LFA Seminar 6

ex 1

Scrieți CFG-uri pentru:

i) $\{ a^n b^n \mid n \geq 0 \}$

ii) $\{ a^n b^n \mid n \geq 1 \}$

iii) $\{ w \in \{a,b\}^* \mid w$ conține cel puțin 3 de $a \}$

iv) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$

v) $\{ w \in \{a,b\}^* \mid w = w^R \}$

vi) $\{ w w^R \mid w \in \{a,b\}^* \}$

ex 2

Scrieți PDA-uri pentru:

i) $\{ a^n b^n \mid n \geq 0 \}$ (și CFG)

ii) $\{ w \in \{ "(", ")" \}^* \mid w$ bine-parantezat $\}$

iii) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$

iv) $\{ w w^R \mid w \in \{a,b\}^* \}$

ex 1

Scrieți CFG-uri pentru:

i) $\{ a^n b^n \mid n \geq 0 \}$

ii) $\{ a^n b^n \mid n \geq 1 \}$

iii) $\{ w \in \{a,b\}^* \mid w \text{ conține cel puțin 3 de } a \}$

iv) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$

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Sol:

CFG = context free grammar

$S \longrightarrow S + S \mid S \times S \mid 0 \mid 1 \mid \dots \mid 9$

" S se rescrie ca S + S

" Gramatica o să genereze ... "

exemplu

3 + 4 + 7

$S \longrightarrow S + S \longrightarrow S + S + S \longrightarrow 3 + 4 + 7$

Obs. Orice expresie regulată poate fi simulată de o gramatică

i) $\{ a^n b^n \mid n \geq 0 \}$

$S \to a S b \mid \varepsilon$

$S = a a b b$

$S \to a S b \to a a S b b \to a a \varepsilon b b = a a b b$

□

ii) $\{ a^n b^n \mid n \geq 0 \}$

sau

$S \to a S b \mid a b$

□

iii) $\{ w \in \{a,b\}^* \mid w \text{ conține cel puțin 3 de } a \}$

Expresia regulata:

$(a \cup b)^* a (a \cup b)^* a (a \cup b)^* a (a \cup b)^*$

Gramatica

$S \to B a B a B a B$

$B \to a \mid b \mid \varepsilon \mid BB$

□

iv) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$

$S \to aSb \mid bSa \mid abS \mid baS \mid Sab \mid Sba \mid \varepsilon \mid$

exemplu

$aaabbbba$

$\to SbA$

$\to aSb$

$\to aSb$

$\to aSb$

$\to \varepsilon$

v) $\{ w \in \{a,b\}^* \mid w = w^R \}$

- $S \to aSb \mid bSa \mid abS \mid baS \mid Sab \mid Sba \mid \varepsilon \mid SS$

exemplu

$bbaa|aabb$

$bSa \quad aSb$

$aaSbb$

$\downarrow \varepsilon$

$aabb$

$bSa \quad aSb$

$bbSaa \quad aaSbb$

$\downarrow \varepsilon \quad \downarrow \varepsilon$

$S \to SS$

- $S \to aSa \mid bSb \mid \varepsilon \mid a \mid b$

vi) $\{ w w^R \mid w \in \{a,b\}^* \}$ ← palindrom de lungime pară

$S \to a S a \mid b S b \mid \varepsilon$

□

ex 1

Scrieți PDA-uri pentru:

i) $\{ a^n b^n \mid n \geq 0 \}$ (și CFG)

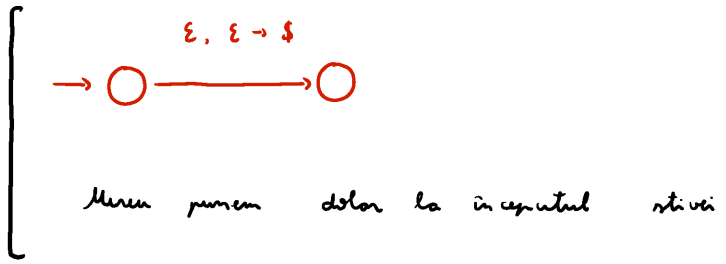
ii) $\{ w \in \{ "(", ")" \}^* \mid w$ bine-parantezat $\}$

iii) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$

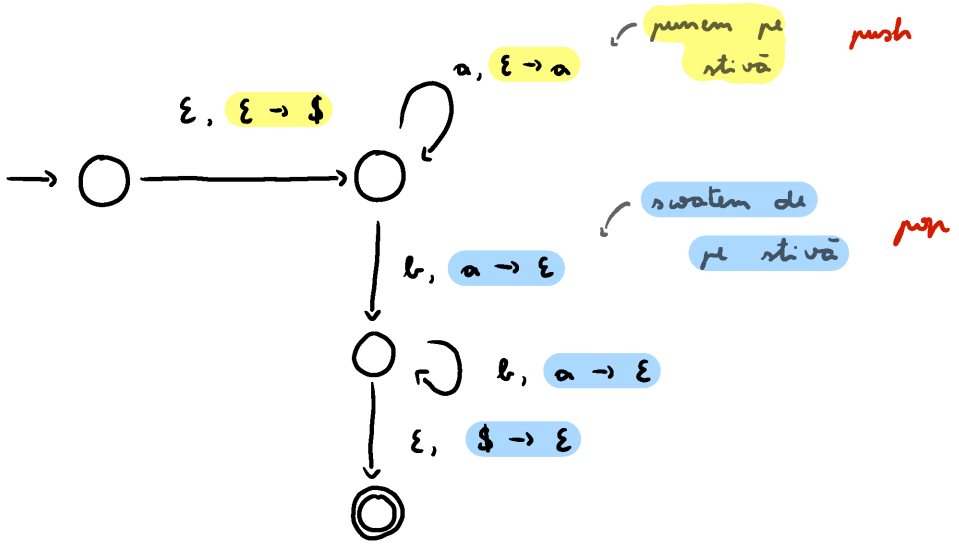
iv) $\{ w w^R \mid w \in \{a,b\}^* \}$

Sol:

i) $\{ a^n b^n \mid n \geq 0 \}$



Mereu punem dolar la începutul stivei



□

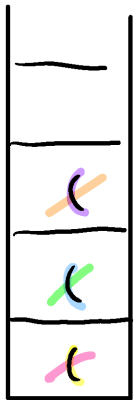
ii) $\{ w \in \{ "(", ")" \}^* \mid w$ bine-parantezat $\}$

( ) ( )

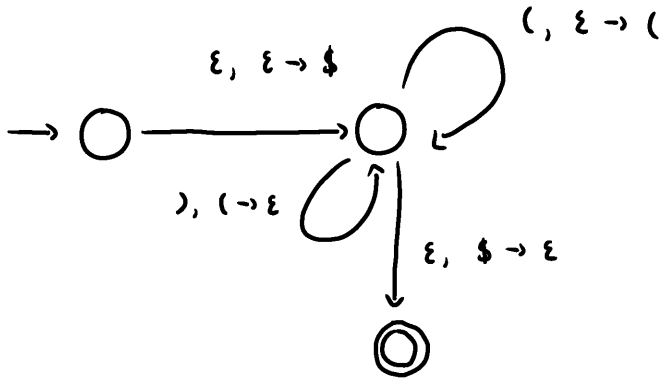
Gramatica

$S \to (S) \mid SS \mid \varepsilon$

(( )) ( )

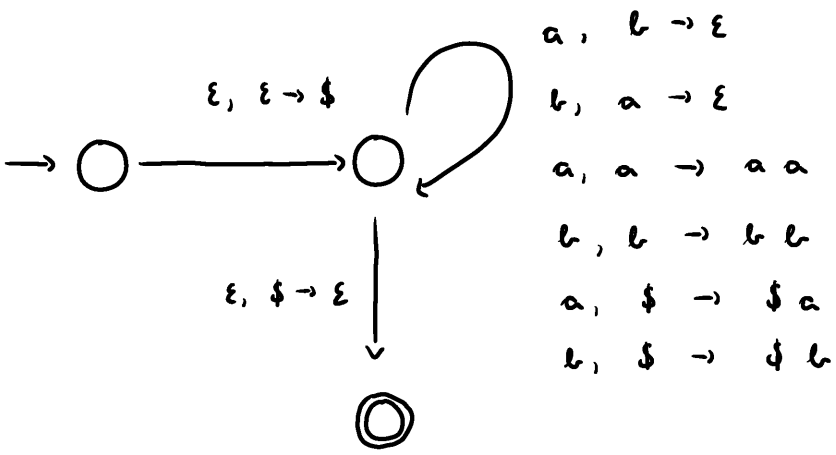


PDA



□

iii) $\{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$



a, b → ε dacă primește a ieșe în vf stivei e b ⇒ pop b

a, a → a a dacă primește a ieșe în vf stivei e a

⇒ pop a , push a , push a

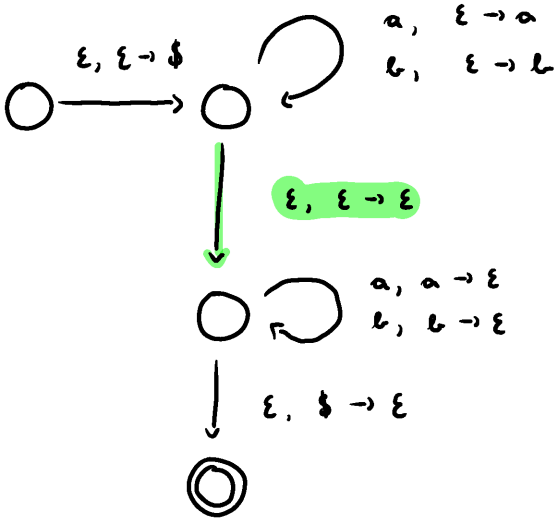
(adaugi doar unul) □

iv) $\{ w w^R \mid w \in \{a,b\}^* \}$

PDA → nondeterminist

⇒ presupunem că fiecare

loc e mijlocul cuvântului



□