Private Overborrowing and International Reserves *

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October 2018

Abstract

In the last two decades there has been an increase in the issuance of international liabilities by the private sector as a share of GDP in emerging economies. This fact was accompanied by a decrease in the issuance of public debt and an increase of international reserves as a share of GDP. This paper explores one mechanism that could explain these facts using a dynamic stochastic general equilibrium model with financial constraints. In the presence of an occasionally binding credit constraint, which depends on general equilibrium prices, the decentralized equilibrium with no government intervention will feature a level of private borrowing above what is constrained efficient and a lower welfare. A benevolent government can decentralize the constrained efficient consumption allocations by accumulating international reserves. This intervention, however, pushes the economy to an even higher amount of international private debt. The model is calibrated to the economies of Brazil and Mexico. It explains on average 26% of the observed reserves and 70% of the observed foreign asset position in the past two decades.

Keywords: Financial Frictions, International Reserves, Overborrowing, Constrained Efficiency

JEL Codes: E13, E32 , E44 ,F34,F41

*Department of Economics, University of Minnesota. This paper is based on my third year paper. I am indebted to my main advisors Manuel Amador, Javier Bianchi, Juliana Salomao, and Tim Kehoe. I am also grateful to Monica Tran Xuan, Camilo Alvarez, and Stelios Fourakis, and participantas at several seminars and conferences.

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1 Introduction

Until the 1990s, the typical external position of an emerging market economy was characterized by a low level of international reserves, and few international issuance of private debt. Since then, however there has been a surge in the issuance of private international liabilities and an accumulation of international reserves. While these two issues have been studied independently in the literature this article proposes a quantitative exercise to analyze them jointly.

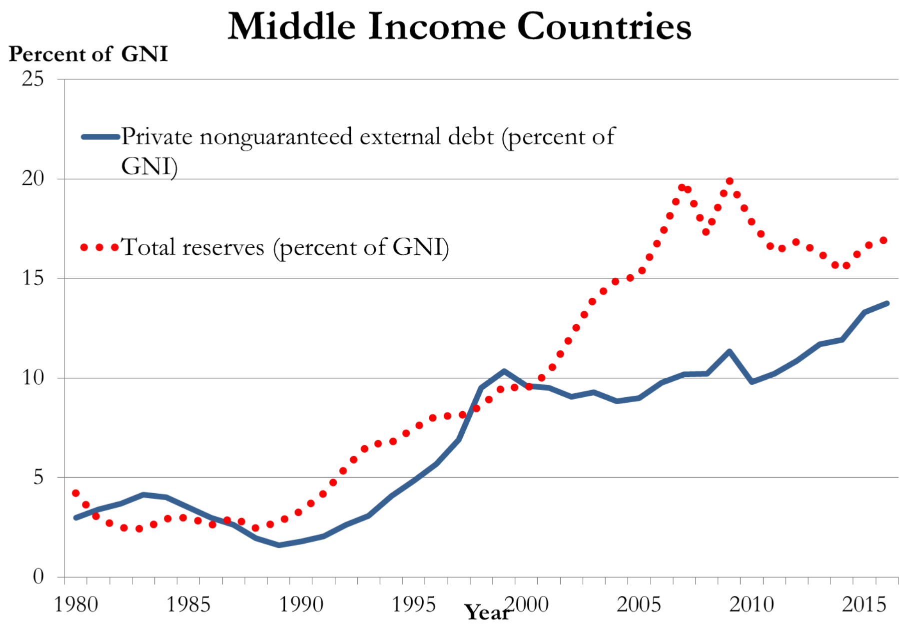
The increase in private liabilities has raised concerns about a private sector that is more dependent on international lending, especially in the aftermath of the 2008 financial crisis and more recently during the Turkish currency crisis of 2018 when these international flows reversed course. The argument frequently encountered is that excessive borrowing by the private sector leaves emerging economies vulnerable to a sudden stop, similar to the ones experienced by the public sector in the 1980s and 1990s. The first aspect of this debate is to understand how optimal borrowing decision at the individual level in the private sector can generate an aggregate level of debt that is not socially optimal, and what are the welfare consequences of this distortion. Models of "overborrowing", have been studied by Bianchi (2011), Benigno et al. (2013) and Korinek (2018) among others. The common idea of this literature is that in the presence of an occasionally biding credit constraint that depends on general equilibrium prices that private agents take as given, the aggregate level of debt will be above what a social planner, who is also subject to the same credit constraint but internalizes the price effects would chose. In addition, these authors have also shown how a government could decentralize the constrained efficient allocations using borrowing taxes or tightening of the credit constraint margins.

Simultaneously, the increase in international reserves has raised a different set of concerns. The most commonly encountered is the costly nature of this accumulation. Indeed, most emerging economies keep the vast majority of their international reserves in low return assets while at the same time borrow internationally at a premium over the risk free rate. Several explanations have been proposed for the raise of international reserves. For instance, Hur and Kondo (2016) explore the implications of a model in which reserves decrease the probability of a sudden stop of debt to the public sector. More recently, Bianchi et al. (2018) explore how reserves can be used as costly policy tool to protect against rollover risk of public debt. This paper presents a different rational for reserves by showing how they can correct the welfare distortions caused by excessive issuance of liabilities by the private sector instead of the traditional focus on public debt. This is significantly more relevant now than it was in the past since publicly-guaranteed international debt in the average middle income country has decreased from 23.4% of gross national income (GNI) in 1986 to 9.3% in 2016, while the share of privately guaranteed debt has increased from 2.6% in 1996 to 9.6% of GNI in 2016[1].

Section 2 present some empirical facts about the evolution of international liabilities and reserves in emerging economies using the World Bank International debt statistics data set. The focus will be on both the average middle income country and the two Latin American countries to which the model will be calibrated later. In Section 3, the paper will present a dynamic stochastic general equilibrium (DSGE) model that adds international reserves to an environment similar to the one studied in Mendoza (2002), Bianchi (2011), and Korinek (2018). In this environment, the article will demonstrate how a benevolent government can implement the constrained efficient outcome with only access to lump-sum taxes and international reserves. Finally Section 4 will calibrate the model to Brazil and Mexico and will show quantitatively how much of the increase of in reserves and private foreign debt can be explained by this mechanism.

[1]These figures are calculated for the average gross domestic product (GDP) weighted middle income country as defined by the World Bank, and contrary to Figure 1 they include China

2 Empirical Facts



Source: World Bank, International Debt Statistics Statistics (2015)

Figure 1: International reserves, and private debt, middle income countries excluding China

This section will first document how private debt and international reserves have evolved in the average GDP-weighted middle income countries[2]. Nevertheless, Figure 1 excludes China from the sample[3]. This is because of the large size of its economy today and because the private sector access to international capital markets is fundamentally different in China than in most other middle income countries. It is apparent in this figure that until the middle 1990s, both international reserves and private debt were low, below 5% of total GNI. Conversely in 2016, the former stands at 17% of GNI while the latter represents 13.8% of GNI. This sharp raise of private debt contrasts with the tepid increase of international public debt in the sampled countries over the same time period. According to the International Debt Statistics database, publicly guaranteed debt (PGD) increased from 12.1% in 1980 to 15.6% of GNI in 2016.

[2]The same trends are visible in an equally weighted average

[3]Including China in the sample does not change the different trends observed.

	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves
Private Debt	0.124***	0.118***		0.147***	0.148***	
	(9.70)	(9.41)		(11.34)	(11.65)	
Publicly Guaranteed Debt (PGD)		-0.264***	-0.190***		-0.243***	-0.173***
		(-10.31)	(-10.35)		(-9.54)	(-9.28)
Constant	-2.081***	-2.458***	-2.743***	-2.005***	-2.329***	-2.724***
	(-43.32)	(-41.30)	(-100.73)	(-43.56)	(-41.22)	(-107.38)
Observations	2396	2394	4470	2396	2394	4470
Countries	124	124	124	124	124	124
Pooled/Panel FE	Pooled	Pooled	Pooled	FE	FE	FE

	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves	Foreign Reserves
Private Debt	0.268***	0.262***		0.219***	0.214***	
	(17.47)	(17.01)		(14.33)	(14.14)	
Publicly Guaranteed Debt (PGD)		-0.0937**	-0.116***		-0.173***	-0.171***
		(-3.05)	(-3.53)		(-5.22)	(-5.04)
Constant	-1.685***	-1.842***	-2.676***	-1.830***	-2.102***	-2.758***
	(-33.01)	(-25.47)	(-48.57)	(-36.88)	(-29.35)	(-50.56)
Observations	1351	1351	1395	1351	1351	1395
Countries	35	35	35	35	35	35
Pooled/Panel FE	Pooled	Pooled	Pooled	FE	FE	FE

t statistics in parentheses

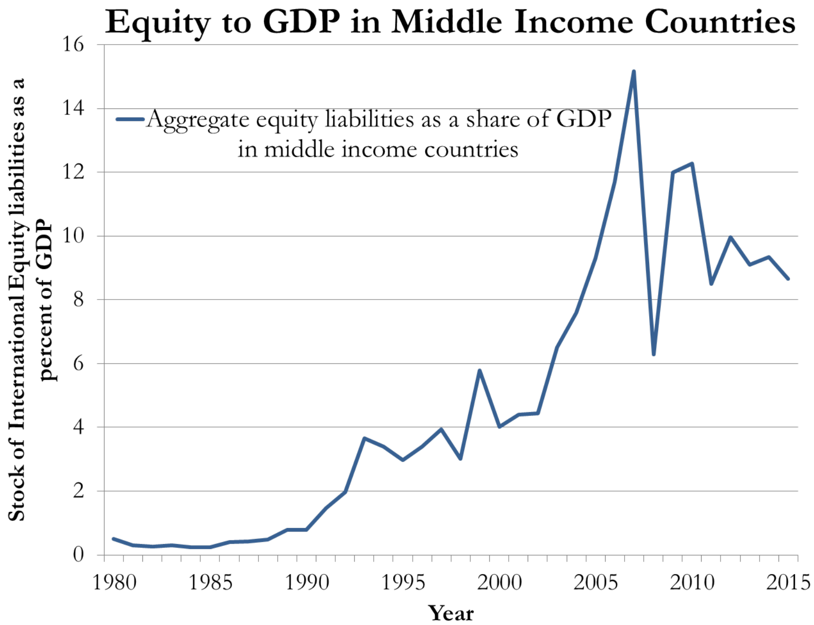
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: World Bank, International Debt Statistics

Table 1: International reserves, and private debt, middle income countries excluding China

To further document the relations between international reserves, publicly guaranteed debt, and private non-publicly guaranteed debt, a battery of regressions are calculated in Table 1. The use of publicly guaranteed debt instead of just central government public debt is motivated by the fact that government-owned companies are still important issuers of international debt in some countries in the sample. Two country samples are used in the regressions: a complete list of the 124 middle income countries, as defined by the World Bank, in the database, and a sub-sample of 35 middle income countries for which data is available for at least 20 years and had positive levels of private debt in 1995 and 2015 [4]. The regressions are run on the logs of foreign reserves, private debt, and PGD, as a share of GNI.

[4] The countries in the sub-sample are: Argentina, Bolivia, Brazil, Bulgaria, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Honduras, India, Indonesia, Jamaica, Kazakhstan, Macedonia, Malaysia, Mauritius, Mexico, Moldova, Morocco, Papua New Guinea, Paraguay, Peru, Philippines, Romania, Solomon Islands, South Africa, Sri Lanka, Thailand, Turkey, Ukraine, Uzbekistan, Venezuela, and Zimbabwe

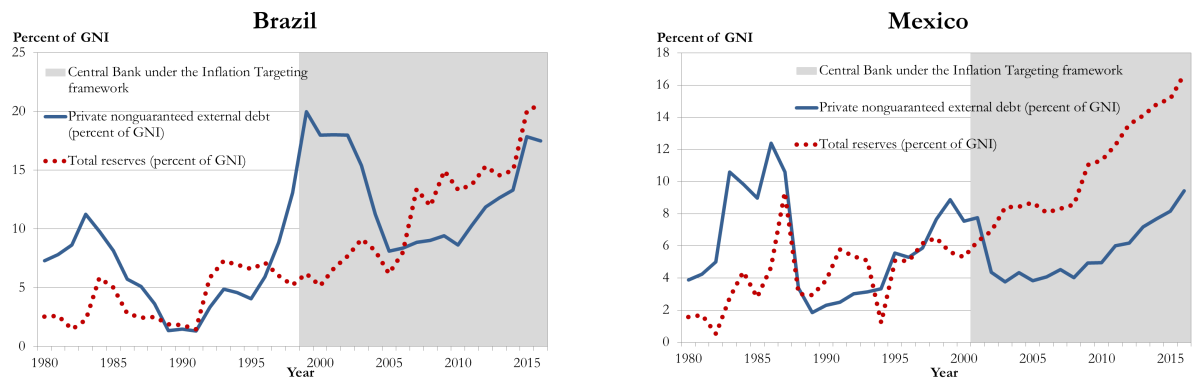


Source: Lane and Milesi-Ferretti (2018)

Figure 2: Equity, middle income countries excluding China

Furthermore, the results also hold in levels. The first three columns of each table represent pooled OLS regressions while the last three are fixed effects regression. The conclusions are consistent and significant across tables in which larger reserves are associated with higher levels of private debt and to a lesser extent with lower levels of PGD.

International private debt is however, only one of the tools that private agents have at their disposal to issue liabilities internationally. The other main set of assets is equity. Using a variety of national and international sources, Lane and Milesi-Ferretti (2018) have constructed a panel data set of the aggregate external position for most countries decomposed by type of asset. Figure 2 shows the aggregate equity liabilities as a share of GDP in middle income countries. Again, there is a substantial increase in the last two decades. In 1980, equity liabilities represented only 0.5% of GDP. At their peak, in 2007, they had reach 15.2% and they have since stabilized at 8.6% of GDP. Unfortunately, Lane and Milesi-Ferretti (2018) do not decompose their stocks by type of issuer (public/private). As a result, it cannot be asserted that all of the increase in equity issuance is taking place in the private



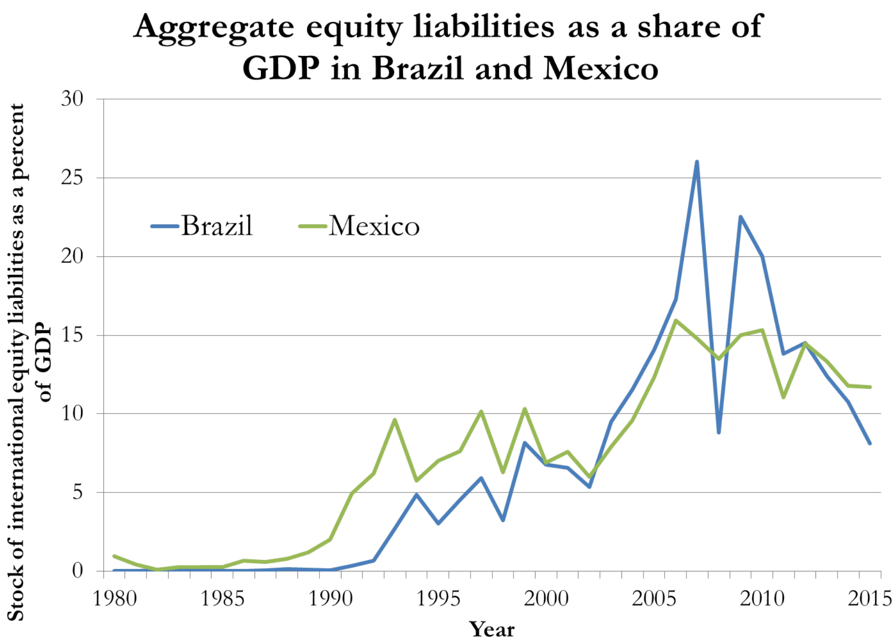
Source: World Bank, International Debt Statistics

Figure 3: International Reserves and Private Debt in Brazil and Mexico

sector. That being said, given that all countries in the sample issue publicly guaranteed debt but not all of them have large public companies that issue equity overseas, and how private and public debt evolved over this time period, it is likely that a significant share of this increase is attributable to the private sector.

While the fact that these trends hold for middle income countries overall shows that the mechanism presented in this paper could be applied to a broad set of countries, in the quantitative analysis the focus will be on the two largest economies of Latin America, Brazil and Mexico. The emphasis is in these two countries, first because the trends identified for the aggregate middle income country are relevant for them. Additionally, the region that they belong to has been to the object of study of a large body of literature on "sudden stops" (Eichengreen et al. (2008)) and overborrowing (Bianchi (2011)). Finally, in the late 1990s and early 2000s, central banks in both of these countries joined the IMF inflation target framework, a measure frequently associated with more central bank independence in developing countries. This incident coincided with the increase in international reserves accumulation as it can be see in Figure 3. Similarly, the share of equity liabilities over GDP has increased in both of these countries as shown in Figure 4.

More generally, once all types of assets are combined, two key differences between the



Source: Lane and Milesi-Ferretti (2018)

Figure 4: Equity as a share of GDP

period preceding the inflation target regime, 1970-1998 for Brazil and 1970-2000 for Mexico, and the present period, 1999-2015 for Brazil and 2001-2015 for Mexico, are apparent. The net foreign assets position deteriorated while the international reserves increased. On average the net foreign assets position of Brazil decreased from -28.2% of GDP for the 1970-1998 period, to -35.2% of GDP for the 1999-2015 period. At the same time, international reserves increased from from an average 5.1% of GDP in the former period to 10.8% of GDP in the latter. Similarly, in Mexico during the 1970-2000 period, the net foreign asset position was -32.0% of GDP and international reserves stood at 3.0% of GDP. In contrast, during the 2001-2015 period, the first term decreased to -36.2% while the second one increased to 10.3% of GDP.

These observations combined with the previously observed trends in private debt and the stock of equity liabilities support the claim that Brazil and Mexico are good candidates for this study. In both countries, the level of private sector international borrowing and international reserves increased in the last 15 years, immediately after their central banks gained independence. The next section will construct a model where this type of transition could be the result of a benevolent planner operating in a constrained economy. Later, the paper will show quantitatively how much of the trends observed in the data could be rationalized by this motive.

3 The Dynamic Stochastic General Equilibrium model

3.1 Environment

Consider a continuum of identical households in a small open economy with a tradable goods sector (c^T) and a nontradable goods sector (c^N). The former goods can be traded internationally while the latter have to be produced and consumed domestically. Both goods are combined to produced a composite good c in an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta+1)$ such that:

$$c = [\omega(c^T)^{-\eta} + (1-\omega)(c^N)^{-\eta}]^{\frac{-1}{\eta}}$$

Where $\eta > -1$ and $\omega \in (0,1)$. The representative agent is infinitely-lived, is exposed to endowment shocks every period, and has constant-relative risk aversion (CRRA) preferences over her consumption of the composite good, given by:

$$\mathbb{E}_0[\sum_{t=0}^{\infty} \beta \frac{c_t^{1-\sigma}}{1-\sigma}] \tag{1}$$

At time t, the agent receives e_t^T units of the tradable good and e_t^N units of the nontradable good. It is assumed that the vector of endowments given by $(e^T, e^N) \in E \subseteq \mathbb{R}^2$ follows a first-order Markov process, and this is the only source of uncertainty. There is only one type of asset in the economy, a one period, non-state contingent bond denominated in units of tradables that pays an exogenous fixed gross interest rate R, such that $\beta R < 1$. The government collects its international reserves ($F_{t+1} > 0$) via lump-sums from the households.

At the same time, the government reverts the proceedings of last period savings to the households (RF_t). Private agents, by contrast, can save, and borrow subject to a credit constraint. The price of tradables is normalized to one and the relative price of nontradables at time t is p_t. The household budget constraint is therefore:

$$b_{t+1} + c_t^T + p_t c_t^N + F_{t+1} = Rb_t + e_t^T + pe_t^N + RF_t \tag{2}$$

Where b_{t+1} denotes the bond holdings chosen in period t. The convention that a positive value denotes assets is maintained. It is assumed that foreign lenders restrict private borrowing so that the total amount of individual debt does not exceed a fraction θ of total current income in terms of tradables. The credit constraint is then:

$$b_{t+1} \geq -\theta(e_t^T + p_t e_t^N) \tag{3}$$

As it is common in the literature, the credit constraint arises from informational and institutional frictions on the credit market, such as imperfect judicial systems, limited enforcement of contracts, monitoring costs, and asymmetric information. This is the case if it is assumed that in case of default the external creditors can also seize a fraction of the nontradable endowment and sell it in the domestic economy. This assumption is grounded on the empirical evidence that external credit market booms in nontradable sectors such as construction are common in emerging market (Tornell and Westermann (2002)). Another feature of the constraint is that the collateral is in the form of present income, as opposed the present value of all future income as in Kehoe and Levine (1993). This assumption is justified by pointing at the major role current income plays in determining market access at the individual level (Jappelli (1990)), and by the fact that it is a major element when determining loan access in the housing market. The form of the credit constraint is crucial for the main mechanism of the paper. Finally, it is important to repeat that it is also assumed that each private agent in the model is of measure zero, and takes the relative price of nontradables as given when facing its credit constraint.

3.2 Sequential Decentralized Equilibrium (SDE)

Definition *Given an initial level of debt ($b_0 = 0$) and reserves ($F_0 = 0$), a sequential decentralized equilibrium is a sequence of allocations and prices $\{c_t^T, c_t^N, b_{t+1}, F_{t+1}, p_t\}_{t\geq 0}$ such that:*

i *Households choose $\{c_t^T, c_t^N, b_{t+1}\}_{t\geq 0}$ to maximize (1), subject to (2) and (3) and taking b_0, and $\{p_t, F_{t+1}\}_{t\geq 0}$ as given.*

ii *The government chooses $\{F_{t+1}\}_{t\geq 0}$ to maximize (1), subject to the resource constraints taking the policy functions of the household problem as given.*

iii *The resources constraints hold at all $t \geq 0$. Namely:*

$$c_t^T + F_{t+1} + b_{t+1} = e_t^T + RF_t + Rb_t \tag{4}$$

$$c_t^N = e_t^N \tag{5}$$

The sequential equilibrium, can be also characterized by its optimality conditions. Denote by λ_t and by μ_t the weakly positive Lagrange multipliers at time t associated with the budget constraint and credit constraint respectively. From the household first-order conditions:

$$b_{t+1} + \theta(e_t^T + p_t e_t^N) \geq 0 \;\; w.e \; if \;\; \mu_t > 0 \tag{6}$$

$$[c_t^N] \;\; p_t = \frac{u_N(t)}{u_T(t)} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1} \tag{7}$$

$$[c_t^T] \;\; \lambda_t = u_T(t) = \omega\left(c_t^T\right)^{-\eta-1}\left[\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}\right]^{\frac{-(1-\sigma)-1}{\eta}} \tag{8}$$

$$[b_{t+1}] \;\; \lambda_t = R\beta\mathbb{E}_t[\lambda_{t+1}] + \mu_t \tag{9}$$

The first equation (6) is the complementary slackness condition of the credit constraint.

The next equation (7), equates the relative price of nontradables with the ratio of marginal utility of consumption of the two types of goods. This intratemporal substitution is key for the mechanism, the households do not internalize that the relative price of nontradables is an increasing function of the consumption of tradables. The final two conditions, equate the present shadow value of wealth (λ_t), first with the marginal utility of consumption of tradables (8) and then with the present value of the expected shadow value of wealth next period and the current shadow price of relaxing the credit constraint (μ_t) (9). Taken together, these two conditions will yield the intertemporal substitution that governs the dynamics of private debt for the consumers:

$$u_T(t) = R\beta\mathbb{E}_t[u_T(t+1)] + \mu_t \tag{10}$$

The rest of the optimality conditions could be derived from the government problem and the resource constraints. This paper will not however characterize the solution to the problem with these conditions. Instead it will use the second welfare theorem. A proposed sequence of allocations will be constructed using the solutions to a planner problem. These allocations will satisfy the consumer problem, and the government restrictions. They will also give the household the same level of welfare as the one obtained by a constrained social planner, and therefore will have to be one of the solutions to the problem given above. The constrained planner, can allocate consumption and debt but is still subject to the same budget, resource, and credit constraints as the decentralized economy. It will later be shown how this allocation can be implemented and what are the consequences for prices, taxes, and reserves.

3.3 Sequential Centralized Equilibrium (SCE)

Definition *Given an initial level of debt ($b_0 = 0$) and reserves ($F_0 = 0$), a sequential centralized equilibrium is a sequence of allocations $\{\tilde{c}_t^T, \tilde{c}_t^N, \tilde{b}_{t+1}\}_{t\geq 0}$ that maximize (1), subject to:*

$$\tilde{b}_{t+1} + \tilde{c}_t^T + \tilde{p}_t \tilde{c}_t^N = R\tilde{b}_t + e_t^T + \tilde{p}_t e_t^N$$

$$\tilde{p}_t = \frac{1-\omega}{\omega} \left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N} \right)^{\eta+1}$$

$$\tilde{b}_{t+1} \geq -\theta(e_t^T + \tilde{p}_t e_t^N)$$

$$\tilde{c}_t^T + \tilde{b}_{t+1} = e_t^T + R\tilde{b}_t \tag{11}$$

$$\tilde{c}_t^N = e_t^N$$

To simplify notation, a price variable ($\tilde{p}_t$) is introduced but it is clear from the equations above that the problem can be written using only allocations. Most of the optimality conditions of the centralized problem are analogous to the ones of the decentralized problem[5]. The crucial difference of the centralized problem compared to the previous problem is that the planner takes into account the general equilibrium effects on prices instead of taking them as given. This has consequences for the intertemporal substitution, as you can see in two of its optimality conditions:

$$\tilde{\lambda}_t = \tilde{u}_T(t) + \tilde{\mu}_t \tilde{\psi}_t \tag{12}$$

$$\tilde{\lambda}_t = R\beta \mathbb{E}_t[\tilde{\lambda}_{t+1}] + \tilde{\mu}_t \tag{13}$$

Where $\tilde{\psi}_t = \theta(\eta+1)\frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^{\eta}$ is the derivative of the debt limit with respect to an increase in the consumption of tradables and $\tilde{\lambda}_t$, and $\tilde{\mu}_t$ are the positive Lagrange multipliers associated with the budget and credit constraints in the centralized problem. The wedge in (12) arises because the planner internalizes the effect that an extra unit of consumption of tradables partially relaxes the credit constraint through its effect on the relative price of nontradables. Furthermore, combining (12) and (13) gives the Euler equation of the centralized problem that reveals the trade-offs of issuing debt:

[5]The complete list of optimality conditions for the planner problem can be found in the appendix B

$$\underbrace{\tilde{u}_T(t) - R\beta\mathbb{E}_t[\tilde{u}_T(t+1)]}_{\text{Welfare gain of consumption}} + \underbrace{\psi_t\tilde{\mu}_t}_{\text{Appreciates collateral}} = \underbrace{\tilde{\mu}_t}_{\text{Tightens constraint}} + \underbrace{R\beta\mathbb{E}_t[\tilde{\psi}_{t+1}\mu_{t+1}]}_{\text{Depreciates future collateral}} \tag{14}$$

Here you can see all the mechanisms at play. The left hand side terms are the total benefits for the social planner of issuing an extra unit of debt today and paying it back tomorrow. This process increases consumption today and decreases it tomorrow. The net welfare gain is the first term. Moreover, increasing consumption today increases the relative price of nontradables which in turn relaxes the current credit constraint. The second term captures this appreciation of the collateral.[6]. The costs are listed on the right hand side terms and are twofold. First, issuing more debt tightens the credit constraint. The second effect, crucial to the propositions of the paper, captures how extra debt today decreases the present value of next period wealth, which in turn tightens the future credit constraint through the price effect. The differences between (10) and (14) are crucial for the two propositions of the article and will be explored in more detail in the proofs. It should be noted at this point however, that the shadow value of the credit constraint next period (μ_{t+1}) only appears in the planner's Euler equation.

3.4 Definitions of Recursive Equilibrium

To further emphasis the difference between these two problems it is useful to present them in recursive form[7]. This is because it frames the problem in terms of the two crucial state variables, aggregate bonds (B), and reserves (F). In the decentralized problem, the representative household takes the aggregate private asset position of the economy (B) as given, and forecasts its value next period using function $\Gamma(.)$. At the same time, she chooses her private level of borrowing (b') subject to her credit constraint. Similarly, the agent takes

[6] Note that $\psi_t = \frac{\partial\theta[e_t^T + p_t(c_t^T)e_t^N]}{\partial c_t^T}$

[7] This is also the way the problem is solved numerically in the quantitative section

transfers from the government (RF) as given, and forecasts its the future values (F') using function $\Lambda(.)$. The other relevant state variables are the private current bond holdings and the present endowments. Following the convention that current variables have no superscript while next period variables carry a prime superscript, the household recursive problem in the decentralized economy is:

$$V(b, B, F, e_N, e_T) = \max_{b', c_T, c_N} \frac{c(c_T, c_N)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{e'_N, e'_T | e_N, e_T} V(b', B', F', e'_N, e'_T) \tag{15}$$

subject to

$$b' + c_T + p(B, F, e_N, e_T) c_N + F' = e_T + p(B, F, e_N, e_T) e_N + Rb + RF$$

$$b' \geq -\theta(e_T + p(B, F, e_N, e_T) e_N)$$

$$B' = \Gamma(B, F, e_N, e_T)$$

$$F' = \Lambda(B, F, e_N, e_T)$$

Where $p(B, F, e_N, e_T) = \frac{1-\omega}{\omega}(\frac{e^T + RB + RF - B' - F'}{e^N})^{\eta+1}$ is the relative price of nontradables. Note that the household is aware of the laws of motion, and in equilibrium she will correctly forecast them, however she has no control over the aggregate variables governing them. The government problem is to solve for the level of reserves that maximizes the household welfare taking her policy functions as given. In a rational expectations environment, the government policy function should coincide with the law of motion used by the household. By the same logic, the law of motion of the aggregate bond holdings should be consistent with private agent's expectations.

Definition *A recursive decentralized equilibrium is a pricing function $p(B, F, e_N, e_T)$, a law of motion of aggregate debt $\Gamma(B, F, e_N, e_T)$, and a law of motion of reserves $\Lambda(B, F, e_N, e_T)$, decision rules for household $\{\hat{b}'(B, F, e_N, e_T), \hat{c}_N(B, F, e_N, e_T), \hat{c}_T(B, F, e_N, e_T)\}$, and for the government $\{\hat{F}'(B, F, e_N, e_T)\}$, and a value function for the household $\hat{V}(B, F, e_N, e_T)$ such* *that:*

i *Given* p, Γ, Λ, *the value and policy functions,* $\hat{V}, \hat{b}', \hat{c}_N$ *and* $\hat{c}_T$, *solve the household problem (15)*

ii *The household policy function is consistent with the law of motion,* $\hat{b}'(B, F, e_N, e_T) = \Gamma(B, F, e_N, e_T)$

iii *The government policy function solves the government problem and is consistent with the household's expectation* $\hat{F}'(B, F, e_N, e_T) = \Lambda(B, F, e_N, e_T)$

iv *Markets clear :* $e_N = \hat{c}_N(T, b, B, e_N, e_T)$ *and* $e_T + RB + RF = \Gamma(B, F, e_N, e_T) + \hat{c}_T(B, F, e_N, e_T) + \Lambda(B, F, e_N, e_T)$

As mentioned before, this article will not solve that problem directly. Instead a solution will be proposed based on a sequence of allocations that deliver the constrained efficient welfare. To achieve this, one has to solve the recursive problem of a constrained planner, called here the centralized problem. In recursive form the centralized problem is simpler since it does not deal with distinctions between aggregates and private bond holdings. Also, substituting the resource constraints (11), and (5), and the pricing formula (7), the constrained planner recursive problem is simply:

$$V^{CE}(b, e_N, e_T) = \max_{b'} \frac{c(c_T, e_N)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{e'_N, e'_T | e_N, e_T} V^{CE}(b', e'_N, e'_T) \tag{16}$$

subject to

$$b' + c_T = e_T + Rb$$

$$b' \geq -\theta(e_T + \frac{1-\omega}{\omega}(\frac{c_T}{e_N})^{\eta+1} e_N)$$

Definition *A recursive centralized equilibrium, or constrained efficient allocations, is a set of decision rules* $\{\tilde{b}'(b, e_N, e_T), \tilde{c}_T(b, e_N, e_T)\}$, *that solve the recursive centralized problem (16)*

3.5 Propositions

The first property of this article will motivate the need for government intervention in the decentralized equilibria. As such it deals with a **laissez-faire economy**, defined as a solution to the decentralized problem when the government sets the reserves at zero at all times.

Proposition 1 *The laissez-faire economy is not constrained efficient in general.*

The formal proof of this proposition can be found in the Appendix C, but an analogous version of this proof is already present in Bianchi (2011). The main intuition of the proof can be found in the differences between the Euler equations of the decentralized and centralized problem when the constraint is not binding at time t. As you can see in (14), the planner acknowledges that one of the channels by which current aggregate debt affects the probability of a binding credit constraint the following period ($\mu_{t+1}\psi_{t+1}$) is through the future relative price of nontradables. In decentralized problems, households choose their level of private borrowing taking future aggregate debt as given so they ignore this channel. This leads them to choose a higher level of borrowing at time t. This overborrowing is inefficient because it increases the probability of facing a binding credit constraint next period relative to what a constrained planner would have done.

Before turning to the problem of how to implement the constrained efficient allocation it is important to mention a significant caveat encountered frequently in this type of models:

Lemma 1 *If $\exists t$ such that $\tilde{\psi}_t > 1$ then the equilibrium in the decentralized problem may not be unique*

The formal proof of this lemma in the case of the decentralized problem can be found in the appendix D. The main idea behind it, is that when $\tilde{\psi}_t > 1$, an increase in the consumption of tradables appreciates the collateral so much that it can be financed entirely by debt even when the constraint binds. The multiple equilibria issue affects a wide variety of similar models (Mendoza (2005)),Bianchi (2011), and Jeanne and Korinek (2010)) and is usually simply assumed away (Korinek and Mendoza (2014), Jeanne and Korinek (2010)). It is acknowledged that this is problematic since it involves making assumptions about an endogenous variable. Unfortunately this article at the moment does not provide an ideal solution to this problem, that is, a set of restrictions that have to be imposed on the primitives of the model to guarantee uniqueness. [8] That being said, given that this condition is also crucial for the proposed implementation to be valid, something weaker than $\forall t\ \tilde{\psi}_t \leq 1$ is assumed instead. The following will nevertheless be sufficient to guarantee uniqueness and the validity of the results, assume that:

(i) $\eta > 0$

(ii) The set of endowments is such that $\forall t > 0\ \frac{e_t^T}{e_t^N} > 0$ *and there exists a* $k = min_t \frac{e_t^T}{e_t^N}$*. Define* y *as the largest real number such that,* $k(1+\theta) = y - \theta y^{1+\eta}$*. Assume that:*

$$\theta(\eta+1)\frac{1-\omega}{\omega}y^{\eta} \leq 1$$

(iii) $\beta R < 1$ *and* $b_0 = 0$

(iv) $\tilde{b}_t \leq 0\ \forall t$

These assumptions still rely on restrictions on endogenous variables *(iv)*, but assumption *(iii)* is usually enough to guarantee this. Nevertheless, it is hoped that future versions of this paper will provide a better answer to this problem. Having mentioned this caveat[9] the main theoretical result can now be stated :

Proposition 2 *Suppose that assumptions (i)-(iv) hold. Then the constrained efficient welfare can be decentralized by setting :*

$$F' = \tilde{b}' + \theta(e_T + \frac{(1-\omega)}{\omega}(\frac{\tilde{c}_T}{e_N})^{\eta+1}e_N)$$

[8] Mendoza (2005) finds restrictions for the deterministic case (an exogenous debt limit that is never binding) and Jeanne and Korinek (2010) find one for one of their finite horizon examples. However none of those are translatable directly to the stochastic infinite horizon case at hand

[9] In the quantitative section, it is checked that $\psi_t < 1$ always and this holds for a wide range of parameters

$$b' = -\theta(e_T + \frac{(1-\omega)}{\omega}(\frac{\tilde{c}_T}{e_N})^{\eta+1}e_N)$$

Where $\tilde{b'} = \tilde{b'}(e_T, e_N, b), \tilde{c}_T = \hat{c}_T(e_T, e_N, b)$ *are the policy functions of the constrained social planner problem.*

An economy in which this policy is implemented is called a **Regulated Economy**. Again, the formal proof of this proposition, can be found in the Appendix F. The intuition this time arises from the resource and budget constraints. For small levels of reserves, an unconstrained household will off set one to one the increases in government reserves with increases in private debt. Above a certain point the credit constraint binds and the household has to reduce her consumption of tradables. The government decentralizes the constrained efficient consumption by raising the reserves to the point where the households consumption of tradables equals the constrained efficient value. Note that this means that in the regulated economy the constraint binds at all times and therefore:

Corollary 1 *In the regulated economy, the household has the constrained efficient consumption allocations but a weakly higher level of debt relative to the centralized problem*

When the constraint binds in the centralized problem reserves are set at zero. This is because when the primitives satisfy the restrictions that make this implementation valid, anytime the constrained binds in the centralized economy it also binds in the laissez-faire economy. In fact it is true that $b_t'^{LF}(B, B, e_t^N, e_t^T) \leq \tilde{b'}_t(B, e_t^N, e_t^T)\ \forall t \forall B$, where $b_t'^{LF}(B, B, e_t^N, e_t^T)$ is the policy function of the lassez-faire economy and $\tilde{b'}_t(B, e_t^N, e_t^T)$ is the constrained efficient one. Note that this does not mean that the debt is always higher in the regulated economy relative to the laissez-faire economy, in fact this will not be the case in the quantitative section for every point but on average at the steady state.

4 Quantitative Exercise

4.1 Calibration strategy

The model will now be applied to the economies of Brazil and Mexico. As documented in the first section, over the last two decades there was an increase in reserves and private debt in both countries. The beginning of this increase coincides with the moment when the central banks in those countries joined the IMF inflation targeting program, which can be interpreted as a proxy for gaining independence. The calibration will therefore assume the following hypothesis:

- From the start of the sample (1970), to the moment each country joined the IMF program, their economies behaved like the laissez-faire economy described of the previous section. This time period is used to calibrate the country parameters, by matching steady state averages to the relevant macro moments from the data.
- In the second half of the sample, from joining the program to the end (2015), their economies have behaved like the regulated economy defined of the previous section. The country parameters, as well the income process are assumed to have remained the same. We can then track what the theoretical model predicts should be the optimal level of reserves and debt. Since these model generated aggregates were not used in the calibration, they can be compared to the ones observed in the data.

4.2 Parameters

Having established the strategy, the next step is to present the list of variables that will be taken from the literature and will remain constant for both countries. They are listed in Table 2. Both the risk aversion and the real international interest rate are commonly found at those values in the literature. The elasticity of substitution between tradables and nontradables, comes from the calibration of Mendoza (2002) when using Mexican data, one

Parameters	Values	Source
Risk free interest rate	$R = 1 + .04$	Standard, Bianchi(2011)
Elasticity of substitution tradables and nontradables	$\frac{1}{1+\eta} = 0.83$	Bianchi(2011), Mendoza(2002)
Risk Aversion	$\sigma = 2$	Standard

Table 2: Parameters taken from the literature

the two countries studied here. It is in the upper bound of the values frequently encountered in the literature that range from (0.5) in Stockman and Tesar (1995)) and González-Rozada et al. (2004) to (0.83) in Mendoza (2005), Bianchi (2011), Korinek and Mendoza (2014). Since this article focuses on issues of overborrowing, as the problems studied in the latter set of papers, the parameter is picked from this literature.

4.3 Income Process

The endowment shocks are modeled as a first-order bivariate autoregressive process:

$$\log \begin{bmatrix} e^T_{t+1} \\ e^N_{t+1} \end{bmatrix} = \rho \log \begin{bmatrix} e^T_t \\ e^N_t \end{bmatrix} + \begin{bmatrix} \epsilon^T_t \\ \epsilon^N_t \end{bmatrix}$$

Where ρ is a 2x2 matrix of autocorrelation coefficients and $\epsilon_t = [\epsilon^T_t, \epsilon^N_t]'$ follows a bivariate normal distribution with zero mean and contemporaneous variance covariance matrix V. To estimate ρ and V, the article uses HP-filtered cyclical components of tradables and nontradables GDP from the World Development Indicators (WDI (2017)) for the 1970-2015 period, the longest time series available. What is meant by tradables in this context is primary products and industrial production[10] and classify the rest of GDP as nontradables.

[10]Since the mining and energy sectors are important in both Brazil and Mexico this approach is favored instead of only manufacturing production

The estimates for each country are:

$$\rho^{Brazil} = \begin{bmatrix} 0.61 & -0.05 \\ 0.04 & 0.65 \end{bmatrix} \quad V^{Brazil} = \begin{bmatrix} 0.0013 & 0.0007 \\ 0.0007 & 0.0006 \end{bmatrix}$$

$$\rho^{Mexico} = \begin{bmatrix} 0.51 & -0.07 \\ 0.39 & 0.48 \end{bmatrix} \quad V^{Mexico} = \begin{bmatrix} 0.0010 & 0.0006 \\ 0.0006 & 0.0027 \end{bmatrix}$$

Moreover, in both countries the correlation between tradables an nontradables is positive and so are the autocorrelation. The volatility of the cyclical fluctuations in each sectors are also similar in both countries. The complete list of correlations is provided in the appendix G. The vector of shocks is then discretized into a first-order Markov process, with four grid points for each shock, using the Tauchen and Hussey (1991) quadrature based approach. As a normalization, the mean endowment in each sector is set to one.

4.4 Calibration and targets

The remaining parameters (ω, β, θ) are calibrated to match historical averages from the pre-inflation targeting period. For Brazil this corresponds to the 1970-1998 period and for Mexico this corresponds to the 1970-2000 period. These three parameters are sufficient to equate exactly three steady states averages with three empirical moments. A summary of the targets and the calibrated values of the primitives of the model can be found in Table 3. The share of tradables in the CES aggregator, ω, is set to match the average share of tradable production in GDP in the data. The model counterpart of this object is the share of consumption expenditure on tradables over output measured in units of tradables $(\frac{c_T}{e_T + pe_N})$. This approach is reasonable since the parameter ω influences the equilibrium relative price between tradables and nontradables in the model. For Brazil the target is 29.8% of GDP, while for Mexico it is 44.7%. The values of ω delivered by this method are inline with the Mendoza (2005) estimates for Mexico and with Kumhof et al. (2007) for Brazil.

Parameters	Values	Target
Brazil		
Share of Tradables	$\omega = 0.30$	29.8% (Agriculture+Industry)/GDP average 1970-1998
Discount factor	$\beta = 0.91$	-28.2% NFA average 1970-1998
Credit Constraint	$\theta = 0.30$	5.5% Probability of SS Eichengreen et al. (2008)
Mexico		
Share of Tradables	$\omega = 0.45$	44.7% (Agriculture+Industry)/GDP average 1970-2000
Discount factor	$\beta = 0.94$	-32.0% NFA average 1970-2000
Credit Constraint	$\theta = 0.35$	5.5% Probability of SS Eichengreen et al. (2008)

Table 3: Calibrated parameters and targets

Given that the model only has one type of asset it is natural to pick the average net foreign assets (NFA) position as a share of GDP as a target for the debt to output ratio in the model ($\frac{b'}{e_T + pe_N}$). To calibrate the remaining parameters, the paper follows Bianchi (2011) strategy and targets the probability of a sudden stop as defined by Eichengreen et al. (2008). In the model, a sudden stop in the laissez-faire economy is defined as a period in which the constraint bind and this leads to a reduction in debt of more than one standard deviation. This definition is consistent with the one proposed by the authors, where a sudden stop is characterized as current account reversal. They estimate that for the average emerging economy has 5.5% annual probability of suffering a sudden stop, so β, θ are calibrated to make sure that this happens with the same probability on the steady state of the laisse-faire economy and to match the average NFA position. The values obtained, are reported in Table 3, and are within range of the ones found in the literature with annual data.

4.5 Policy functions and steady state distributions of debt

Before turning to the quantitative results, some of the features of the different versions of the model introduced in the theoretical section are explored. This subsection will show the debt accumulation decisions, the optimal reserves accumulation, and simulate the model to observe the empirical distributions at the steady state of debt under the three alternative versions of the model: The laissez-faire economy, the regulated economy and the centralized economy.

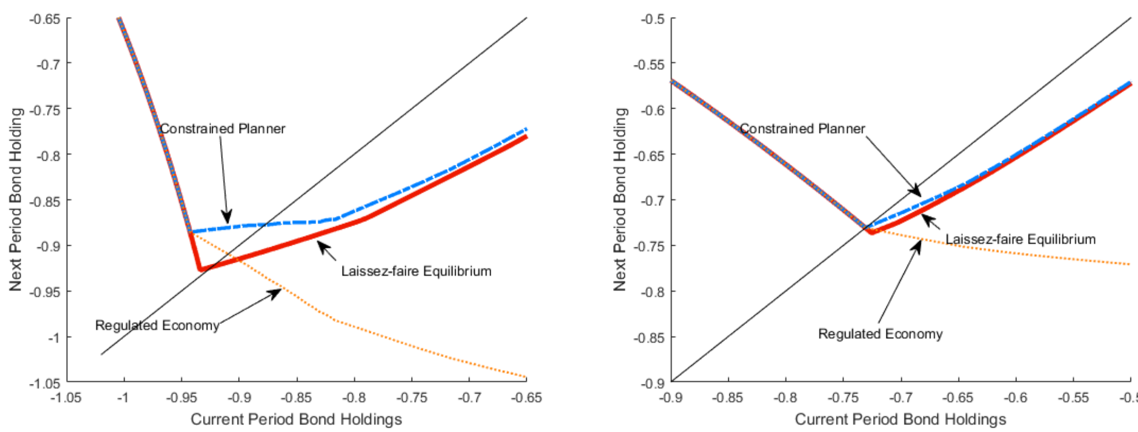


Figure 5: Brazil and Mexico Policy functions respectively

The decision rules of bond accumulation next period as function of the current bond holdings at the median endowment of tradables and nontradables are reported on Figure 5 as well as the 45 degree line. In the absence of endogenously binding credit constraints, one should expect the next period bond holdings to be an increasing function of current bond holdings. This is indeed observed for low values of debt in the laissez-faire and regulated economy. However, as it has been documented in other models with endogenously binding credit constraints (Mendoza (2005), Bianchi (2011), and Bianchi and Mendoza (2018)), the slope of the policy functions changes at the point at which the credit constraint is satisfied with equality but is not binding. Beyond this point the credit constraint binds, so the the lower the level of the initial bond holdings the higher the future bond holdings position. Interestingly, this change monotonicity does not occur in the regulated economy. This is because, as explained in the theoretical section the constraint is always binding in this economy. Therefore, future bond holdings are always a decreasing function of current bond holdings, i.e next period debt is increasing in current debt.

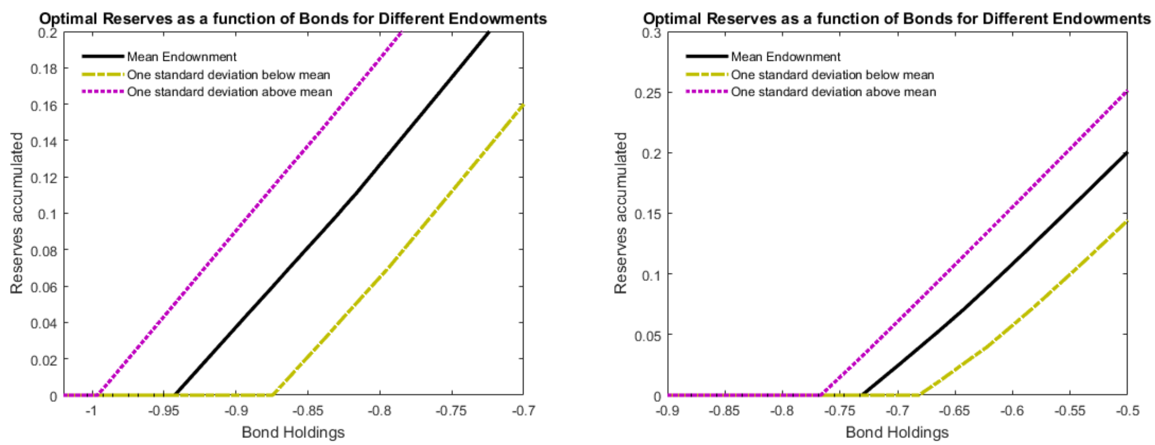


Figure 6: Brazil and Mexico, optimal reserves as function of current bond holdings in the regulated economy

To explore this regulated economy, unique to this paper, in more detail Figure 6, shows the optimal reserves as a function of current bond holdings for the median endowment and for one standard deviation shocks above and below it. Note that reserves are only strictly positive below a certain threshold of debt. This is the point at which the debt is binding in the centralized planner economy. One could say that the policy intervention is ”prudential”, in the sense that the intervention happens in periods where the debt is unconstrained, in the centralized planner problem, and disappears when the constraint binds. This could also lead to collapses in reserves during a sudden stop, as it has been widely documented in literature on financial crisis in emerging economies for example in Cole and Kehoe (2000)), and Manasse and Roubini (2009) among many others.

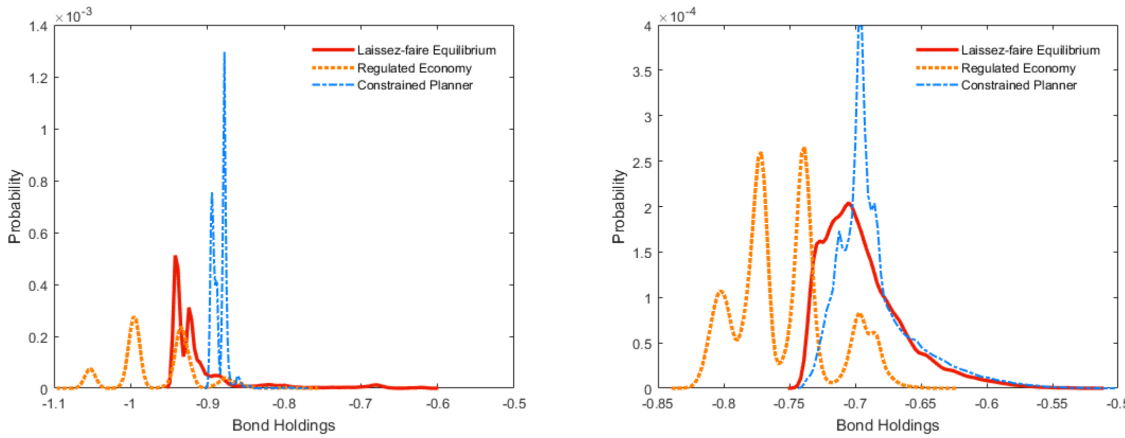


Figure 7: Brazil and Mexico Policy distributions of debt at the steady state

Finally, by simulating the three economies with the same parameters one can compute the probability distributions at the steady state, showed in Figure 7. As it can be inferred from this figure, the average debt is higher in the regulated economy, than in the laissez-faire equilibrium and the centralized economy. Note that, it is also possible to see in this figure the overborrowing result discussed in proposition 1, the average debt is higher in the laissez-faire economy relative to the centralized planner.

4.6 Results and other untargeted moments

Having verified empirically some of the claims made in the theoretical section this article can now turn to its main quantitative results. For each country, Table 4 reports two weighted averages at the steady state of their respected regulated economies. The weights being the probability of each of the ergodic states. To provide further evidence against misspecificaiton other untargeted moments are also reported subsequently.

Brazil	Model	Data
Variable		
International Reserves	2.4%	10.8%
	Weighted mean at the ergodic steady state	As a share of GDP average 1999-2015
NFA net of Reserves	-29.9%	-46.1%
	Weighted mean at the ergodic steady state	As a share of GDP average 1999-2015
Mexico		
Variable		
International Reserves	3.1%	10.3%
	Weighted mean at the ergodic steady state	As a share of GDP average 2001-2015
NFA net of Reserves	-34.7%	-46.5%
	Weighted mean at the ergodic steady state	As a share of GDP average 2001-2015

Table 4: Estimated results and untargeted moments

The first line of each country table is the average level of reserves necessary to implement the constrained efficient consumption allocation. The second one is the average debt to output measured in units of tradables ($\frac{b'}{e_T+pe_N}$). The relevant empirical counterparts of these objects are the average level of international reserves to GDP, and the average net foreign assets position excluding reserves. Both of these averages are calculated for the post-IMF inflation targeting program periods for each country. As one can see in Table 4, the model predicts 22% of the level of reserves observed in Brazil in the post-IMF program period, and 30% for Mexico. Similarly, the model predicted net foreign asset position is 65% of what it is in the data for Brazil and 75% of its value in the Mexican data.

Finally, as an additional validation of the proposed calibration hypothesis Table 5 shows the volatility of the model generated macro economic aggregates of consumption, real exchange rate and trade balance and compare them to their data counterparts. This is done at the steady state for the laissez-faire and regulated economy and compared to the average

Brazil	Model		Data	
Standard Deviation	Laissez-Faire	Regulated	1970-1998	1999-2015
Consumption	4.4	3.4	3.1	3.5
Real Exchange Rate	6.5	2.8	10.9	11.2
Current Account/GDP	2.4	1.6	2.3	2.0
Trade Balance/GDP	2.5	1.7	2.6	2.0
Correlation with GDP	Laissez-Faire	Regulated	1970-1998	1999-2015
Consumption	0.9	0.9	0.7	0.9
Real Exchange Rate	0.9	0.8	0.6	0.0
Current Account/GDP	−0.8	−0.5	−0.3	−0.5
Trade Balance/GDP	−0.3	−0.3	−0.4	−0.5
Mexico	Model		Data	
Standard Deviation	Laissez-Faire	Regulated	1970-1998	1999-2015
Consumption	4.5	4.4	4.0	2.3
Real Exchange Rate	3.8	3.6	12.6	5.1
Current Account/GDP	1.0	1.4	2.9	0.7
Trade Balance/GDP	1.0	1.4	2.2	2.8
Correlation with GDP	Laissez-Faire	Regulated	1970-1998	1999-2015
Consumption	0.7	0.7	0.9	0.8
Real Exchange Rate	0.5	0.4	0.0	0.2
Current Account/GDP	0.0	−0.4	−0.3	−0.3
Trade Balance/GDP	−0.1	−0.4	−0.1	−0.2

Table 5: Untargeted second moments and correlations with GDP

volatilities for each of the two studied countries at the two time periods. Furthermore, Table 5 provides the correlations of the aforementioned macroeconomic aggregates with output both in the data and in the model. One should compare the steady state averages with their empirical counter parts, namely the laissez-faire values with the first time period and the regulated economy with the second.

The difference between the model and the data are reasonably small for most variables. The fit is not satisfactory for the real exchange rate,[11] measured as the real effective exchange rate in the data. This however, may have to do with the choice of empirical counterpart, and it is hoped that with another measure of the real exchange rate the fit will improve.

[11] The real exchange rate is calculated in the model as $[\omega^{\frac{1}{1+\eta}} + (1-\omega)^{\frac{1}{1+\eta}} p^{\frac{\eta}{1+\eta}}]^{\frac{-\eta}{1+\eta}}$

5 Conclusion

This article has presented a theoretical framework and a quantitative evaluation that can partially explain the documented increase in foreign reserves and private debt in emerging economies over the last two decades.

In the model, private households in a small open economy suffer from impatience and an endogenously binding credit constraint. They accumulate precautionary savings to smooth consumption but fail to internalize the general equilibrium consequences of their borrowing decisions on the price of their collateral. Under certain conditions, this will in turn lead them to borrow above what is socially optimal and decreases welfare. A benevolent government, recognizing this problem intervenes by accumulating foreign reserves to the point where the net foreign asset position of the country is socially optimal. This delivers the constrained efficient welfare, but leads the household to her borrowing limit at all times.

In the quantitative section, the model is calibrated to the economies of Brazil and Mexico. At those parameter values, an outside observer, who watches an economy transition from a steady state with no intervention to a steady state with government reserves set optimally would see the average level of both reserves and private debt increase as a share of GDP. It can then be measured quantitatively, how much of the presently observed level of reserves and debt can be explained by this mechanism. For the two countries studied, the paper manages to explain an average of 27% of the reserves and 70% of the foreign assets position.

On the theoretical side, future work will improve on the set of restrictions necessary to discard multiple equilibria. The quantitative fit of this model could also be improved by introducing a stochastic credit constraint parameter as in Bianchi and Mendoza (2018), a venue that future versions of this article will explore. Finally, another direction of further research in this topic will be to introduce firms to the model and allow them to issue debt and equity internationally. One could then evaluate the effects of their overborrowing and explore ways macroprudential policies could address them.

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A Characterization of the SDE

Assume that $\{b_0 = 0, F_0 = 0\}$. An equilibrium sequential decentralized equilibrium is a sequence of allocations for the household $\{c_t^N, c_t^T, b_{t+1}\}_0^\infty$, allocations for the government $\{F_{t+1}\}_0^\infty$ and prices $\{p_t\}_0^\infty$. Such that

- Taking vector of prices and government polices $\{p_t, F_{t+1}\}_0^\infty$, and the initial conditions $\{b_0 = 0, F_0 = 0\}$, as given the households allocations solve:

$$\max_{\{c_t^N, c_t^T, b_{t+1}\}_0^\infty} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta \frac{c_t^{1-\sigma}}{1-\sigma}] \tag{17}$$

subject to $\forall t \geq 0$

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{\frac{-1}{\eta}}$$

$$b_{t+1} + c_t^T + p_t c_t^N + F_{t+1} = e_t^T + p_t e_t^N + RF_t + Rb_t$$

$$b_{t+1} \geq -\theta(e_t^T + p_t e_t^N)$$

- The resource constraints are satisfied every period, $\forall t \geq 0$:

$$c_t^T + F_{t+1} + b_{t+1} = e_t^T + RF_t + Rb_t$$

$$c_t^N = e_t^N$$

- Taking the household policy functions and the resource constraints as given, the government chooses $\{F_{t+1}\}_{t\geq 0}$ to maximize (17)

Alternatively, given that the household problem is convex the equilibrium can be characterized by its first order conditions and the resource constraints. For all $t \geq 0$, call λ_t the weakly positive Lagrange multiplier associated with the budget constraint and μ_t the weakly positive Lagrange multiplier associated with the credit constraint. Therefore, one has to find $\{c_t^N, c_t^T, b_{t+1}, F_{t+1}, p_t, \lambda_t, \mu_t\}_0^\infty$ that satisfy $\forall t \geq 0$:

$$p_t = \frac{u_N(t)}{u_T(t)} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1} \tag{18}$$

$$\lambda_t = u_T(t) = \omega\left(c_t^T\right)^{-\eta-1}\left[\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}\right]^{\frac{-(1-\sigma)-\eta}{\eta}} \tag{19}$$

$$\lambda_t = R\beta\mathbb{E}_t[\lambda_{t+1}] + \mu_t \tag{20}$$

$$b_{t+1} + \theta(e_t^T + p_t e_t^N) \geq 0 \;\; w.e \; if \;\; \mu_t > 0 \tag{21}$$

$$c_t^T + F_{t+1} + b_{t+1} = e_t^T + RF_t + Rb_t \tag{22}$$

$$c_t^N = e_t^N \tag{23}$$

$$\lambda_t \geq 0 \tag{24}$$

$$\mu_t \geq 0 \tag{25}$$

B Characterization of the SCE

Assume that $\{\tilde{b}_0 = 0\}$. An sequential centralized equilibrium is a sequence of allocations $\{\tilde{c}_t^N, \tilde{c}_t^T, \tilde{b}_{t+1}\}_0^\infty$. Such that, the planner solves:

$$\max_{\{\tilde{c}_t^N, \tilde{c}_t^T, \tilde{c}_{t+1}\}_0^\infty} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}]$$

subject to $\forall t \geq 0$

$$\tilde{c}_t = [\omega(\tilde{c}_t^T)^{-\eta} + (1-\omega)(\tilde{c}_t^N)^{-\eta}]^{\frac{-1}{\eta}}$$

$$\tilde{p}_t = \frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^{\eta+1}$$

$$\tilde{b}_{t+1} + \tilde{c}_t^T + \tilde{p}_t\tilde{c}_t^N = e_t^T + \tilde{p}_t e_t^N + R\tilde{b}_t$$

$$\tilde{b}_{t+1} \geq -\theta(e_t^T + \tilde{p}_t e_t^N)$$

$$\tilde{c}_t^T + \tilde{b}_{t+1} = e_t^T + R\tilde{b}_t$$

$$\tilde{c}_t^N = e_t^N$$

Alternatively, given that the constrained efficient problem is convex the equilibrium can be characterized by its first order conditions and the resource constraints . For all $t \geq 0$, call $\tilde{\lambda}_t$ the weakly positive Lagrange multiplier associated with the budget constraint and $\tilde{\mu}_t$ the weakly positive Lagrange multiplier associated with the credit constraint. The objective is therefore to find $\{\tilde{c}_t^N, \tilde{c}_t^T, \tilde{b}_{t+1}, \tilde{p}_t, \tilde{\lambda}_t, \tilde{\mu}_t\}_0^\infty$ that satisfy $\forall t \geq 0$:

$$\tilde{p}_t = \frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^{\eta+1} \tag{26}$$

$$\tilde{\lambda}_t = \tilde{u}_T(t) + \tilde{\mu}_t\tilde{\psi}_t = \omega\left(\tilde{c}_t^T\right)^{-\eta-1}\left[\omega(\tilde{c}_t^T)^{-\eta} + (1-\omega)(\tilde{c}_t^N)^{-\eta}\right]^{\frac{-(1-\sigma)-\eta}{\eta}} + \tilde{\mu}_t\theta(\eta+1)\frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^{\eta} \tag{27}$$

$$\tilde{\lambda}_t = R\beta\mathbb{E}_t[\tilde{\lambda}_{t+1}] + \tilde{\mu}_t \tag{28}$$

$$\tilde{b}_{t+1} + \theta(e_t^T + \tilde{p}_t e_t^N) \geq 0 \;\; w.e \; if \;\; \tilde{\mu}_t > 0 \tag{29}$$

$$\tilde{c}_t^T + \tilde{b}_{t+1} = e_t^T + R\tilde{b}_t \tag{30}$$

$$\tilde{c}_t^N = \tilde{e}_t^N \tag{31}$$

$$\tilde{\lambda}_t \geq 0 \tag{32}$$

$$\tilde{\mu}_t \geq 0 \tag{33}$$

C Proof of Proposition 1

Suppose for a contradiction that, the laissez faire allocations and the constrained efficient allocations coincide for all t. Then $\forall t \; \tilde{u}_T(t) = u_T(t)$, where a tilde denotes the solutions to the planner problem. This means you can combine (19) and (27):

$$\lambda_t = \tilde{\lambda}_t - \tilde{\mu}_t \tilde{\psi}_t$$

If you re-write this one period ahead and take conditional expectations:

$$R\beta\mathbb{E}_t[\lambda_{t+1}] = R\beta\mathbb{E}_t[\tilde{\lambda}_{t+1}] - R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}]$$

However, when you combine (20) and (28), in a period where the credit constraint binds (i.e $\mu_t = \tilde{\mu}_t = 0$) you get instead:

$$R\beta\mathbb{E}_t[\lambda_{t+1}] = R\beta\mathbb{E}_t[\tilde{\lambda}_{t+1}]$$

If the constraint binds with positive probability at time $t+1$, (so $R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}] > 0$) this two conditions contradict each other.

D Proof of Lemma 1

Suppose for a contradiction that at time t you have that $\tilde{\psi}_t > 1$. Suppose also that at time t you have $\tilde{\mu}_t > 0$. This means that at time t, the credit constraint binds in the planners problem. Consider the following perturbation to the consumption of tradables and borrowing to the constrained efficient allocation $\{\tilde{c}_t^N, \tilde{c}_t^T, \tilde{b}_{t+1}, \tilde{p}_t, \tilde{\lambda}_t, \tilde{\mu}_t\}_0^\infty$:

$$c_t^{T,\epsilon} = \tilde{c}_t^T + \epsilon$$

$$b_{t+1}^{T,\epsilon} = \tilde{b}_{t+1}^T - \epsilon$$

Where $\epsilon > 0$ is a small perturbation, financed entirely by debt. Is this perturbation implementable? Note that, by definition:

$$\tilde{\psi}_t = \frac{\partial(\theta[e_t^T + \tilde{p}_t(c_t^T)e_t^N])}{\partial c_t^T}$$

$$\tilde{\psi}_t = e_t^N \theta \frac{\partial(\tilde{p}_t(c_t^T))}{\partial c_t^T}$$

Now check that the small perturbation respects the budget constraint of the planners problem:

$$\tilde{b}_{t+1} - \epsilon + \tilde{c}_t^T + \epsilon + (\tilde{p}_t + \frac{\tilde{\psi}_t \epsilon}{\theta e_t^N})\tilde{c}_t^N = e_t^T + (\tilde{p}_t + \frac{\tilde{\psi}_t \epsilon}{\theta e_t^N})e_t^N + R\tilde{b}_t$$

$$\frac{\tilde{\psi}_t \epsilon}{\theta e_t^N}\tilde{e}_t^N = \frac{\tilde{\psi}_t \epsilon}{\theta e_t^N}e_t^N$$

It also trivially satisfies the resource constraint of tradables. Since it is assumed that at time t the credit constraint binds $\tilde{b}_{t+1} = -\theta[e_t^T + \tilde{p}_t e_t^N]$, you can start from:

$$\tilde{\psi}_t > 1$$

And add the optimal optimal debt to obtain the new the credit constraint:

$$\tilde{b}_{t+1} - \epsilon > -\theta[e_t^T + \tilde{p}_t e_t^N] - \tilde{\psi}_t \epsilon$$

From the optimality conditions of the centralized problem it is known that the net welfare gains of such perturbation are:

$$\tilde{u}_T(t) - R\beta\mathbb{E}_t[\tilde{u}_T(t+1)] = R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}]$$

This shows that if the probability that the constraint binds the next period is zero, this perturbation leaves the consumer indifferent between the two allocation. Moreover, assuming indeed that the constraint does not bind the next period under the initial allocation, it is always possible to pick ϵ small enough such that this is still the case in the perturbed economy so this policy is implementable and gives the consumer the same welfare. It is a another equilibrium.

E Restating Proposition 2

Suppose that $\{\tilde{c}_t^N, \tilde{c}_t^T, \tilde{b}_{t+1}, \tilde{p}_t, \tilde{\lambda}_t, \tilde{\mu}_t\}_0^\infty$ are a constrained efficient equilibrium, namely they satisfy equation (26-33). Assume that:

1. $\eta > 0$
2. The set of endowments is such that $\forall t > 0\ \frac{e_t^T}{e_t^N} > 0$ and there exists a $k = min_t \frac{e_t^T}{e_t^N}$. Define y as the largest real number such that, $k(1+\theta) = y - \theta y^{1+\eta}$. Assume that:

$$\theta(\eta+1)\frac{1-\omega}{\omega}y^\eta \leq 1 \tag{34}$$

3. $\beta R < 1$ and $b_0 = 0$,
4. $\forall t\ \tilde{b}_t \leq 0$

Then the following sequence of allocations satisfy the decentralized equilibrium characterization equations (2-9), $\forall t \geq 0$:

$$c_t^N = \tilde{c}_t^N \tag{35}$$

$$c_t^T = \tilde{c}_t^T \tag{36}$$

$$b_{t+1} = -\theta(e_t^T + \tilde{p}_t e_t^N) \tag{37}$$

$$F_{t+1} = \tilde{b}_{t+1} + \theta(e_t^T + \tilde{p}_t e_t^N) \tag{38}$$

$$p_t = \tilde{p}_t \tag{39}$$

$$\lambda_t = \tilde{u}_T(t) \tag{40}$$

$$\mu_t = R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}] + \tilde{\mu}_t(1-\tilde{\psi}_t) \tag{41}$$

F Proof of Proposition 2

The equations trivially satisfied are:

- Equation (26),(35), and (36) imply that (18) is satisfied.
- Equation (35), (36), and (40) imply that (19) is satisfied
- Using (37) you can see that the credit constraint always binds, therefore (21) is satisfied
- Replacing (35), (37), and (38) in (30) implies that (22) is satisfied
- Equations (31), and (35) imply that (23) is satisfied
- Equation (35), and (36), and the concavity of the utility function implies that consumption is always positive and therefore (24) is satisfied because we know that (19) is satisfied

This leaves two more equations ((20), and (25)), to verify.

Combine equations (27) and (28), this gives:

$$\tilde{u}_T(t) + \tilde{\mu}_t\tilde{\psi}_t = R\beta\mathbb{E}_t[\tilde{u}_T(t+1) + \tilde{\mu}_{t+1}\tilde{\psi}_{t+1}] + \tilde{\mu}_t$$

This can be re-written, as:

$$\tilde{u}_T(t) = R\beta\mathbb{E}_t[\tilde{u}_T(t+1)] + R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}] + \tilde{\mu}_t(1-\tilde{\psi}_t)$$

And replacing with the proposed solutions (40), and (41)

$$\lambda_t = R\beta\mathbb{E}_t[\lambda_{t+1}] + \mu_t$$

This verifies equation (20). Finally,using the definition proposed in (41), you know that (25) will be satisfied if $\forall t$:

$$\mu_t = R\beta\mathbb{E}_t[\tilde{\mu}_{t+1}\tilde{\psi}_{t+1}] + \tilde{\mu}_t(1-\tilde{\psi}_t) \geq 0 \tag{42}$$

You know by equation (33), that $\forall t \tilde{\mu}_t \geq 0$, and by definition:

$$\tilde{\psi}_t = \theta(\eta+1)\frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^\eta = \theta(\eta+1)\tilde{p}_t\frac{\tilde{c}_t^N}{\tilde{c}_t^T}$$

The strict concavity of the utility functions implies that consumption will not be negative and therefore $\forall t\, \tilde{\psi}_t > 0$. Also it is clear that when the constraint does not bind $\tilde{\mu}_t = 0$, the proposed Lagrange multiplier verifies equation (25). The rest of the assumptions are set to make sure that $\tilde{\psi}_t \leq 1\ \forall t$[12] when the credit constraint binds. To bound the ratio of $\frac{\tilde{c}_t^T}{e_t^N}$, first use the resource constraint:

$$\frac{\tilde{c}_t^T}{e_t^N} = \frac{e_t^T - \tilde{b}_{t+1} + R\tilde{b}_t}{e_t^N} \tag{43}$$

Now use assumption (4), [13] $\forall t$, $\tilde{b}_t \leq 0$, and the credit constraint to the set an upper bound to the ratio:

$$\frac{\tilde{c}_t^T}{e_t^N} \leq \frac{e_t^T(1+\theta) + \theta(\frac{\tilde{c}_t^T}{e_t^N})^{1+\eta}e_t^N}{e_t^N}$$

For simplicity call $y_t = \frac{\tilde{c}_t^T}{e_t^N}$ and $k_t = \frac{e_t^T}{e_t^N}$, the above equation can be re-written as:

$$y_t \leq k_t(1+\theta) + \theta y_t^{1+\eta}$$

$$y_t - \theta y_t^{1+\eta} \leq k_t(1+\theta)$$

Using assumption (1), it is trivial to show that the function $f(x) = x - \theta x^{\eta+1}$ increases from 0 to reach a maximum when $x = \frac{1}{\theta(1+\eta)}$, and then declines to $-\infty$, crossing zero when

[12] On the quantitative section this conditions is always verified and holds for a wide range of parameters

[13] Assumption (3), $\beta R < 1$ and $\tilde{b}_0 = 0$ is necessary to guarantee that assumption (4) holds. It is also usually sufficient.

$x = \theta^{\frac{-1}{\eta}}$.[14] By assumption (2), for all t you have that $0 < k \leq k_t$, this means that:

$$y_t \leq y$$

$$\tilde{\psi}_t = \theta(\eta+1)\frac{1-\omega}{\omega}\left(\frac{\tilde{c}_t^T}{\tilde{c}_t^N}\right)^{\eta} \leq \theta(\eta+1)\frac{1-\omega}{\omega}y^{\eta} \leq 1$$

[14]This means that $y < \theta^{\frac{-1}{\eta}}$ and that an even stronger sufficient condition is $(1+\eta)\frac{1-\omega}{\omega} \leq 1$

G Appendix of tables

Correlation and Volatility	Brazil	Mexico
Autocorrelation of the tradable sector	0.58	0.47
Autocorrelation of the nontradable sector	0.68	0.56
Standard deviation of the tradable sector	0.045	0.037
Standard deviation of the nontradable sector	0.034	0.065
Correlation between tradables and nontradables	0.82	0.38

Table 6: Sectoral correlations and volatilities