Fie $\Sigma, \Delta \in \mathcal{P}(E)$ si $\alpha, \beta, \gamma \in E$, astfel incat:

$$\Sigma \vdash \alpha \vee (\beta \to \gamma),\ \Delta \vdash \gamma \to \alpha.$$

Conform Teoremei de Completitudine Tare (TCT), rezulta:

$$\Sigma \vDash \alpha \vee (\beta \to \gamma),\ \Delta \vDash \gamma \to \alpha.$$

Fie $h: V \to L_2 = \{0, 1\}$ a.i. $h \vDash \Sigma \cup \Delta. \Leftrightarrow h \vDash \Sigma$ si $h \vDash \Delta$

$$\left.\begin{array}{l} \Sigma \vDash \alpha \vee (\beta \to \gamma) \\ h \vDash \Sigma \end{array}\right\} \Rightarrow h \vDash \alpha \vee (\beta \to \gamma) \Leftrightarrow$$

$$\Leftrightarrow 1 = \tilde{h}(\alpha \vee (\beta \to \gamma))$$

$$= \tilde{h}(\alpha) \vee [\tilde{h}(\beta) \to \tilde{h}(\gamma)]$$

$$= \tilde{h}(\alpha) \vee \overline{\tilde{h}(\beta)} \vee \tilde{h}(\gamma)$$

⇔ h~(α)=1 sau $\overline{\text{h\~{}}(\beta)}$=1 sau h~(γ)=1

⇔ h~(α)=1 sau h~(β)=0 sau h~(γ)=1.

$$\left.\begin{array}{l} \Delta \vDash \gamma \to \alpha \\ h \vDash \Delta \end{array}\right\} \Rightarrow h \vDash \gamma \to \alpha$$

⇔ 1= h~(γ->α) = h~(γ)-> h~(α)

⇔ h~(γ) ≤ h~(α).

<u>Caz 1:</u> Daca h~(β)=0 => h~(β->α) = h~(β)->h~(α) = 0->h~(α)=1.

<u>Caz 2:</u> Daca h~(β)=1 => h~(α)=1 sau h~(γ)=1.

Presupunem prin absurd ca h~(α)≠1. => h~(γ)=1 => 1 ≤ h~(α) => h~(α)=1; contradictie. => h~(α)=1.

=> h~(β->α) = h~(β)->h~(α) = h~(β)->1 = 1.

In ambele cazuri, => h |= β->α.

⇨ $\Sigma \cup \Delta \vDash \beta \to \alpha$ $\Sigma \cup \Delta \vdash \beta \to \alpha$