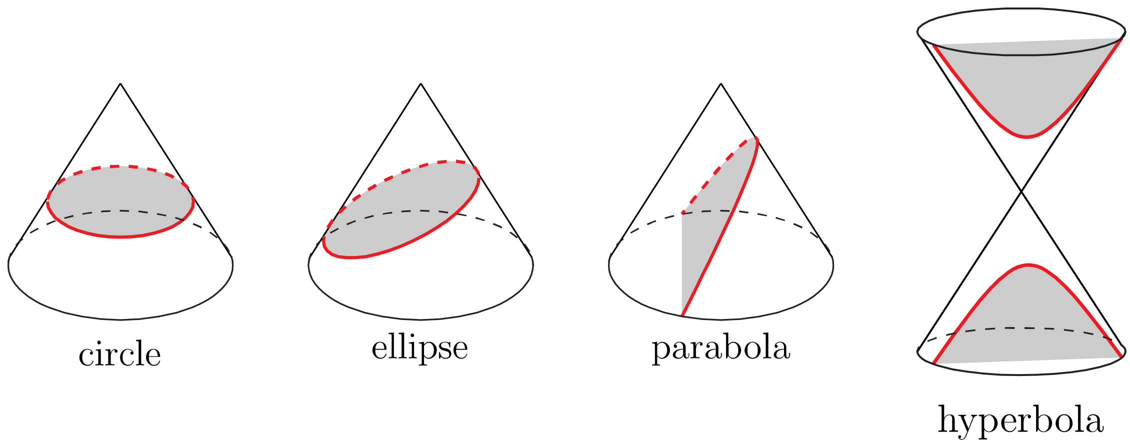
A.8 Conic Sections and Quadric Surfaces

A conic section is the curve of intersection of a cone and a plane that does not pass through the vertex of the cone. This is illustrated in the figures below.



An equivalent[1]
(and often used) definition is that a conic section is the set of all points in the xy-plane that obey $Q(x,y)=0$ with

$$Q(x,y)=Ax^2+By^2+Cxy+Dx+Ey+F=0$$

being a polynomial of degree two[2]
. By rotating and translating our coordinate system the equation of the conic section can be brought into one of the forms[3]

- $\alpha x^2+\beta y^2=\gamma$ with $\alpha,\beta,\gamma>0$, which is an ellipse (or a circle),
- $\alpha x^2-\beta y^2=\gamma$ with $\alpha,\beta>0$, $\gamma\neq 0$, which is a hyperbola,
- $x^2=\delta y$, with $\delta\neq 0$ which is a parabola.

The three dimensional analogs of conic sections, surfaces in three dimensions given by quadratic equations, are called quadrics. An example is the sphere $x^2+y^2+z^2=1$.

Here are some tables giving all of the quadric surfaces.

name	elliptic cylinder	parabolic cylinder	hyperbolic cylinder	sphere
equation in standard form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$y=ax^2$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$x^2+y^2+z^2=r^2$
$x=$constant cross-section	two lines	one line	two lines	circle
$y=$constant cross-section	two lines	two lines	two lines	circle

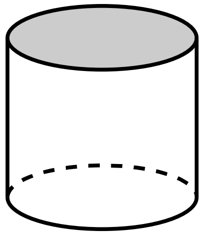
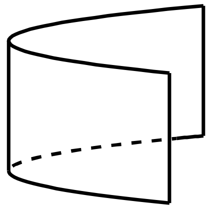
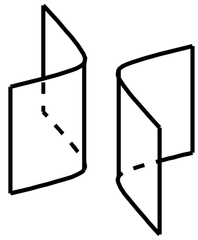
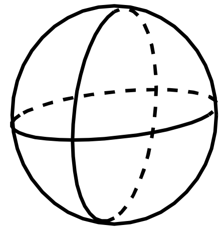
$z =$constant cross-section	ellipse	parabola	hyperbola	circle
sketch				

Figure A.8.1. Table of conic sections

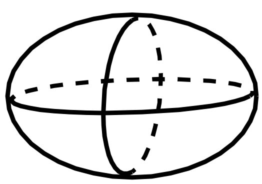
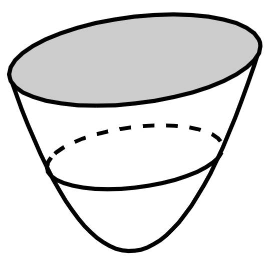
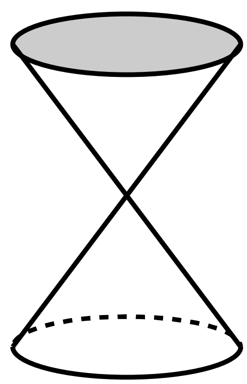
name	ellipsoid	elliptic paraboloid	elliptic cone
equation in standard form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$
$x=$ constant cross-section	ellipse	parabola	two lines if $x=0$, hyperbola if $x\neq 0$
$y=$ constant cross-section	ellipse	parabola	two lines if $y=0$,hyperbola if $y\neq 0$
$z=$ constant cross-section	ellipse	ellipse	ellipse
sketch			

Figure A.8.2. Table of quadric surfaces-1

name	hyperboloid of one sheet	hyperboloid of two sheets	hyperbolic paraboloid
equation in standard form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}$
$x=$ constant cross-section	hyperbola	hyperbola	parabola

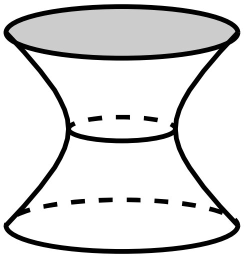
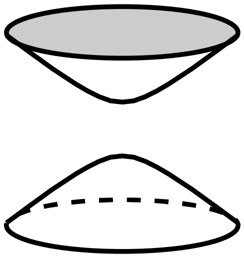
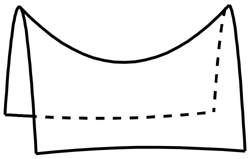
$y =$ constant cross-section	hyperbola	hyperbola	parabola
$z =$ constant cross-section	ellipse	ellipse	two lines if $z = 0$, hyperbola if $z \neq 0$
sketch			

Figure A.8.3. Table of quadric surfaces-2