisis

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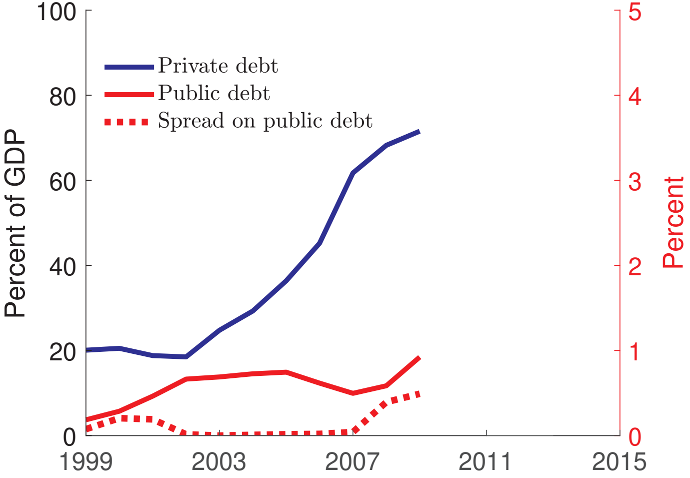
Motivation: The road to the 2012 Spanish debt crisis



Private and public net foreign liabilities and difference between Spanish and German 6-year T-bonds

Source: Bank of Spain, Eurostat, and Bloomberg

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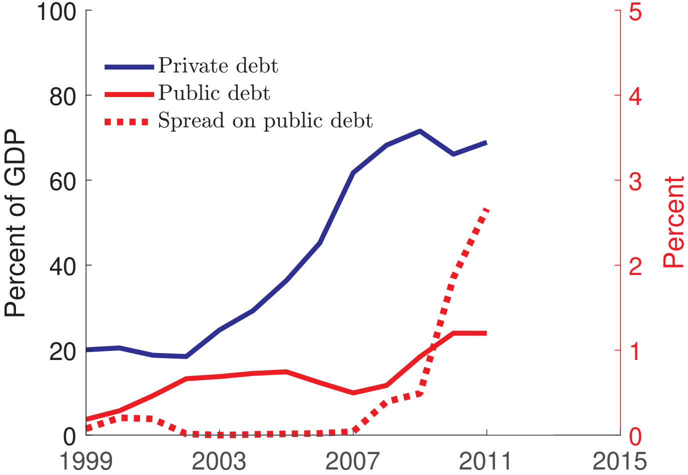

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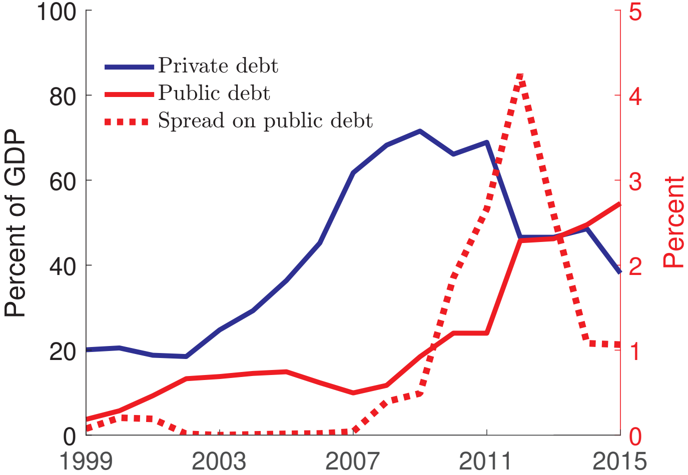

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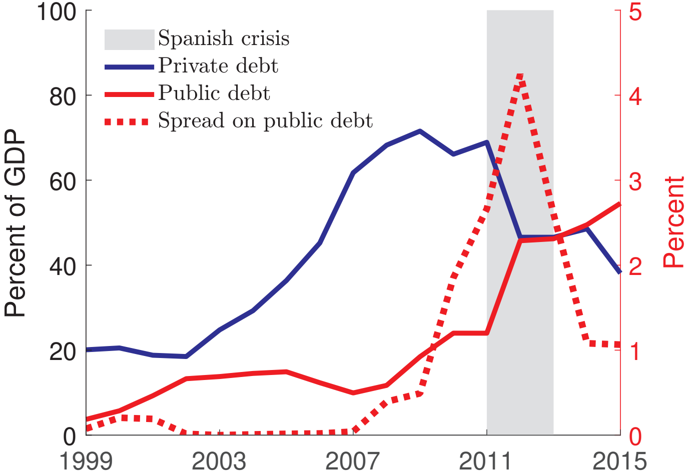

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Source: Bank of Spain, Eurostat, and Bloomberg Total debt: Composition matters

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Contribution 2: Measure overborrowing and its impact on crises

(i) Private debt was 5% of GDP above the socially optimal level

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Policy implication $\rightarrow$ More restrictive prudential policies if sovereign risk

This paper: one model, two versions

Model with two types of international bonds

- Private debt, constrained by *market value of income* (credit friction)
- Public debt, strategically defaultable

Baseline version: Competitive households issue private debt. Government makes transfers and public debt decisions

Social planner (SP) version: Planner makes all aggregate borrowing decisions and faces the same constraints and also lacks commitment

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$\rightarrow$ **Policy implementation**: Government sets taxes on private debt

Related literature

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- Sovereign default and private debt

Kaas, et al. (2020); Arellano, et al. (2020); Kim and Zhang (2011); Mendoza and Yue (2009); Jeske (2006); Wright (2006)

- Financial crisis and government's intervention

Jeanne and Korinek (2019); Davila and Korinek (2018); Bianchi (2016); Kester (2016); Chari and Kehoe (2016); Bianchi (2011)

- Doom loops between the sovereign and financial sectors

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$\rightarrow$ Inefficient international private debt during sovereign defaults

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$\rightarrow$ Bailouts financed with long term defaultable debt

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$\rightarrow$ Quantify the channel from financial vulnerabilities to higher spreads

Model

Model: Small open endowment economy

Goods: Tradable (y^T) and nontradable (y^N). All bonds denominated in units of tradables

Real exchange rate (p^N): Relative price of nontradables

Shocks: Income (y^T) , financial (κ) , and private default (π).

Agents: Government, households, and risk-neutral foreign lenders

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Timing:

1. Shocks realize, summarized in $s = (y^T, \kappa, \pi)$
2. Government's default, borrowing, and tax decisions (*strategic*)
3. Households' consumption and borrowing decisions (*competitive*)
4. Lenders price both assets (*competitive*)
5. Markets clear

Asset structure

Public debt (L):

1. Perpetuity with exponential decay δ
2. Strategically defaultable
3. Issued at price Q

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1. Perpetuity with exponential decay δ
2. Strategically defaultable
3. Issued at price Q

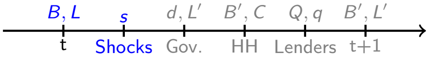
Private debt (B):

1. One period
2. Fraction π exogenously defaulted each period
3. Issued at price q
4. Constrained by current income:

$$q(\pi)B' \leq \kappa[y^T + p^N y^N]$$

Baseline: Government's problem

Government state space is $S = (s, B, L)$



Government takes households and lenders best responses as given

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B, L	s	d, L'	B', C	Q, q	B', L'
t	Shocks	Gov.	HH	Lenders	t+1

Government takes households and lenders best responses as given

Government chooses default d, and public debt L' to solve :

$$W(S) = \max_{d \in \{0,1\}} \underbrace{d}_{\text{Default}} \underbrace{W^D(S)}_{\text{Value in default}} + [1 - d] \underbrace{W^R(S)}_{\text{Repayment} \to \text{access to L}'}$$

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Period budget constraint that pins down transfers

$$\underbrace{T}_{\text{Transfers}} = (1-d) \times \Big[\underbrace{Q^{BR}}_{\text{Public debt price}} \times \underbrace{[L' - (1-\delta)L]}_{\text{Public debt issuance}} - \underbrace{\delta L}_{\text{Repayment}} \Big]$$

where Q^{BR} is the lenders best response to government policies

Repayment Problem | Default Problem

Baseline: Household's problem

Households face agg. state $S_G = (S, d, L')$

$$\begin{array}{cccccc} B, L & s & d, L' & B', C & Q, q & B', L' \\ \text{t} & \text{Shocks} & \text{Gov.} & \text{HH} & \text{Lenders} & \text{t+1} \end{array}$$

$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V(S'_G, b')]$$

subject to

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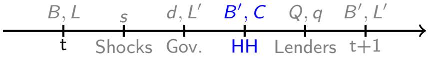

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$$\underbrace{q(\pi) b'}_{\text{Market value of new debt}} \leq \kappa \underbrace{(y^T + p^N y^N)}_{\text{Market value of current income}}$$

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$$\underbrace{q(\pi) b'}_{\text{Market value of new debt}} \leq \kappa \quad (y^T + \underbrace{p^N}_{\text{Pecuniary aggregate demand externality}} y^N)$$

Taking aggregate prices, laws of motion and public policies as given

$$\underbrace{p^N = p^N(S_G)}_{\text{Real exchange rate}} \; ; \quad \underbrace{T = \mathcal{T}(S_G)}_{\text{Transfer function}} \; ; \quad S'_G = \mathcal{S}'_G \Big(\underbrace{s', \mathcal{B}'(S_G), L'}_{S'}, \underbrace{\mathbf{d}(S'), \mathcal{L}'(S')}_{\text{Next period policies}} \Big) \; ;$$

Baseline: Definition of equilibrium

Decision rules and prices such that:

(i) Government optimizes

(ii) Households optimize

(iii) Foreign lenders optimally price private and public debt ▸ Details

(iv) Market of tradable and nontradable goods clear ▸ Details

Formal definition

Baseline: Equilibrium price of non tradables

Households' intratemporal optimality condition pins down p^N

The MRS between T and NT equals the relative price

$$p^N(S_G) = f\left(\frac{\mathcal{C}^T(S_G)}{y^N}\right)$$

Where $f(\cdot)$ is an increasing function

Functional form Euler equation

Social planner (SP) problem

Agents: Planner and risk-neutral foreign lenders

Shocks: Same as baseline (y^T, κ, π)

Assets (B, L)**:** Same structure, with credit constraint:

$$qB' \leq \kappa \left[y^T + f\left(\frac{c^T}{y^N}\right) y^N \right],$$

Planner chooses aggregates but takes pricing schedules as given

Foreign lenders optimally price both assets

Recursive planner problem | Euler equation planner

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Individually optimal private borrowing decisions ignore two GE effects

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- *Government's response to a binding collateral constraint → Bailouts*
- Bailouts ↑ public debt and ↑ the frequency of sovereign defaults
- Lenders internalize this → ↑ cost of public debt in baseline

Why bail out households when the credit constraint binds?

Bailouts increase consumption via two channels ▸ Illustration

- Bailouts ↑ current consumption C^T [1^{st} channel]
- Higher tradable consumption → ↑ price of nontradables $f(\frac{C^T}{y^N})$
- Higher price of nontradables → ↑ value of collateral $y^T + f(\frac{C^T}{y^N})y^N$
- Higher collateral → ↑ borrowing limit → ↑ borrowing b'
- Higher private borrowing b' ↑ consumption C^T [2^{nd} channel]

Summary of the main mechanism

In the baseline relative to the socially planned economy

- Private debt is higher
- Credit constraint binds more often
- More frequent public debt-financed bailouts and sovereign defaults

Mechanism: Households overborrow $\rightarrow$ Bailouts $\rightarrow$ High spreads

Theoretical finding: Decentralization of the SP problem

Proposition I

The consumption, borrowing, and default rules of the social planner coincide with the solutions of a decentralized equilibrium in the same environment where the government can also set state contingent taxes on private debt that satisfy:

$$1 - \boldsymbol{\tau} = \frac{\beta \mathbb{E}_s \left[(1 - \pi') \left(u_T^{SP}(\mathcal{C}^{SP,T}{}', y^{N}{}')) \right) \right] + \mu^{SP} q^{SP}}{q^{SP} u_T(\mathcal{C}^{SP,T}, y^N)}$$

Household problem with taxes

Quantitative analysis

Calibration

Calibrate baseline model to the pre-crisis years (1999-2011), assuming

- Discretized AR 1 process for income, private default, financial shock
- Default costs linear on the log of income
- Dynamic discrete choice method *à la* Dvorkin, Sanchez, Sapriza, and Yurdagul (2020) ▸ Details

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Quantitative exercises: Use the calibrated model to

1. Measure overborrowing and its costs at the ergodic distribution
2. Simulate the 2012 Spanish debt crisis
3. Construct optimal macroprudential policies

Calibrated parameters

Parameter	Value	Moments	Target (in %)	Model
Discount factor	$\beta = .92$	Av. Total Debt	56	56
Scale ϵ	$v = .020$	Std. Total Debt	4.8	5.0
Financial shock	$\bar{\kappa} = .45$	Av. Private Debt	42	42
$\kappa_{t+1} = (1-\rho)\bar{\kappa} + \rho\kappa_t + \varepsilon_t^k$	$\sigma^\kappa = .020$	Std. Private Debt	7.1	5.8
Default Cost	$\phi_0 = .31$	Av. Spread	.45	.45
$\phi(y^T) = \phi_0 + \phi_1 \log(y^T)$	$\phi_1 = 1.9$	Std. Spread	.61	.61

Overborrowing in the policy functions

Effect on crisis at the ergodic

Effect on spreads

Parameters estimated outside of the model

Results: Baseline vs. social planner at the ergodic

Average (in %)	Data	Baseline
Private debt	42	42
Public debt	14	15
Spread	.45	.45
Probability of a financial crisis	-	2.5
Probability of default	-	.46
Welfare gains (increase in c^T)	-	-

Welfare Calculations Macroprudential policies at the ergodic

Results: Baseline vs. social planner at the ergodic

Average (in %)	Data	Baseline	Social planner
Private debt	42	42	37
Public debt	14	15	12
Spread	.45	.45	.034
Probability of a financial crisis	-	2.5	.10
Probability of default	-	.46	.030
Welfare gains (increase in c^T)	-	-	.41

Welfare Calculations Macroprudential policies at the ergodic

Simulating the 2008-2015 Spanish crisis

Exercise I: Dynamics and counterfactual

Feed exogenous shocks from the data $\rightarrow$ compute models' dynamics

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Estimation method of κ_t: Particle filter on the baseline model ▸ Details

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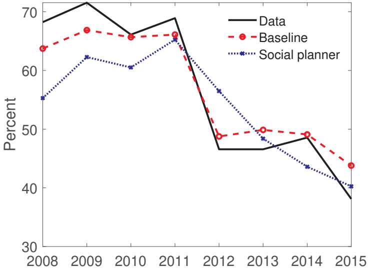
Endogenous dynamics:

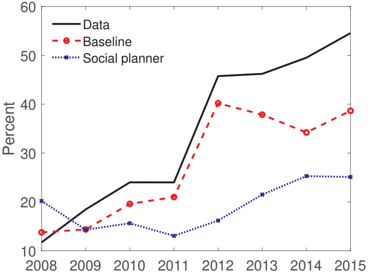
1. Public debt
2. Private debt
3. Interest rate spread on public debt

Exercise I: Endogenous private debt, public debt and spreads

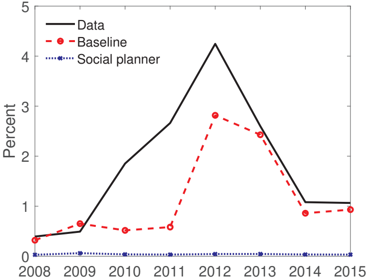
(a) Private debt



(b) Public debt



(c) Spreads



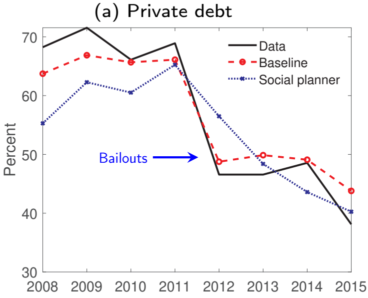
Income shock

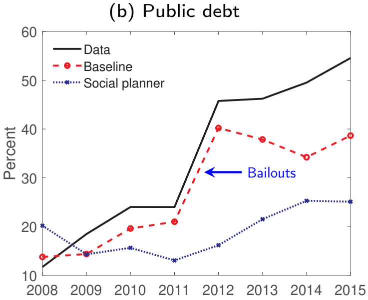
Private default shock

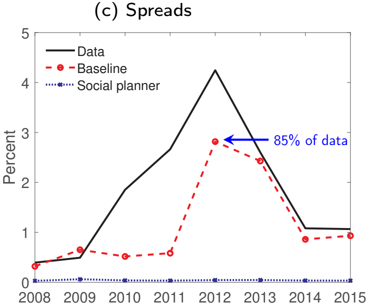
Financial shock

Optimal macroprudential tax

Exercise I: Endogenous private debt, public debt and spreads







Income shock

Private default shock

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Optimal macroprudential tax

Exercise II: Effect of private debt on spreads

In Exercise I social planner avoids entirely the increase in spreads

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Planner achieves this by combining:

1. Lower private debt before crisis

2. Lower public debt during the crisis

Exercise II: Effect of private debt on spreads

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Estimate κ_t using the particle filter on the baseline model. Still endogenous:

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2. Interest rate spread on public debt

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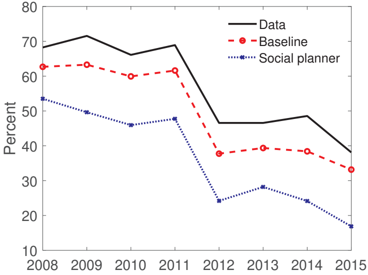
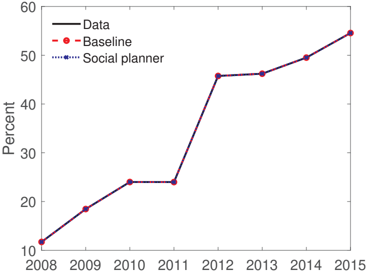
Effect of private overborrowing: $spread^{SP} - spread^{data}$

Exercise II: Endogenous private debt and spreads

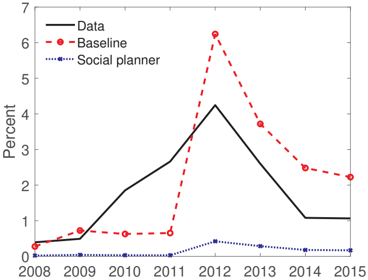
(a) Private debt



(b) Public debt



(c) Spreads



Income shock

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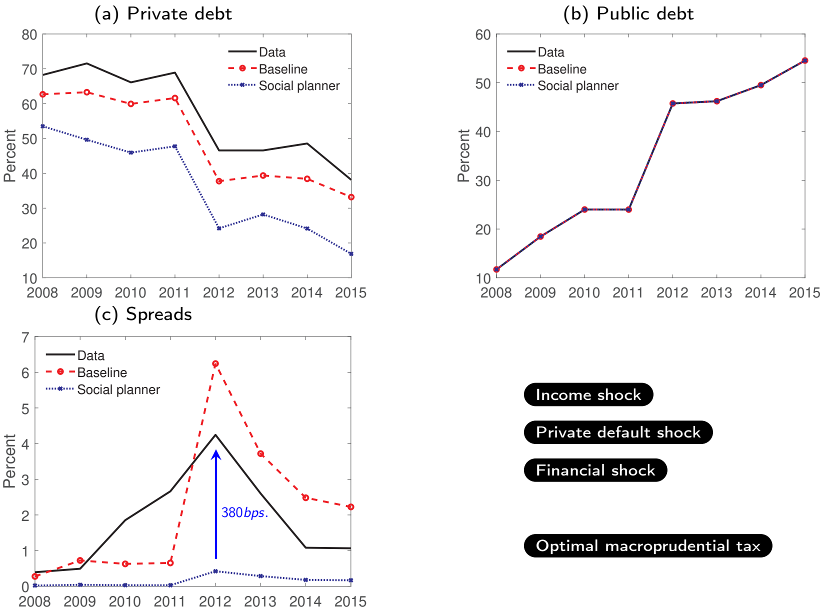
Financial shock

Optimal macroprudential tax

Exercise II: Endogenous private debt and spreads



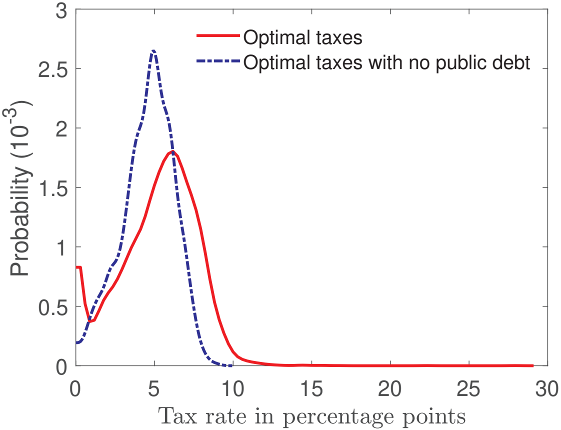
Income shock

Private default shock

Financial shock

Optimal macroprudential tax

New policy recommendation : Higher taxes on private debt



Higher taxes than a model with no public debt

The average tax rate increases from 4.6% to 5.3%

Household problem with taxes

Simulations at the ergodic

Policy functions

Extensions

A model with no public debt Details

A model with no private debt Details

A model with an exogenous credit constraint Details

Conclusion

Model of how financial crises lead to sovereign debt crises

Conclusion

Model of how financial crises lead to sovereign debt crises

1. Private borrowing above socially optimal levels
2. **Financial crisis**: Private debt is constrained. Contraction
3. Symmetric evolution of private and public debt
4. **Sovereign debt crisis**: Public interest spikes follow bailouts

Conclusion

Model of how financial crises lead to sovereign debt crises

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Quantitatively consistent with the Spanish data

Conclusion

Model of how financial crises lead to sovereign debt crises

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Quantitatively consistent with the Spanish data

Measures the level of excessive private debt and its cost

- Private borrowing by 5.0% of GDP
- The probability of a financial crisis by 240 bps.
- The spread in 2012 by 380 bps.

Conclusion

Model of how financial crises lead to sovereign debt crises

1. Private borrowing above socially optimal levels
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Quantitatively consistent with the Spanish data

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- Private borrowing by 5.0% of GDP
- The probability of a financial crisis by 240 bps.
- The spread in 2012 by 380 bps.

Policy recommendations:

- If sovereign risk is an issue, then stricter macroprudential policies
- Optimal policy response combines preventive and remedial measures

Thank you

Appendix

Households

Choose b_{t+1}, c_t^T, c_t^N to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(c(c^T, c^N)) = \frac{c(c^T, c^N)^{1-\sigma}}{1-\sigma}$,
$c(c^T, c^N) = [\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}]^{-\frac{1}{\eta}}$ $\eta > -1$, $\omega \in (0,1)$,

subject to budget constraint

$$c_t^T + p_t^N c_t^N + b_t = q b_{t+1} + y_t^T + p_t^N y^N + T_t,$$

and credit constraint

$$q b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N \right)$$

Government

Picks debt L^j_{t+1} from a fixed grid $\{L^1, ...L^J\}$ and makes default decision d_t to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(\mathcal{C}_t) + d_t \big(\epsilon_t^{J+1} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t^j],$$

subject to budget constraint

$$T_t = (1-d_t)\bigg(Q_t \big[L^j_{t+1} - (1-\delta)L^i_t\big] - \delta^i_t \bigg).$$

Foreign Lenders

Are risk neutral and lend to both the government and the households

$$Q_t = q\mathbb{E}_t\Big[(1 - d_{t+1})\big(\delta + (1-\delta)Q_{t+1}\big)\Big]$$

Market clearing conditions

Market clearing for non-tradables

$$\mathcal{C}^N(s, L, B) = y^N$$

Resource constraint for tradables

$$\mathcal{C}^T(s, L, B) + (1-\pi)B = y^T + q(\pi)\mathcal{B}'(s, L, B) + \Big\{1 - \boldsymbol{d}(s, L, B)\Big\}\times \\ \Big\{Q(s, L, B)\Big[\mathcal{L}'(s, L, B) - (1-\delta)L\Big] - \delta L\Big\}.$$

Back

Equilibrium relative price of nontradables

Household's CRRA on CES utility function:

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega (c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right]^{-\frac{1-\sigma}{\eta}}$$

Solution to the intratemporal household optimization problem:

$$p^N = \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1}$$

$$p^N(s, L, B) = \frac{1-\omega}{\omega} \left(\frac{\mathcal{C}^T(s, L, B)}{y^N}\right)^{\eta+1}$$

Back

Functional forms: Utility

Household's CRRA on CES utility function:

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right]^{-\frac{1-\sigma}{\eta}}$$

Elasticity of substitution between tradables and nontradables is $\frac{1}{1+\eta}$

Share of tradables in consumption is ω

Risk aversion coefficient is σ

Back to equilibrium conditions

Recursive Equilibrium

A Markov recursive competitive equilibrium is defined by,

1. value functions V, W, W^R, and W^D
2. policy functions for the private sector $\{\hat{b}, \hat{c}^T, \hat{c}^T\}$
3. pricing functions p^N and Q
4. perceived laws of motion $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N\}$
5. policy functions for the public sector $\{\mathcal{D}, \mathcal{L}'\}$

Such that,

Recursive Equilibrium II

1. Given $\{p^N, Q\}$, and $\{\mathcal{B}', \mathcal{L}'\}$, the private policy functions, $\{\hat{b}', \hat{c}^T, \hat{c}^T\}$ and V, solve the HH problem
2. Given Q, and $\{\mathcal{B}', \mathcal{C}^T, \mathcal{C}^N\}$, the public policy functions, $\{\mathcal{D}, \mathcal{L}'\}$ and W, W^R, and W^D, solve the government's problems
3. Rational expectations: $\{\mathcal{B}' = \hat{b}, \mathcal{C}^T = \hat{c}^T, \mathcal{C}^N = \hat{c}^N\}$
4. Given all policies Q satisfies the lenders problems
5. Markets clear: $y^N = \mathcal{C}^N$ and $\mathcal{C}^T + (1-\pi)B = y^T + q\mathcal{B}' + (1-\mathcal{D})\{Q[\mathcal{L}' - (1-\delta)L] - \delta L\}$

Back

Recursive government problem I

Discrete and bounded choices of public debt

$$L' \in \Lambda = \{L_1, ..., L_J\}$$

Each period the government draws a taste vector shock $\boldsymbol{\epsilon}$

$$\boldsymbol{\epsilon} = [\epsilon_1, ..., \epsilon_J, \epsilon_{J+1}]$$

First Jth elemets are associated with a public debt level and the $J+1$st with the choice of default

$\boldsymbol{\epsilon}$ follows a multivariate generalized extreme value with location $m = -v\gamma$, scale $v > 0$, shape $\xi = 0$

Where γ is the Euler constant

Back to calibration Back to Government problem

Recursive government problem II

Taking the household policy functions as given the government solves:

$$W(s, L_i, B, \boldsymbol{\epsilon}) = \max_{d \in \{0,1\}} [1-d] W^R(s, L_i, B, \boldsymbol{\epsilon}) + d W^D(s, L_i, B, \epsilon_{J+1})$$

Where the value of default is:

$$W^d\big(s, L_i, B, \epsilon_{J+1}\big) = u\Big(\mathcal{C}^D(s, B)\Big) - \phi(y^T) + \epsilon_{J+1} +$$

$$\beta \mathbb{E}_{s'|s} \mathbb{E}_{\boldsymbol{\epsilon}'} \Big[W\big(s', 0, \mathcal{B'}^D(s, B), \boldsymbol{\epsilon}'\big) \Big]$$

Subject to the resource constraints, $\mathcal{C}^{\mathcal{N}} = y^N$ and :

$$\mathcal{C}^{T,D}(s, B) = y^T + q\mathcal{B'}^D(s, B) - B,$$

Recursive government problem III

And the value of repayment is:

$$W^c(s, L_i, B, \epsilon) = \max_{j \in [\![1,J]\!]} u\big(\mathcal{C}(s, L_i, B)\big) + \epsilon_j +$$

$$\beta \mathbb{E}_{s'|s} \mathbb{E}_{\epsilon'} [W(s', L_j, \mathcal{B}'(s, L_i, B), \epsilon')]$$

Subject to the resource constraints, $\mathcal{C}^{\mathcal{N}} = y^N$ and:

$$\mathcal{C}^{\mathcal{T}}(s, , L_i, B) = y^T + q\mathcal{B}'(s, L_i, B) - B +$$

$$Q(s, L_i, \mathcal{B}')[L_j - (1-\delta)L_i] - \delta L_i$$

Back to Government problem Back to calibration

Recursive government problem III

And the value of repayment is:

$$W^c(s, L_i, B, \boldsymbol{\epsilon}) = \max_{j \in [\![1,J]\!]} u\big(\mathcal{C}(s, L_i, B)\big) + \epsilon_j +$$

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$$\underbrace{Q(s, L_i, \mathcal{B}')[L_j - (1-\delta)L_i] - \delta L_i}_{T(s, L_i, B')}$$

Back to Government problem Back to calibration

Foreign lenders pricing in equilibrium

For all $j \in [\![0, J]\!]$:

$$Q(s, L_j, B') = q\mathbb{E}_{s'|s}\mathbb{E}_{\epsilon'}\left[\left[1 - d(s', L_j, B', \boldsymbol{\epsilon}')\right]\times\right.$$

$$\left.\left[\delta + (1-\delta)Q\Big(s', L'(s', L_j, B', \boldsymbol{\epsilon}'), \mathcal{B}'(s, L_j, L', B')\Big)\right]\right]$$

Recursive equilibrium Solution method Back to Government problem

Recursive problem of the government : properties

The Government expected values can be re-written:

$$\boldsymbol{W}(s, L_i, B) = E_{\boldsymbol{\epsilon}}[W(s, L_i, B, \boldsymbol{\epsilon})]$$

$$\boldsymbol{W}(s, L_i, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{W^c(s, L_i, B, \boldsymbol{\epsilon}); W^d(s, L_i, B, \epsilon_{J+1})\big\}\Big]$$

$$\boldsymbol{W}(s, L_i, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{\max_{j \in [\![1,J]\!]}\{u(\mathcal{C}_{i,j}) + \beta \mathbb{E}_{s'|s}\boldsymbol{W}(s', L_j, \mathcal{B}'_{i,j}) + \epsilon_j\};$$

$$u(\mathcal{C}_{0,0}) - \phi(y^T) + \beta \mathbb{E}_{s'|s}\boldsymbol{W}(s', 0, \mathcal{B}'_{0,0}) + \epsilon_{J+1}\big\}\Big]$$

Subject to the resource constraints:

$$\mathcal{C}^T(s, L_i, L_j, B) = y^T + q\mathcal{B}'(s, L_i, L_j, B) - B + T(s, L_i, L_j, \mathcal{B}'_{i,j})$$

$$\mathcal{C}^N(s, L_i, L_j, B) = y^N$$

Recursive problem of the government : properties II

We assume that the vector ϵ is i.i.d over time and follows a multivariate Generalized Extreme Value distribution with joint cumulative distribution function:

$$F(\boldsymbol{x}) = \exp\Big[- \Big(\sum_{j=1}^{J} \exp\Big(- \frac{x_j - m}{pv} \Big) \Big)^p - \exp\Big(- \frac{x_{J+1} - m}{v} \Big) \Big]$$

It is useful to define:

$$\Upsilon_{i,j} = u(\mathcal{C}_{i,j}) + \beta \mathbb{E}_{s'|s} \boldsymbol{W}(s', L_j, \mathcal{B}'_{i,j})$$

$$\Upsilon_{i,def} = u(\mathcal{C}_{0,0}) - \phi(y^T) + \beta \mathbb{E}_{s'|s} \boldsymbol{W}(s', 0, \mathcal{B}'_{0,0})$$

Recursive problem of the government : properties III

The Government value can be re-written:

$$\boldsymbol{W}(s, L_i, B) = \Upsilon_{i,def} + v \log \left[1 + \left(\sum_{k \in [\![1,J]\!]} \exp\left(- \frac{\Upsilon_{i,def} - \Upsilon_{i,k}}{pv} \right) \right)^p \right]$$

There are also close form solutions for the probability of default:

$$\boldsymbol{d}(s, L_i, B) = \mathbb{E}_\epsilon[d(s, L_i, B, \epsilon)]$$

$$\boldsymbol{d}(s, L_i, B) = \frac{1}{1 + \left(\sum_{k \in [\![1,J]\!]} \exp\left(- \frac{\Upsilon_{i,def} - \Upsilon_{i,k}}{pv} \right) \right)^p}$$

Recursive problem of the government : properties IV

The probability of issuance for each level of government debt conditional on repayment :

$$\boldsymbol{G}_{s,L_i,B}(L_j) = \mathbb{P}_{\epsilon}\big(L' = L_j | d(s, L_i, B, \epsilon) = 0\big)$$

$$\boldsymbol{G}_{s,L_i,B}(L_j) = \frac{1}{\sum_{k \in [\![1,J]\!]} \exp\left(\frac{\Upsilon_{i,k} - \Upsilon_{i,j}}{pv}\right)}$$

The price of debt:

$$Q(s, L_j, B') = q\mathbb{E}_{s'|s}\Bigg[\big[1 - \boldsymbol{d}(s', L_j, B')\big]\big[\delta +$$

$$+(1-\delta)\sum_{k \in [\![1,J]\!]} Q\big(s', L_k, , \mathcal{B}'(s', L_j, L_k, B')\big)\boldsymbol{G}_{s',L_j,B'}(L_k)\big]\Bigg]$$

Back

Baseline: Foreign lenders problem

When lending to households, they maximize recursive profits: :

$$\chi(s) = \max_{q} -qB' + \frac{B'}{1+r}\mathbb{E}_{s'|s}\big[1-\pi'\big]$$

When lending to the government they maximize recursive profits:

$$\Pi(s, L', B') = \max_{Q} -QL' + \frac{L'}{1+r}\mathbb{E}_{s'|s}\Bigg[\Big\{1-d'\Big\}\times$$

$$\Big\{\delta + (1-\delta)Q\Big(s', L'', B''\Big)\Big)\Big\}\Bigg]$$

Where,

$$B'' = \mathcal{B}'(s', L', B', d', L'')$$

$$L'' = \mathcal{L}'(s', L', B')$$

$$d' = \boldsymbol{d}'(s', L', B')$$

Back

SP: Foreign lenders problem

When lending private bonds, they maximize recursive profits: :

$$\chi(s) = \max_{q} -qB' + \frac{B'}{1+r}\mathbb{E}_{s'|s}\left[1-\pi'\right]$$

When lending to the public bonds they maximize recursive profits:

$$\Pi(s, L', B') = \max_{Q} -QL' + \frac{L'}{1+r}\mathbb{E}_{s'|s}\Bigg[\Big\{1 - d\big(s', L', B'\big)\Big\}\times$$

$$\Big\{\delta + (1-\delta)Q\Big(s', L'', B''\Big)\Big\}\Bigg]$$

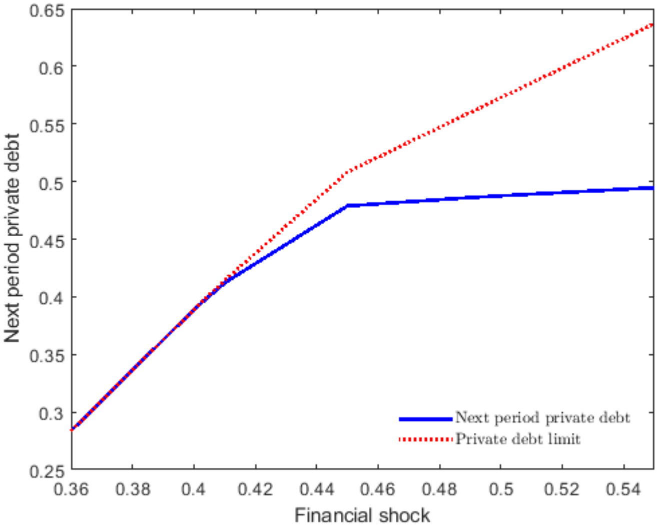
Where,

$$B'' = \mathcal{B}^{SP\prime}(s', L', B')$$

$$L'' = \mathcal{L}^{SP\prime}(s', L', B')$$

Back

Private debt as function of the financial shock



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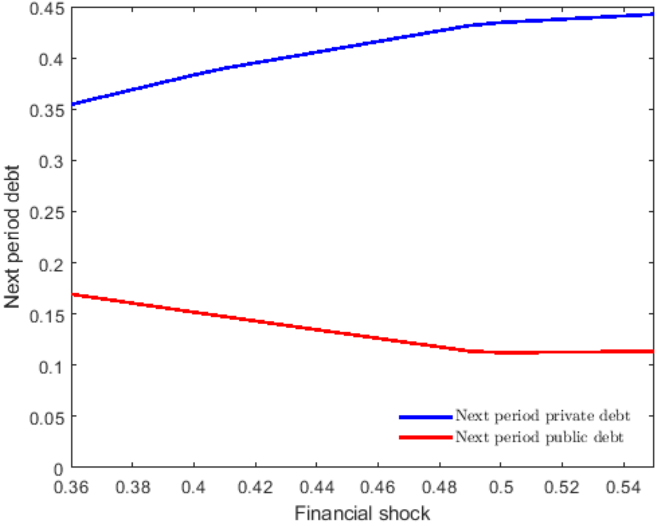
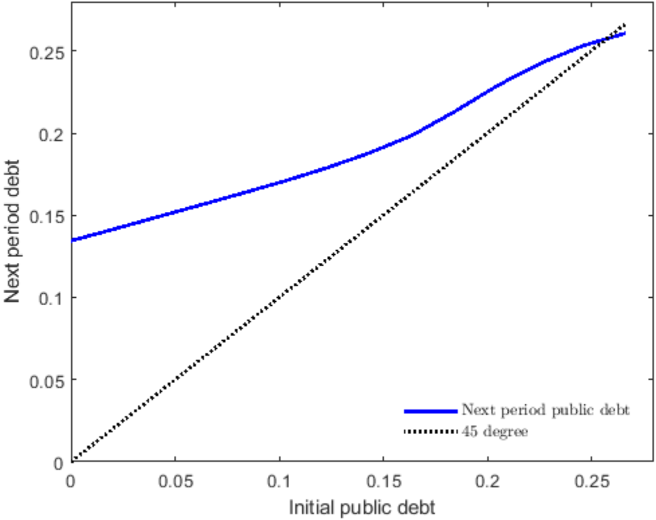
Public debt as function of the financial shock



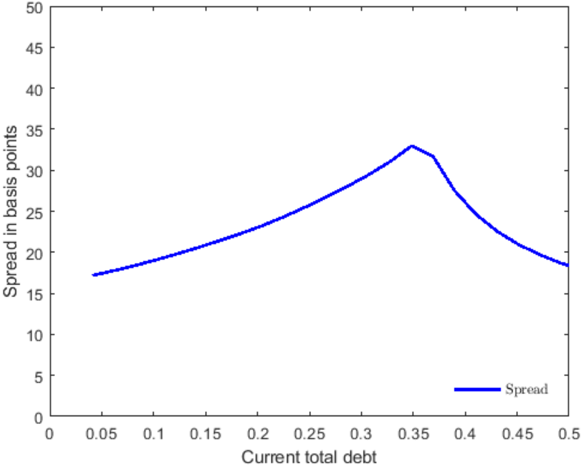
Back

Next period public debt as a function of current private debt



Back

Spreads as a function of current private debt



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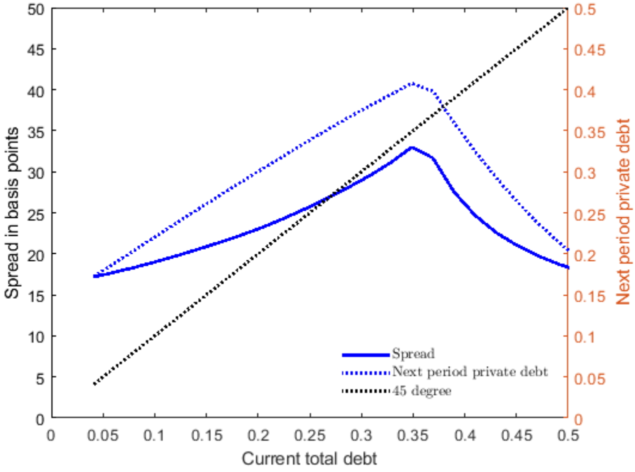
Spreads as a function of current private debt



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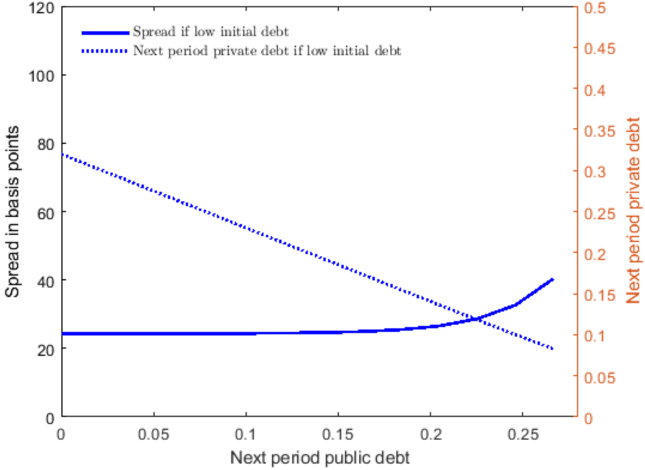
Spreads as a function of next period public debt



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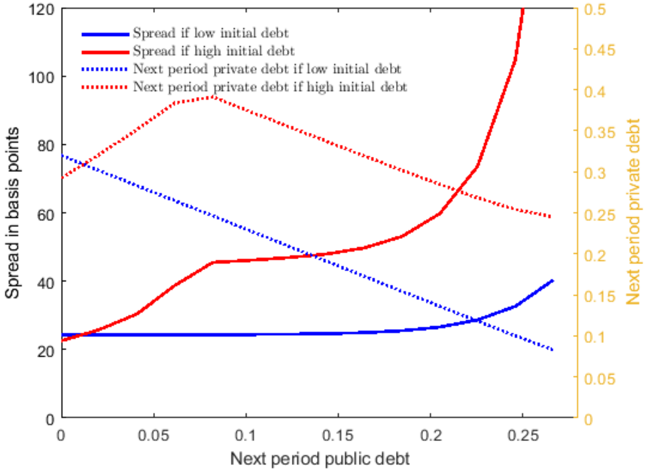
Spreads as a function of next period public debt



Back

Euler equations in the baseline

Intertemporal Euler equation for private bonds

$$qu_T(c) = \beta\mathbb{E}[(1-\pi')u_T(c')] + \mu q$$

Where μ is the Lagrange multiplier associated with the credit constraint

Back to equilibrium conditions

Household problem with taxes

Households solve:

$$V^{\tau}(s, L, B, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V^{\tau}(S'_G, b')]$$

subject to

$$c^T + p^N c^N + (1-\pi)b = y^T + p^N y^N + (1 - \tau(S_G))q(\pi)b' + T(S_G)$$

$$q(\pi)b' \leq \kappa[p^N y^N + y^T],$$

$$p^N = p^N(S_G),$$

$$B' = \mathcal{B}'(S_G),$$

$$L' = \mathcal{L}'(S_G)$$

Government problem with taxes | Euler equations with taxes | Back to equivalence result | Back to densities

Planner's Euler equation of private bonds

Intertemporal equation for the private bonds

$$(u_T(c_T^{SP}) + \mu\psi)(q^{SP} + Q_{B'}^{SP}(L' - (1-\delta)L))$$

$$= \beta\mathbb{E}[(1-\pi')(u_T(c^{SP\prime}_T) + \mu^{SP\prime}\psi')] + q^{SP}\mu^{SP}$$

μ^{SP} is the Lagrange multiplier associated with the credit constraint

$$\psi = \kappa(1+\eta)\frac{(1-\omega)}{\omega}\left(\frac{c_T^{SP}}{y_N}\right)^{\eta}$$

Marginal effect of an extra unit of private debt on the credit constraint

$Q_{B'}^{SP}$ marginal effect of an extra unit of private debt on the price of L

Back planner's problem Back to main mechanism

Euler in the baseline model with taxes on b'

$$(1-\tau)qu_T(c) = \beta\mathbb{E}[(1-\pi')u_T(c')] + \mu q$$

When the constraint does not bind:

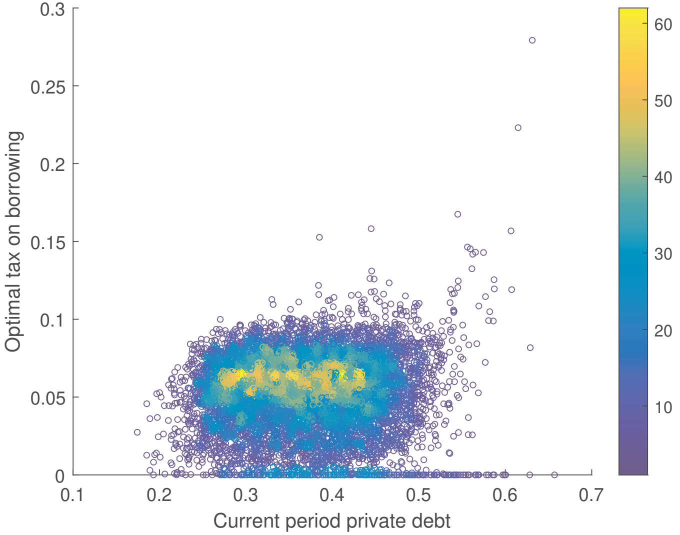
$$(1-\tau) = \frac{\beta\mathbb{E}[(1-\pi')u_T(c')]}{qu_T(c)}$$

Otherwise

$$(1-\tau) = \frac{\beta\mathbb{E}[(1-\pi')u_T(c')]}{qu_T(c)} + \frac{\mu q}{qu_T(c)}$$

Back

Optimal tax on borrowing

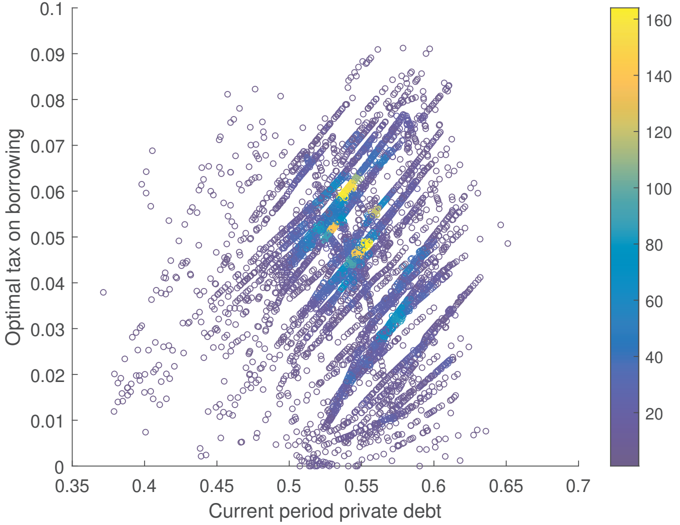


Back to Ex. I | Back to results | Back to densities | With respect to total debt | With no L

Optimal tax on borrowing relative to total debt

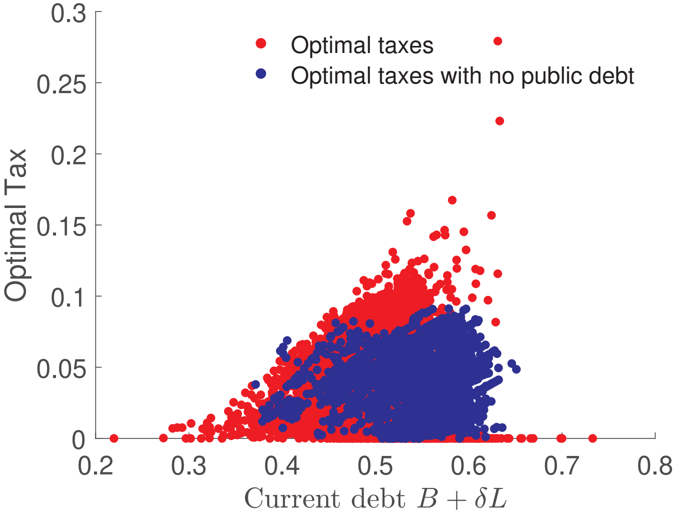
Back

Taxes in a model with no public debt L



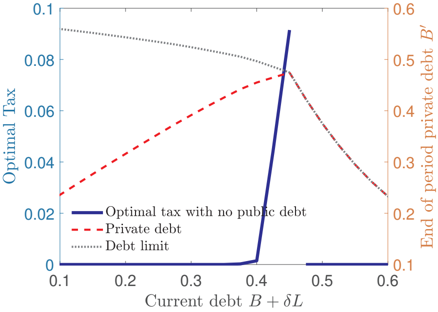
Back
Compare models at the ergodic

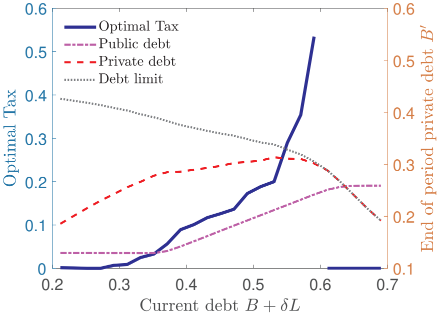
Taxes in models with and without L



Back to no public debt Back to densities

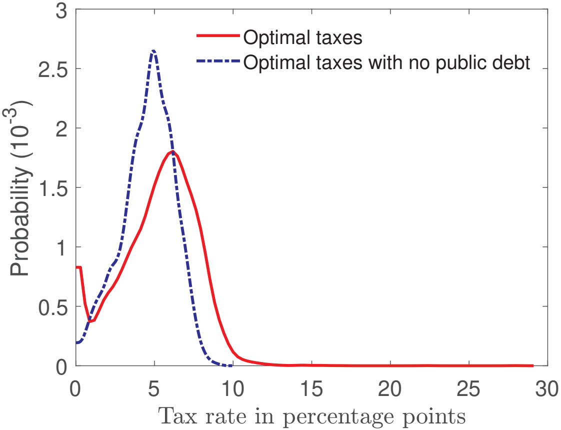
Optimal taxes as function of private debt





Back

Policy recommendation : Higher taxes on private borrowing



Higher taxes than a model with no public debt

Simulations at the ergodic

The average tax rate increases from 4.6% to 5.3%

Policy functions

Welfare calculations

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1-\sigma}$$

Where:

$$C(c^T, c^N) = \left((c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right)^{\frac{-1}{\eta}}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$\theta(s_0, L_0, B_0) = \left(\frac{W^{SP}(s_0, L_0, B_0) * (1-\sigma) * (1-\beta) + 1}{W(s_0, L_0, B_0) * (1-\sigma) * (1-\beta) + 1}\right)^{\frac{1}{1-\sigma}} - 1$$

Back

Alternative Welfare calculations

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1-\sigma}$$

Where:

$$C(c^T, c^N) = \left((c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right)^{\frac{-1}{\eta}}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$\theta(s_0, L_0, B_0, L_0') = \left(\frac{V^{SP}(s_0, L_0, B_0, L_0') * (1-\sigma) * (1-\beta) + 1}{V(s_0, L_0, B_0, L_0') * (1-\sigma) * (1-\beta) + 1}\right)^{\frac{1}{1-\sigma}} - 1$$

Back

One Period Welfare gains

The household utility function:

$$u(c^T, c^N) = \frac{C(c^T, c^N)^{1-\sigma} - 1}{1-\sigma}$$

Welfare gains in terms of permanent consumption under the baseline are:

$$W^{SP}(s, L, B) = \Big(1 - \hat{d}(s, L, B)\Big)\Big[(1 + \theta(s, L, B))u(\hat{c}(s, L, B)) +$$

$$+\beta \mathbb{E} V(s', \hat{B}', \hat{L}')\Big] + \hat{d}(s, L, B)\Big[(1 + \theta(s, L, B)) \times$$

$$\times u(\hat{c}(s, 0, B)) + \beta \mathbb{E} V(s', \hat{B}', 0)\Big]$$

Back

Models of sovereign debt

Average (in %)	Data	Baseline	SP	$\delta = .14$	$\delta = 1$
Private debt	42	42	37	-	-
Public debt	14	15	12	13	15
Spread	.45	.45	.034	.080	.28
Probability of a financial crisis	-	2.5	.10	-	-
Probability of default	-	.46	.030	.040	.35
Welfare gains	-	-	.41	2.4	2.7
				Hatchondo &Martinez 2009	Arellano 2008

For the all models welfare gains are calculated relative to baseline

Back

Models of private debt

Average (in %)	Data	Baseline	SP	Decen- -tralized	Efficient
Private debt	42	42	37	44	43
Public debt	14	15	12	-	-
Spread	.45	.45	.034	-	-
Probability of a financial crisis	-	2.5	.10	6.4	1.8
Probability of default	-	.46	.030	-	-
Welfare gains (rel to Baseline)	-	-	.41	6.4	6.2
Welfare gains (rel to Bianchi DE)	-	-	-	-	.17
				Bianchi 2011	Bianchi 2011

Back

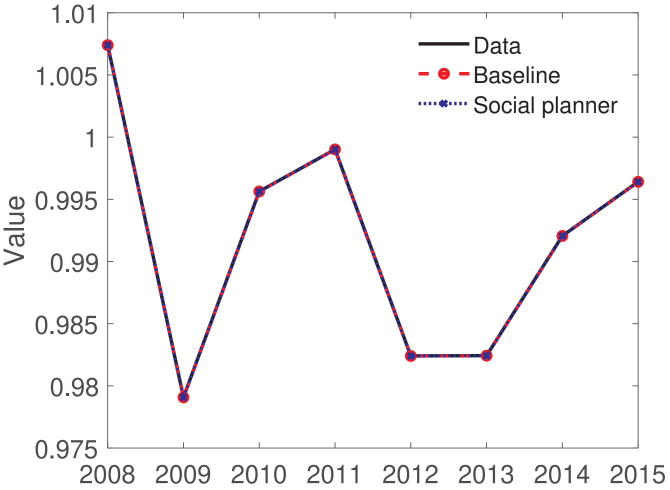
Model with an exogenous constraint

$$qB' \leq \kappa(y^N + y^T)$$

Average (in %)	Data	Baseline	SP	Decent.	Planner
Private debt	42	42	37	33	32
Public debt	14	15	12	12	12
Spread	.45	.45	.034	.038	.036
Probability of a financial crisis	-	2.5	.1	0	0
Probability of default	-	.46	.030	.030	.030
Welfare gains	-	-	.0041	-	-

Back

Exercise I: Exogenous dynamics of income



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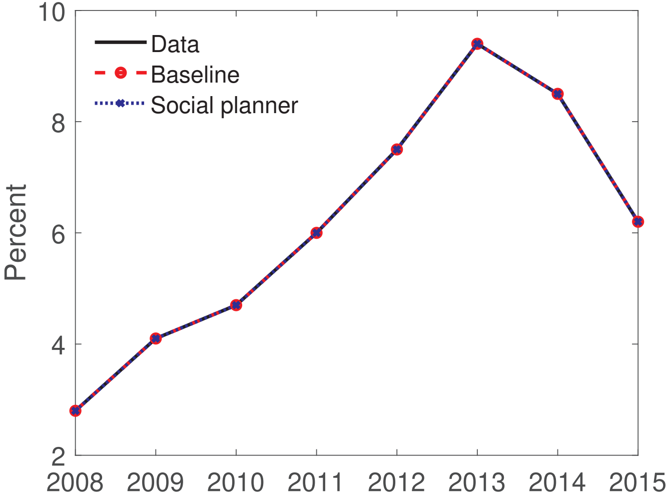
Exercise I: Exogenous dynamics of private default



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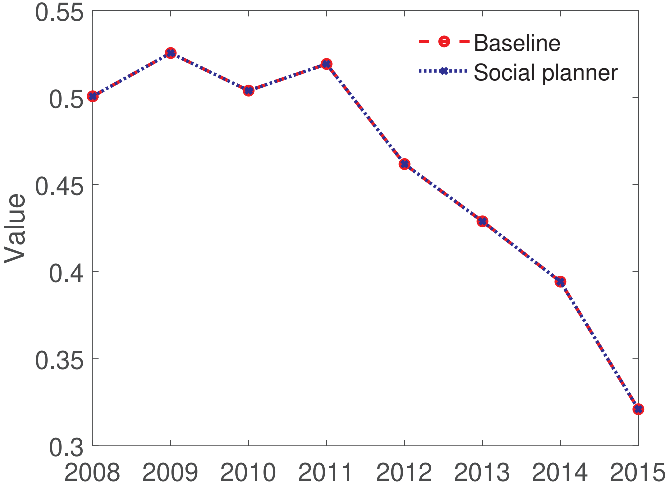
Exercise I: Exogenous financial shock



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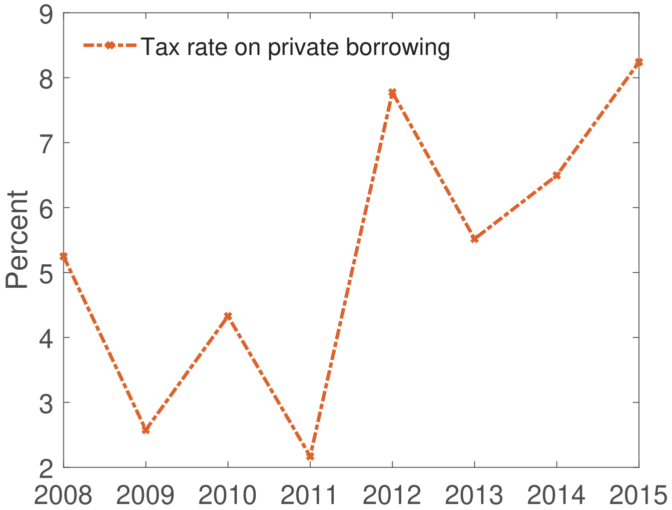
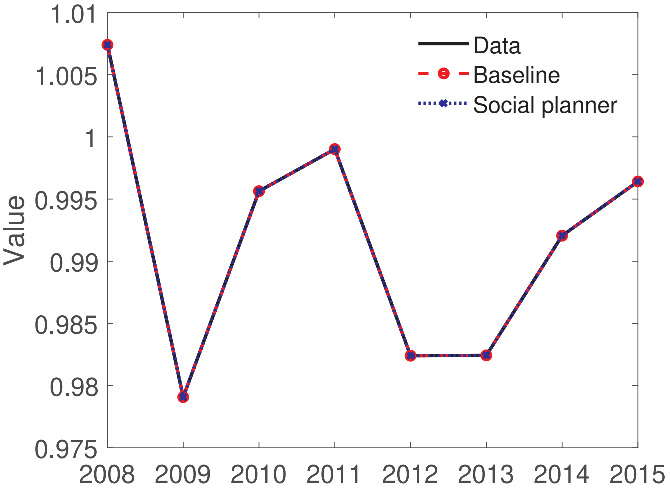
Exercise I: Optimal tax dynamics



Back Taxes at the ergodic

Exercise II: Exogenous dynamics of income



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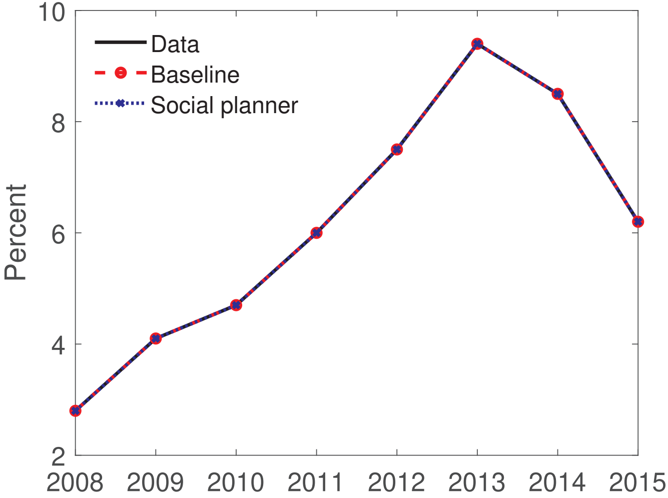
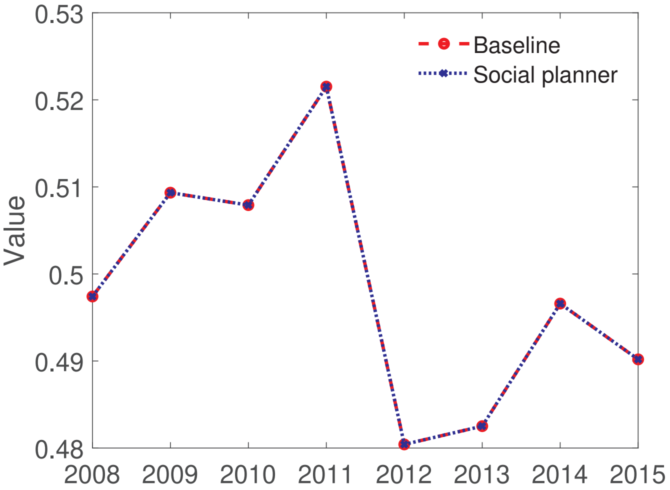
Exercise II: Exogenous dynamics of private default



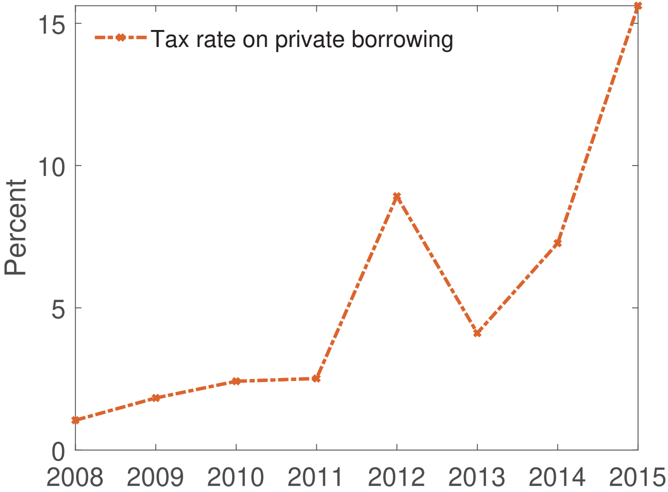
Back

Exercise II: Exogenous financial shock



Back

Exercise II: Optimal tax dynamics



Back Taxes at the ergodic

Why do households overborrow?

Households do not pick B', thereby ignore two GE effects

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1. Effect on next period credit constraint

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$$B' \uparrow \Rightarrow \mathrm{C}^{T\prime} \downarrow \Rightarrow \mathrm{p}^N(C^{T'}) \downarrow$$

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   High debt today increases $\mathbb{P}$ of binding credit constraint tomorrow

2. Effect on the price of public debt

$$B' \uparrow \Rightarrow \mathrm{C}^{T\prime} \downarrow \Rightarrow \mathbb{P} \text{ of } d' \uparrow \Rightarrow \mathrm{Q} \uparrow$$

Why do households overborrow?

Households do not pick B', thereby ignore two GE effects

1. Effect on next period credit constraint

$$B' \uparrow \Rightarrow C^{T\prime} \downarrow \Rightarrow p^N(C^{T\prime}) \downarrow$$

   High debt today increases $\mathbb{P}$ of binding credit constraint tomorrow

2. Effect on the price of public debt

$$B' \uparrow \Rightarrow C^{T\prime} \downarrow \Rightarrow \mathbb{P} \text{ of } d' \uparrow \Rightarrow Q \uparrow$$

High private debt increases the cost of public debt

Planner incorporates these effects on its borrowing decision $\Rightarrow B' > B'^{SP}$

▸ Back ▸ Euler Equations

Why does the government respond with more public debt?

Private and public debt can be complements or substitutes

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Private and public debt can be complements or substitutes

When the credit constraint binds

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Private and public debt can be complements or substitutes

When the credit constraint binds

$$L' \uparrow \Rightarrow \mathrm{T} \uparrow \Rightarrow \mathrm{C}^T \uparrow \Rightarrow \mathrm{p}^N(C^T) \uparrow \Rightarrow \mathrm{B'} \uparrow$$

Why does the government respond with more public debt?

Private and public debt can be complements or substitutes

When the credit constraint binds

$$L' \uparrow \Rightarrow \mathrm{T} \uparrow \Rightarrow \mathrm{C}^T \uparrow \Rightarrow \mathrm{p}^N(C^T) \uparrow \Rightarrow \mathrm{B'} \uparrow$$

Public and private debt are complements

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Public and private debt are complements

When the credit constraint binds does not bind

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When the credit constraint binds

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When the credit constraint binds does not bind

$$L' \uparrow \Rightarrow \mathrm{T} \uparrow \Rightarrow \mathrm{B'} \downarrow$$

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$$L' \uparrow \Rightarrow \mathrm{T}\uparrow \Rightarrow \mathrm{C}^T \uparrow \Rightarrow \mathrm{p}^N(C^T) \uparrow \Rightarrow \mathrm{B'}\uparrow$$

Public and private debt are complements

When the credit constraint binds does not bind

$$L' \uparrow \Rightarrow \mathrm{T}\uparrow \Rightarrow \mathrm{B'}\downarrow$$

Public and private debt are substitutes

Planner faces a biding constraint less frequently $\Rightarrow$L'>L'^{SP}

Back

Mechanism: Baseline relative to planner

Households overborrow because they don't internalize:

1. The probability of a binding credit constraint next period
2. The probability of default next period
3. Future issuances of public debt

The government responds by issuing more public debt because:

1. When the constraint doesn't bind, public and private debt are imperfect substitutes
2. When the constraint binds, public debt relaxes the constraint by depreciating the real exchange rate

This combination increases the frequency of crises and spreads

SP: Planner problem I

$$W^{SP}(s, L, B) = \max_{d \in \{0,1\}} [1-d] W^{SP,R}(s, L, B) + d W^{SP,D}(s, L, B)$$

Where the value of default is:

$$W^{SP,D}(s, L, B) = \max_{B'} u\Big(c\Big) - \phi(y^T) + \beta \mathbb{E}_{s'|s}\Big[W^{SP}\Big(s', 0, B'\Big)\Big]$$

Subject to the resource constraints :

$$c^T = y^T + q(\pi)B' - (1-\pi)B,$$

$$q(\pi)B' \leq \kappa \left[\frac{(1-\omega)}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} + y^T\right],$$

$$c^N = y^N$$

SP: Planner problem II

And the value of repayment is:

$$W^{SP,R}(s, L, B) = \max_{L'>0, B'} u\left(c\right) + \beta \mathbb{E}_{s'|s}\left[W^{SP}\left(s', L, B'\right)\right]$$

Subject to the resource constraints:

$$c^T = y^T + q(\pi)B' - (1-\pi)B + Q\left(s, L', B'\right)\left[L' - (1-\delta)L\right] - \delta L$$

$$q(\pi)B' \leq \kappa\left[\frac{(1-\omega)}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} + y^T\right],$$

Back

$$c^N = y^N$$

SP: Equilibrium pricing

With zero profits: :

$$q(s) = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[1-\pi'\right]$$

With zero profits:

$$Q(s,L',B') = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[\left\{1 - d^{SP}(s',L',B')\right\}\times\right.$$

$$\left.\left\{\delta + (1-\delta)Q\Big(s',L'',B''\Big)\right\}\right]$$

Where,

$$B'' = \mathcal{B}'^{SP}(s',L',B')$$

$$L'' = \mathcal{L}'^{SP}(s',L',B')$$

Foreign Lenders Problem Back

Government problem with taxes I

Taking the household policy functions as given the government solves:

$$W^{\tau}(s, L, B) = \max_{d \in \{0,1\}} [1 - d] W^{\tau,R}(s, L, B) + d W^{\tau,D}(s, L, B)$$

In default:

$$W^{\tau,D}\left(s, L, B\right) = \max_{\tau} u\left(\mathcal{C}^{T,N}_{L=0,\tau}\right) - \phi(y^T) + \beta \mathbb{E}_{s'|s}\left[W^{\tau}\left(s', 0, \mathcal{B}'_{L=0,\tau}\right)\right]$$

subject to

$$\mathcal{C}^{N}_{L=0,\tau} = y^N$$

$$\mathcal{C}^{T}_{L=0,\tau} = y^T + (1 - \tau) q(\pi) \mathcal{B}'_{L=0,\tau} - (1 - \pi) B + \underbrace{\tau q(\pi) \mathcal{B}'_{L=0,\tau}}_{T(s, 0, B, \tau)}$$

Government problem with taxes II

In repayment, the government solves:

$$W^R(s,L,B) = \max_{L'>0,\tau} u\big(\mathcal{C}(s,L,B,\tau)\big)+\beta\mathbb{E}_{s'|s}\Big[W\Big(s',L,\mathcal{B}'(s,L,B,\tau)\Big)\Big]$$

Subject to:

$$\mathcal{C}^{\mathcal{N}} = y^N$$

$$\mathcal{C}^{\mathcal{T}} = y^T + (1-\tau)q(\pi)\mathcal{B}' - (1-\pi)B +$$
$$\underbrace{Q\Big(s,L,B,\tau\Big)\Big[L' - (1-\delta)L\Big] - \delta L + \tau q(\pi)\mathcal{B}'}_{T(s,L,B,\tau)}$$

Back

Baseline: Recursive government problem, default

In default:

$$W^D(s,B) = u(\mathcal{C}^{BR}(\mathcal{C}(s,0,B;L'=0,d=1)) - \phi(y^T) + \beta\mathbb{E}_{s'|s}\left[W\left(s',0,\mathcal{B}'^{BR}(s,L'=0,d=1)\right)\right]$$

Subject to

$$\mathcal{C}^{N,BR}(s,0,B;L'=0,d=1) = y^N$$

$$\mathcal{C}^{T,BR}(s,0,B;L'=0,d=1) = y^T + q(\pi)\mathcal{B}'^{BR}(s,0,B;L'=0,d=1) - (1-\pi)B$$

Back

Baseline: Recursive government problem, repayment

In repayment, the government solves:

$$W^R(s, L, B) = \max_{L'>0} u\big(\mathcal{C}(s, L, B; L', d)\big) + \beta \mathbb{E}_{s'|s}\left[W\left(s', L', \mathcal{B}'^{BR}(s, L, B; L', d)\right)\right]$$

Subject to

$$\mathcal{C}^{N,BR}(s, L, B; L', d) = y^N$$

$$\mathcal{C}^{T,BR}(s, L, B; L', d) = y^T + q(\pi)\mathcal{B}'^{BR}(s, L, B; L', d) - (1-\pi)B$$

$$+ \underbrace{Q^{BR}\left(s, L', \mathcal{B}'^{BR}(s, L, B; L', d)\right)\left[L' - (1-\delta)L\right] - \delta L}_{T(s,L,B;L',d)}$$

Solution method Back

Baseline: Foreign lenders pricing in equilibrium

Price of private debt is:

$$q(\pi) = \frac{\mathbb{E}_{s'|s}\left[1 - \pi'\right]}{1 + r}$$

Price of public debt is:

$$Q(s, L', B') = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[\left\{1 - d'\right\} \times \left(\delta + (1-\delta)Q'(s', L'', B'')\right)\right]$$

Where,

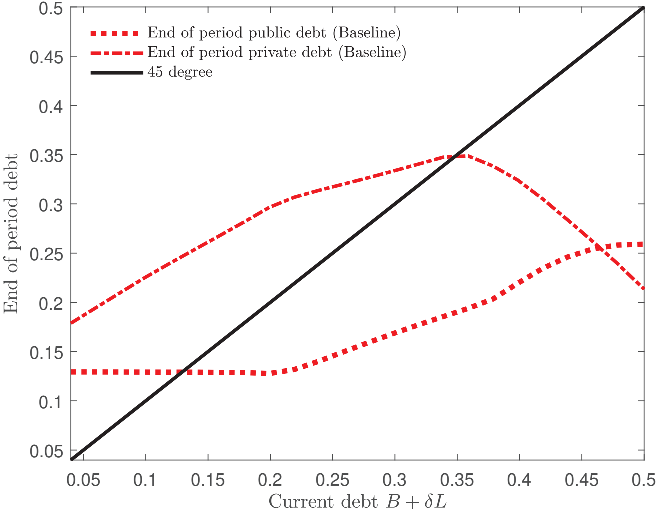
$$B'' = \mathcal{B}'(s', L', B', d', L'')$$

$$L'' = \mathcal{L}'(s', L', B')$$

$$d' = \boldsymbol{d}'(s', L', B')$$

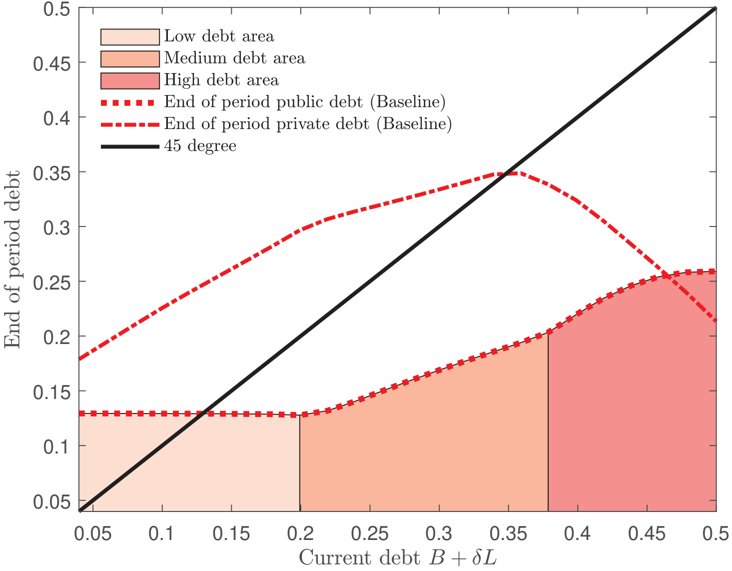
Foreign Lenders Problem Back

Public debt $\mathcal{L}'$ as a function of current debt



Back

Public debt $\mathcal{L}'$ as a function of current debt



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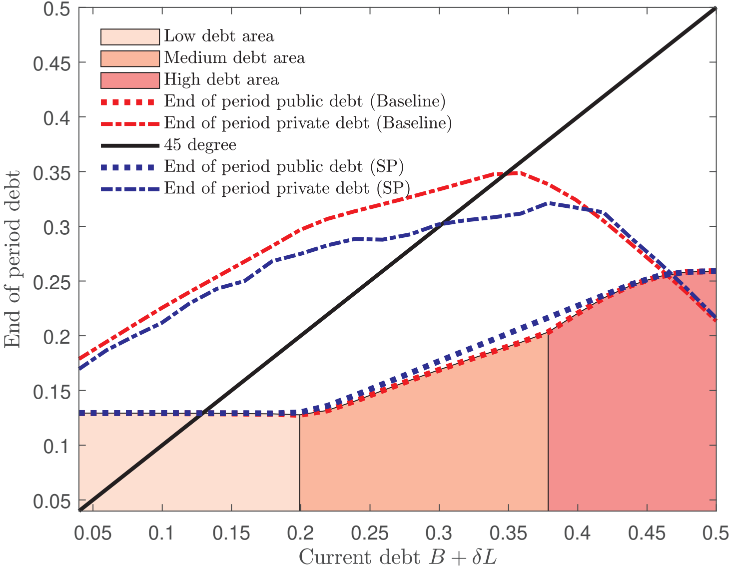


Back

Use particle filter to get κ_t

Write the problem in terms of state vectors $S_t = [B_{t+1}, L_{t+1}, y_t^T, \kappa_t, \pi_t]$.
And a vector of observable data
$Y_t = [PrivateD_t, PublicD_t, Output_t, Nonperforming_t, Spread_t]$

$$Y_t = g(S_t) + \eta_t \quad \text{with } \eta \sim N(0, \Sigma)$$

$$S_t = f(S_{t-1}, \epsilon_t) \quad \text{with } \epsilon_t \sim N(0, I)$$

Denote by $p(S_t|Y^t)$ the conditional distribution of the state vector given observations up to t. Approximate using $\{S_t^i, w_t^i\}_{i=1}^N$:

$$\frac{1}{N}\sum_{i=1}^{N} f(S_t^i) w_t^i \to \mathbb{E}[f(S_t)|Y^t]$$

Use particle filter algorithm to get w_t^i. Run counterfactual experiment with these shocks.

Simplifications

No measurement error in $\{y_t^T, \pi_t, L_{t+1}, \}$ (taken directly from the data).
Now $Y_t = [PrivateD_t, Spread_t]$

$$PrivateD_t = \frac{b_{t+1}}{p_t^N y_t^N + y_t^T} + \eta_t^{11} \quad \text{with } \eta \sim N(0, \sigma^\eta)$$

$$Spread_t = \frac{\delta - \delta Q_t}{Q_t} - r + \eta_t^{22} \quad \text{with } \eta \sim N(0, \sigma^\eta)$$

σ^η set to 1% sample variance of $[PrivateD_t, Spread_t]_{2008}^{2015}$

We use Kitagawa 1996 algorithm to calculate $p(Y_t|S_t)$

Algorithm for the particle filter (Kitagawa 1996)

1. Set $t = 1$. Set $w_i = 1/N$ for $i = 1...N$. Draw $\{S_0^i\}_{i=0}^N$ from ergodic
2. For each draw compute $S_{t|t-1}^i$ using the model (For us $b'(S_{t-1}^i)$)
3. Construct particle weights as:

$$\tilde{w}_t^i = p(Y_t | S_{t|t-1}^i) w_{t-1}^i$$

4. Scale weights $\tilde{w}_t^i$ so they add up to 1, now called w_t^i . Sample N values with replacement from $\{S_{t|t-1}^i, w_t^i\}$. Name the draws S_t^i. Reset $w_t^i = 1/N$ for all i

Use the particle filter

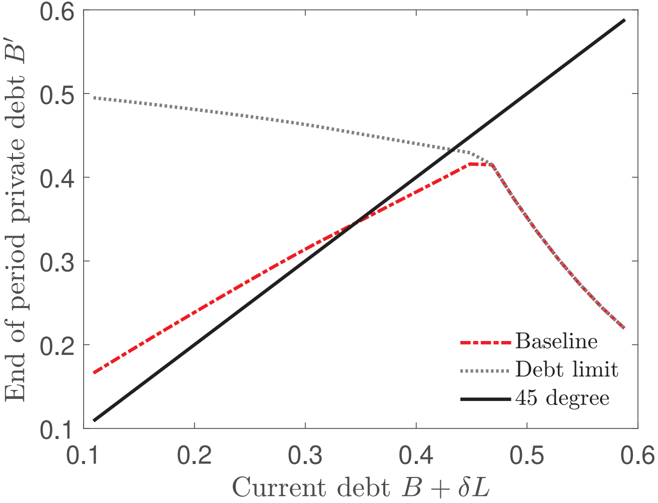
For each $t = 2008..2015$. We can compute the likely path of financial shocks

$$\hat{\kappa}_t^{filtered} = \sum_i^N w_t^i \kappa_t(S_t^i)$$

We use this series of financial shock to compute the dynamics

Back

Private debt $\mathcal{B}'$ as a function of current debt



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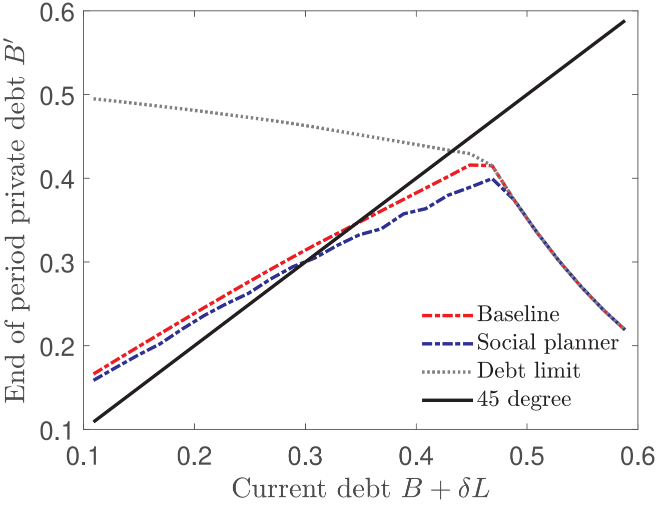
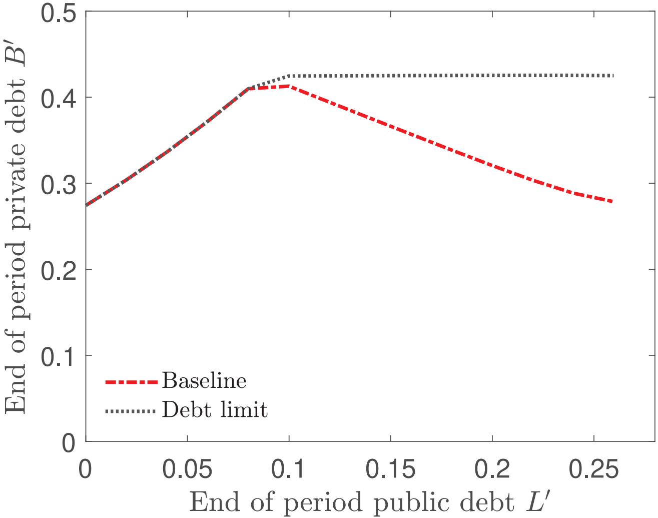
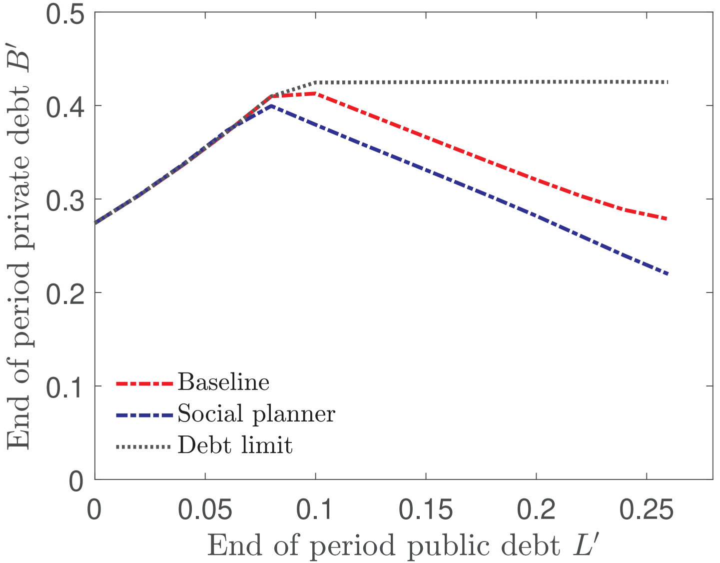


Private debt $\mathcal{B}'$ as a function of public debt $\mathcal{L}'$

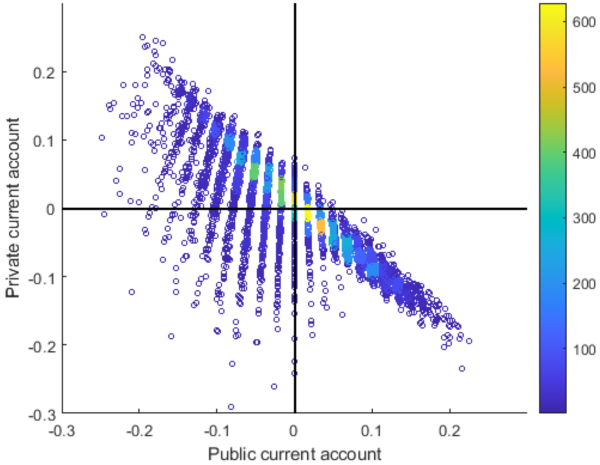


Private debt $\mathcal{B}'$ as a function of public debt $\mathcal{L}'$

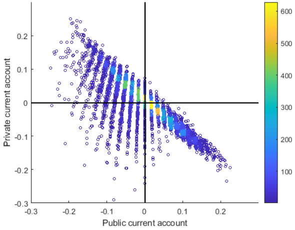


Public debt L' | Spreads $Q(B)$ | Spreads $Q(L')$ | Back to calibration

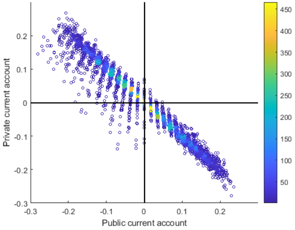
Current accounts: $B' - B$ and $L' - L$



Current accounts: $B' - B$ and $L' - L$



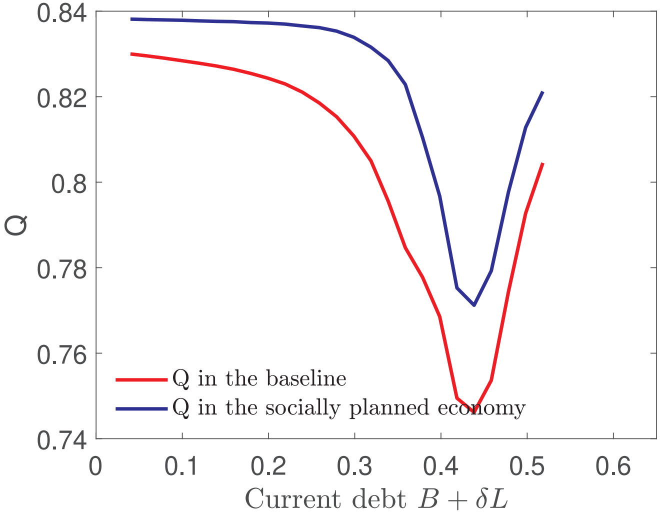
Baseline



Social planner

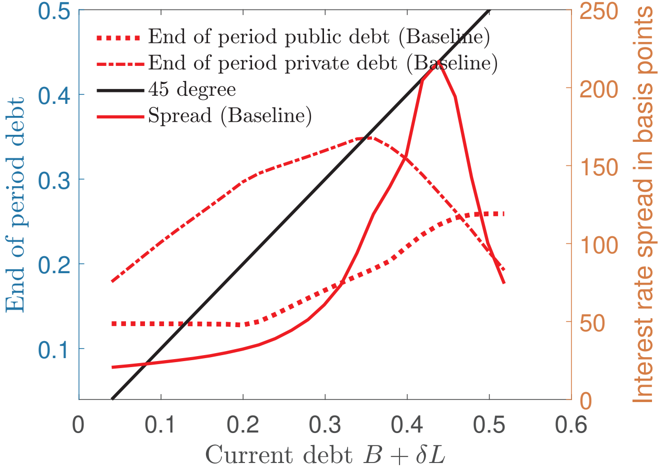
Back to calibration

Interest rate spread schedule



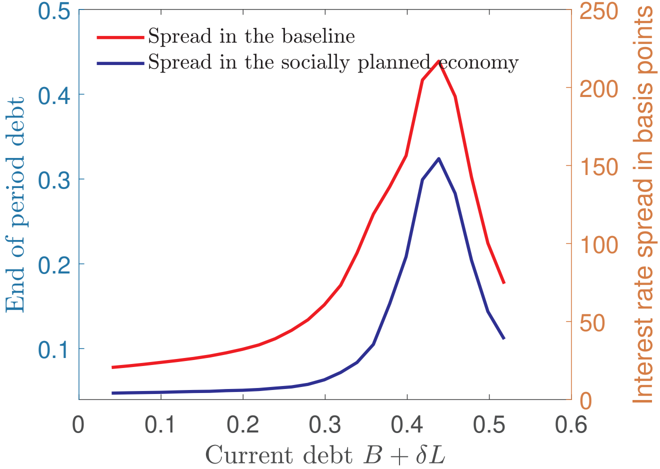
Back to calibration Spreads relative to policies

Interest rate spread on public debt



Back to calibration

Interest rate spread on public debt



Back to calibration

Parameters estimated outside of the model

Parameter	Value	Source/Transition
Risk aversion	$\sigma = 2.0$	Standard value
Elasticity of substitution	$1/(1+\eta) = .83$	Standard value
Decay	$\delta = .14$	Avg. maturity 99-11 6 years
Risk free rate	$r = .027$	GER 1Year T-bill 99-11
Weight tradables in CES	$\omega = .39$	Tradable share 95-18
Income shock	$\rho = .75$	Estimated from
$\log(y^T_{t+1}) = \rho \log(y^T_t) + \varepsilon_t$	$\sigma^y = .010$	GDP 95-18
Private default shocks	$\bar{\pi} = .021$	Estimated from
$\log(\pi_{t+1}) - (1-\rho^{pi}) \log(\bar{\pi})$	$\rho^\pi = .82$	Nonperforming loans
$= \rho^\pi \log(\pi_t) + \varepsilon^\pi_t$	$\sigma^\pi = .33$	95-18

Back

Baseline: Household's problem

Let $s = (y^T, \kappa, \pi)$. Households solve:

Back to households problem in equilibrium

$$V^{BR}(s, L, B, b, L', d) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}[V^{BR}(s', L', B', b', L'', d')]$$

subject to

$$\underbrace{c^T + p^N c^N}_{\textbf{Consumption}} + (1-\pi) \underbrace{b}_{\textbf{Current private debt}} = \underbrace{y^T + p^N y^N}_{\textbf{Endowment}} + q(\pi) \underbrace{b'}_{\textbf{End-of-period private debt}} + \underbrace{T}_{\textbf{Transfers}},$$

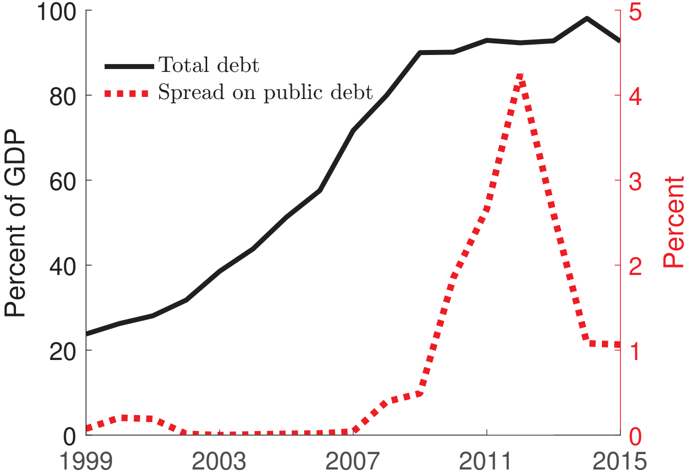
$$\underbrace{q(\pi) b'}_{\textbf{Market value of new debt}} \leq \kappa \underbrace{(y^T + p^N y^N)}_{\textbf{Market value of current income}}$$

Taking aggregate prices, laws of motion and public policies as given

$$\underbrace{d' = \boldsymbol{d}^{BR}(s, L, B, L', d)}_{\textbf{Default}}$$

$$\underbrace{p^N = p^{BR}(s, L, B, L', d')}_{\textbf{Real exchange rate}} ; \underbrace{B' = \mathcal{B}^{BR\prime}(s, L, B, L', d')}_{\textbf{Aggregate private debt}} ; \underbrace{L'' = \mathcal{L}^{BR\prime}(s, L, B, L', d)}_{\textbf{Aggregate public debt}} ; \underbrace{T = \mathcal{T}^{BR}(s, L, B, L', d)}_{\textbf{Transfers}}$$

Total debt and spreads: Composition matters

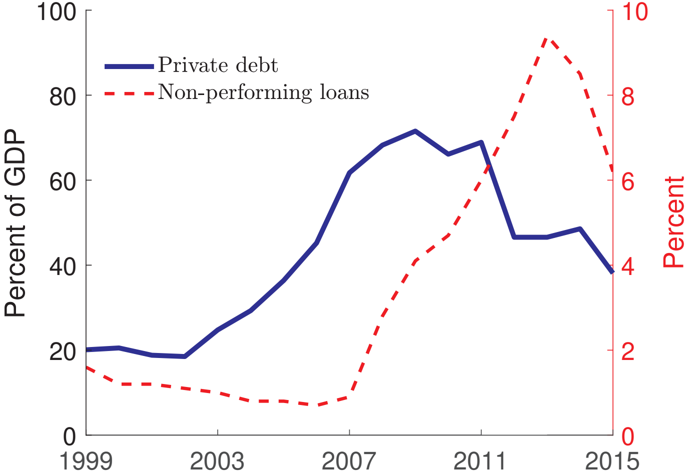


Net foreign liabilities and difference between Spanish and German 6-year T-bonds

Source: Bank of Spain, Eurostat, and Bloomberg

Back

Private origins of the crisis: Nonperforming loans

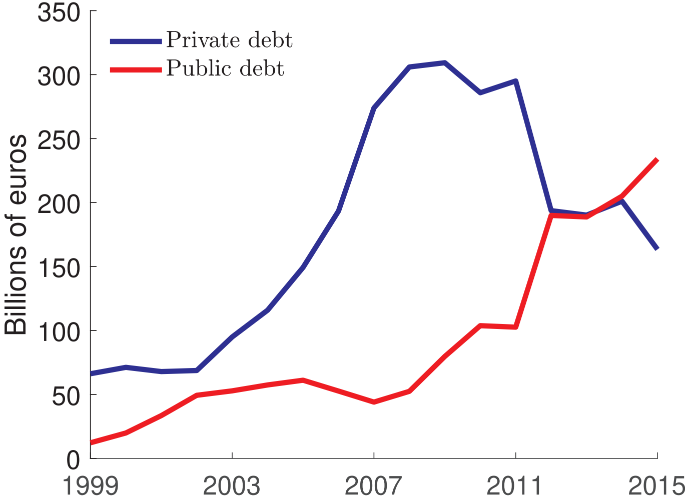


Private net foreign liabilities and non-performing loans as a percent of gross loans

Source: Bank of Spain, Eurostat, and Bloomberg

Back

Motivation: The road to the 2012 Spanish debt crisis



Private and public net foreign liabilities

Source: Bank of Spain, Eurostat, and Bloomberg Back Private origins of the crisis

Baseline: Equilibrium real exchange rate

The utility function is CRRA on the CES composite

$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega (c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right]^{-\frac{1-\sigma}{\eta}}$$

Euler equation Back

Baseline: Equilibrium real exchange rate

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$$u(c^T, c^N) = \frac{1}{1-\sigma} \times \left[\omega (c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}\right]^{-\frac{1-\sigma}{\eta}}$$

Elasticity of substitution between tradables and nontradables is $\frac{1}{1+\eta}$

Share of tradables in consumption is ω

Risk aversion coefficient is σ

Euler equation Back

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Elasticity of substitution between tradables and nontradables is $\frac{1}{1+\eta}$

Share of tradables in consumption is ω

Risk aversion coefficient is σ

Households' intratemporal optimality condition in equilibrium,

$$p^N(S_G) = \frac{1-\omega}{\omega} \left(\frac{\mathcal{C}^T(S_G)}{y^N}\right)^{\eta+1}$$

Euler equation Back

Next Steps

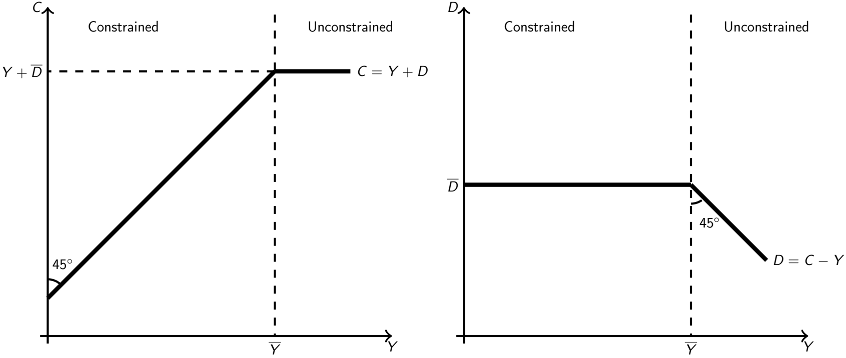
1. Cross country evidence and regression analysis

2. Solve a simple rule for macroprudential policies under sovereign risk of the form:

$$\tau_t^b = \gamma_0 + \gamma_1 B_t + \gamma_2 L_t + \gamma_3 y_t + \gamma_4 \pi_t + \gamma_5 \kappa_t$$

Start from the OLS regression coefficients at the ergodic and search on a grid around those parameters

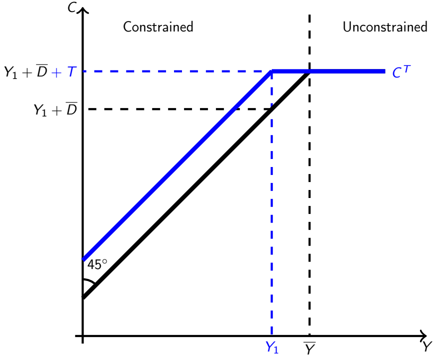
Why bail and when bail out?

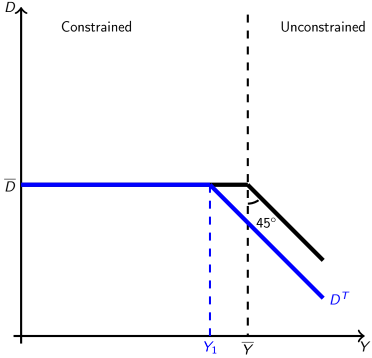


Exogenous credit constraint

$$\underbrace{C}_{\textit{Consumption}} = \underbrace{Y}_{\textit{Income}} + \underbrace{D}_{\textit{Debt}} \text{ and } D \leq \underbrace{\overline{D}}_{\textit{Debt limit}}$$

Exogenous constraint: Fiscal bailouts

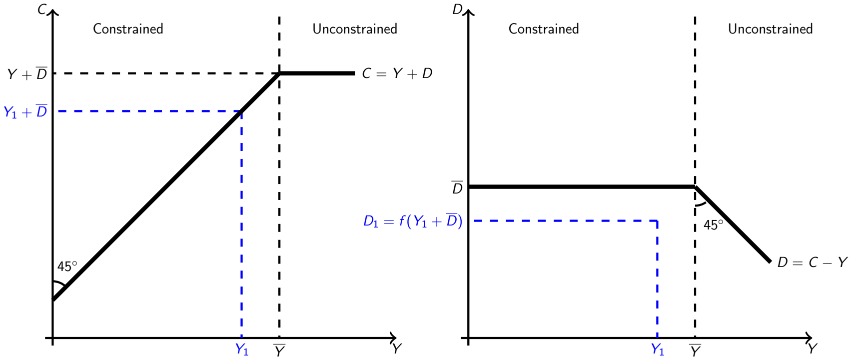




$$C^T = Y + D^T + T \text{ and } D^T \leq \overline{D}$$

$$C^T = C^\star + T$$

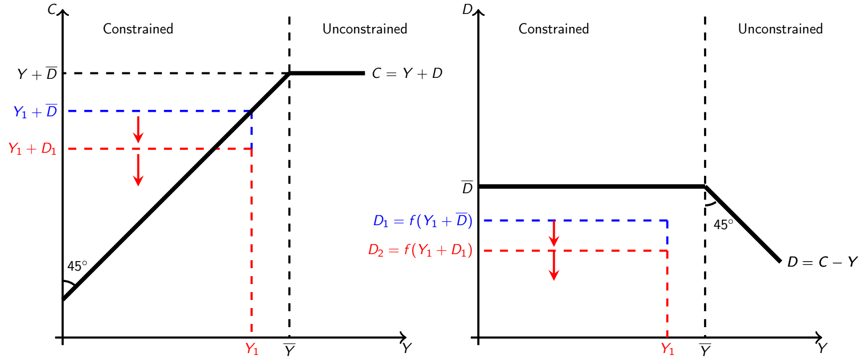
Endogenous credit constraint



$$C = Y + D \text{ and } D \leq f(C), \text{ with } 0 < f' < 1$$

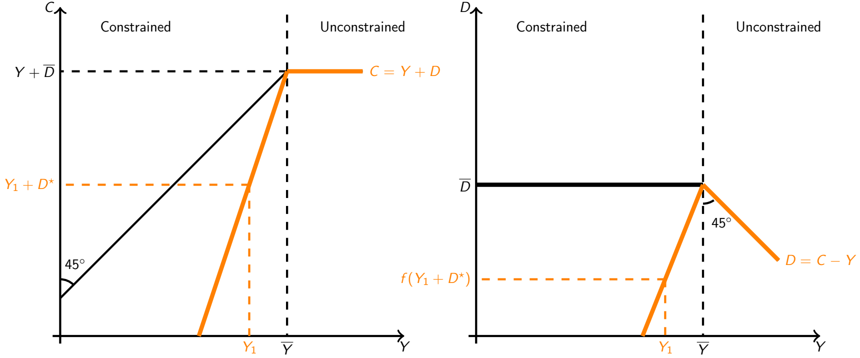
Assume $f(\overline{Y} + \overline{D}) = \overline{D}$

Fisherian debt deflation



$$C = Y + D \text{ and } D \leq f(C), \text{ with } 0 < f' < 1$$

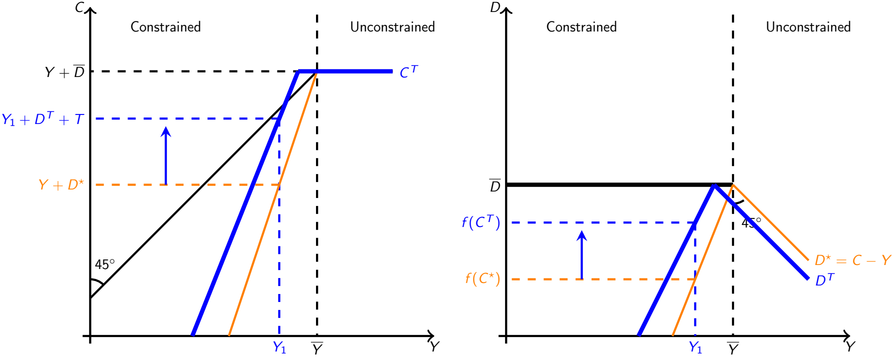
Endogenous credit constraint: *Laissez-faire*



$C^\star = Y_1 + D^\star$ and $D^\star \leq f(C^\star)$, such that

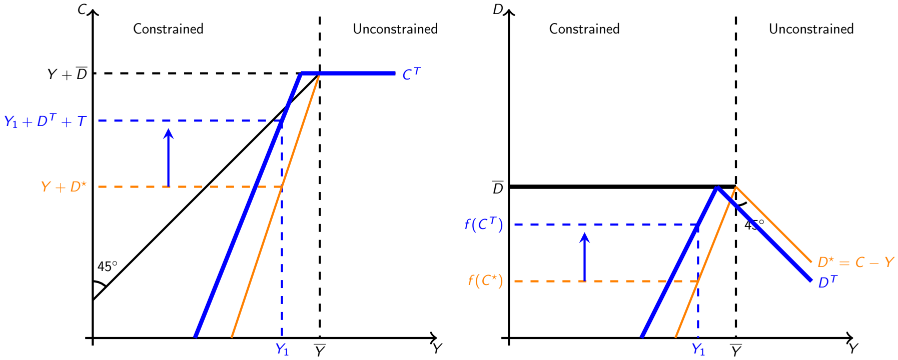
$$D^\star = f(Y_1 + D^\star)$$

Endogenous constraint: Fiscal bailouts



$$C^T = Y + D^T + T \text{ and } D \leq f(C), \text{ such that } D^T = f(Y_1 + D^T + T)$$

Endogenous constraint: Fiscal bailouts



$$C^T = Y + D^T + T \text{ and } D \leq f(C), \text{ such that } D^T = f(Y_1 + D^T + T)$$

Financial amplification $D^T > D^\star$ and $C^T > C^\star + T$

Back

Main mechanism in a simple three period model

Period 1

$$C_1 = \bar{Y} + D_1$$

$$D_1 \leq f(C_1)$$

Period 2

$$C_2 = Y_2 + D_2 - RD_1 + QT$$

$$D_2 \leq f(C_2)$$

$$T \in \{0, \bar{T}\}$$

$$Y_2 \sim \Psi(\cdot)$$

Period 3

$u(C_3^R)$ or $u(C_3^D) - \phi$

Repayment

$$C_3^R = \bar{Y} - RD_2 - T$$

Default

$$C_3^D = \bar{Y} - RD_2$$

$$\phi \sim \Phi(\cdot)$$

Main mechanism in a simple three period model

Period 1

$$C_1 = \bar{Y} + D_1$$

$$D_1 \leq f(C_1)$$

Period 2

$$C_2 = Y_2 + D_2 - RD_1 + QT$$

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$$T \in \{0, \bar{T}\}$$

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Period 3

$u(C_3^R)$ or $u(C_3^D) - \phi$

Repayment

$$C_3^R = \bar{Y} - RD_2 - T$$

Default

$$C_3^D = \bar{Y} - RD_2$$

$$\phi \sim \Phi(\cdot)$$

Social planner problem

Period 1

Planner

$$\max_{C_1} u(C_1) + \beta\mathbb{E}[u_2 + \beta u_3]$$

$$C_1 = \bar{Y} + D_1$$

$$D_1 \leq f(C_1)$$

Period 2

Planner

$$\max_{C_2, T} u(C_2) + \beta\mathbb{E}[u_3]$$

$$C_2 = Y_2 + D_2 - RD_1 + QT$$

$$D_2 \leq f(C_2)$$

$$T \in \{0, \bar{T}\}$$

$$Y_2 \sim \Psi(\cdot)$$

Period 3

Planner

$$\max\{u(C_3^R); u(C_3^D) - \phi\}$$

Repayment

$$C_3^R = \bar{Y} - RD_2 - T$$

Default

$$C_3^D = \bar{Y} - RD_2$$

$$\phi \sim \Phi(\cdot)$$

Social planner solution with high income

Period 1	Period 2	Period 3
Planner	*Planner*	*Planner*
$C_1^{SP} = \bar{Y} + D_1^{SP}$	$C_2^{SP} = Y_2^H + D_2^{SP} - RD_1^{SP}$	**Repayment**
$D_1^{SP} \leq f(C_1^{SP})$	$D_2^{SP} < f(C_2^{SP})$	$C_3^R = \bar{Y} - RD_2^{SP} - T^{SP}$
	$T^{SP} = 0$	**Default**
		$C_3^D = \bar{Y} - RD_2^{SP}$
		$\phi \sim \Phi(\cdot)$

Social planner solution with low income

Period 1

Planner

$C_1^{SP} = \bar{Y} + D_1^{SP}$

$D_1^{SP} \leq f(C_1^{SP})$

Period 2

Planner

$C_2^{SP} = Y_2^L + D_2^{SP} - RD_1^{SP} + Q\bar{T}$

$D_2^{SP} = f(C_2^{SP})$

$T^{SP} = \bar{T}$

Period 3

Planner

Repayment

$C_3^R = \bar{Y} - RD_2^{SP} - T^{SP}$

Default

$C_3^D = \bar{Y} - RD_2^{SP}$

$\phi \sim \Phi(\cdot)$

Decentralized problem

Period 1

Households

$$\max_{c_1} u(c_1) + \beta \mathbb{E}[u_2 + \beta u_3]$$

$$c_1 = \bar{Y} + d_1$$

$$d_1 \leq f(C_1)$$

Period 2

Households

$$\max_{c_2} u(c_2) + \beta \mathbb{E}[u_3]$$

$$c_2 = Y_2 + d_2 - Rd_1 + QT$$

$$d_2 \leq f(C_2)$$

Government

$$\max_{T} u(C_2) + \beta \mathbb{E}[u_3]$$

$$T \in \{0, \bar{T}\}$$

$$Y_2 \sim \Psi(\cdot)$$

Period 3

Government

$$\max\{u(C_3^R); u(C_3^D) - \phi\}$$

Repayment

$$C_3^R = \bar{Y} - RD_2 - T$$

Default

$$C_3^D = \bar{Y} - RD_2$$

$$\phi \sim \Phi(\cdot)$$

Decentralized solution with high income

Period 1

Solution

$$C_1^B = \bar{Y} + D_1^B$$

$$D_1^B \leq f(C_1^B)$$

$$D_1^B \geq D_1^{SP}$$

Period 2

Solution

$$C_2^B = Y_2^H + D_2^B - R D_1^B + Q \bar{T}$$

$$D_2^B = f(C_2^B)$$

$$T^B = \bar{T}$$

Period 3

Solution

Repayment

$$C_3^R = \bar{Y} - RD_2^B - T^B$$

Default

$$C_3^D = \bar{Y} - RD_2^B$$

$$\phi \sim \Phi(\cdot)$$

Decentralized solution with high income

Period 1

Solution

$$C_1^B = \bar{Y} + D_1^B$$

$$D_1^B \leq f(C_1^B)$$

$$D_1^B \geq D_1^{SP}$$

Overborrowing

Period 2

Solution

$$C_2^B = Y_2^H + D_2^B - R D_1^B + Q \bar{T}$$

$$D_2^B = f(C_2^B)$$

$$T^B = \bar{T}$$

Crises

Period 3

Solution

Repayment

$$C_3^R = \bar{Y} - RD_2^B - T^B$$

Default

$$C_3^D = \bar{Y} - RD_2^B$$

$$\phi \sim \Phi(\cdot)$$

Back