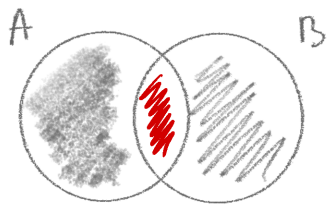
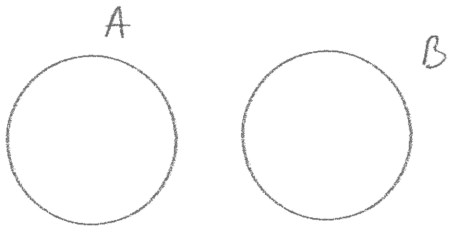
Seminar 3

Principiul includerii și excluderii

Fie A, B mulțimi finite

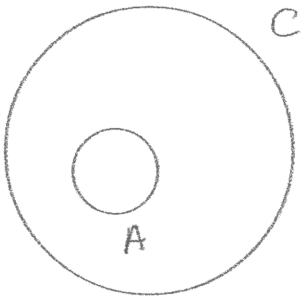


$A \cap B \neq \emptyset$



$A \cap B = \emptyset$

$$|A \cup B| = |A| + |B| - |A \cap B|$$



↙ $|C|$ e cunoscută

Dacă aflu $|C \setminus A|$ aflu și A

ex 1

$A = \{1, 2, \ldots, 1000\}$

$B = \{a \in A \mid 3 | a \text{ sau } 5 | a \text{ sau } 7 | a\}$

Calculați $|B|$

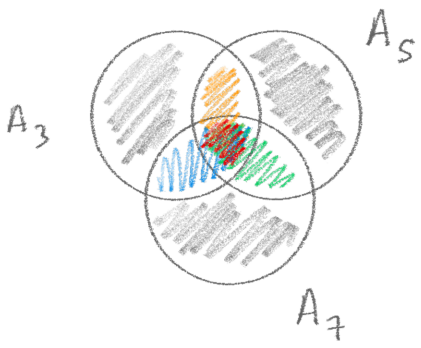
$A \cup B \ni a \iff a \in A$ sau $a \in B$

$A \cap B \ni a \iff a \in A$ și $a \in B$

$A \setminus B \ni a \iff a \in A$ și $a \notin B$

Sol:

$\Rightarrow B = A_3 \cup A_5 \cup A_7$



$$|A_3 \cup A_5 \cup A_7| = |A_3| + |A_5| + |A_7| - |A_3 \cap A_5| - |A_3 \cap A_7| - |A_5 \cap A_7| + |A_3 \cap A_5 \cap A_7|$$

$$= |A_3| + |A_5| + |A_7| - |A_{15}| - |A_{21}| - |A_{35}| + |A_{105}|$$

$$= \left[\frac{1000}{3}\right] + \left[\frac{1000}{5}\right] + \left[\frac{1000}{7}\right] - \left[\frac{1000}{15}\right] - \left[\frac{1000}{35}\right] - \left[\frac{1000}{21}\right] + \left[\frac{1000}{105}\right]$$

□

ex 2

Câte numere naturale mai mici sau egale cu un milion (10^6) sunt de forma x^2

nu de forma x^3 nu de forma x^5, unde $x \in \mathbb{N}$.

Sol:

$Y = \{ n \in \mathbb{N} \mid n \leq 10^6, \ n \neq x^2 \text{ nu } n \neq x^3 \ n \neq x^5, \ x \in \mathbb{N} \}$

$Y_2 = \{ n \in \mathbb{N} \mid n \leq 10^6 \text{ a.î. } n = h^2 \text{ pt } h \in \mathbb{N} \}$

$Y_3 = \{ n \in \mathbb{N} \mid n \leq 10^6 \text{ a.î. } n = h^3 \text{ pt } h \in \mathbb{N} \}$

$Y_5 = \{ n \in \mathbb{N} \mid n \leq 10^6 \text{ a.î. } n = h^5 \text{ pt } h \in \mathbb{N} \}$

$Y, Y_2, Y_3, Y_5 \subseteq A = \{ 0, 1, \dots, 10^6 \}$

prin def $Y_2, Y_3, Y_5 \subset A$

$n \in Y \iff \cancel{n \in A}, \ n \notin Y_2$ ($\iff n \in \complement_A Y_6$) nu

$n \notin Y_3$ ($\iff n \in \complement_A Y_3$) nu $n \notin Y_5$ ($\iff n \in \complement_A Y_5$)

$\iff n \in \complement_A Y_2$ nu $n \in \complement_A Y_3$

nu $n \in \complement_A Y_3$

$\iff n \in \complement_A Y_2 \cup \complement_A Y_3 \cup \complement_A Y_5$

$\iff n \in \complement_A (Y_2 \cap Y_2 \cap Y_5)$