5. Determine a série de Fourier trigonométrica da função f definida por

$$f(t) = \begin{cases} 0 & -2 < t < -1 \\ t & -1 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

sendo k constante positivo, com $f(t+4) = f(t)$. Apresente ao menos 4 termos não nulos da série.

$$c = \frac{1}{T}\int_{\alpha}^{\beta} f(x)\,\mathrm{d}x,\quad a_k = \frac{2}{T}\int_{\alpha}^{\beta} f(x)\cos\left(\frac{2\,k\,\pi\,x}{T}\right)\mathrm{d}x\ ,\ b_k = \frac{2}{T}\int_{\alpha}^{\beta} f(x)\sin\left(\frac{2\,k\,\pi\,x}{T}\right)\mathrm{d}x$$

```
> restart;
> T := 4;
```

$$T := 4 \tag{1.1}$$

```
> f := piecewise(`and` (-2 < t, t < -1),0,`and`(-1 < t, t < 1),t,
  `and` (1 < t, t < 2),0);
```

$$f := \begin{cases} 0 & -2 < t \textbf{ and } t < -1 \\ t & -1 < t \textbf{ and } t < 1 \\ 0 & 1 < t \textbf{ and } t < 2 \end{cases} \tag{1.2}$$

Como esta função é ímpar, de modo que c=0, $a_k = 0$. Os outros coeficientes são dados por

```
> b := 2*(int(f*sin(2*n*Pi*t/T), t = -2 .. 2))/T;
```

$$b := -\frac{2\left(-2\sin\left(\frac{1}{2}\,n\,\pi\right) + \cos\left(\frac{1}{2}\,n\,\pi\right)n\,\pi\right)}{n^2\,\pi^2} \tag{1.3}$$

```
> assume(n, integer);
> b := unapply(b, n)
```

$$b := n\sim \rightarrow -\frac{2\left(-2\sin\left(\frac{1}{2}\,n\sim\pi\right) + \cos\left(\frac{1}{2}\,n\sim\pi\right)n\sim\pi\right)}{n\sim^2\,\pi^2} \tag{1.4}$$

```
> FS := (t, l) → Sum(b(i) * sin(2 * i * π * t/T), i = 1 ..l);
```

$$FS := (t, l) \rightarrow \sum_{i=1}^{l} b(i)\,\sin\left(\frac{2\,i\,\pi\,t}{T}\right) \tag{1.5}$$

Os primeiros termos da série são então dados por

```
> value(FS(t, 8))
```

$$\frac{4\sin\left(\frac{1}{2}\,\pi\,t\right)}{\pi^2} + \frac{\sin(\pi\,t)}{\pi} - \frac{4}{9}\,\frac{\sin\left(\frac{3}{2}\,\pi\,t\right)}{\pi^2} - \frac{1}{2}\,\frac{\sin(2\,\pi\,t)}{\pi} + \frac{4}{25}\,\frac{\sin\left(\frac{5}{2}\,\pi\,t\right)}{\pi^2} + \frac{1}{3}\,\frac{\sin(3\,\pi\,t)}{\pi} - \frac{4}{49}\,\frac{\sin\left(\frac{7}{2}\,\pi\,t\right)}{\pi^2} - \frac{1}{4}\,\frac{\sin(4\,\pi\,t)}{\pi} \tag{1.6}$$