Private Overborrowing under Sovereign Risk *

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Abstract

This paper examines how systemic credit externalities in international private borrowing increase the severity and frequency of sovereign debt crisis. I propose an open economy model of international private debt subject to a collateral constraint and international public debt issued without commitment. General equilibrium effects exacerbate collateral financial frictions in the private sector constraining its ability to rollover its debt. Mitigating this financial amplification provides an incentive for government bailouts financed by risky public liabilities which in turn can lead to a sovereign debt crisis. The article shows that without restrictions on private borrowing, private agents borrow more than a financial regulator who internalizes the general equilibrium effects of private borrowing. The regulator optimal allocations can be implemented with state contingent macroprudential taxes on private debt. Optimal macroprudential policies, not only reduce the occurrence and magnitude of financial crisis, but also the need for public bailouts, and the average sovereign spread paid on public debt. Using Spanish data from 1999 to 2015, I show that optimal macroprudential policies would have reduce the stock of private debt by 5% of GDP on average, and cut the annual probability of experiencing a financial crisis by 240 basis points. Finally, had macroprudential policies been in place in 2012 the interest rate spread on public bonds would have peaked 380 basis points below the observed level.

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1 Introduction

A feature of the 2010-2015 European Debt Crisis is that large spikes on interest rate spreads also affected countries that had pursued fiscally frugal policies in the lead up to the crisis. Among those countries was Spain, the largest economy in the Eurozone in uninterrupted compliance of the budgetary and public debt limits set by the Stability and Growth Pact from the introduction of the Euro in 1999 up to the of global financial crisis in 2008.[1] However, in the 1999-2008 time period, Spain accumulated a large stock of international private debt, primarily in its banking sector.[2] Once the financial turmoil got started, the government funded multiple rounds of bailouts to highly indebted financial institutions which caused an abrupt increase in public debt and its interest rate spreads. These events have lead researchers to examine the problem of the optimal response of a sovereign with defaultable debt to systemic vulnerabilities in international private credit.[3] A joint analysis of private debt and sovereign risk is necessary to provide accurate policy prescriptions. Assuming a sovereign with full commitment could lead to policy prescriptions that are not sustainable in reality, while assuming that bailouts have to be financed only with taxes raised within period would lead to sub-optimal policies that do not incorporate the gains from smoothing costs overtime.

This paper sheds light on this problem by providing quantitative answers to the following three questions. First, can credit frictions affecting individual private borrowers generate the private and public debt and public interest rate spread patterns observed during the Spanish Debt Crisis? Second, if it is so, was the Spanish private sector excessively indebted in the lead up to the crisis and by how much? Third, what is the optimal design of macroprudential policies aimed at reducing private borrowing in this context and how effective are these policies in reducing the frequency, and magnitude of crisis, and in improving social welfare?

I find that systemic externalities in private credit regulated by a sovereign who can borrow internationally without commitment are sufficient to rationalize the patterns observed in the Spanish data. The baseline model features low public debt and and near zero interest rate spreads alongside private debt build-ups above the socially optimal level. Moreover, when confronted with the productivity and external shocks taken from the Spanish data in 2008-2015 the government in this model finds it optimal to provide a large transfer to the private sector financed with external public debt. This response in turn leads to a sudden decrease in private debt and a rise in the public interest rate spread

[1] Morris et al. (2006) discusses the reform of the Pact in 2005 and distinguishing Spain for its compliance. Schuknecht et al. (2011) describes the evolution of deficits and sovereign debt in the post-reform period and documents Spanish compliance up until the 2008 recession.

[2] Lane (2013) and Chen et al. (2013) discuss current account imbalances of periphery European countries. Hale and Obstfeld (2016) and Hobza and Zeugner (2014) analyze capital flows within the Eurozone and document the flow in the form of debt instruments from 'core' countries towards financial institutions in the periphery. In't Veld et al. (2014) and Ratto and Roegera (2015) link the increase in capital flows to Spanish banks financing a boom in the construction sector.

[3] The feedback loop between sovereigns and the domestic financial sector in this context is referred to as "doom loop" or "lethal embrace" and is described in Acharya et al. (2014) and Farhi and Tirole (2018). More details can be found in the literature review.

commensurate with the increase observed in Spain in 2012.

Furthermore, I argue that the effects of systemic credit externalities and subsequent government bailouts are quantitatively important. I quantify the excessive private debt stock from 1999 to 2011 to be 5% of gross domestic product (GDP) on average. I also find that the optimal credit market intervention could have taken the form of state dependent taxes on borrowing. Adopting optimal macroprudential policy would have reduced the annual probability of experiencing private financial crisis, defined as a contraction of more than one standard deviation below the mean of the current account of the private sector, from 2.5% to 0.1% and reduced the annual probability of observing a sovereign default from 0.46% to 0.03%. I can also estimate that the social welfare gains of implementing these measures are equivalent to those obtained by an increase of 0.41% in aggregate consumption during this time period. Finally, I show that even when taking the fiscal response of the government as given, the adoption of prudential policies at the onset of the crisis in 2008 would have reduced the spread paid on government debt by 380 basis points (bps.) at the peak of the crisis in 2012.

To compute these answers I construct a model that embeds the insights of the macroprudential literature on financial crisis caused by collateral debt constraints developed by Kehoe and Levine (1993), Mendoza (2002) and Bianchi (2011), within a sovereign debt structure in the tradition of Eaton and Gersovitz (1981), Arellano (2008), and Aguiar and Gopinath (2006). In the baseline version of the model, I consider a dynamic stochastic general equilibrium model (DSGE) that combines a continuum of identical households with a government who of taxes them in a lump-sum fashion and has access to strategical defaultable international public debt. The government can use this lump sum to transfer resources to the households (bailouts) or alternatively to tax them to honor its international obligations. Households consume tradable and nontradable goods and can smooth their consumption of the tradable good by borrowing internationally subject to a collateral credit constraint. Their borrowing capacity is assumed to be a stochastic fraction of their current endowment of tradables and nontradables evaluated at market prices. I latter extend this framework to allow the government to impose state dependent taxes on private borrowing. I prove that conceptually this extension is equivalent to letting a benevolent planner make all borrowing decisions and transfer the proceeds to households who make all consumption decisions subject the same market prices, and resource and credit constraints.

The implications of a collateral constraint that depends on market determined prices are the key to understand the mechanisms of the model. Consider the situation of a representative household that enters the period with a large amount of private debt and faces an adverse shock, either in the form of a productivity or external financial shock. Without government intervention, the household is unable to rollover its debt without violating the collateral constraint and is therefore forced to deleverage and reduce its consumption of tradables. Since all households are assumed to be identical, the reduction in aggregate consumption of tradables induces a decline in the market price of the nontradable goods that causes a decline in the value of collateral. Thus, the credit constraint tightens even more, calling upon an even greater contraction in consumption. Since the engine of this feedback loop is a general equilibrium price that competitive households take as given, the optimal borrowing decision from at the individual level are frequently above the socially optimal level.[4] This exposes them to more frequent and severe credit boom and bust cycles relative to a planner who incorporates the general equilibrium effects on its decisions making. This financial amplification mechanism is described in Mendoza (2002) and Bianchi (2011) and is referred to sometimes as a Fisherian deflation, after Irving Fisher's classic debt-deflation effect.

The novel mechanism of this paper is the exploration of how this financial amplification interacts with the government's borrowing and default decisions. Assuming that the government is benevolent and a strategic player implies that it will use international public debt to mitigate the most negative consequences of this systemic vulnerability. To fix ideas, I divide the government responses into ex-ante episodes, decisions made during periods where the credit constraint is not binding even in the absence of government interventions, and ex-post episodes, decisions made during periods when government inaction implies a binding constraint and a contraction in consumption. In all cases the government evaluates the benefits of providing a positive transfer to households financed with external public debt against the expected costs of a future with either higher taxes or the dead-weight loses of a sovereign default.

Relative to an ex-ante episode an ex-post scenario provides an additional benefit to government bailouts. A positive transfer can limit the decline in consumption and therefore boost the market value of private collateral. Accordingly, government transfers during these episodes allow the households to issue a higher level of private debt further facilitating consumption smoothing. However, fiscal bailouts achieve this gains at the cost of increasing overall indebtedness. This increases the risk of default in the future, which translates into an increase on the interest rate spread paid on public debt in the current period. The multiplicative benefits of these interventions are sufficiently large to justify the cost of significant increases in spreads. This is the main channel that allows the model to replicate the patterns observed during the peak years of the crisis in 2012.

Conversely in an ex-ante episode public and private debt interact in a different way. When households are not facing a biding credit constraint a positive transfer is subject to the classic consumption smoothing and Ricardian equivalence effects. Households, now at their Euler equations, respond to government transfers by decreasing their private borrowing, both to reduce consumption volatility and because they anticipate higher future taxes. Private and public debt therefore act as imperfect substitutes. Depending on the government expectations of future default costs,[5] the regulator can find it optimal to substitute some private liabilities with defaultable public debt. Since the main con-

[4]Under different calibrations, Schmitt-Grohé and Uribe (2019) or assumptions Benigno et al. (2013), show that this mechanism can also lead to underborrowing. This case is never encountered in the numerical cases considered in this paper.

[5]The assumption of persistent income cost is commonly found in the sovereign default literature

sequence of high private debt is an increasing probability of a binding credit constraint, the incentive to intervene in this fashion increase with the level of private debt. As a result, these bailouts are commonly seen in the periods immediately presiding a crisis and when the default cost are high.[6] Finally, since the benefits from these interventions are quantitatively smaller they are not usually associated with significant increases in interest rate spreads. This mechanism helps the model fit the patterns observed in the data in the years immediately presiding 2012.

Prudential policies, equated in this paper to taxes on private borrowing,[7] allow the government to restore the socially efficient level of private borrowing without resorting to a public debt intervention. The benefits in this context are twofold. First, by decreasing the level of private borrowing prudential policies decrease the severity, measured in terms of drops in consumption, and frequency of private financial crisis. Secondly, and as a consequence of the first benefit, taxes on private borrowing reduce the need for government bailouts in both ex-ante and ex-post episodes. This translates into lower public debt in general and therefore a smaller probability of a sovereign default. The combination of these two factors leads to a lower interest rate spreads on public debt.

A positive, baseline, version of the model without macroprudential policies is calibrated to pre-crisis Spanish data from 1999 to 2011. In particular the calibration targets the mean and the volatility of the private and total investment position and the same moments for the interest rate spread on public debt during this time period. Additionally, using the calibrated parameters I solve the model with optimal state dependent taxes. The comparison between the behavior of this counterfactual regulated economy and the baseline model at their respective ergodic distribution allows me to compute the level of excessive debt, the welfare gains, and the change in the probabilities of experiencing a crisis. Finally, I conduct two numerical exercises to obtain the dynamic responses of the model. The first exercise is an out of sample validation of the modeling approach. I feed to the calibrated baseline model the exogenous domestic shocks taking directly from the Spanish data from 2008 to 2015, during the crisis years and is imitate aftermath. To infer from the data the unobserved exogenous financial shock I use the particle filter approach proposed in Bocola and Dovis (2019). The model endogenously replicates the dynamics of private and public debt, bailouts and spikes on interest rates during the period of interest. The second exercise follows the same steps but I further restrict to the model to the path of public borrowing observed in data. By feeding these shocks to the regulated version of the model I can measure the reduction in spreads attributable to macroprudential policies alone.

Related Literature This paper builds upon the literature on sovereign debt as well as the literature on pecuniary externalities and macroprudential policies. It is most closely related to the literature analyzing the relation between sovereign debt and the domestic private financial sector.

[6] In practice, since default costs are assumed be increasing in output this can episodes can coincide with output booms.

[7] Bianchi (2011), Bianchi and Mendoza (2018), and Arce et al. (2019) discuss alternative measures to implement constrained efficiency in these models such as capital reserve requirements and international reserves.

Following the theoretical framework of sovereign defaultable debt introduced in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008) developed quantitative models of sovereign debt and business cycles. A growing literature has emerged extending this framework, in particular by introducing long-term public debt. Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016) highlight the importance of long term debt in generating dynamics of the interest rate spread that are consistent with the data.[8] The model presented here incorporate these findings by assuming a long term structure for public debt while keeping, for simplicity, the short term maturity in private debt.[9] This paper is however most closely related to the branch of the sovereign debt literature that focuses on the link between sovereign debt and the private economy. This paper distinguishes itself from Mendoza and Yue (2009), Arellano et al. (2017), by giving private agents to access to international credit markets even during a sovereign default episode. In this regard this paper is most closely related to Kaas et al. (2020). The main difference with this recent work is that private debt in my model is inefficiently high from a social perspective and this increases the incidence of credit busts. As a result, the frequency of public bailouts, in response to reductions in the borrowing capacity in the private sector, is an endogenous outcome in this paper.[10]

The paper also contributes to the literature on credit frictions and macroprudential policies. In particular it is related to the literature on systemic credit risk (see Lorenzoni (2008), Bianchi (2011), and Dávila and Korinek (2018)) and how to manage them with taxes on private borrowing (see Bianchi and Mendoza (2018), Farhi and Werning (2016) and Jeanne and Korinek (2019)). The paper contributes to this literature by showing that in the absence of capital controls, government bailouts financed with external defaultable debt can partially mitigate the effects of crisis caused by the pecuniary externality on private credit.[11] Although the role of bailouts in the model is similar to the one found in Bianchi (2016), Keister (2016), Chari and Kehoe (2016) and my paper distinctly assumes that these bailouts can be paid for with long term strategically defaultable debt which can therefore lead to another type of crisis.[12]

By analyzing the interaction between sovereign and private financial crisis the article also con-

[8]These papers, along with Aguiar et al. (2019) and Hatchondo et al. (2016) discuss the issue of "debt dilution", the time inconsistency problem that emerges when public defaultable debt is long term. Additional papers discussing responses to the trade offs involved on maturity structure with long term debt include among many others Broner et al. (2013), Chatterjee and Eyigungor (2013) and Bianchi et al. (2018).

[9]The presence of multiple maturities links the paper to literature studding the role optimal debt maturity structure such as in Arellano and Ramanarayanan (2012) and Sanchez et al. (2018). This paper differentiates itself from this literature by assuming that the government will not be able to fully control the issuances of short term private debt.

[10]Additionally in the baseline version of the model public debt is sometimes used as in imperfect substitute to private debt during output booms.

[11]Another recent strand of related litterateur studies the implications of this pecuniary externality for exchange rate policy, e.g. Fornaro (2015), Ottonello (2015), and Benigno et al. (2016)

[12]The literature also deals extensively with the issue of moral hazard that the expectation of government bailouts induces. This concern is not addressed in this paper since households take as given that government policies are functions of aggregate states and not their individual actions. Additional research on the issue can be found in Nosal and Ordoñez (2016), Stavrakeva (2020) and Pasten (2020).

tributes to the growing literature on "doom loops". Theoretical analysis of this issue are presented in Korinek (2012), Brunnermeier et al. (2016) and Farhi and Tirole (2018).[13] The simpler model presented in this paper distinguishes itself by providing a quantitative study of a country during such an episode. In doing this, the paper is more closely related to other quantitative analyses of the relationship between sovereigns and private borrowers,in particular Perez (2015), Bocola (2016), and Sosa-Padilla (2018). The analysis in these papers focuses on the role of sovereign debt in the balance sheet of domestic banks and on how the increase in sovereign spreads exacerbates domestic credit vulnerabilities. This paper complements this view by instead highlighting how preexisting private credit vulnerabilities generates incentives for the government to increase its debt even at the expense of increasing the risk of default and therefore the interest rate spread.

Finally, methodologically this paper applies dynamic discrete choice methods to solve a sovereign debt model drawing from the contributions of Sanchez et al. (2018).[14]. The method is used to from smooth the government policy functions and reduce computational errors. Additionally, to construct a quantitative counterfactual of the Spanish debt crisis the paper uses the nonlinear particle filter method proposed by Kitagawa (1996). This technique uses likelihood functions to construct a numerical approximation of an unobserved stochastic shock and was first applied to quantitative business cycle models in Bocola (2016) and Bocola and Dovis (2019).

Layout. The paper is organized as follows. Section 2 outlines the motivating empirical facts in the Spanish data. Section 3 presents the model and the main theoretical result. Section 4 details the calibration and discusses the main mechanisms though which private and public debt interact in the model. Section 5 summarizes the the quantitative results of the paper, first it compares the positive and normative versions of the model at their respective ergodic distributions, and second it provides counterfactual exercises to disentangle the effects of private borrowing, and government bailouts during the 2008-2015 Spanish crisis. Finally, Section 6 concludes.

2 Motivation: The path of debt and spreads in Spain 1999-2015

This section documents the evolution of international private and public debt in Spain from the creation of the Eurozone in 1999 to the end of the Spanish sovereign debt crisis in 2015. The evolution of these assets, combined with the evolution of their underlying default risks, serves as an illustration of the intertwined relationship between private vulnerabilities and sovereign debt crisis. However as noted by Reinhart and Rogoff (2011), Lane (2013), and Gennaioli et al. (2018) similar patterns similar to the one observed in Spain have been present in crisis in other countries.

[13]Other relevant theoretical papers on this issue include Uhlig (2014) and Cooper and Nikolov (2018)

[14]Other models using this techniques include Mihalache (2020). A review of the method and an alternatives can be found in Gordon (2019).

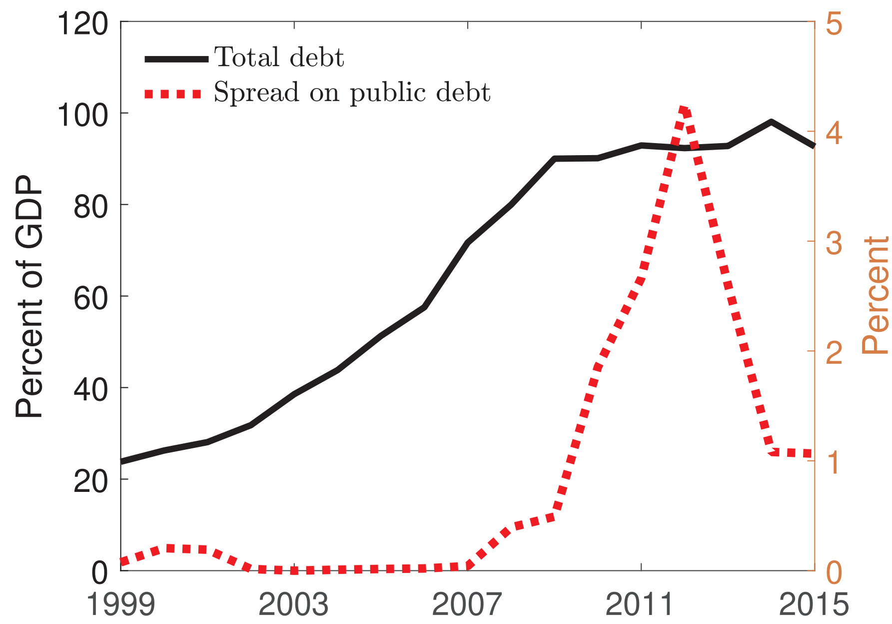

Figure 1: Total international debt and sovereign spread
Note: Total debt corresponds to the inverse of the international investment positions. Spreads correspond to the average difference between the in interest rate paid on a Spanish 6 year treasury bill and its German equivalent. Data source for debt is the Bank of Spain while the interest rate data is form Bloomberg. More details can be found in appendix C.

Figure 1 presents a summary of the Spanish debt crisis and exposes the difficulty of studying external debt without distinguishing between private and public liabilities. The left axis plots the evolution of the international investment position[15] as a percent of gross domestic product (GDP) in Spain from 1999 to 2015 (solid line) in an inverted scale[16]. Although this measure combines all assets for simplicity I will refer to it as total debt throughout the paper. The right axis plots the sovereign spread (dotted line), calculated as the difference between a 6-year treasury bond issued by Spain and its German counterpart.[17] Total debt in figure shows a period of accumulation of external debt between 1999 and 2008, followed by a period where total debt remained constant at around 92% of GDP. These dynamics are juxtaposed with the evolution of the sovereign spread, that remains close to zero up to 2009, and then experiences a spike with a peak in 2012. Looking at this figure through the prism of the standard sovereign debt model makes it hard to reconcile a period with rapidly increasing debt but low spreads (1999 to 2008), and a period of significant movement in the spread while total debt remains constant (2009 to 2015). Indeed alternative explorations of the sovereign debt crisis in Spain such us Hatchondo et al. (2016) and Bianchi and Mondragon (2018) use data on spreads only from the latter period.

Next, I summarize in Figure 2 the evolution of the private international liabilities during this time

[15] Annualized data from the Bank of Spain, more details can be found in appendix C

[16] A positive number represents net liabilities

[17] This maturity is chosen because it corresponds to the average maturity of public debt in Spain during this period. For more details see Section 4 and Appendix C.

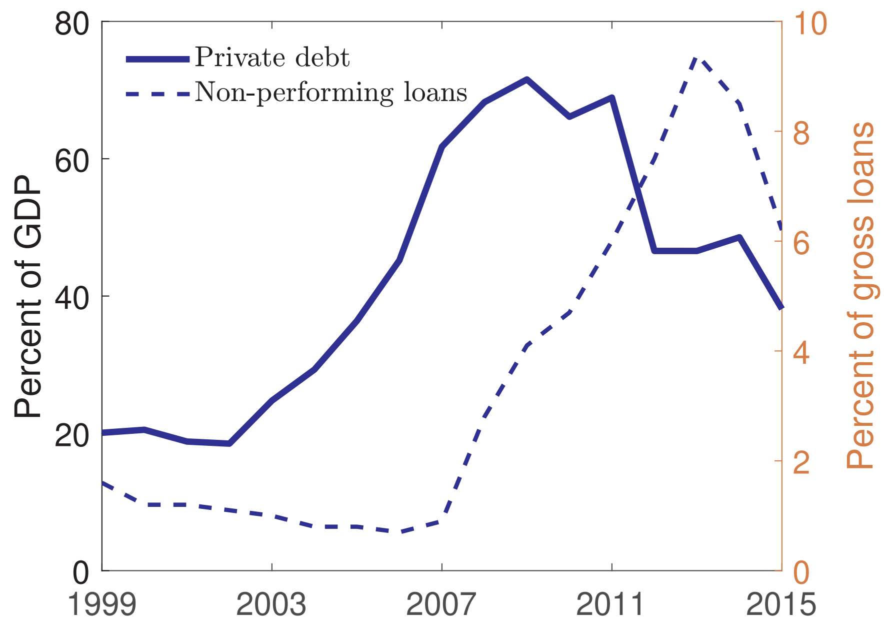

Figure 2: Private debt and nonperforming loans

Note: Private debt corresponds to the inverse of the international investment positions of the financial, and non financial private sector. Nonperforming loans are computed as a share of total gross loans. Data source for debt is the Bank of Spain while the loans data is form Bloomberg. More details can be found in appendix C.

period. The right axis corresponds to international net debt position of the private sector as a percent of GDP[18] (solid line) while the right axis corresponds to non-performing loans as a percent of gross loans (dashed line). As before the evolution of private debt displays two distinct periods. Net liabilities in the private sector grew from 20% of GDP in 1999 to 70% of GDP in 2009. From that point on they remained stable or declined, with the biggest yearly drop encountered in 2012 amounting to 22% of GDP. As noted by several observers (e.g. International Monetary Fund (2012), International Monetary Fund (2014), and Martin et al. (2019)), Spanish external private debt build up was primarily driven by a banking sector that was financing a construction boom. When housing prices dropped and mortgages started going unpaid, private debt became increasingly more difficult to rollover abroad. For this reason, I use the percent of nonperforming loans as a proxy measure of aggregate risk in private sector. Figure 2 also shows that the rapid increase in private debt stopped at the same time as the share of nonperforming loans started increasing. Moreover, the large drop in private liabilities took place when the share of private default was at high point. On average 7.5% of gross loans where nonperforming between 2011 and 2015.

Finally in Figure 3 complements this analysis by showing the joint evolution of public and private debt. This figure shows once again the evolution of private debt during our period of interest but plots alongside the evolution of public liabilities. Combined these two positions add up to total debt

[18]To compute the position of the private sector I subtract from total debt assets held by public administration and the bank of Spain. See details in Appendix C.

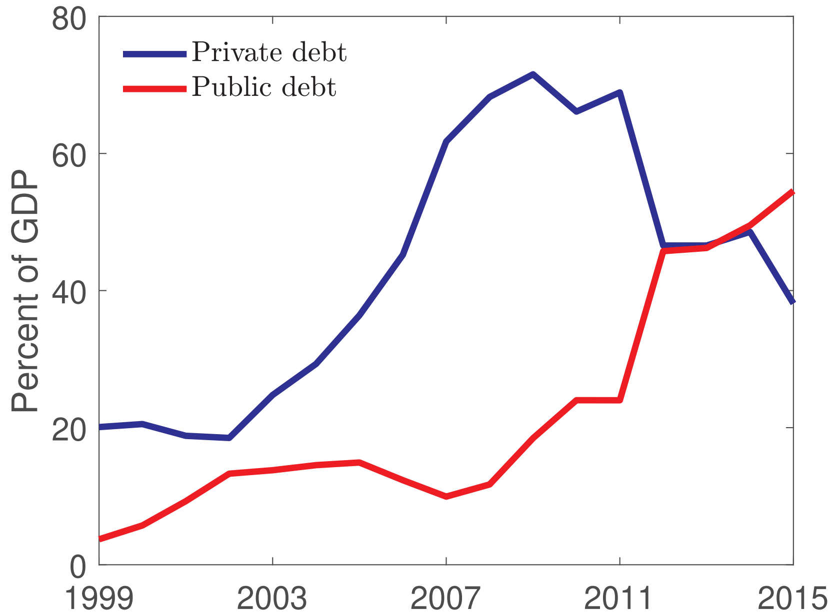

Figure 3: Private and public debt

Note: Private debt corresponds to the inverse of the international investment positions of the financial, and non financial private sector. Public debt corresponds to the inverse of the international investment position of the Bank of Spain and other public administrations. Data source is the Bank of Spain. More details can be found in appendix C.

presented in Figure 1. The stinking symmetry between these two aggregates stresses the importance of the decomposition that this paper argues for. From 1999 to 2008, public external debt in Spain was below 20%. Nevertheless, from 2008 to 2015 public external debt increased from 11% to 55% of GDP. Of particular importance for this paper is the fact that the largest yearly increase was also 2012, when public liabilities increased by 22% of GDP, exactly mirroring the drop in private liabilities. This evolution is of course not a coincidence, as noted in Banco de España (2017) between 2008 and 2012 the Spanish government provided multiple rounds of bailouts to its financial institution with the largest bailout occurring in 2012 to BFA-Bankia (18 Billion) at time the third largest lending institution.

I conclude this section by summarizing the sequence of events observed in Spain. In the pre-crisis years, 1999-2007, large build ups of private debt coexist with low public debt and public spreads close to zero. This period is followed by a private financial crisis, corresponding in the data to years 2008 to 2011, characterized by an increase in nonperforming loans in the private sector, and a moderate private deleveraging. Throughout this second period, public debt and spreads increase but remain relatively low. The final period, from 2012 to 2015, corresponds to the sovereign debt crisis and is characterized by large public bailouts that reduce net liabilities in the private sector but are financed with issuances of public debt. This shift in debt ownership coincides with significant increases in the spread paid on public debt. The next section propose a theory that sheds light on this interplay between private and public external debt. In the model, both types of debt are endogenous and their dynamics will be consistent with the aforementioned facts.

3 A model of private and public debt crises

This section presents a dynamic small open-economy model with one-period international private bonds subject to an occasionally binding borrowing constraint as in Bianchi (2011) and long term, strategically defaultable, international public bonds as in Hatchondo and Martinez (2009). The first subsection presents the environment for each of the agents in the economy. The second subsection defines and characterizes the baseline unregulated decentralized competitive equilibrium where the government only has access to public debt and lump-sum transfers. The third shows the optimal policy problem of a social planner (SP) who makes all borrowing decisions in both assets but is unable to commit to future policies. The last subsection demonstrates the SP's allocations are equivalent to those of a regulated competitive equilibrium where the government gains access to state-contingent taxes on private debt.

3.1 Environment

Time is discrete and indexed by $t \in \{0, 1, ..\infty\}$. The model consists of a continuum of identical households of unit measure, a benevolent domestic government, and a continuum of risk neutral competitive foreign creditors who lend to both domestic agents via two different assets. The focus is on real values as opposed to nominal ones because most Spanish debt was denominated in Euros.[19] The timing of events within the period is as follows. First, all aggregate shocks are realized, then the government makes default, public borrowing, and tax decisions. Finally, given shocks and government policies, households choose private borrowing and consumption, and lastly goods and credit markets clear.

3.1.1 Households

Preferences The representative household has an infinite life horizon and preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + D_t]. \tag{1}$$

Where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is the subjective discount factor; D_t is an additive preference shifter that the depends entirely on government decisions, and exogenous shocks. Households take it as given; the period utility $u(\cdot)$ takes constant-relative-risk-aversion (CRRA) form ; and the consumption basket c is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta + 1)$ between tradable goods c^T

[19] The interaction of sovereign default and the inability to inflate away the debt in the context of the European Debt Crisis is studied in Aguiar et al. (2014) and Aguiar et al. (2015). For the specific case of Spain, Bianchi and Mondragon (2018) explore this issue in an environment with nominal rigidities.

and non-tradable goods c^N, given by

$$c = \left[\omega\left(c^T\right)^{-\eta} + (1-\omega)\left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1).$$

Endowments Each period the economy receives a stochastic endowment of tradable goods $y^T \in \mathbb{R}^+$ and non-tradable goods $y^N \in \mathbb{R}^+$. It is assumed that both endowments follow first-order Markov process independent of each other and of all other stochastic shocks in the model. The tradable good is used as the numeraire.

Private Debt Households can borrow by issuing one-period non-state-contingent debt denominated in units of tradables. Following the standard convention lowercase b denotes the individual level of private debt and while uppercase B denotes the aggregate level. Each period a stochastic fraction π_t of these bonds is defaulted on. Like the endowment shocks, the fraction of defaulted private bonds is drawn from a first-order Markov process independently from all the other stochastic shocks in the model. In light of this, private debt is issued in international competitive capital markets at price q_t which in equilibrium depends on the exogenous default shocks. In addition, private bonds are subject to a collateral credit constraint that requires the market value private debt issuances $q_t b_{t+1}$ to be below a fraction $\kappa_t \geq 0$ of the market value of current income:

$$q_t b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N\right), \tag{2}$$

where p_t^N is the equilibrium price of non-tradable goods in units of tradables. This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit-market access. [20] Theoretically, the constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the endowment owned by a defaulting household. Non-tradable goods enter the collateral constraint because even though foreign creditors do not value the non-tradable good, it is assumed that they can seize these goods in the event of default and sell them in exchange for tradable goods in the domestic market.[21] Given the importance of mortgage lending, where collateral constraints are common, in the private credit build up in Spain in our period of interest, this assumption is particularly relevant here. Additionally, while private debt is explicitly modeled as issued internationally by the households, this credit could also be provided by a competitive domestic financial system that has unrestricted access to world capital markets and faces the same enforcement friction. As noted Section 2 this interpretation is more in line with the events that unfolded in Spain where

[20] See Jappelli (1990).

[21] The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks. See Bianchi and Mendoza, 2018, for a derivation of a similar constraint.

most of the international private borrowing was undertook by banks and mortgage lending institutions that then channeled these funds to households. The assumption that private debt is issued with a short term maturity is consistent with the empirical literature documenting a reduction in the maturity of private bonds issued in advanced economies during the period of interest.[22]

The fraction of market income required as collateral κ_t is stochastic, and drawn from a first - order Markov process and referred to as the financial shock. Stochastic changes in the borrowing capacity can be viewed as shocks that lead creditors to adjust collateral requirements on borrowers. Financial shocks of this form have been shown to be capable of accounting for the dynamics of private financial crisis in advanced economies (e.g. Jermann and Quadrini (2012), and Boz and Mendoza (2014)) as well as balance of payment crisis in emerging economies (e.g. Mendoza (2002), and Coulibaly (2018))). From a modeling perspective these shocks generate fluctuations in private borrowing that are not directly caused by fluctuations in output, nor private default. This is consistent with recent empirical work by Forbes and Warnock (2020) that documents that shocks in international volatility, monetary policy, or sudden stop crisis in similar and/or neighboring countries can cause fluctuations in the lenders' perceptions about the private sector's solvency. In the specific context of Spain during the European Debt Crisis these shocks allow the model to account for a change in investors behaviour towards Eurozone banks in the wake of the Greek Sovereign Debt crisis.

The private default shocks, π, will allow the model to capture the dynamics of nonperforming loans in Spain and imply a more realist cost of borrowing for private debt. However, neither the existence of the financial amplification mechanism nor the government best responses presented later rely on κ_t or π_t being stochastic.[23] These shock will nevertheless generate fluctuations in private borrowing that operate independently from income fluctuations and will impact government policies.

Households' budget constraint Each period, individual households face the following budget constraint:

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t b_{t+1} + y_t^T + p_t^N y^N + T_t. \tag{3}$$

Where T_t is a lump-sum transfer that the households receive from the government. A positive transfer indicates a bailout while a negative of one denotes a lump-sum tax. This transfer is the primary link between the households and and the governments and will be available to the government in all versions of the model. Access to this instrument allows the government to directly modify the household's cash-in-hand without introducing additional distortions. As a result, the interactions that will arise between private and public debt are not a consequence of a restrictive set of tax instruments. In other words, if the government only had access to distortionary taxes the main mechanism of the paper will still be present but will interact with the distortions introduced by the assumed functional

[22] See for instance Gorton et al. (2020) and Chen et al. (2019).

[23] Models with a constant κ and no private default such as Mendoza (2010) are also able to produce private crisis dynamics with realistic business cycle features.

form of the tax instrument. The last subsection will consider the implications of giving the government an additional tax instrument, a linear tax on private borrowing, τ_t as a proxy for macroprudential policies.

3.1.2 Government

Public debt The government can borrow by issuing without commitment a long term bond ($L \geq 0$) on international capital markets *à la* Eaton and Gersovitz (1981).[24] Each period the sovereign chooses either to default ($d \in \{0, 1\}$) or to keep its credit market access by paying its obligations and reissuing new ones. As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), it is assumed that a bond issued in period t promises in case of repayment a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. In particular, a one unit issued in in the current period promises to pay a fraction $(1 - \delta)$ of all remaining debt each following period. This payment structure has the advantage of condensing all future payments obligations into a one-dimensional state variable that is proportional to the quantity of long-term coupon obligations that mature in the current period. Hence the debt dynamics can be summarized by:

$$L_{t+1} = (1 - \delta)L_t + i_t, \tag{4}$$

where L_t is the number of public bonds due at the beginning of period t, and i_t is the amount of bonds issued in period t. As in common in the literature, sovereign debt is assumed to only take values in a a finite and bounded support with $\mathcal{J}$ points[25]. The grid of potential long term debt positions can be summarized by a vector Λ, where L_j is the *jth* element.

$$\Lambda = \Big[L_1, L_2, ..L_{\mathcal{J}}\Big]^T$$

Default Default brings immediate financial autarky and an additive utility cost that is an increasing function of tradable output $\phi(y_t^T)$[26]. For simplicity it is assumed that after one period of exclusion from markets the government returns to international markets with zero debt[27]. More importantly for our purposes, sovereign default does not imply default on private debt, nor an exclusion of private

[24] In related work Arce et al. (2019) show that a regulator that can accumulate a risk free international assets and lump sum tax households can implement the same constrained efficient welfare proposed in Bianchi (2011).

[25] The assumption of a discrete and bounded support is usual in the sovereign default literature with long term debt, see Chatterjee and Eyigungor (2012).

[26] Utility losses from default in sovereign debt models are also used in Aguiar and Amador (2013), Bianchi and Sosa-Padilla (2018), and Roch and Uhlig (2018) among others. An alternative often used is output costs from default. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications.

[27] Assuming an exogenous probability of reentry into financial markets, as in Arellano (2008), would not change the results but require to keep track of an additional state.

agents from financial markets, and this is in contrast to other papers with both public and private international debt such as Mendoza and Yue (2009). This assumption is made for both empirical and conceptual reasons. Empirically, Kalemli-Ozcan et al. (2018), Gennaioli et al. (2018) and Bottero et al. (2020) find that although private borrowing declines during a sovereign default crisis it does not disappear. Conceptually, this paper focuses on endogenizing the interaction between the two types of debt and assuming joint default imposes a direct effect between them.

Government's preferences The sovereign is benevolent, and therefore has the same utility and discount factors as the households. Furthermore, to make the problem computationally tractable, and following Sanchez et al. (2018), each period the government draws a random vector $\boldsymbol{\epsilon}$ of size $\mathcal{J}+1$ of additive preference shocks. One element of the vector is associated with the choice of default while the remaining $\mathcal{J}$ elements are associated with each debt choice on Λ in case of repayment. The elements of the vector are labeled:

$$\epsilon(L_j) = \epsilon_j$$
$$\epsilon^{Def} = \epsilon_{\mathcal{J}+1}$$

$\boldsymbol{\epsilon}$ is drawn from a multivariate generalized extreme value distribution and is i.i.d. over time and independent within the grid of public debt, with mean m and variance $v > 0$.[28] Shocks affecting more directly the default decisions are now common in the literature, see for instance Arellano et al. (2017) , Aguiar et al. (2019) and Aguiar et al. (2020). They are considered an alternative to the i.i.d. income shocks also encountered in the literature (e.g. Chatterjee and Eyigungor (2012)). In particular, the shocks allow the government to break ties between similar portfolio positions. An interpretation of these shocks is that they capture additional costs or benefits of default, such as the perceptions of policy makers of the costs of default. At the same time, as noted by Sanchez et al. (2018), provided that the variance of these shocks is small enough they will have small quantitative consequences in aggregate moments. To summarize,the government's flow utility at time t is therefore:

$$u(C_t) + d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}),$$

where d_t represents the government default decision, C_t is private consumption, and $\phi(y_t)$ is the utility cost of default, and ϵ_t is the additive preference shock that depends on the default and public borrowing decisions. This equation provides an explicit formulation of the additive preference term in the household preferences (1):

$$D_t = d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1})$$

[28]For additional information concerning the distribution of taste shocks see Appendix A.

Government's budget constraint Each period the government's budget constraint is given by government default decision d_t, the public debt dynamics (4), the lump sum transfers T_t.[29] Hence the government faces the following budget constraint:

$$T_t = (1 - d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big] \tag{5}$$

where L_t is the long term debt of the government at the beginning of time t, and L_{t+1} is the long term debt at the end of period t. Finally, Q_t is the price at which the government issues these bonds, which in equilibrium depends on the government's and household's portfolio decisions and the exogenous shocks.

3.1.3 International lenders

Private and sovereign bonds are traded with a continuum of risk-neutral, competitive foreign lenders. Lenders have access to a one-period risk-free security paying a net interest rate r. By a no-arbitrage condition equilibrium bond prices for the private sector are:

$$q_t = \frac{\mathbb{E}_t[1 - \pi_{t+1}]}{1+r}$$

Similarly, equilibrium bond prices when the government repays are given by:

$$Q_t = \frac{\mathbb{E}_t}{1+r}\Big[(1 - d_{t+1})\big(\delta + (1-\delta)Q_{t+1}\big)\Big]$$

In equilibrium, an investor has to be indifferent between investing in a risk-free security and buying a private bond at price q_t. Since private debt is only held for one period and is exogenously defaulted on lenders price it using the probability of default next period. Similarly. an investor has to be indifferent between investing in a risk-free security and buying a government bond at price Q_t, bearing the risk of default. In case of default no public debt is recovered. In case of repayment next period, the payoff is given by the coupon δ plus the market value Q_{t+1}, of the non maturing fraction of the bonds. This pricing equation of public bonds is similar to the one encountered in the sovereign default literature with bonds with multiple maturities bonds such as Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016).

[29] The last subsection will modify this constraint by granting the government access to taxes on private debt.

3.1.4 Resource constraints

Since both types of debt are denominated in tradables the market clearing conditions are:

$$c_t^N = y_t^N, \tag{6}$$

$$c_t^T + (1-\pi_t)b_t = y_t^T + q_t b_{t+1} + (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big]. \tag{7}$$

3.2 Unregulated competitive equilibrium (Baseline)

This subsection defines and characterizes the baseline problem in recursive form. The focus is on the Markov perfect equilibrium where policy rules and prices that expressed as functions of payoff-relevant state variables. In all cases, I denote with a prime end of period levels of private and public debt. The first subsection presents the problem of the private sector taking government policies as given. The second one presents the recursive problem of the government and the formal definition of competitive equilibrium.

Households For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the current level of aggregate private debt B, and the current level of its own debt b. Households take as given the price of non-tradables $p^N(s, L, B)$, the equilibrium price of price bonds $q(s)$, and government's decisions regarding public debt $\mathcal{L}'$, the lump-sum transfer $\mathcal{T}$, and the preference shock $\mathcal{D}$. In addition, in order to form expectations on future states, households need a "perceived" law of motion of aggregate private debt $\mathcal{B}'$. The household's optimization problem in recursive form is:

$$V(s, L, B, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V(s', L', B', b')] \tag{8}$$

subject to

$$c^T + p^N(s, L, B)c^N + (1-\pi)b = y^T + p^N(s, L, B)y^N + q(s)b' + T,$$

$$q(s)b' \leq \kappa[p^N(s, L, B)y^N + y^T],$$

$$T = \mathcal{T}(s, L, B),$$

$$D = \mathcal{D}(s, L, B),$$

$$B' = \mathcal{B}'(s, L, B),$$

$$L' = \mathcal{L}'(s, L, B)$$

In equilibrium, $p^N(s, L, B')$ is the price of nontradables, and $q(s)$ is the price of private bonds. while $\mathcal{T}$, and $\mathcal{L}'$ are solutions to the government's problem, and $\mathcal{D}$ is function of government policies. The solution to the household problem yields decision rules for individual bond holdings $\hat{b}'(s, L, B, b)$, tradable consumption $\hat{c}^T(s, L, B, b)$, and non-tradable consumption $\hat{c}^N(s, L, B, b)$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings, $\mathcal{B}'$, to an actual law of motion, given the representative agent's choice $\hat{b}'(s, L, B, B)$. In a rational expectations equilibrium these two laws of motion must coincide. Let $\mathcal{B}'(s, L, B)$ and $\{C^i(s, L, B)\}_{i=T,N}$ denote the aggregate policy functions for the entire private sector. The solutions to the household problem solve the optimality conditions. These include the budget constraint (3), the credit constraint (2), and the first order conditions of the households' problem. In particular, the household's intratemporal optimality condition pin's down the equilibrium price of nontradables:

$$p^N(s, L, B) = \frac{1-\omega}{\omega}\left(\frac{C^T(s, L, B)}{y^N}\right)^{\eta+1}, \tag{9}$$

Condition (9) is a static optimality condition equating the marginal rate of substitution between tradable and non-tradable goods to their relative price. The equation implies that the price of nontradables is an increasing function of c^T. Consequently, a reduction of c^T causes in equilibrium a reduction in the collateral value (2). In states where the credit constraint binds, this triggers the financial amplification mechanism, whereby a drop in consumption induces a contraction of private borrowing which in turn drives consumption further down. Because of standard consumption-smoothing effects, consumption increases with the cash-in hands of the households. Since the government has the ability to increase the cash-in-hands of the households via the fiscal transfer, mitigating the amplification mechanism is an important incentive for government bailouts.

Government With respect to the private sector and international lenders the government behaves in the current period as a strategic player subject to the resource constraints in addition to its budget constraint. However, since the focus is on Markov perfect equilibrium, the government cannot commit to future default, public borrowing and transfer policies. One could interpret this environment as a game where the government makes intratemporal decisions while taking as given the best response functions of the other players (households and foreign lenders) and also the strategies of future governments who decide policies in the future. [30] Thus, the government takes into account the general equilibrium effects of its policies on the aggregate choices of the private sector (consumption and private borrowing), and prices (the price of nontradables and price of bonds) but cannot choose those functions.

[30]For concision, I equate in the discussion the solutions to the current government policy functions with the strategies of future government. Alternatively one could impose this equality as an equilibrium condition as in Bianchi and Mendoza (2018).

The focus on a Markov perfect equilibrium is important. As noted before, strategically defaultable long term bonds where the government cannot commit to future debt issuances is subject to a time inconsistency problem known as debt dilution. The solutions to the recursive, time consistent problem, do not coincide with the solutions to the sequential problem with commitment. Throughout the paper the focus is on the time consistent policies.[31] Consequently, government default, borrowing and transfer strategies each period will only depend on payoff-relevant states, namely, the exogenous aggregate shocks (s), and the initial levels of private (B) and public debt (L).

Let the best response functions of the private sector be $\tilde{B}'(s, L, B, d, L', T)$, and $\{\tilde{C}^i(s, L, B, d, L', T)\}_{i=T,N}$. Let $\tilde{Q}(s, L', B')$ be the lenders best response. These functions are solutions to the households and lenders problem for a general value of the current policies taking all future policies as given. In a rational expectations equilibrium it must be the case that:

$$\mathcal{B}'(s, L, B) = \tilde{B}'(s, L, B, \boldsymbol{d}(s, L, B), \mathcal{L}'(s, L, B), \mathcal{T}(s, L, B)), \tag{10}$$

$$C^T(s, L, B) = \tilde{C}^T(s, L, B, \boldsymbol{d}(s, L, B), \mathcal{L}'(s, L, B), \mathcal{T}(s, L, B)), \tag{11}$$

$$C^N(s, L, B) = \tilde{C}^N(s, L, B, \boldsymbol{d}(s, L, B), \mathcal{L}'(s, L, B), \mathcal{T}(s, L, B)) \tag{12}$$

$$Q(s, L, B) = \tilde{Q}(s, \mathcal{L}'(s, L, B), \mathcal{B}'(s, L, B)), \tag{13}$$

Using the above notation, the government's problem is:

$$W(s, L, B) = \max_{d \in \{0,1\}} [1 - d] W^R(s, L, B) + d W^D(s, B) \tag{14}$$

Where the default decision is denoted d and the value of the government under default $W^D(s, L, B)$ is:

$$W^D(s, B) = u\left(\tilde{C}^T, \tilde{C}^N\right) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\left[W\left(s', 0, B'\right)\right] \tag{15}$$

$$\text{subject to}$$

$$\begin{aligned}
\tilde{C}^T(s, 0, B, 1, 0, 0) + (1 - \pi)B &= y^T + q(s)B' \\
\tilde{C}^N(s, 0, B, 1, 0, 0) &= y^N \\
T &= 0 \\
D &= \epsilon^{Def} - \phi(y^T) \\
B' &= \tilde{B}'(s, 0, B, 1, 0, 0)
\end{aligned}$$

In default, the government looses access to public borrowing. In contrast to most sovereign debt models, consumption is not pinned down by the endowment shock during a default because house-

[31] For a discussion of policies that remedy debt dilution see Hatchondo et al. (2016) and Aguiar et al. (2019).

holds maintain access to financial markets and are still liable for their obligations. As a consequence, a sovereign default can still leave the economy highly leveraged, albeit in private bonds. In case of repayment the value is:

$$W^R(s,L,B) = \max_{T,L' \in \Lambda} u\big(\tilde{C}^T, \tilde{C}^N\big) + \epsilon(L') + \beta \mathbb{E}_s[W(s',L',\tilde{B}')] \tag{16}$$

subject to

$$\begin{aligned}
\tilde{C}^T(s,L,B,0,L',T) + (1-\pi)B &= y^T + q(s)B' + \tilde{Q}(s,L',B')[L' - (1-\delta)L] - \delta L, \\
\tilde{C}^N(s,L,B,0,L',T) &= y^N, \\
\tilde{B}' &= \tilde{B}'(s,L,B,0,L',T) \\
T &= \tilde{Q}(s,L',B')[L' - (1-\delta)L] - \delta L \\
D &= \epsilon(L')
\end{aligned}$$

The solution to the government's problem yields decision rules for default $\boldsymbol{d}(s,L,B)$, public borrowing $\mathcal{L}'(s,L,B)$ and transfers $\mathcal{T}(s,L,B)$. The preference shifter D is also pinned down by these decisions. The default rule is equal to 1 if the government defaults and is equal to 0 otherwise. In a Markov perfect rational expectations equilibrium as defined below, lenders use these decision rules to price debt contracts. Accordingly, the solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{17}$$

and for public debt:

$$Q(s,L,B) = \frac{1}{1+r} \times \mathbb{E}_s\left[\Big[1-d'\Big] \times \Big[\delta + (1-\delta)Q\Big(s',L',B'\Big)\Big]\right], \tag{18}$$

Where:

$$\begin{aligned}
B' &= \mathcal{B}'(s,L,B), \\
L' &= \mathcal{L}'(s,L,B), \\
d' &= \boldsymbol{d}(s',L',B')
\end{aligned}$$

Definition 1. *A Markov unregulated competitive equilibrium is defined by, a set of value functions* $\{V, W, W^R, W^D\}$*, policy functions for the private sector* $\{\hat{b}, \hat{c}^T, \hat{c}^N\}$*, policy functions for the public sector* $\{\boldsymbol{d}, \mathcal{L}', \mathcal{T}\}$*, a pricing function for nontradable goods* p^N*, pricing functions for public debt* Q *and private debt* q*, best response pricing and allocation functions* $\{\tilde{B}', \tilde{C}^T, \tilde{C}^N, \tilde{Q}\}$ *and aggregate laws of motion* $\{\mathcal{B}', C^T, C^N\}$ *such that*

1. *Given prices* $\{p^N, q\}$*, government policies* $\{\boldsymbol{d}, \mathcal{L}', \mathcal{T}\}$*, and perceived law of motion* $\mathcal{B}'$*, the private*

policy functions $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ *and value function* V *solve the household's problem (8)*

2. *Given bond prices* $\{Q, q\}$ *and aggregate laws of motion* $\{\mathcal{B}', C^T, C^N\}$*, the public policy functions* $\{\boldsymbol{d}, \mathcal{L}', \mathcal{T}\}$ *and value functions* W*,* W^R*, and* W^D*, solve the Bellman equations (14)–(16)*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion* $\{\mathcal{B}'(s, L, B) = \hat{b}'(s, L, B, B), C^T(s, L, B) = \hat{c}^T(s, L, B, B), C^N(s, L, B) = \hat{c}^N(s, L, B, B)\}$

4. *Best response functions* $\{\tilde{B}', \tilde{C}^T, \tilde{C}^N, \tilde{Q}\}$ *evaluated at optimal government policies* $\{\boldsymbol{d}, \mathcal{L}', \mathcal{T}\}$ *are consistent with actual laws of motion* $\{\mathcal{B}', C^T, C^N\}$ *and* Q *, i.e. they satisfy (10)–(13)*

5. *The private bond price function* $q(s)$ *satisfies (17)*

6. *Given public* $\{\boldsymbol{d}, \mathcal{L}'\}$*, and private* $\{\mathcal{B}'\}$*, policies the public bond price* $Q(s, L, B)$ *satisfies (18)*

7. *Goods market clear:*

$$
\begin{aligned}
C^N(s, L, B) &= y^N \\
C^T(s, L, B) + (1 - \pi)B &= y^T + q(s)\mathcal{B}'(s, L, B) + \Big\{1 - \boldsymbol{d}(s, L, B)\Big\}\Big\{Q(s, L, B)\Big[\mathcal{L}'(s, L, B) - (1 - \delta)L\Big] - \delta L\Big\}
\end{aligned}
$$

3.3 Recursive social planner's problem

This section formulates the problem of social planner in the same environment who chooses aggregate allocations subject to resource, implementability and collateral constraints. This formulation is similar to the ”primal approach” to optimal policy analysis. The planner lacks the ability to commit to future defaulting and borrowing decisions. The planner does not set prices and takes optimal pricing functions as given, but in contrast to the households in the previous section, it internalizes how its consumption, and borrowing decisions affect them. Thus the equilibrium price of nontradable goods (p^N) and bonds (q, Q) will enter the SP problem as implementability constraints.[32]

As before, the focus is on the Markov perfect stationary equilibrium. The planner cannot commit to future policy rules, and therefore at any given period it chooses allocations taking as given the strategies of future planners. Equilibrium is characterized by a fixed point of these policy rules, where the policy rules of future planners are consistent with the solutions to the optimization problem of the planner in the current period. Thus, the planner has no incentives to deviate from the future policy rules, thereby making these policies time consistent.

The social planner's optimization problem consists of maximizing the utility of the households (1), subject to the credit constraint (2), the resource constraints (6), (7). Denote by $\{\mathcal{L}^{SP\prime}, \mathcal{B}'^{SP\prime}\}$ the public

[32]This formulation is equivalent to let a planner make all borrowing decisions and transfer the proceeds to competitive households who make all consumption decisions taking prices as given. For more details see Klein et al. (2008) and Bianchi and Mendoza (2018).

and private borrowing and by $\boldsymbol{d}^{SP}$ the default decisions of future planers that SP takes as given. The planning problem is:[33]

$$W^{SP}(s, L, B) = \max_{d \in \{0,1\}} [1-d] W^{SP,R}(s, L, B) + d W^{SP,D}(s, B), \tag{19}$$

where default value of the planner $W^{SP,D}(s, B)$ is:

$$\begin{aligned}
W^{SP,D}(s, B) &= \max_{c^T, B'} u\left(c^T, y^N\right) - \phi(y^T) + \epsilon_{Def} + \beta \mathbb{E}_s\left[W^{SP}(s', 0, B')\right] \\
c^T + B(1-\pi) &= y^T + q^{SP}(s) B', \\
q^{SP}(s) B' &\leq \kappa \left(\frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N + y^T \right), \\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned} \tag{20}$$

and value of the planner under repayment $W^{SP,R}(s, L, B)$ is:

$$\begin{aligned}
W^{SP,R}(s, L, B) &= \max_{c^T, B', L' \in \Lambda} u\left(c^T, y^N\right) + \epsilon(L') + \beta \mathbb{E}_s[W^{SP}(s', L', B')] \\
c^T + B(1-\pi) + \delta L &= y^T + q^{SP}(s) B + Q^{SP}(s, L', B')[L' - (1-\delta)L], \\
q^{SP}(s) B' &\leq \kappa \left(\frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N + y^T \right), \\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r},
\end{aligned}$$

$$Q^{SP}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\left[\left[1 - \boldsymbol{d}^{SP}(s', L', B')\right] \times \left[\delta + (1-\delta) Q^{SP}\left(s', \mathcal{L}^{SP\prime}(s', L', B'), \mathcal{B}^{SP\prime}(s', L', B')\right)\right]\right]$$

Contrary to the government in the baseline version, the planner directly controls the level of private borrowing B'. Moreover, the planner chooses aggregates. As a result, its decisions take into account the general equilibrium effects of all equilibrium prices. This includes the effect of the price of nontradables (9) on the private debt limit (2) and the equilibrium best response of the foreign lenders.

Definition 2. *A Markov stationary socially planned equilibrium is defined by a set of value functions* $\{W^{SP}, W^{SP,R}, W^{SP,D}\}$*, policy functions for allocations* $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \mathcal{B}^{SP\prime}\}$*, and defaulting* $\boldsymbol{d}^{SP}$*, and pricing functions for public* Q^{SP} *and private* q^{SP} *debt, that solve (19) given conjecture future policies* $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \boldsymbol{d}^{SP}\}$

[33] For concision the equilibrium price of nontradables (9) and the resource constraint of nontradables (6) are already incorporated in this formulation. Similarly the price of public bonds Q^{SP} is equated with the equilibrium best response of competitive risk neutral lenders.

3.4 Decentralization with macroprudential policies

This section considers another competitive version of the model where the government gains access to state contingent linear taxes on private borrowing. It also shows that, the allocations that solve this competitive equilibrium problem coincide with the solutions of the socially planned problem presented in the last subsection. The new tax instruments replaces the households' budget constraint (3) with,

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t(1-\tau_t)b_{t+1} + y_t^T + p_t^N y^N + T_t, \tag{21}$$

where τ_t now represents government taxes collected on private borrowing. The introduction of taxes does not modify credit constraint (2). As with all other government policies, tax on private debt are taken as given by the households. At the same time, the government still rebates all taxes to the households using the lump-sum transfer, its budget constraint (5) is modified to:

$$T_t = (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big] + \tau_t q_t B_{t+1} \tag{22}$$

Note that the tax instruments available to the government are not a function of its default decisions. Accordingly, in this formulation bailouts are still possible when the government defaults its public debt. The complete recursive formulation of the problem with taxes can be found in Appendix A, here I stress the main characteristic of the problem.

Proposition 1. *The solutions to the socially planned equilibrium can be decentralized with a state-contingent tax on debt that satisfies:*

$$1 - \boldsymbol{\tau}(s,L,B) = \frac{\beta \mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(s',L',B'), C^{SP,N}(s',L',B'))\Big)\Big] + \mu^{SP}(s,L,B)q^{SP}(s)}{q^{SP}(s)u_T(C^{SP,T}(s,L,B), y^N)} \tag{23}$$

where μ^{SP} corresponds to the Lagrange multiplier associated with the credit constrained in the planner problem (19)

Proof:See Appendix (B)

The proof is done in two steps. First, I show that the planning problem can be formulated as a relaxed version of the competitive equilibrium with taxes. Second, I show that solutions to the planning problem are sufficient to construct policies that satisfy the additional constraints of the competitive equilibrium problem with taxes.

3.5 Mechanism

This subsection explains the intuition behind the main mechanism of the model. It achieves this by comparing the intertemporal optimality conditions of the baseline and planner problems presented before.

Consider the intertemporal optimality conditions of the household's problem (8): [34]

$$q(s)u_T(C^T(s,L,B)) = \beta\mathbb{E}_s[(1-\pi')u'_T(C^{T\prime}(s,L,B))] + \mu q(s), \tag{24}$$

$$0 \le \kappa(p^N(s,L,B)y^N + y^T) - q(s)\mathcal{B}'(s,L,B) \quad \text{with equality if } \mu > 0, \tag{25}$$

where $u_T(.)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$, the marginal utility of consumption of tradables, and μ denotes the Lagrange multiplier on the borrowing constraint. Condition (24) is the household's Euler equation for private debt and (25) is the complementary slackness condition. If $\mu > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit of private debt and repaying next period.

The crucial difference between the baseline model and the planning problem is visible when comparing in the Euler equation of private bonds of each problem.[35] Using the same notation as before and denoting by (SP) the planner policies:[36]

$$\left(u_T^{SP}(C^{SP,T})+\mu^{SP}\psi^{SP}\right)\left(q^{SP}+Q_{B'}^{SP}(\mathcal{L}^{SP\prime}-(1-\delta)L)\right) = \beta\mathbb{E}_s\left[(1-\pi')\left(u_T^{SP}(C^{SP,T\prime})+\mu^{SP\prime}\psi^{SP\prime}\right)\right]+\mu^{SP}q^{SP}, \tag{26}$$

As before the prime notation is used to denote future states and the marginal utility of consumption and Lagrange multipliers are denoted by u_T^{SP} mu^{SP}. Contrary to the Euler equation of the baseline model (24), the planners' Euler equation also includes the marginal effect on the collateral value of an additional unit of in tradable consumption $\psi^{SP} = \kappa(1+\eta)\frac{(1-\omega)}{\omega}(\frac{C^{SP,T}}{y^N})^{\eta}$, public borrowing policies $\mathcal{L}^{SP\prime}$, and marginal effect on the price of public bonds of an additional unit of in tradable consumption $Q_{B'}^{SP}$. These terms capture the additional effects that the planner takes into account when deciding its level of private borrowing. While the first term is common in the Fisherian debt deflation literature, the latter two are encountered in the sovereign debt maturity management literature. I now briefly discuss each of them.

The term ψ^{SP} appears in Bianchi (2011). It captures that, relative to the households in the baseline

[34] These expressions are obtained by assuming that the policy and value functions are differentiable and then applying the standard envelope theorem to the first-order conditions of the household problem and assuming that rational expectations hold

[35] The complete characterization of the optimality conditions of the planning problem is discussed in Appendix (A).

[36] As before these first order conditions are obtained by assuming differentiability and the standard envelope conditions. In addition, it also assumed that the equilibrium price of bonds is differentiable.

model, the planner considers the marginal benefit of an extra unit of private borrowing on the current and future real exchange rate. In the current period, additional borrowing increases the consumption of tradables and therefore the price of nontradable, which in turn relaxes the credit constraint ($\mu^{SP}\psi^{SP}$). At the same, additional private borrowing decreases expected cash-in-hand next period, depressing the expected future price of nontradables ($\mu^{SP\prime}\psi^{SP\prime}$). As a result, additional borrowing increases the probability of facing a binding constraint next period. The absence of this trade-off present in the competitive households problem is leads to private overborrowing.[37]

The terms $\mathcal{L}^{SP\prime}$ and $Q^{SP}_{B'}$ are seen in Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) in models where the government has access to public bonds of different maturities. While the private bond discussed here has a short term maturity it is not strategically defaultable and is instead subject to the collateral constraint. Nevertheless the same trade-off described in their apply here. Private borrowing changes the marginal utility of consumption by changing the value of public debt $Q^{SP}_{B'}$. Keeping all other things equal, an extra unit of private bonds decreases expected wealth next period. This increases the probability of public default, and provides additional incentives for public borrowing in case of repayment. Consequently, private lenders demand a higher premium to for the same level of public debt. Since this effect is not taken into account in the baseline problem spreads are higher.

4 Quantitative analysis

Having defined equilibrium for the two versions of the model I solve both versions numerically. The baseline is solved using time iteration for the private equilibrium and value function iteration for the government problem. The socially planned economy can be solved by value function iteration. More details regarding the numerical solution methods are described in Appendices D and E.

4.1 Calibration

The baseline version of the model is calibrated using Spanish macroeconomic data from 1999 to 2011 and assuming that one period in the model corresponds to one year in the data. I assume that Spain was at the ergodic distribution of the baseline version of the model during this period. The calibration consists of selecting a set of parameters so that the ergodic distribution averages coincide with the relevant macroeconomic moments in the data.

The starting year is chosen to coincide with the creation of the Eurozone. Before this, most Spanish public debt was in domestic currency and therefore its nominal value was subject to government

[37] If $\psi^{SP} < 1$ in some states this can instead lead to underborrowing. In all quantitative specifications considered in the paper this case is never encountered. For specification where this is violated see Schmitt-Grohé and Uribe (2019) and Benigno et al. (2013).

choices. The end year is chosen for two reasons. First, to insure that the large bailouts of 2012 are out of sample. This will allow me to use the comparison between the model and the data responses to the large financial shock as an out of sample validation. Second, choosing 2011 as the end year keeps out of sample the significant European policies introduced in 2012 that conflict with some of the fundamental assumptions underlying the baseline version of the model. In particular, although Spain did not have significant Eurozone prudential policies in 1999-2011, this changed in June 2012. During that month, heads of state and government proposed the creation of the Single Supervisory Mechanism to supervise bank debt within the union. Additionally, in the same month EU leaders agreed to allow the European Stability Mechanism to offer direct help to Spanish Banks, specifically to substitute domestic bailouts. Finally, one moth later in July 2012 then president of the European Central Bank (ECB) Mario Dragui famously signalled the commitment of the institution to do "whatever it takes to preserve the Euro". That statement was interpreted at time as a commitment from the ECB to buy Eurozone public bonds of distressed countries.[38] Given that the baseline version of the model assumes no macroprudential policies, and that the last two mechanisms of supranational bailouts are not explicitly modeled I restrict the sample to the year prior to their introduction.

Functional forms. The utility function is assumed to have a Constant Relative Risk Aversion on the composite CES good

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad \text{with } \sigma > 0$$

The default utility cost is parameterized as follows:

$$\phi(y^T) = \max\{0, \phi_0 + \phi_1 \ln y^T\}$$

As Arellano (2008) and Chatterjee and Eyigungor (2012) discuss, a non-linear specification of the default costs allows the model to reproduce the levels and the standard deviation of spreads in the data. In particular, I follow Bianchi et al. (2018) in specifying the default cost function in terms of utility.

Estimated parameters Table 1 shows the set of parameters that are estimated outside of the model. The risk aversion and elasticity of substitution between tradables and tradables, σ and $1/(\eta+1)$, are set at values frequently encountered in the similar setups.[39] For simplicity and to reduce the state space, the endowment of nontradables, y^N is set to one. I assume the endowment of tradables is drawn from first-order log-normal autoregressive (AR (1)) process. I estimate this process using the

[38]For a discussion of how beliefs can be crucial for sovereign default incentives see Cole and Kehoe (2000), Conesa and Kehoe (2017) and Aguiar et al. (2020).

[39]See for instance Garcia-Cicco et al. (2010), and Bengui and Bianchi (2018)

Table 1: Parameters estimated outside of the model

Description	Parameter	Value
Risk aversion	σ	2.0
Elasticity of substitution	$1/(1+\eta)$	.83
Share of tradables	ω	.39
Persistence of tradables	ρ^y	.75
Volatility of tradables	σ^y	.010
Mean private default rate	$\bar{\pi}$	.021
Persistence private default rate	ρ^π	.82
Volatility private default rates	σ^π	.33
Risk free interest rate	r	.027
Duration of long term bonds	δ	.14

Note: The risk aversion and elasticity of substitution between tradables and non tradables are standard in the literature. The share of tradables is the average share of value added of agriculture, manufacturing and tradable services on GDP. The risk free rate is average yield of one year German bonds. The duration parameter is chosen to match the average bond duration of 6 years of Spanish bonds. The tradable income shock and private default parameters are estimated by fitting a first order autoregressive process on the logs of the tradable share of GDP and share of nonperforming gross loans respectively. All public bonds and yield data is from 1999 to 2011, and the GDP and nonperforming loans process are estimated using the longest available series. The data source for bond yields and nonperforming loans is Bloomberg, while the sectoral GDP series are taken from Eurostat. For details see Appendix C.

cyclical component of linearly detrended tradable GDP for Spain. Since the focus is on fluctuations around the business cycle I use the cyclical component of the linearly detrended share of tradable output.[40] The estimated values for persistence and volatility respectively are, $\rho^y = .75$ and $\sigma^y = .01$. The recursive specification is:

$$\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y \quad \text{with } \varepsilon_t^y \sim N(0, \sigma^y)$$

The value of ω is chosen to replicate the share of nontradable GDP in the data, which is 60%.[41] To compute the model counterpart of this object at the steady state, I use the mean value of external private and public liabilities of $\bar{b}$ and $\bar{L}$ at their calibrated values.[42] The value of ω is then set so that $\frac{\bar{p}^N y^N}{\bar{p}^N y^N + y^T} = 0.60$ where $\bar{p}^N = \frac{1-\omega}{\omega} \frac{y^T - r\bar{b} - \delta r \bar{L}}{y^N}$. Since the average tradable and non-tradable endowments are one, this implies $\omega = 0.39$.

Similarly, I assume that the the exogenous share of private bonds defaulted on each period follows a log normal AR (1) process. The parameters of this process are estimated using the gross share of

[40]Details and sources in Appendix C

[41]Tradable GDP is computed using the value added shares of agriculture, manufacturing and tradable services. More details in Appendix C.

[42]In the baseline calibration described below $\bar{b} = 0.42$ and $\frac{\delta}{1+\frac{1-\delta}{1+r}} \bar{L} = .14$

nonperforming loans as a percent of total loans.[43] The estimation yields an average private default rate $\bar{\pi} = 2.1\%$, a persistence term $\rho^{\pi} = .82$, and a volatility $\sigma^{\pi} = .33$. The recursive specification of the process is:

$$\ln \pi_t = (1 - \rho^{\pi})\bar{\pi} + \rho^{\pi} \ln \pi_{t-1} + \varepsilon_t^{\pi} \quad \text{with } \varepsilon_t^{\pi} \sim N(0, \sigma^{\pi})$$

Two interest rate parameters are estimated outside of the model, r and δ. The risk free interest rate is set to the average yield of the one year German treasury bill over the calibration period, $r = 2.7\%$. One year bonds are chosen as a benchmark to reproduce the maturity of the short term private bond in the model. The duration parameter δ is chosen so that average duration in the model corresponds to the average maturity of Spanish bonds in the data. Using Bank of Spain data I find an average maturity of public debt of 6 years during the period of interest. This calculation is in line with previous estimates of Spanish maturity such as Hatchondo et al. (2016) and Bianchi and Mondragon (2018). The Macaulay definition of duration of a bond given the coupon structure of the model is:

$$D = \frac{1 + \bar{i_L}}{\delta + \bar{i_L}}$$

Where $\bar{i_L}$, is the constant per-period yield delivered by a long term bond held to maturity (forever) with no default.[44] The implied duration is then $\delta = .14$.

Table 2: Calibrated parameters

Description	Parameter	Value	Moment	Target	Model
Discount factor	β	.92	Mean total debt	.56	.56
Volatility taste shock	v	.020	Volatility total debt	.048	.050
Mean financial shock	$\bar{\kappa}$	.45	Mean private debt	.42	.42
Volatility financial shock	σ^{κ}	.020	Volatility private debt	.071	.058
Default Cost	ϕ_0	.31	Mean spread	.0045	.0045
Default Cost	ϕ_1	1.9	Volatility spread	.0061	.0061

Note: Total and private debt are computed using the international investment position as in Section 2, Spreads correspond to the difference between the interest rate paid by Spanish 6-year bonds and their German equivalents. All moments are computed using data from 1999 to 2011. For additional details, see appendix C.

Calibrated parameters. A second subset of six parameters are calibrated to match six aggregate moments from the Spanish data. The calibrated parameters are the two constants in the default cost function ϕ_0 and ϕ_1, the discount factor β, the standard deviation of the taste shocks σ^{ϵ}, and the constants determining the process of the financial shocks $\bar{\kappa}$ and $^{\kappa}$. A summary of all the targets and model counterparts is shown in table 2.

[43] Details and sources in Appendix C.

[44] In the baseline calibration it corresponds to the targeted spread plus the risk free rate, $\bar{i_L} = 3.1\%$

The parameters associated with the default costs ϕ_0 and ϕ_1 are identified using the difference in returns between the average 6-year Spanish bond and the average German bond of the same maturity. The targeted moments are the average and the standard deviation of this spread, and their model counterparts are the average and standard deviation of the spread paid by the long term bond L_t. To compute the sovereign spread in the model that is implicit in a bond price Q in the model I use the definition of the constant per-period yield. Given the coupon structure the yield satisfies:

$$Q = \sum_{j=1}^{\infty} \delta \frac{(1-\delta)^{j-1}}{(1+\bar{i_L})^j}$$

The average targeted spread is 0.45% with a standard deviation of 0.47%, and this implies values for the default cost parameters of $\phi_0 = .3$ and $\phi_1 = 1.9$. The targets are low, when compared to the related literature since they are computed using data from 1999-2011.[45] As mentioned before, this paper deviates from other quantitative models of the sovereign debt crisis in Europe in that it include in the calibration the years 1999-2007 where interest rate spread of Spanish government debt was very close to zero. Since the aim of the paper is to study the link between the build up of private debt during those years and the subsequent sovereign debt crisis, it is important for the model to simultaneously match both the years with zero spreads and large spikes observed during the crisis. To achieve this, I calibrate the model so that the average spread at the ergodic distribution matches the near zero environment and in the next section I see what the model predicts for the latter years when confronted with the same shocks.

The discount factor β and the volatility of the taste shocks σ^ϵ are identified by targeting the average and standard deviation of the total debt in Spain during this period. To compute the model counterparts of these measure I first calculate the international positions of the public and private sectors. The stock of public debt as a percent of output at time t in the model is calculated for our coupon structure as the present value of future payment obligations discounted at the risk-free rate, that is $\frac{\delta}{1+\left(\frac{1-\delta}{1+r}\right)} \times \frac{L_t}{(p_t^N y_t^N + y_t^T)}$. By contrast, the international position of the private sector as a percent of output at time t is simply $\frac{B_t}{(p_t^N y_t^N + y_t^T)}$. At the calibrated targets $\beta = .92$ adn $\sigma^\epsilon = .02$.

Finally, given that an important aspect of the model is the build up in private debt in the years leading up to the crisis, the last two targeted aggregated moments are the average and standard deviations of the private debt in the Spanish data. As noted in the motivation section an important feature of the Spanish data is that the volatility of the private and public positions is higher than the volatility of the total debt. It is therefore important that model matches not only the aggregate positions but also some of its decomposition. This is used to identify the process of financial shocks κ_t. As with the other exogenous shocks of the model, I assume that the financial shock follows a first order normal

[45] For other quantitative studies of the Spanish sovereign spread see Hatchondo et al. (2016) and Bianchi and Mondragon (2018).

AR(1) process of the form:

$$\kappa_{t+1} = (1-\rho^{\kappa})\bar{\kappa} + \rho^{\kappa}\kappa_t + \varepsilon_t^{\kappa} \quad \text{with } \varepsilon_t^{\kappa} \sim N(0, \sigma_{\kappa}).$$

For simplicity, I assume that the persistence parameter coincides with the persistence of tradable income $\rho^{\kappa} = \rho$, while the mean ($\bar{\kappa}$) and volatility parameters (σ_{κ}) are estimated within the model. The model is able to replicate the average debt of the private sector and a higher volatility for the private position relative to the aggregate. However it is not fully capable to replicate the large difference seen in the data. At the baseline calibration $\bar{\kappa} = .45$ and $\sigma^{\kappa} = .02$.

4.2 Policy functions of private and public debt

To shed light on the workings of the model this section shows an analysis of the policy functions for public and private debt accumulation. Both of these variables are functions of the exogenous shocks of the model and of the initial portfolio composition. To fix ideas, this section will first show how do the accumulation of private and public debt varies with respect to the main two exogenous shocks, income and financial shocks, and then how it varies with the endogenous states. Since the government acts first, the issuances of private debt are a function of both the beginning of period debt of the country and the newly issued public debt. Taking into account this best response from the households the government chooses issuances of public debt optimally. For simplicity the initial level of public debt has been set to zero in all the policy functions plots, making all initial debt private. Nevertheless, all of the implications follow through with a strictly positive level of initial public debt. Unless otherwise specified all stocks are expressed as a share of mean output at the ergodic.

Policy functions of private debt Figure 4 depicts the optimal private debt accumulation policy as a function of the income and financial shock. In panel (a), the amount of private debt is shown as a function of the endowment of tradables shock, for the mean value of κ and π_t and for two possible values of beginning-of-period debt. In panel (b), the amount of private debt is shown as a function of the financial shock, for the mean value of y^T, again for two possible values of debt. The figure shows that households end of period private debt are most sensitive to the exogenous shocks when the households are facing a binding credit constraint. If the initial level of debt is low, represented by the dashed line in the plot, next period private debt increases only slightly when income is low or the borrowing capacity is larger (smaller y^T or higher κ). However, if the beginning of period debt is high enough households borrow up to their credit constraint. As a result increases in the endowment of tradables or the value of the financial shock (higher y^T or higher κ) are met with equivalent increases in private borrowing.

Focusing now in the endogenous states, Figure 5 plots the law of motion of next period private

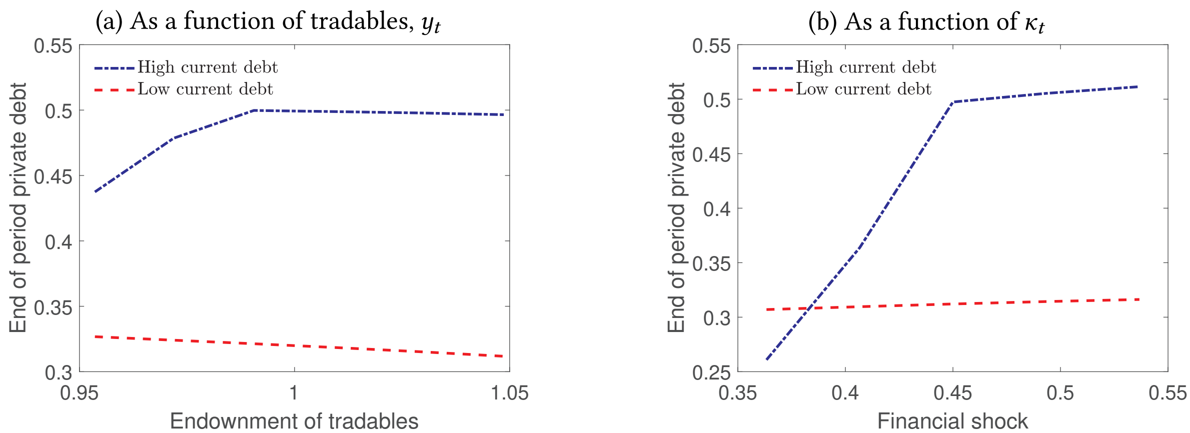

Figure 4: Policy function for private debt relative to the exogenous states

debt relative to the initial level of debt, panel (a), and to next period public debt, panel (b). To help visualize the importance of the credit constraint, the total borrowing capacity of the private sector (debt limit) is plotted alongside the policy functions. In both of these panels the exogenous shocks are kept constant. In the first panel the level of next period public debt is set at zero and in the second panel the starting level of debt one standard deviation above the mean. Panel (a) shows that for low levels of initial debt, the credit constraint slacks and end of period private debt increases with current total debt. The change in the sign of the slope of the policy function indicates the point at which the credit constraint is satisfied with equality but is not binding. Beyond this point, higher levels of initial debt imply a lower level of tradable consumption. This in turn lowers the price of nontradables p^N and further restrict the borrowing capacity of the economy. This is therefore an illustration of the Fisherian debt deflation mechanism discussed in the previous section. As a result similar policy functions can be seen Bianchi (2011) and Bianchi and Mendoza (2018). In contrast, panel (b) depicts the private sector response to the government's end of period debt and is novel to this paper. Low levels of end of period public debt imply a reduction in the fiscal transfer received by the household. At the plotted values, without substantial government assistance (above 8% of output), private borrowing will be constrained. Given the financial amplification mechanism described before, in this constrained area higher government borrowing increase the consumption of tradables, the price of non-tradables, the borrowing limit of the private sector, and private borrowing. This process comes to an halt once government assistance is large enough to ensure that the households will not borrow up to their limit. Further government borrowing, continues to increase the transfer received by the households, but they now respond by borrowing less themselves. In this region, it can be said that private and public debt are imperfect substitutes or alternatively that public debt crowds-out private debt.

Policy functions of public debt The government chooses the level of public borrowing taking into account the household's best responses. Since the choice of public debt is also conditional on the default decision and on the value of taste shock, in what follows I plot the expected level of next

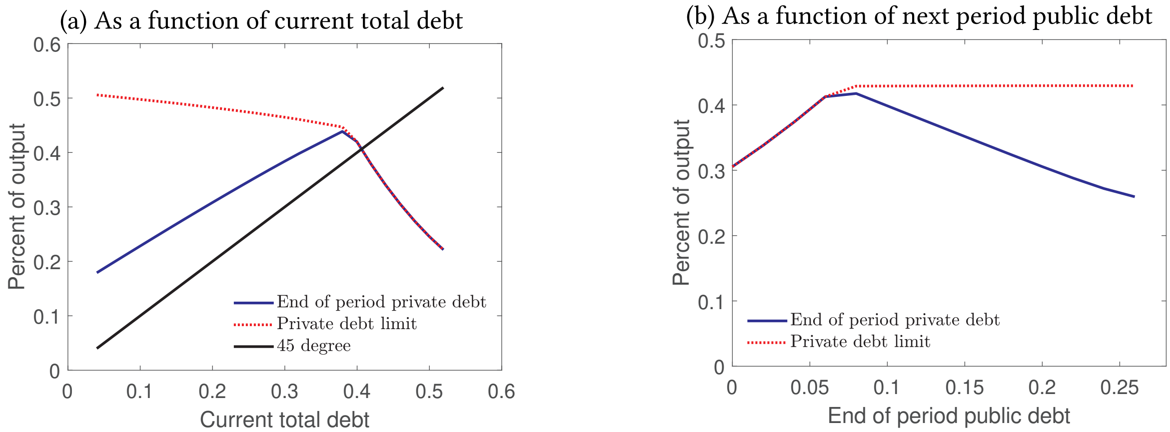

Figure 5: Policy function of private debt relative to the endogenous states

period public debt conditional on repayment. All values are plotted as a share of output. I start by showing public debt as a function of the same exogenous shocks and then show how it changes with initial debt. Figure 6 shows the optimal public debt accumulation policy as a function of the income (panel (a)) and financial shock (panel (b)). Given that the long term public bond provides roll-over benefits relative to the short term bond, as in Arellano and Ramanarayanan (2012), the government finds it optimal to always have strictly positive level of public debt, even when the households are unconstrained. This is can be seen when initial debt is low, when the endowment is high and the financial shock κ is large. Alternatively when the endowment of tradables is low, and the private sector is already highly indebted, public borrowing is limited by the risk of default that translates into higher spreads for public debt. Finally, when the economy is hit by an adverse financial shock (low κ) and the private sector is highly indebted the government issues more public debt. As explained above, public debt in these cases has a twofold beneficial effect. It allows for higher consumption when the households are constrained and therefore not on their Euler equation and it simultaneously relaxes the credit constraint by depreciating the real exchange rate, and this allows for higher private borrowing.

Finally Figure 7 shows optimal level of end of period public debt, conditional on repayment, to a given level of current private debt (blue line). To help visualize the situation of the households the figure also shows the expected end of period private debt. All values are plotted as a share of output, and all exogenous shocks and the initial level of public debt are kept at constant values. Three regions seem to emerge from this figure. To simplify the discussion, I refer to them as the roll-over, precautionary and crisis zones.

When the initial level of debt is low, issuances of public debt are kept relatively constant and low. Public debt is issued in this cases because of its hedging benefits. Long term debt allows the government to partially insure the households against transitory fluctuations in all exogenous shocks. Private debt is monotonously increasing in initial debt while public debt is almost constant. If the

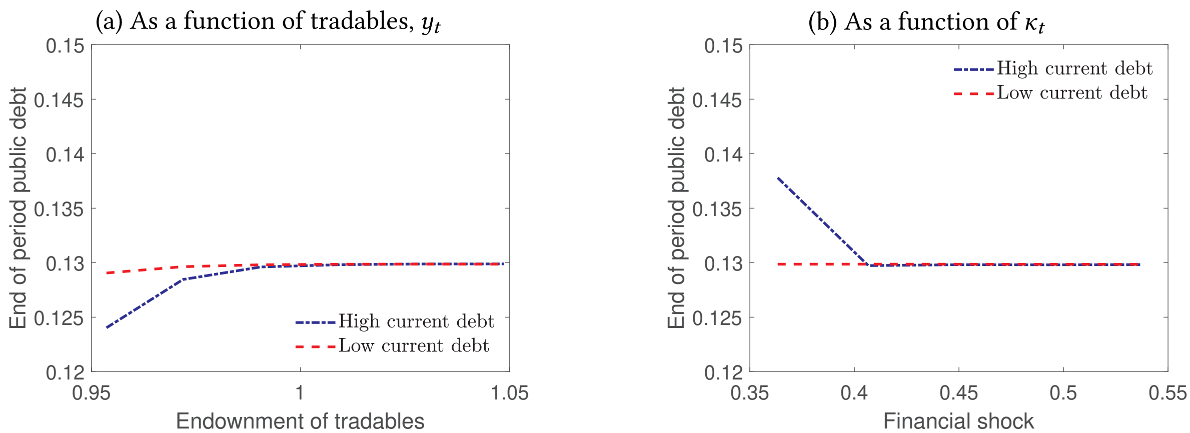

Figure 6: Policy function of public debt relative to the exogenous states

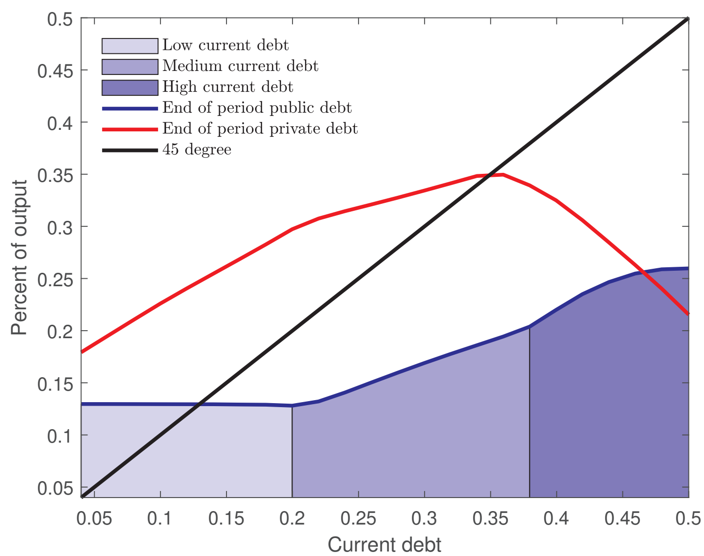

Figure 7: Expected end of period public and private debt as function of initial debt

initial debt is large enough however the constraint for the private sector will bind if the government end of period debt is zero. For this levels of initial debt, households are not expected to face a credit constraint on average but will if the government draws a high enough taste shock for low levels of public debt and both private and public debt are increasing in the initial level of debt. The slope of private debt accumulation is smaller than in the previous region because public debt is expected to crowd out some of the private borrowing of unconstrained households. Finally, if the initial level of debt is very high, it is never optimal to provide a large enough bailout that would prevent the the households from facing a binding constraint. In this region, issuances of public debt are at their highest. This is because in these states the households are households are at their credit constraint, as such public debt always increase the borrowing capacity of the private sector. Conversely, the higher the initial level of debt the more constrained the households are expected to be, even after receiving transfers, and therefore lower the level of end of period private debt.

Comparison with the socially planned economy A social planner who perfectly controls the issuance of both types of assets would also have similar policy functions. In this subsection, we compare those policies to those presented in the baseline model discussed above. Figure 8, compares the evolution of end of period private debt in the baseline and socially planned economy as function of the initial stock of private (panel (a)) and end of period public debt (panel (b)). In both panels, overborrowing in the baseline economy is present only when the constraint does not bind. When the constraint binds, private borrowing is pinned down by the resource constraints and therefore there is no room for disagreement between the models. The sources of private overborrowing in both panels however are different. In the first panel, households overborrow for low levels of initial private debt because they don't internalize the marginal effect of their debt on the probability of facing a binding constraint next period. Once again, this figure is similar to the one observed in other models of private overborrowing with a credit constraint that is increasing in the price of nontradables, such as Benigno et al. (2013) and Bianchi (2011). In contrast in the second panel, overborrowing is now also caused by a smaller private borrowing in response to government issuances of public debt. Contrary to the planner the households do not internalize that higher government debt increases the probability of sovereign default next period and that this also increases the premium paid by the government to issue debt.

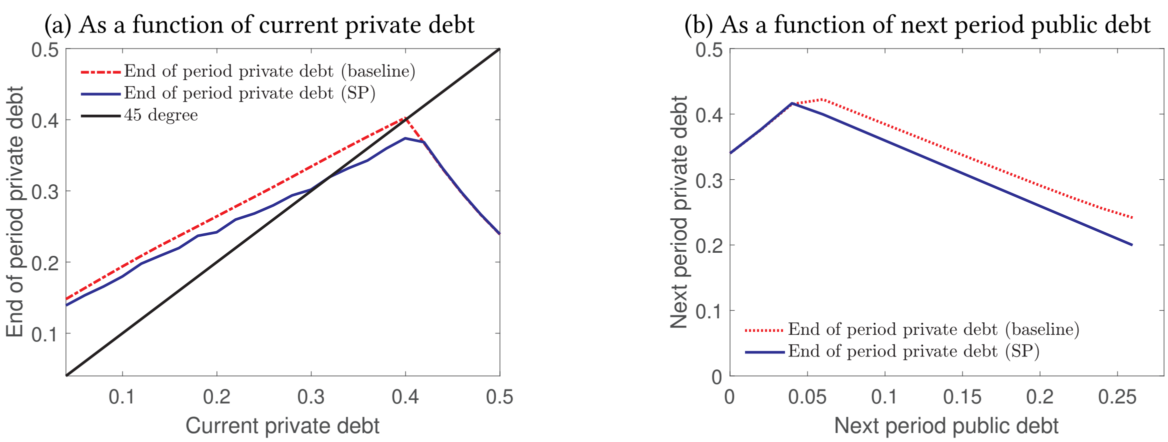

Figure 8: Policy function of private debt, baseline vs SP

Figure 9 compares the expected optimal level of public borrowing in the baseline and socially planned economies as a function of the initial stock of private debt conditional on repayment. As before the households private debt responses are plotted alongside. This figure shows again private overborrowing in the baseline model when the constraint does not bind. Additionally, it also shows that public borrowing is higher in the planned economy then initial debt is small or medium. In these areas the planner internalizes that it is approaching it's borrowing capacity on the private bond and substitute some of that borrowing with the public bond. The government in the decentralized economy would like to implement the same policy but does not control the issuances of the private bond. Correctly predicting that the household will not reduce private borrowing at the same rate as a planner would do, the government decides to issue less public debt. The differences in public borrowing are however quantitatively smaller than the differences in private borrowing.[46] As shown in the next section, when we compare the ergodic distributions, the small differences in public borrowing will not be enough to quantitatively compensate for the fact that the baseline economy faces the credit constraint more often than the planned one, causing the government to more frequently issue public debt to relieve the households.

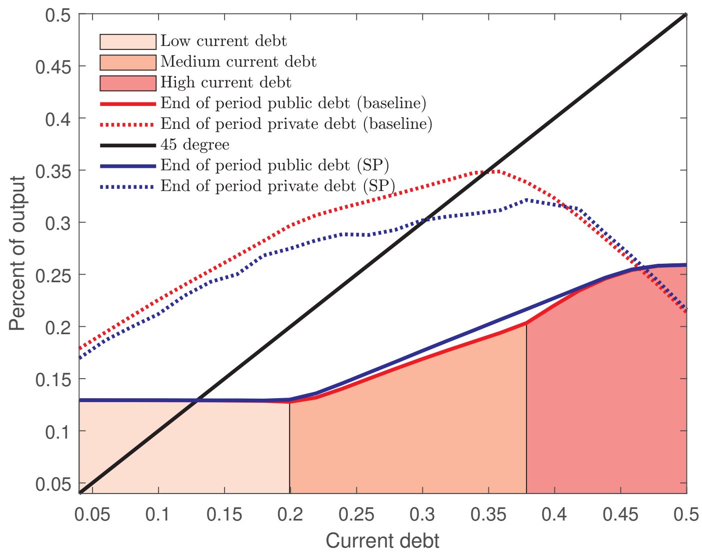

Figure 9: Expected end of period public and private debt as function of initial debt

Finally, I conclude the subsection by comparing the evolution of the expected interest rate spreads paid on public debt in both economies conditional on repayment. Figure 10 plots the expected spreads as function of the initial private debt. These spreads are computed at the same states than Figure 9. This allows us to see that the spreads peak when the debt enters the high debt zone. The shape of this plot shows that the interest rate spreads are mostly driven by the evolution of total end of period debt. Default is more likely in a more indebted economy. Up until the moment the constraint binds both private and public debt are increasing with initial debt. Beyond this point however, the private sector deleverages at a rate that outpaces the increase in public borrowing. As a result, total indebtedness decreases. This reduces the probability of default and the spread. In all cases the spreads are higher in the baseline economy. This is the case even though figure 10 shows that for moderate or high levels of debt areas the planner is expected to issue more public debt. The gap in interest rates exists because total debt is higher in the baseline economy due to household overborrowing. Anticipating this, foreign lenders demand a higher spread from the government.

[46]When the constraint binds, this figure also shows a small amount of underborrowing in the baseline economy. This is caused by fact that the planner faces a more favorable price schedule and therefore can relax the constraint a little bit more.

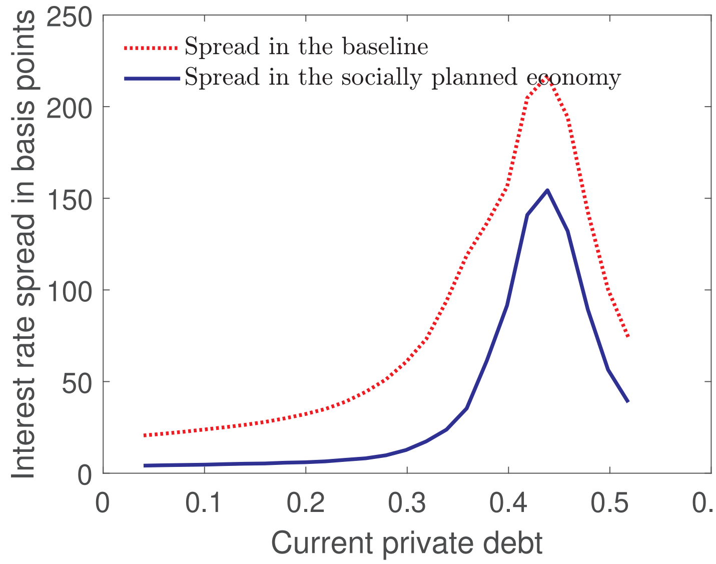

Figure 10: Expected spreads on public debt as function of initial debt

4.3 Business cycle properties

This subsection evaluates the model's quantitative performance by comparing untargeted moments from the data with moments from the model at the ergodic distribution. I compute the model's moments by simulating the exogenous processes for 10000 periods and eliminating the first 500 observations. The moments form the data are computed with annual data for the sample period 1999-2017. The longer sample period is chosen to avoid small sample bias. Similar results are obtained when restricting the sample to 1999-2011. Real GDP is the measure of output and consumption corresponds to total final consumption expenditure and is measured in real terms.[47] The current account and trade balance are computed as percent of GDP. All data is from Eurostat and additional details can be found in Appendix C.

Table 3 compares the unconditional second moments in the Spanish data with the moments computed using the ergodic distribution calibrated baseline model. The model successfully captures the volatility of consumption, the current account and trade balance and overestimates the volatility of output. Nevertheless, the model correctly predicts that the volatility of output will exceed the volatility of consumption. This is in contrast to traditional sovereign default models where the opposite is true.[48] This suggest that access to the international private debt is important to simultaneously achieve a volatility of consumption and net capital flows consistent with the Spanish data. Table 3 also computes correlations between output and the other bossiness cycles statistics. The model correctly predicts the sign of all the correlations.

[47] As before GDP and consumption data are detrended.

[48] Neumeyer and Perri (2005) find that consumption is more volatile than output in emerging economies while the opposite is true in advanced economies. Spain is listed by the IMF as an advanced economy.

Table 3: Untargeted business cycles statistics

Statistic	Data	Calibration
Volatility		
Output	.032	.062
Consumption	.031	.037
Current account	.041	.046
Trade balance	.034	.040
Correlations		
Output - Consumption	.97	.99
Output - Current account	-.59	-.91
Output - Trade balance	-.54	-.94
Output - Spread on public debt	-.46	-.10
Public debt - Spread on public debt	.53	.28

Note: Output corresponds to real gross domestic product and consumption corresponds to real final consumption expenditure, both series are detrended. Current account and trade balance are measured as a percent of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish 6-year bonds and their German equivalents. For additional details, see Appendix C.

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5 Results

This section summarizes the quantitative finds of the paper. The first subsection details the results obtained by comparing the baseline and regulated economy. The second subsection details the quantitative exercises conducted to simulate the counterfactual dynamics during the Spanish Debt Crisis.

5.1 Social planner and baseline economies at the ergodic

Table 4 presents the first set of quantitative results of the paper. The table shows the values of the calibrated aggregate moments at the ergodic for the data, the baseline, and the centralized economy. The baseline version of the model is calibrated to matching the moments from the data, the socially planned economy is not. Instead, I use the calibrated parameters of the baseline calibration to compute the ergodic distribution of the planned problem. The average private debt at the ergodic distribution for the social planner is 36% of output, while in the baseline case it is 41%. This difference of 5% of output is the estimate of the total amount of excessive private debt in Spin before the crisis. The table also shows that the increase in private debt, in the baseline relative to the planner, is insufficient to explain the increase in overall indebtedness. The baseline economy accumulates on average more public debt, around 2% of output. The explanation for this can be seen in the bottom half of the table. In this part I compute four measures of aggregate well-being for the baseline and planned economy, namely the probability of a binding credit constraint, the probability of a financial crisis, the probability of a sovereign default, and a measure of welfare gains. The credit constraint binds more frequently under the baseline. As explained in the previous section optimal government borrowing is higher when the constraint binds. As a result, average public debt is higher under the baseline because the government has to respond more often to crisis. Related literature usually defines a financial crisis as an episode with a binding constraint and contraction of more than one standard deviation below the mean of the current account of the private sector.[49] Following this definition I find that adopting macroprudential policies reduces the probability of a financial crisis by 240 bps on average.

Table 4: Baseline and social planner aggregate moments at the ergodic

Moment	Data	Baseline	Social planner
Total debt	.56	.56	.49
Private debt	.42	.42	.37
Mean spread	.0045	.0045	.00034
Volatility debt	.048	.050	.027
Volatility private debt	.071	.058	.071
Volatility spread	.0061	.0061	.00030
Probability of a binding constraint	-	.099	.024
Probability of a financial crisis	-	.025	.0010
Probability of default	-	.0046	.00030
Welfare gains	-	-	.0041

Note: All calibrated parameters are kept constant in the computation of the socially planned economy. A financial crisis is defined as a episode in which the credit constrained binds and the current account of the private sector contracts by more than one standard deviation below the mean. Welfare gains are calculated as the proportional increase in permanent consumption under the baseline. Debt levels in the data are calculated using the international investment positions more detail in Appendix C.

Similarly, table 4 shows information about the spreads paid on public debt relative to the risk free rate. In the planned economy spreads are on average an order of magnitude below their baseline counterparts. The reduction in the spread is caused both by the fact that the planner borrows less in general and that it faces less often a binding constraint. The result is also consistent with the smaller average probability of sovereign default in the regulated economy relative to the baseline. Finally, table 4 also displays in its last line the welfare gains of moving from the baseline to the planned economy. The welfare gains are calculated as the proportional increase in consumption for all possible future states that would make the households indifferent between staying in the baseline and moving to the centralized equilibrium. This measure explicitly incorporates the cost of lower consumption in

[49] See for instance Bianchi (2011) and Bengui and Bianchi (2018).

the transition to the ergodic state of planned economy. Taking advantage of the homoscedasticity of the utility function, the expected welfare gain in state (s_0, L_0, B_0) are:

$$\boldsymbol{\theta}(s_0, L_0, B_0) = \left(\frac{\boldsymbol{W}^{SP}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1}{\boldsymbol{W}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1 \tag{27}$$

On average at the ergodic state, households would need to receive a permanent increase of 0.36% in consumption to be indifferent between the two economies. These welfare gains are larger than the ones encountered in the literature. In Bianchi (2011), the welfare gains from correcting the overborrowing externality are around 0.13%. To further illustrate this point I use the calibrated parameters to solve alternative versions of the model that are closer to the existing literature and compare the welfare gains from transitioning to our baseline framework in the extensions section of the paper.

5.2 Simulating the 2012 debt crisis

This section uses the data, and the calibrated models to provide counterfactual to the events that unfolded in Spain between 2008 and 2015 and shed light on what optimal macroproduential policies would have looked like and could have achieved. The idea is to feed to the model the exogenous shocks that affected Spain during this period and contrast the endogenous response in terms of debt and spreads of the baseline and socially planned version with their data counterparts. Two exercises are conducted. In the first one, only the three fundamental exogenous shocks are fed, namely the income shock, the private default shock, and the financial shock. The public and private debt, as well as the spread on public bonds are then allowed to respond endogenously to these shocks. The second exercise disentangles the effect of macroprudential policies on public spreads, by also taking the evolution of public debt directly from the data. I then compute for this sequence of shocks the model-predicted evaluations of private debt and interest rate spread. This second exercise therefore corresponds to the endogenous response of competitive agents (households and lenders) when fixing government policies to their data counterparts.

In both exercises, the exogenous income shock, y_t is taken directly from the Spanish GDP in tradable data. Similarly, the share of private bonds defaulted on, π_t matches exactly the data on gross nonperforming loans during this time period. The taste shocks, ϵ_t, are all set to zero in the first exercise and selected to perfectly match the evolution of the public debt in the second. The financial shock κ_t is always unobserved in the data. To circumvent this problem, the particle filter method proposed by Bocola and Dovis (2019) is applied to this problem. Additional details about the particle filter method can be found in appendix F, here I present a summary of the methodology.

The baseline model defines a nonlinear state-space system

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t$$
$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t)$$

where $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$ is the state vector, and ε_t the vector collecting all the innovations in the three structural exogenous shocks. The vector of observables, $\boldsymbol{Y}_t$, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds.[50] The vector e_t represents uncorrelated Gaussian measurement errors, and is equal to the difference between the data aggregates $\boldsymbol{Y}_t$ and their model counterparts $g(\boldsymbol{S}_t)$. The functions $g(\cdot)$ and $f(\cdot)$ come from the calibrated numerical solutions of the baseline model. The realizations of the state vector are estimated by applying the particle filter to this system of equations and data from 2008 to 2015. The process yields a path of financial shocks and a set of initial endogenous states that are then feed to the social planner policy functions $f^{SP}(\cdot)$ to generate the allocations of debts and spreads that would have emerged had macroprudential policies been in place. Note that the social planner functions are not used to estimate the system, and are only used ex-post to generate counterfactual policies. Finally, the implied tax on borrowing, necessary to implement the planner allocations in a competitive equilibrium are also constructed.

In the first exercise, I assume that only tradable output and nonperforming private loans are observed with no error. This leaves three observable variables not perfectly fitted in $\boldsymbol{Y}_t$, public debt, private debt, and spreads. To match them there are three stochastic variables in $\boldsymbol{S}_t$, namely B_t, L_t, and κ_t. By setting the variance of all measurement errors to 1% of their sample variance, one can compute the filtered path of these three stochastic variables that is consistent with the data. Figure 11 summarizes the results of this exercise.

The baseline model (plotted in dashed red lines) is able to capture the main events of the crisis. In particular, the magnitude of the 2012 public bailout, around 12 % of GDP, financed by an equivalent increase in public debt. This leads to an increase in the interest rate spread on public bonds to around 3%, equivalent to 80% of the increase observed in the data. The baseline model is less successful at tracking the evolution of public debt after 2012, predicting a lower indebtedness than what is observed in the data. Similarly the interest rate spread increase in the model before 2012 is below its data counterparts. Two observations could partially explain these discrepancies. First, while the model captures some of the fluctuations in the external conditions for borrowing via the financial shock it may be the case that this is not enough to fully replicate the uncertainty around government bonds of Eurozone countries around the worst years of the Greek debt crisis, Secondly, there is no model counterpart to the Mario Draghi speech of June 2012, at the effect it had on public spreads. Accordingly, the model

[50] As in the calibration the linearly detrended cyclical component of tradable output is used. Public debt is initialized at zero and initial private debt is adjusted to match the composition in the data.

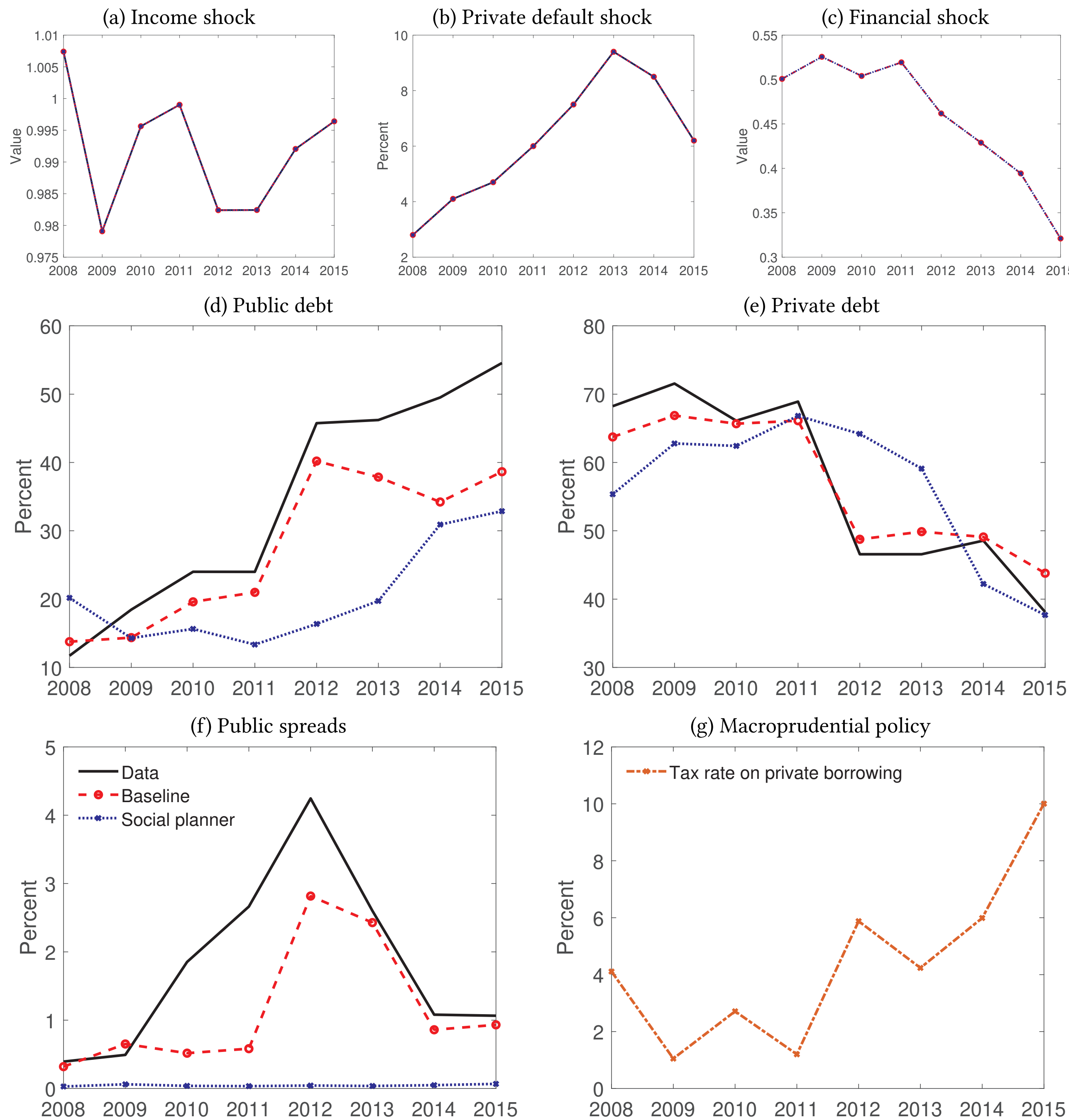

Figure 11: Evolution of debts, taxes, spreads and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding exactly observed income shocks, nonperforming loans and the most likely path of financial shocks from the particle filter. Public debt, private debt and spreads are the particle filtered weighted averages. Both debt series are expressed as a percent of output while nonperforming loans are expressed as a percent of gross loans. Taxes and interest rate spreads are expressed in percent. Data sources can be found in appendix C while details on the particle filter can be found in appendix F.

expects less public debt than the data to replicate the drop in spreads observed in the 2013-2015 period. All things considered the baseline model predicts a pattern of public debt, private debt, and spreads that is consistent with the data and validates the approach of the paper.

Having validated the positive approach I now turn to the normative counterfactual. In contrast to the baseline case, the economy with macroprudential policies is characterized by a smooth transition from private to public debt. The main bailout is delayed to 2014, coinciding with an uptick in tradable income and therefore an increase in default costs. This transition allows the government to maintain the interest rate spread close to zero throughout the period and cuts in half the size of the largest yearly bailout to around 10% of GDP. It is important to note that private debt is lower in the planned economy outside of the 2011-2013 crisis window. This is in part due to the presence of a macroprudential tax on private borrowing that are on average 4% during this time period. Similarly, public debt in the economy with macroprudential policies is significantly below the levels observed in the data for most of the period, and importantly even after the bailouts take place.

As noted above in the first exercise the 2012 spike on public debt spread would have been completely avoided if the government would have managed public borrowing and macroprudential policies optimally. In order to disentangle how much of the difference is caused by lower public borrowing and how much is caused by macroprudential policies a second counterfactual exercise is proposed. Taking advantage of the probabilistic framework of the model, I can select the taste shocks ϵ_t such that the path of public debt coincides exactly with the one observed in the data in both the baseline and planned economies. The particle filter is then conducted to back out the implied financial shock and the filtered endogenous evolution of private debt and the sovereign spread. As before I then feed the path of exogenous shocks to the planner policy functions to compute the counterfactual private debt, spreads and taxes. The result of the second exercise are presented in figure 12.

The model once again predicts a drop of 20% of GDP in private borrowing close in magnitude to the one observes in the data. Overall private debt is however around 5% below the one observed in the data for most of the period. The spread on public debt is close to zero before 2008, then it raises to a peak and 2012, followed by a return to low spreads from 2013 onward. The magnitude of the increase between 2008 and 2012 is not the same under the baseline and the data however, with the model experiencing a larger raise in 2012. The small mismatch in private debt and the larger spread are both consequences of the requirement to fit public debt exactly in this exercise. Nevertheless I argue that the baseline model is capable of replicating the patterns of interest.

The final quantitative finding of the paper comes from comparing the evolution of the data and the socially planned economy. Indebtedness in the planned economy is still lower than in in the baseline. Private debt is always below its baseline and data counterparts. In this exorcise, the data on the evolution of public debt imposes that the main bailout takes place in 2012. As a result, the public spread in the planned economy now also peaks in 2012. The peak value is .4%, or 380 basis points below the spread observed in the Spanish data. This is the estimate of the increase in the severity of the sovereign debt crisis caused by lack of macroprudential policies. It should be restated here that this estimate is obtained while keeping the path of public debt at their data values. The reduction in the spread is therefore not a consequence of lower public borrowing but of macroprudential policies.

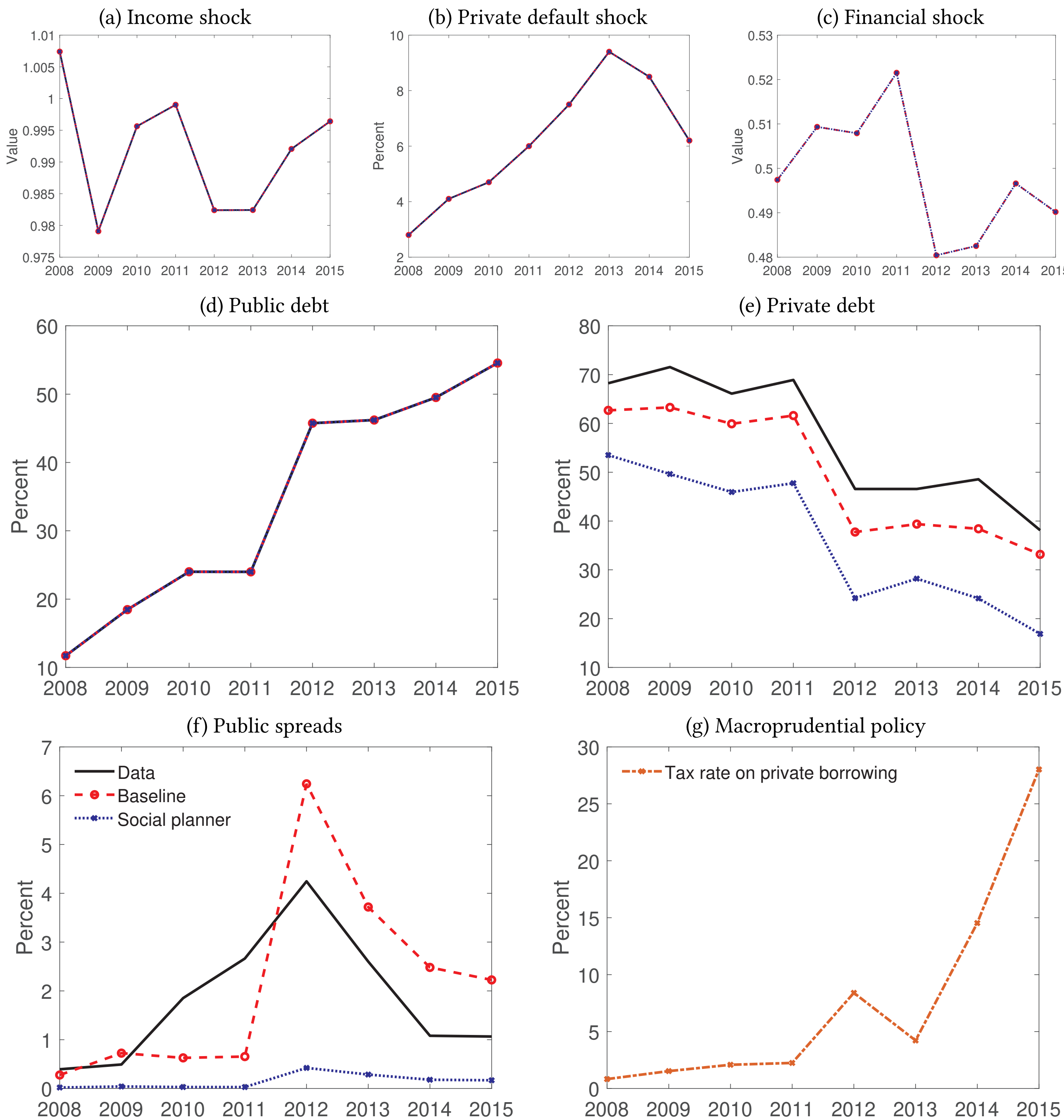

Figure 12: Evolution of debts, taxes, spreads and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding exactly observed income shocks, nonperforming loans and taste shocks to match exactly public debt, and the most likely path of financial shocks from the particle filter. Private debt and spreads are the particle filtered weighted averages. Both debt series are expressed as a percent of output while nonperforming loans are expressed as a percent of gross loans. Taxes and interest rate spreads are expressed in percent. Data sources can be found in appendix C while details on the particle filter can be found in appendix F.

In the planned economy the lenders internalize that the regulator will pair the increase in public borrowing with high taxes on private borrowing (on average 8% over the period) leading to a reduction in private debt, and therefore limiting the probability of a future sovereign default.

6 Conclusions

This paper provides a simple framework to understand the pattern of debt and spreads in Spain that culminated in the 2012 crisis. The model focuses on the interaction between systemic externalities in private borrowing and sovereign default. The combination of decentralized private actors constrained in their international borrowing by a fraction of the market value of their current income and a benevolent government capable of assisting them with public funds creates a cycle of private and public crises. The cycle begins with a private build up of debt when financial conditions are favorable. During this time the government issues low public debt and faces a low spreads. As the private sector accumulates more and more debt it becomes increasingly vulnerable to a financial crisis. Eventually an adverser exogenous shocks materializes and the households face a tight borrowing limit. In the model I allow for a crisis to be triggered by the following exogenous factors: slowdowns in output, contractions of domestic credit, and shock to international financial markets. When confronted with this imminent painful private deleveraging, the government responds with fiscal transfers financed by new issuances of public debt. Bailouts have a multiplicative positive effect in these circumstances. A positive transfer causes an appreciation of the value of current income, and through this channel they increase the borrowing capacity of the private sector. To this, the the credit constrained households respond with additional private borrowing that enables higher consumption. Unfortunately these gains come at the expense of raising the spectrum of a sovereign default. In all cases spreads paid on government debt increase, and in some particularly adverse circumstances default materializes. Once the public debt has been repaid or defaulted upon the cycle can restart.

The paper also shows how a benevolent government can impose macroprudential policies, to restrict private borrowing and break the cycle. An important contribution of the paper is to quantify the costs associated with not having macroprudential policies in place. The case study is Spain both before and during the European Debt crisis 2012-2015. I estimate that in the lead up to the crisis 2007-2011, excessive private debt in Spain was equivalent to 5%of GDP. As a result, the annual probability of experiencing a financial crises was 240 bps. above the social desirable level. Finally, the paper argues that the lack of macroprudential policies exacerbated increase in the spread paid on Spanish debt in 2012 by 380 bps.

Several interesting avenues for future research remain open. It will be fruitful to investigate the quantitative consequences of adding moral hazard into the motivations for private overborrowing. Alternatively, one could also explore how budgetary covenants or other fiscal limits could simultaneously deal with the incentives for bailouts and with public debt dilution as in Hatchondo et al. (2016) and in Aguiar and Amador (2018). A final extension would be to investigate how a monetary response to private overborrowing would interact with the fiscal response presented here.

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Appendices

A Recursive competitive problem with taxes

For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the initial level of aggregate private debt B, and the initial level of its own debt b. At the same time, households take as given the price of non-tradables $p^{N\tau}(s, L, B)$, the equilibrium price of price bonds $q^\tau(s)$, and government's decisions regarding public debt $\mathcal{L}^\tau$, the lump-sum transfer $\mathcal{T}^\tau$, taxes $\boldsymbol{\tau}$, , and the preference shock $\mathcal{D}^\tau$. With perceived law of motion of aggregate private debt $\mathcal{B}^{\tau\prime}$. The household's optimization problem in recursive form is:

$$V^\tau(s, L, B, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V^\tau(s', L', B', b')] \tag{28}$$

$$\text{subject to}$$

$$c^T + p^{N,\tau}(s, L, B)c^N + (1 - \pi)b = y^T + p^{N,\tau}(s, L, B)y^N + q^\tau(s)(1 - \tau)b' + T,$$

$$q^\tau(s)b' \leq \kappa[p^{N,\tau}(s, L, B)y^N + y^T],$$

$$T = \mathcal{T}^\tau(s, L, B),$$

$$D = \mathcal{D}^\tau(s, L, B),$$

$$B' = \mathcal{B}^{\tau\prime}(s, L, B),$$

$$L' = \mathcal{L}^{\tau\prime}(s, L, B)$$

$$\tau = \boldsymbol{\tau}(s, L, B)$$

Using the same notation as in the baseline case for the off-equilibrium allocation functions of the private sector $\tilde{B}^{\tau\prime}(s, L, B, d, L', \tau, T)$, and $\{\tilde{C}^{i,\tau}(s, L, B, d, L', \tau, T)\}_{i=T,N}$, and public bond pricing $\tilde{Q}^\tau(s, L', \tau, B')$ function. The government's problem is:

$$W^\tau(s, L, B) = \max_{d \in \{0,1\}} [1 - d]W^{R,\tau}(s, L, B) + dW^{D,\tau}(s, B) \tag{29}$$

Where the default decision is denoted d and the value of the government under default $W^{D,\tau}(s, L, B)$

is given by:

$$W^{D,\tau}\big(s,B\big) = \max_{\tau} u\Big(\tilde{C}^T, \tilde{C}^N\Big) + \epsilon^{Def} - \phi(y^T) + \beta\mathbb{E}_s\Big[W^{\tau}\big(s',0,B'\big)\Big] \tag{30}$$

subject to

$$\begin{aligned}
C^{\tilde{T},\tau}(s,0,B,1,0,\tau,0) + (1-\pi)B &= y^T + q^{\tau}(s)(1-\tau)B' \\
C^{\tilde{N},\tau}(s,0,B,1,0,\tau,0) &= y^N \\
T &= \tau B' \\
D &= \epsilon^{Def} - \phi(y^T) \\
B' &= \tilde{B}^{\tau\prime}(s,0,B,1,0,0)
\end{aligned}$$

In case of repayment the value is:

$$W^{R,\tau}(s,L,B) = \max_{T,\tau,L'\in\Lambda} u\big(\tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}\big) + \epsilon(L') + \beta\mathbb{E}_s[W^{\tau}(s',L',\tilde{B}')] \tag{31}$$

subject to

$$\begin{aligned}
C^{\tilde{T},\tau}(s,L,B,0,L',\tau,T) + (1-\pi)B &= y^T + q^{\tau}(s)B' + \tilde{Q}^{\tau}(s,L',\tau,B')[L'-(1-\delta)L] - \delta L, \\
C^{\tilde{N},\tau}(s,L,B,0,L',\tau,T) &= y^N, \\
\tilde{B}' &= \tilde{B}^{\tau\prime}(s,L,B,0,L',T) \\
T &= \tilde{Q}^{\tau}(s,L',\tau,B')[L'-(1-\delta)L] - \delta L \\
D &= \epsilon(L')
\end{aligned}$$

The solution to the government's problem yields decision rules for default $\boldsymbol{d}^{\tau}(s,L,B)$, public borrowing $\mathcal{L}^{\tau\prime}(s,L,B)$, taxes $\boldsymbol{\tau}(s,L,B)$ and transfers $\mathcal{T}^{\tau}(s,L,B)$. The preference shifter D^{τ} is also pinned down by these decisions. The solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q^{\tau}(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{32}$$

and for public debt:

$$Q^{\tau}(s,L,B) = \frac{1}{1+r} \times \mathbb{E}_s\left[\left[1-d'\right] \times \left[\delta + (1-\delta)Q^{\tau}\left(s',L',B'\right)\right]\right], \tag{33}$$

Where:

$$\begin{aligned}
B' &= \mathcal{B}^{\tau\prime}(s,L,B),\\
L' &= \mathcal{L}^{\tau\prime}(s,L,B),\\
d' &= \boldsymbol{d}^{\tau}(s',L',B')
\end{aligned}$$

Definition 3. *A Markov regulated competitive equilibrium with taxes is defined by, a set of value functions $\{V^{\tau}, W^{\tau}, W^{R,\tau}, W^{D,\tau}\}$, policy functions for the private sector $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$, policy functions for the public sector $\{\boldsymbol{d}^{\tau}, \boldsymbol{\mathcal{L}}^{\tau\prime}, \boldsymbol{\tau}, \boldsymbol{\mathcal{T}}^{\tau}\}$, a pricing function for nontradable goods $p^{N,\tau}$, pricing functions for public debt Q^{τ} and private debt q^{τ}, best response pricing and allocation functions $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}, \tilde{Q}^{\tau}\}$ and aggregate laws of motion $\{\mathcal{B}^{\tau\prime}, C^{T,\tau}, C^{N,\tau}\}$ such that*

1. *Given prices $\{p^{N,\tau}, q^{\tau}\}$, government policies $\{\boldsymbol{d}^{\tau}, \boldsymbol{\mathcal{L}}^{\tau\prime}, \boldsymbol{\tau}, \boldsymbol{\mathcal{T}}^{\tau}\}$, and perceived law of motion $\mathcal{B}^{\tau\prime}$, the private policy functions $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$ and value function V solve the household's problem (28)*
2. *Given bond prices $\{Q^{\tau}, q\}$ and aggregate laws of motion $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}$, the public policy functions $\{\boldsymbol{d}^{\tau}, \boldsymbol{\mathcal{L}}^{\tau\prime}, \boldsymbol{\tau}, \boldsymbol{\mathcal{T}}^{\tau}\}$ and value functions W^{τ}, $W^{R,\tau}$, and $W^{D,\tau}$, solve the Bellman equations (29)–(31)*
3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(s,L,B) = \hat{b}^{\tau\prime}(s,L,B,B), C^{T,\tau}(s,L,B) = \hat{c}^{T,\tau}(s,L,B,B), C^{N,\tau}(s,L,B) = \hat{c}^{N,\tau}(s,L,B,B)\}$*
4. *Best response functions $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}, \tilde{Q}^{\tau}\}$ evaluated at optimal government policies $\{\boldsymbol{d}^{\tau}, \boldsymbol{\mathcal{L}}^{\tau\prime}, \boldsymbol{\tau}, \boldsymbol{\mathcal{T}}^{\tau}\}$ are consistent with actual laws of motion $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}$ and Q^{τ}, i.e satisfy (10)–(13)*
5. *The private bond price function $q^{\tau}(s)$ satisfies (32)*
6. *Given public $\{\boldsymbol{d}^{\tau}, \boldsymbol{\mathcal{L}}^{\tau\prime}, \boldsymbol{\tau}\}$, and private $\{\mathcal{B}^{\tau\prime}\}$, policies the public bond price $Q^{\tau}(s,L,B)$ satisfies (33)*
7. *Goods market clear:*

$$C^{N,\tau}(s,L,B) = y^N$$

$$C^{T,\tau}(s,L,B) + (1-\pi)B = y^T + q^{\tau}(s)\mathcal{B}^{\tau\prime}(s,L,B) + \left\{1-\boldsymbol{d}^{\tau}(s,L,B)\right\}\left\{Q^{\tau}(s,L,B)\left[\mathcal{L}^{\tau\prime}(s,L,B) - (1-\delta)L\right] - \delta L\right\}$$

Similarly to the baseline model the optimality conditions of the households problem are:

$$q^{\tau}(s)(1-\boldsymbol{\tau}(s,L,B)u_T(C^{T,\tau}(s,L,B)) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau\prime}(s,L,B))] + \mu^{\tau}q^{\tau}(s),$$

$$p^{N,\tau}(s,L,B) = \frac{1-\omega}{\omega}\left(\frac{C^{T,\tau}(s,L,B)}{y^N}\right)^{\eta+1},$$

$$0 \leq \kappa(p^{N,\tau}(s,L,B)y^N + y^T) - q^\tau(s)\mathcal{B}^{\tau\prime}(s,L,B) \quad \text{with equality if } \mu^\tau > 0,$$

where μ^τ is the Lagrange multiplier associated with the credit constraint.

B Proof of proposition 1

This is a proof by construction. We will show that the recursive equilibrium with taxes can be written as a government problem that coincides with the planning problem (19). Start from the recursive competitive equilibrium problem with taxes described in Appendix B. This problem is equivalent to the recursive problem of a government given that chooses allocations for the current period while taking future policies and prices as given.

Denote these policies $\{\boldsymbol{d}^\tau(s,L,B), \mathcal{L}^{\tau\prime}(s,L,B), \boldsymbol{\tau}(s,L,B), \mathcal{T}^\tau(s,L,B), C^{T,\tau}(s,L,B), C^{N,\tau}(s,L,B), \mathcal{B}^{\tau\prime}(s,L,B)\}$. This government maximizes utility considering the optimal responses of households and lenders. This is equivalent to let the government choose all policies using the Kuhn-Tucker conditions of households and lenders as constraints. The problem is therefore:

$$W^\tau(s,L,B) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(s,L,B) + dW^{D,\tau}(s,B),$$

where default value $W^{D,\tau}(s,B)$ is:

$$\begin{aligned}
W^{D,\tau}(s,B) &= \max_{c^T,c^N,B',\tau,\mu} u(c^T,c^N) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\left[W^\tau(s',0,B')\right] \\
c^T + B(1-\pi) &= y^T + q^\tau(s)B', \\
c^N &= y^N, \\
q^\tau(s)B' &\leq \kappa\left(p^{N,\tau}c^N + y^T\right), \\
q^\tau(s)(1-\tau)u_T(c^T,c^N) &= \beta E_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau}(s',L',B')] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} \\
(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}$$

and value under repayment $W^{R,\tau}(s, L, B)$ is:

$$
\begin{aligned}
W^{R,\tau}(s, L, B) &= \max_{c^T, c^N, B', \tau, \mu, L' \in \Lambda} u\big(c^T, c^N\big) + \epsilon(L') + \beta \mathbb{E}_s[W^\tau(s', L', B')] \\
c^T + B(1-\pi) + \delta L &= y^T + q^\tau(s) B + Q^\tau(s, L', B')[L' - (1-\delta)L], \\
q^\tau(s) B' &\leq \kappa\left(p^{N,\tau} c^N + y^T\right), \\
q^\tau(s)(1-\tau) u_T(c^T, c^N) &= \beta \mathbb{E}_s[(1-\pi') u_T(C^{T,\tau}, C^{N,\tau}(s', L', B'))] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} \\
(\kappa(p^{N,\tau} c^N + y^T) - q^\tau(s) B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}
$$

$$
Q^\tau(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\left[\left[1 - \boldsymbol{d}^\tau(s', L', B')\right] \times \left[\delta + (1-\delta) Q^\tau\Big(s', \mathcal{L}^{\tau\prime}(s', L', B'), \mathcal{B}^{\tau\prime}(s', L', B')\Big)\right]\right]
$$

Substituting in the resource constraint for non tradables, and the intratemporal conditions that problem can be simplified to:

$$
W^\tau(s, L, B) = \max_{d \in \{0,1\}} [1-d] W^{R,\tau}(s, L, B) + d W^{D,\tau}(s, B), \tag{34}
$$

where default value $W^{D,\tau}(s, B)$ is:

$$
\begin{aligned}
W^{D,\tau}\big(s, B\big) &= \max_{c^T, B', \tau, \mu} u\big(c^T, y^N\big) - \phi(y^T) + \epsilon_{Def} + \beta \mathbb{E}_s\Big[W^\tau\big(s', 0, B'\big)\Big] \\
c^T + B(1-\pi) &= y^T + q^\tau(s) B', \\
q^\tau(s) B' &\leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta} y^N + y^T\Big) \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r} \\
q^\tau(s)(1-\tau) u_T(c^T, y^N) &= \beta \mathbb{E}_s[(1-\pi') u_T(C^{T,\tau}, C^{N,\tau})] + \mu q^\tau(s) \\
0 &= \left[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta} y^N + y^T\Big) - q^\tau(s) B'\right]\mu \\
\mu &\geq 0
\end{aligned}
$$

and value under repayment $W^{R,\tau}(s, L, B)$ is:

$$W^{R,\tau}(s, L, B) = \max_{c^T, B', \tau, \mu, L' \in \Lambda} u\big(c^T, y^N\big) + \epsilon(L') + \beta \mathbb{E}_s[W^\tau(s', L', B')]$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s) B + Q^\tau(s, L', B')[L' - (1-\delta)L] \tag{35}$$

$$q^\tau(s) B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta} y^N + y^T\Big) \tag{36}$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r} \tag{37}$$

$$Q^\tau(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\Bigg[\Big[1 - \boldsymbol{d}^\tau\Big] \times \Big[\delta + (1-\delta) Q^\tau\Big(s', \mathcal{L}^{\tau\prime}, \mathcal{B}^{\tau\prime}\Big)\Big]\Bigg] \tag{38}$$

$$q^\tau(s)(1-\tau) u_T(c^T, y^N) = \beta \mathbb{E}_s[(1-\pi') u_T(C^{T,\tau}, C^{N,\tau})] + \mu q^\tau(s) \tag{39}$$

$$0 = \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta} y^N + y^T\Big) - q^\tau(s) B'\Bigg]\mu \tag{40}$$

$$\mu \geq 0 \tag{41}$$

In this formulation it is apparent that the social planner problem (19) is a relaxed version of problem (34). In problem (34) the government must satisfy three additional constraints (39)–(41) and has access to two additional instruments μ and τ. Crucially, both μ and τ only appear in problem (34) in constraints (39)–(41). As such, problem (19) will be equivalent to problem (34) if we can use the solutions of (19) to construct two functions $\mu(s, L, B)$ and $\tau(s, L, B)$ that satisfy (39)–(41).

Let $\{C^{SP,T}(s, L, B), C^{SP,N}(s, L, B), \mathcal{L}^{SP\prime}(s, L, B), \mathcal{B}^{SP\prime}(s, L, B), \boldsymbol{d}^{SP}(s, L, B), Q^{SP}, q^{SP}(s)\}$ be a solution of problem (19). Additionally let $\mu^{SP}(s, L, B) \geq 0$ be the multiplier on the collateral constraint of the planner problem (19). μ^{SP} corresponds to the shadow value of relaxing the collateral constraint from the planner's perspective. This multiplier is different from μ which corresponds to the shadow value of relaxing the collateral constraint for individual households, and is a variable chosen by the government in (34). The complementary slackness condition of the social planner problem (19) is:

$$0 = \Bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{C^{SP,T}(s, L, B)}{y^N}\big)^{1+\eta} y^N + y^T\Big) - q^{SP}(s) \mathcal{B}^{SP\prime}(s, L, B),' \Bigg]\mu^{SP}(s, L, B). \tag{42}$$

As such by setting:

$$\mu(s, B, L) = \mu^{SP}(s, L, B)$$

$$1 - \boldsymbol{\tau}(s, L, B) = \frac{\beta \mathbb{E}_s\Bigg[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(s', L', B'), C^{SP,N}(s', L', B'))\Big)\Bigg] + \mu^{SP}(s, L, B) q^{SP}(s)}{q^{SP}(s) u_T(C^{SP,T}(s, L, B), y^N)},$$

We can see that (39)–(41) are satisfied and therefore the two problems are equivalent.

C Data Appendix

Gross Domestic Product (GDP): *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64,-*. Corresponds to Total gross value added in all NACE activities. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Non-tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the share of total value added produced in the following industries: public administration, wholesale and retail, construction, and real state. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the complement of nontradable valued added as a share of total value added. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Private debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spanish monetary financial institutions (excluding the Bank of Spain) and other resident sectors. The data series used are 3273771 and 3273777. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Public debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of the Bank of Spain and all public administrations. The data series used are 2386960 and 3273774. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Total debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spain and is calculated as the consolidation of private and public positions. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011.

Risk free rate: *Bloomberg ticker GTDEM1Y Govt*, Corresponds to the average interest rate spread paid on 1 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2011.

Spread on public bonds: *Bloomberg tickers GTESP6YR Govt and GTDEM6Y Govt*, Corresponds to the difference between average interest rate paid on 6 year Spanish treasury bonds and 6 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2015. In the calibration we use data only from 1999 to 2011.

Average Maturity: *Table 5 from the Bank of Spain's economic bulletin Alloza et al. (2019), of March 2019*, Average maturity of the stock of public debt for Spain in years. Annual data from 1999 to 2011.

Nonperforming loans: *Bloomberg ticker BLTLWESP Index*, Nonperforming loans as a share of total gross loans. Annual data from 1999 to 2015.

Consumption: *Eurostat , GDP and main components (output, expenditure and income) nama_10_gdp*. Corresponds to final consumption expenditure. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2017.

Current Account: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to current account as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

Trade Balance: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to the balance of trade on goods and services as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

D Solution Method: The Government's ex-ante problem

Following the approach of Sanchez et al. (2018), I can re-write the government's Bellman equations before the ϵ shocks are realized. From an ex-ante point of view, the shocks ϵ make the default and borrowing decisions stochastic. By taking expectations over these shocks, the decisions can be viewed as probabilistic. If we view the previously defined equilibrium as a game between the private and public sector each period, the ϵ shocks allow the government to play mixed strategies. This makes the computation of this problem using value function iteration possible. We follow this approach to write (14) from a an ex-ante perspective. That is when all the aggregate states have realized except the $\boldsymbol{\epsilon}$. For this we summarize all other exogenous state variables in $z = (y^T, y^N, \kappa, \pi)$. As mentioned in the main text we assume that L' is a finite and bounded grid with $\mathcal{J}$ elements. Denote by $F(\boldsymbol{\epsilon})$ the joint cumulative density function of the taste shocks and by $f(\boldsymbol{\epsilon})$ its joint density function. To simplify notation in what follows, the following operator to denotes the expectation of any function $Z(\boldsymbol{\epsilon})$ with respect to all the elements of ,

$$\boldsymbol{Z} = \mathbb{E}_{\epsilon} Z(\boldsymbol{\epsilon}) = \int_{\epsilon_1} \int_{\epsilon_2} ... \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) f(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) d\epsilon_1, ..d\epsilon_{\mathcal{J}+1} \tag{43}$$

Given this notation we have that:

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}[W(s, L, B)]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\left\{W^R(s, L, B); W^D(s, B)\right\}\Big]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\Big\{\max_{L' \in \Lambda}\{u(C(s, L, B)) + \epsilon(L') + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L', \mathcal{B}'(s, L, B))\};$$

$$u(C(s, 0, B)) - \phi(y^T) + \epsilon^{Def} + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))\Big\}\Big]$$

Subject to the resource constraints:

$$C^T(s, L, B) = y^T + q(s)\mathcal{B}'(s, L, B) - (1-\pi)B + Q(s, L', B')[L' - (1-\delta)\mathcal{B}'(s, L, B)] - \delta \mathcal{B}'(s, L, B)$$

$$C^N(s, L, B) = y^N$$

Furthermore, if its convenient to define the following expected utility objects:

$$\Upsilon_{L,L'}(z, B) = u(C(s, L, B)) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L, \mathcal{B}'(s, L, B))$$

$$\Upsilon_{def}(z, B) = u(C(s, 0, B)) - \phi(y^T) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))$$

Lemma 2. *Suppose that the $\boldsymbol{\epsilon}$ shocks follow a multivariate generalized extreme value distribution with parameters $\{m, v, p\}$ and are i.i,d over time. Where v is the scale parameter and p is the shape parameter and is set to 1. m corresponds to the location parameter and is set to $-v\gamma$ where γ is the Euler constant. Suppose that public debt L is on a grid with $\mathcal{J}$ points. Then the ex-ante value function of the government's recursive problem can be re-written as*

$$\boldsymbol{W}(z,L,B) = \Upsilon_{def} + v\log\left[1 + \left(\sum_{L'\in\Lambda}\exp\left(-\frac{\Upsilon_{def}-\Upsilon_{L,L'}}{pv}\right)\right)^{p}\right] \tag{44}$$

Additionally given this distributional assumptions there are closed form solutions for the ex-ante probability of default and borrowing policy functions conditional on repayment.

Proof. Given our distributional assumptions

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{\mathcal{J}}\exp\left(-\frac{\epsilon_j - m}{v}\right)\right) - \exp\left(-\frac{\epsilon_{\mathcal{J}+1}-m}{v}\right)\right] \tag{45}$$

For $j \in [\![0, \mathcal{J}+1]\!]$ we denote by $F_j(\boldsymbol{\epsilon}) = \frac{\partial F(\boldsymbol{\epsilon})}{\partial \epsilon_j}$, the marginal with respect to element j^{th} element of $\boldsymbol{\epsilon}$.

$$F_j(\boldsymbol{\epsilon}) = \begin{cases} \frac{1}{v}\exp\left[-\left(\sum_{j=1}^{\mathcal{J}}\exp(-\frac{\epsilon_j-m}{v}) - \exp(-\frac{\epsilon^{def}-m}{v})\right)\right] & \exp(-\frac{\epsilon_j-m}{v}) & for\ j = 1..\mathcal{J} \\ \frac{1}{v}\exp\left[-\left(\sum_{j=1}^{\mathcal{J}}\exp(-\frac{\epsilon_j-m}{v}) - \exp(-\frac{\epsilon^{def}-m}{v})\right)\right] & \exp(-\frac{\epsilon^{def}-m}{v}) & for\ j = \mathcal{J}+1 \end{cases}$$

Using this notation ant the dropping the states (z,B) from the previously defined $\Upsilon_{L,L'}(z,B)$ functions we can compute the ex-ante policy functions of the government in close form solutions. Let the probability of default be $\boldsymbol{d}(z,L,B) = \mathbb{E}_{\boldsymbol{\epsilon}}\boldsymbol{d}(z,L,B,\boldsymbol{\epsilon})$. Note that:

$$\begin{aligned} \boldsymbol{d}(z,L,B) &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{def}+\epsilon^{def}-\Upsilon_1,\ldots,\Upsilon_{def}+\epsilon^{def}-\Upsilon_{def})d\epsilon^{def} \qquad (46)\\ &= \int_{-\infty}^{\infty}\frac{1}{v}\exp\left[-\left(\sum_{j=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_{def}+\epsilon^{def}-\Upsilon_j-m}{v}) - \exp(-\frac{\epsilon^{def}-m}{v})\right)\right]\exp(-\frac{\epsilon^{def}-m}{v})d\epsilon^{def} \\ &= \int_{-\infty}^{\infty}\frac{1}{v}\exp\left[-\exp(-\frac{\epsilon^{def}-m}{v})\left(\sum_{j=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_{def}-\Upsilon_j}{v})+1\right)\right]\exp(-\frac{\epsilon^{def}-m}{v})d\epsilon^{def} \end{aligned}$$

Define $\exp(\phi_{def}) = 1 + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_{def}-\Upsilon_h}{v})$. We can use this to rewrite (46) as:

$$
\begin{aligned}
\boldsymbol{d}(z,L,B) &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\Big[- \exp(-\frac{\epsilon^{def}-m}{v}) \exp(\phi_{def}) \Big] \exp(-\frac{\epsilon^{def}-m}{v}) d\epsilon^{def} \\
&= \frac{1}{v \exp(\phi_{def})} \underbrace{\int_{-\infty}^{\infty} \exp\Big[- \exp(-\frac{\epsilon^{def}-m-v\phi_{def}}{v}) \Big] \exp(-\frac{\epsilon^{def}-m-v\phi_{def}}{v}) d\epsilon^{def}}_{=v} \\
&= \frac{1}{1 + \Big(\sum_{L' \in \Lambda} \exp\Big(- \frac{\Upsilon_{def}-\Upsilon_{L,L'}}{v} \Big) \Big)}
\end{aligned} \tag{47}
$$

Where the last equivalence uses the fact that the PDF of the generalized extreme distribution integrates to 1. Similarly, conditional on repayment, the random component $\boldsymbol{\epsilon}$ make the public borrowing decisions random from an ex-ante perspective. Given a set of current aggregate states relevant for the government, it is useful to introduce the probability of choosing an amount of public debt L' *conditional on not defaulting* as:

$$
\boldsymbol{G}_{z,L,B}(L') = \mathbb{P}_{\boldsymbol{\epsilon}}\big(L' | d(z,L,B,\boldsymbol{\epsilon}) = 0\big)
$$

Using the same notation as before we have that for the L' that is the j^{th} element of Λ:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{1-\boldsymbol{d}(z,L,B)} \int_{-\infty}^{\infty} F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \times \\
& \int_{-\infty}^{\infty} \exp\Big[- \exp(-\frac{\epsilon^j - m}{v}) \Big(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \Big) \Big] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j
\end{aligned}
$$

Defining $\exp(\phi_j) = \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v})$, we can simplify:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \int_{-\infty}^{\infty} \exp\Big[- \exp(-\frac{\epsilon^j - m}{v}) \exp(\phi_j) \Big] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v \exp(\phi_j)} \underbrace{\int_{-\infty}^{\infty} \exp\Big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \Big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v} \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B)) \exp(\phi_j)}
\end{aligned}
$$

Finally this can be further simplified to:

$$\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))} \times \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})}{\sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{1}{\sum_{H\in\Lambda} \exp\left(\frac{\Upsilon_{L,H}-\Upsilon_{L,L'}}{v}\right)}
\end{aligned} \tag{48}$$

Finally the value $\boldsymbol{W}(z,L,B)$ is given by:

$$\begin{aligned}
\boldsymbol{W}(z,L,B) =& \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_j + \epsilon_j) F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j \\
=& \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} \frac{\Upsilon_j + \epsilon_j}{v} \times \\
& \exp\left[- \exp(-\frac{\epsilon^j - m}{v}) \left(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \right) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j \\
& + \int_{-\infty}^{\infty} \frac{\Upsilon_{def} + \epsilon_{def}}{v} \times \\
& \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\
=& \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j) \times \\
& \left[\Upsilon_j + m + v\phi_j + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon_j - m - v\phi_j}{v}) \exp\left[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \right] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v\gamma} \right] \\
& + \exp(-\phi_{def}) \times \\
& \left[\Upsilon_{def} + m + v\phi_{def} + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \exp\left[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \right] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def}}_{=v\gamma} \right]
\end{aligned}$$

Where in the last equivalence we have used the fact that for all j:

$$\Upsilon_j + m + v\phi_j = \frac{(\Upsilon_j + m + v\phi_j) \int_{-\infty}^{\infty} \exp\left[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \right] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}{v}$$

The last step (underscored in the above equations) uses one of the integral properties of the Euler constant. We now use the fact we assumed the distribution of shocks to be mean zero, that is $m = -\gamma v$. Using the definition of ϕ_{def} one can see that:

$$\exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] = \frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}$$

The value of the government is then given by:

$$\begin{aligned}
\boldsymbol{W}(z, L, B) &= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j)[\Upsilon_j + v\phi_j] + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z, L, B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j + v\log(\exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v})} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z, L, B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j - \frac{v\Upsilon_j}{v} + v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j}{v})(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v}))} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z, L, B) &= \frac{v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v}))}{\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z, L, B) &= \frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z, L, B) &= \left[\frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}\right]\left[\sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + 1\right] \\
\boldsymbol{W}(z, L, B) &= \Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})) \qquad (49)
\end{aligned}$$

To sum up the distributional assumptions allow us to obtain closed form solutions for the ex-ante value function (49), the policy functions for default (47), the public borrowing conditional on repayment (48), ∎

Note that the functions $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z, L, B)$ are sufficient to express all government decisions. Using the fact that the shocks are i.i.d over time, and assuming a guess $\boldsymbol{Q}$ of next price schedule functions, we can use $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z, L, B)$ to write the pricing equation of public bonds (18):

$$Q(z, L', B') = q(z)\mathbb{E}_{z'|z}\left[\left[1 - \boldsymbol{d}(z', L', B')\right]\left[\delta + (1-\delta)\sum_{L'' \in \Lambda} \boldsymbol{Q}\big(z', L'', \mathcal{B}'(z', L', B')\big)\boldsymbol{G}_{z',L',B'}(L'')\right]\right] \quad (50)$$

In the quantitative section we assume that the shocks are mean zero ($m = -\gamma v$). We also assume that the shape parameter p is one, therefore taste shocks are independent from each other within the period as well. The scale parameter v is calibrated to match the variance of public debt in the data.

E Numerical Solution

In this section we provide more detail about the solution methods we use to solve both the baseline and planner version of the model described in the main text. For both solutions methods we use the closed form ex-ante solutions of the government's problem described in detail in Appendix D.

Baseline. This version is solved in three steps. The first step solves the households problem while taking government policies and bond prices as given using time iteration method. The second step uses the implied policy functions of the private sector from the first step and the assumed bond schedules, and computes the closed form solutions that solve the government's ex-ante problem. Finally using private and public policy functions the schedule of private bonds is updated. Iterate until convergence in private en public policies.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (17).
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}(z, L', B')$.
- Construct the implied transfer function $\boldsymbol{T}(z, B, L, L')$ using the government budget constraint (5).
- Taking all these functions as given find the optimal private borrowing $B'(z, L, B, L')$ and consumption decisions $C'(z, L, B, L')$ using the private sector Euler equation (24) to find the binding and non binding states.
- Given households optimal policies $B'(z, L, B, L')$, and $C'(z, L, B, L')$, and the guess schedule of public bonds $\boldsymbol{Q}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the government using (47) and (48). Update the government policy functions.
- Compute the government ex-ante value function $W(z, L, B)$ using (49).
- Update the schedule of public bonds $\boldsymbol{Q}(z, L', B')$ using (50).

- Repeat until convergence in $W(z,L,B)$,$B'(z,L,B,L')$, and $C'(z,L,B,L')$, and $\boldsymbol{Q}(z,L',B')$ is achieved.

Social planner. This version is solved in three steps. The first step finds optimal private borrowing on a grid (*grid search method*) given an initial guess of public for each potential default and public borrowing decisions. The second step uses this optimal private borrowing policy and the assumed bond schedules to computes the closed form solutions for public borrowing and default and the value function. Finally using private and public borrowing policy functions the schedule of private bonds is updated. Iterate until convergence in private borrowing policies and the value function is achieved.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (17).
- Construct a grid of potential private borrowing choices B'.
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}^{SP}(z,L,B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}^{SP}(z,L,B,L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}^{SP}(z,L',B')$.
- Taking all these functions as given find the optimal private borrowing $B^{SP\prime}(z,L,B,L')$ in the finite grid discarding all choices that violate the credit constraint (20) for each potential public borrowing and default decision.
- Given optimal private borrowig policy $B^{SP\prime}(z,L,B,L')$ and the guess schedule of public bonds $\boldsymbol{Q}^{SP}(z,L',B')$, compute the ex-ante default and borrowing policy functions of the planner using (47) and (48). Update the planner public borrowing and default policy functions.
- Compute the ex-ante value function $W^{SP}(z,L,B)$ using (49).
- Update the schedule of public bonds $\boldsymbol{Q}^{SP}(z,L',B')$ using (50).
- Repeat until convergence in $W^{SP}(z,L,B)$,$B^{SP\prime}(z,L,B,L')$, and $\boldsymbol{Q}^{SP}(z,L',B')$ is achieved.

F Particle filter method

This appendix details the particle filter method used to conduct the counterfactual exercises of section 5. It follows closely the approach presented in Bocola and Dovis (2019). As noted in the main text, the state space representation of the model is:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t \tag{51}$$

$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t). \tag{52}$$

In this formulation, the first equation captures the measurement error e_t, a vector of i.i.d. normally distributed errors with mean zero and a diagonal variance-covariance matrix Σ. The vector of observable, $\boldsymbol{Y}_t$, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds. The second equation describes the law of motion of the baseline model state variables $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$. The vector ε_t corresponds to the innovations in the AR 1 process of the three structural shocks $[y^T_t, \pi_t, \kappa_t]$.

$$\begin{aligned}
y^T_t &= \exp(\rho^y \ln y^T_{t-1} + \varepsilon^y_t) \\
\pi^T_t &= \exp((1-\rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon^\pi_t) \\
\kappa_t &= (1-\rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon^\kappa_t
\end{aligned}$$

Since we did not observe any defaults in the time periods considered we use the repayment policy functions to compute the transitions. Using the notation of section 3 the evolution of private and public debt in the first exercise is then:

$$\begin{aligned}
L_{t+1} &= \mathcal{L}'(s_t, L_t, B_t) = \mathcal{L}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t) \\
B_{t+1} &= \mathcal{B}'(s_t, L_t, B_t) = \mathcal{B}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t)
\end{aligned}$$

In the first exercise all taste shocks are set to to zero. In the second exercise, we still focus on repayment but this time we select the taste shocks to match public debt exactly to it's data counter part and let private debt the respond endogenously:

$$\begin{aligned}
L_{t+1} &== L^{data}_{t+1} \\
B_{t+1} &= \tilde{B}'(y^T_t, \pi_t, \kappa_t, L_t, B_t, 0, L^{data}_{t+1}, \tilde{T}(s_t, L_t, L^{data}_{t+1}))
\end{aligned}$$

These transitions are summarized in function $f(\cdot)$ for each exercise. Similarly we can generate numerical solutions to compute the model counterparts to debt to output ratios and the public spreads and summarize them in $g(\cdot)$.

Let $\boldsymbol{Y}^t = [\boldsymbol{Y}_1, ..\boldsymbol{Y}_t]$, and denote by $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$ the conditional distribution of the state vector given a history of observations up to period t. In general there is no analytical solution for the density function $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$. The particle filter method approaches this density by using the fact that the conditional density of $\boldsymbol{Y}_t$ given $\boldsymbol{S}_t$ is Gaussian. It consists of finding a set of pairs of states and weights $\{S^i_t, \tilde{w}^i_t\}^N_{i=1}$

such that for all function $h(\cdot)$:

$$\frac{1}{N}\sum_{i=1}^{N} h(\boldsymbol{S}_t^i)\tilde{w}_t^i \underset{a.s}{\longrightarrow} \mathbb{E}[h(\boldsymbol{S}_t)|\boldsymbol{Y}^t].$$

This approximation can then be used to obtain the weighted average path of the state vector over the sample. The states selected S_t^i are called particles and $\tilde{w}_t^i$ corresponds to their weight. To construct this set we follow the algorithm proposed by Kitagawa (1996).

Step 1: Initialization Set $t = 1$ and $\forall i\ \tilde{w}_0^i = 1$, draw S_0^i from the ergodic distribution of the baseline model.

Step 2: Transition For each $i = 1..N$ compute the state vector $\boldsymbol{S}_{t|t-1}^i$ given vector $\boldsymbol{S}_{t-1}^i$ by drawing innovations for the fundamental shocks from the calibrated distributions and using the policy functions summa zed in $f(\cdot)$.

Step 3: Filter Assign to each particle $\boldsymbol{S}_{t|t-1}^i$ the weight

$$w_t^i = p(\boldsymbol{Y}|\boldsymbol{S}_{t|t-1}^i)\tilde{w}_{t-1}^i$$

where $p(\boldsymbol{Y}|\boldsymbol{S}_{t|t-1}^i)$ is a multivariate Normal density.

Step 4: Rescale & Resample Rescale the weights $\{w_t^i\}$ so that they add up to one, and denote these new weights $\{\tilde{w}_t^i\}$. Sample with replacement N values of the state vector from the set $\{\boldsymbol{S}_{t|t-1}^i\}$ using $\{\tilde{w}_t^i\}$ as sample weights. Denote this draws $\{\boldsymbol{S}_t^i\}$. Set $\tilde{w}_t^i = 1\ \forall i$. If $t < T$ set $t = t + 1$ and go to Step 2. Otherwise stop.

In both exercises, it is assumed that measurement error associated with y_t^T and π_t is zero, as such the variance of the measurement error is set to zero for these variables in the measurement equation and the innovations ε_t^y and ε_t^π are set to match the empirical counterparts exactly. Since κ_t has no empirical counterpart, the algorithm help us find the most likely path using its effects on debt aggregates and the spreads. As in Bocola and Dovis (2019) the filter is tuned with $N = 100,000$.

Equipped with a set of particles and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ and the policy functions summarized in $g(\cdot)$ one can approximate the model predictions plotted in figures 11 and 12. As an example for all $t = [2008..2015]$ the predicted interest rate spread, $spr_t^{Baseline}$ at time t is:

$$spr_t^{Baseline} = \sum_i^N \tilde{w}_t^i[\frac{\delta - \delta\boldsymbol{Q}(S_t^i)}{\boldsymbol{Q}(S_t^i)} - r]$$

Similar weighted averages are computed for the debt to output ratio and the exogenous shocks. When computing objects for the social planner the function $g^{SP}(\cdot)$ is used instead.