Dorea Fernando - Emanuel
grupa 143

Cuadrice

Definiție

Cuadricele sunt suprafețe algebrice de gradul al doilea, adică suprafețe ale spațiului afin euclidian tridimensional și reprezintă mulțimea soluțiilor unor ecuații de forma: $a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{10}x + 2a_{20}y + 2a_{30}z + a_{00}$, cu $a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23}$ minim unul nenul.

Invariații

La schimbarea de reper, ecuația unei cuadrice: $a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + a_{10}x + a_{20}y + a_{30}z + a_{00} = 0$ se poate rescrie în notație matriceală

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \qquad D = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{10} \\ a_{12} & a_{22} & a_{23} & a_{20} \\ a_{13} & a_{23} & a_{33} & a_{30} \\ a_{10} & a_{20} & a_{30} & a_{00} \end{pmatrix}$$

$\delta = \det A$

$\Delta = \det D$

J = suma minorilor diagonali de ordin 2 din A

L = suma minorilor diagonali de ordin 2 din D

K = suma minorilor dig. de ordin 3 din D

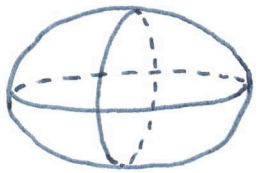
Clasificare

Nr.	δ	Δ	$\lambda_1, \lambda_2, \lambda_3$	$-\frac{\Delta}{\lambda_1\delta}$, $-\frac{\Delta}{\lambda_2\delta}$, $-\frac{\Delta}{\lambda_3\delta}$	K	L	Cuadrica
1.	>0	$\neq 0$		$+ \quad + \quad +$			elipsoid
2.	<0	$\neq 0$		$+ \quad + \quad -$			hiperboloid cu o pânză
3.	>0	$\neq 0$		$- \quad - \quad +$			hiperboloid cu 2 pânze
4.	<0	$\neq 0$		$- \quad - \quad -$			cuadrică vidă
5.	$\neq 0$	0	axele: semn				punct dublu
6.	$\neq 0$	0	$\lambda_1, \lambda_2 > 0, \lambda_3 = 0$				con pătratic
7.	0	$\neq 0$	$\lambda_1, \lambda_2 > 0, \lambda_3 = 0$				paraboloid eliptic
8.	0	$\neq 0$	$\lambda_1, \lambda_2 < 0, \lambda_3 = 0$				paraboloid hiperbolic
9.	0	0	$\lambda_1 > 0, \lambda_2 > 0, \lambda_3 = 0$		>0		cuadrică vidă
10.	0	0	$\lambda_1, \lambda_2 > 0, \lambda_3 = 0$		0		dreaptă dublă
11.	0	0	$\lambda_1 > 0, \lambda_2 > 0, \lambda_3 = 0$		<0		cilindru eliptic
12.	0	0	$\lambda_1, \lambda_2 < 0, \lambda_3 = 0$		>0		cilindru eliptic
13.	0	0	$\lambda_1, \lambda_2 < 0, \lambda_3 = 0$		<0		cuadrică vidă
14.	0	0	$\lambda_1, \lambda_2 < 0, \lambda_3 = 0$		$\neq 0$		cilindru hiperbolic
15.	0	0	$\lambda_1, \lambda_2 < 0, \lambda_3 = 0$		0		plane secante
16.	0	0	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$		0	>0	cuadrică vidă
17.	0	0	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$		0	0	plan dublu
18.	0	0	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$		0	<0	plane paralele
19.	0	0	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$		$\neq 0$		cilindru parabolic

Desene și ecuații

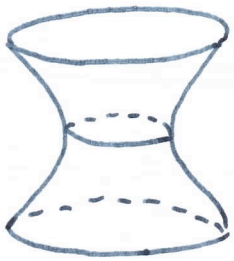
1. Elipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



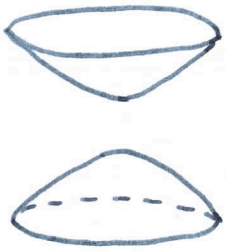
2. Hiperboloid cu o pânză

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



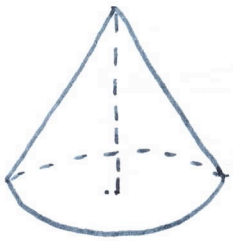
3. Hiperboloid cu 2 pânze

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$



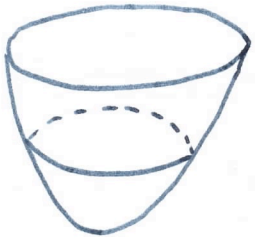
6. Con

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



7. Paraboloid eliptic

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

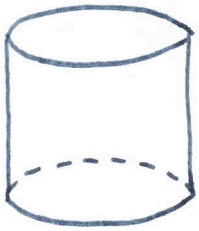


8. Paraboloid hiperbolic

$$\frac{z}{c} = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

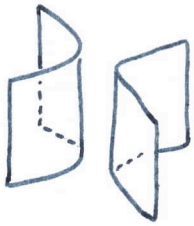
11. Cilindru eliptic

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



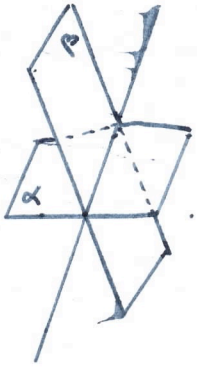
14. Cilindru hiperbolic

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



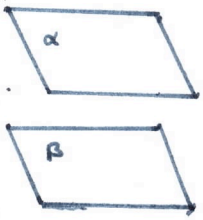
15. Plane secante

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$



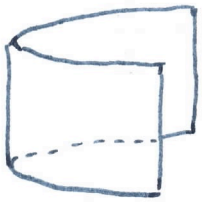
18. Plane paralele

$$x^2 - a^2 = 0$$



19. Cilindru parabolic

$$y = ax^2$$



Bibliografie

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