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STEP POTENTIALS AND BODY CURRENTS NEAR GROUNDS IN TWO-LAYER EARTH

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Abstract - The equations customarily used in safety calculations for grounding electrodes assume that soil conductivity does not vary with depth. A more realistic model assumes that the soil consists of two horizontal layers of different conductivity. This may occur, for example, if a thin surface layer consists of wet soil, thawed soil, or concrete, and a thick layer of underlying soil is frozen, or consists of rock or dry soil. Such conditions markedly change the step voltage, contact resistance, and body current a creature on the surface will experience in the vicinity of a buried ground electrode. Equations are presented for voltage, voltage gradient (step voltage), contact resistance, body current, and electrode resistance in the vicinity of a ground electrode in the shape of a long buried wire in a two-layered earth. Graphs are also presented to give a quantitative and qualitative understanding of the effect of such layering on step voltage and body current.

INTRODUCTION

If a ground electrode is placed in earth having a different conductivity near the surface from the conductivity lower down, the combinations of surface conductivity, basement conductivity and surface layer depth that produce maximum voltage gradient at the surface are not the same as the conditions that produce maximum body current. Since body current is what produces physiological effects, the design of grounding electrodes to meet environmental criteria should take into account the effects of such layering. If this is not done, or if the design is based on meeting specified levels of voltage gradient (step voltage) rather than body current, designs may be used which are either excessively hazardous or excessively costly.

Layering may occur if there is a thin layer of topsoil above a thick layer of underlying rock, if the surface layer is frozen and the underlying soil is not, or if the surface layer has thawed and the underlying soil is still frozen. It also occurs when rain breaks a long dry spell, wetting the surface layer while the underlying soil is still dry. A concrete slab laid on poorly conductive soil is also an example of horizontal layering.

PROBLEM FORMULATION

The problem which will be analyzed assumes that a bare horizontal wire of length L meters and radius a meters is buried at a depth of D meters below the surface of the earth, and is conducting a current of I amps into the earth. The origin of the rectangular coordinate system is at the center of the wire, the wire lying along the x axis and the y axis being horizontal and perpendicular to the wire. It is assumed that the current density leaking from the wire into the earth is the same at every point along the wire. In actuality, the current density is nearly constant over most of the length of the wire, but tends to rise near the ends of the wire. If greater accuracy is wanted than the uniform current density approximation provides, the wire can be broken into several segments and the fields caused by each segment can be computed and summed. The current distribution can be found by forcing all the segments to be at the same voltage. Such refinement may not be warranted, however, since the exact values of soil conductivity in the layers is usually not known accurately. The voltage gradient near the ends can be easily reduced by burying the ends more deeply. This leaves the long center portion of the wire as the main area of interest. Consequently, attention will focus, herein, on the fields near the center of the wire, although equations will be given which are applicable near the ends as well.

The assumption that leakage current density is constant along the wire also implies that the resistance and inductance of the ground wire produce negligible effect compared to the conduction of current into the soil, and that the wire is not long compared to a skin depth. These are usually reasonable assumptions at power frequencies, as will be shown in detail later in this paper.

Graphs are plotted as a function of the conductivity of the top layer of soil. The unit of conductivity is siemens per meter, sometimes called mhos per meter. It is the reciprocal of resistivity in ohm-meters. The symbol for conductivity will be σ; that for resistivity will be ρ. Subscript 1 refers to the top layer of soil, subscript 2 refers to the basement layer. The ground conductor may be buried either in the top or bottom layer. Equations will be given for both cases, and the graphs apply to both cases. The thickness of the top layer is H meters.

While it is true that for any one combination of values of surface conductivity,

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basement conductivity, and surface layer thickness the location of the point at which maximum voltage gradient occurs is also the point at which maximum body current occurs, and the body current is proportional to the voltage gradient, it is also true that a change in the surface layer thickness or conductivity may increase voltage gradient while decreasing body current, or vice-versa. This occurs because the contact resistance between the feet of a person or animal and the earth depends on the conductivity of both the surface and the lower layer and the thickness of the surface layer, if the surface layer is less than about 0.5 meters thick. The voltage gradient at the surface varies differently than contact resistance does with changes in surface layer thickness and conductivity.

As will be seen, the worst case from a body current standpoint occurs when the ground electrode is in a low conductivity bottom layer of soil, and a person stands on a moderately thin surface layer of moderately high conductivity. If the surface layer is very thick or very thin, or its conductivity is very high or very low, the body current is less than with intermediate values.

Another difference between the two-layered case and the homogeneous earth case is that in homogenous earth, if the ground wire electrode is buried at depth D, the maximum voltage gradient and body current will be found at the ends and at distance D beside the wire. In the two-layered case, if the surface layer has substantially higher conductivity than the lower layer, the location at which the maximum gradient and body current occur is found substantially farther away from the buried electrode, and the gradient and body current decrease more slowly with distance. The magnitude of the voltage gradient (but not necessarily the body current) is less than it would be if the conductivity were low all the way to the surface.

While some discussion of grounding in two-layered earth has appeared previously in the literature [1-6], it is hoped the equations and graphs presented herein will provide additional insight and information to readers interested in body current near electrodes in two-layer soil.

Symbols

- b Radius of a disk representing the human foot, meters
- D Depth of electrode below surface of earth, meters
- f Frequency, Hz
- H Thickness of top layer, meters
- I RMS current flowing into earth from electrode, amps
- i_B RMS current going up through one leg and down through other
- K Reflection factor $K = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$
- L Length of buried electrode, meters
- L2 L/2
- R_c Contact resistance between soil and two feet in series, ohms
- R_g Resistance between electrode and distant earth
- s Distance between feet, meters
- V Voltage at surface of earth, relative to a distant point
- y Horizontal distance from center of electrode, perpendicular to conductor
- δ Skin depth, meters
- ρ_1 Resistivity of top layer, ohm-m
- ρ_2 Resistivity of bottom layer, ohm-m
- σ_1 Conductivity of top layer, siemens/meter $\sigma_1 = 1/\rho_1$
- σ_2 Conductivity of bottom layer, siemens/meter $\sigma_2 = 1/\rho_2$
- S/m Siemens/meter

Acknowledgement

I am indebted to C. A. Martin, formerly of RCA, for the development of equations (10) and (11).

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RESULTS

Surface Voltage

Equation (1) gives the voltage at the surface of the earth in the neighborhood of a ground wire buried in the bottom layer (D > H):

$$V(x, y) = \frac{I \rho_1 (1+K)}{2\pi L}\left(A + \sum_{n=1}^{\infty} K^n B\right) \tag{1}$$

where

$$K = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$$

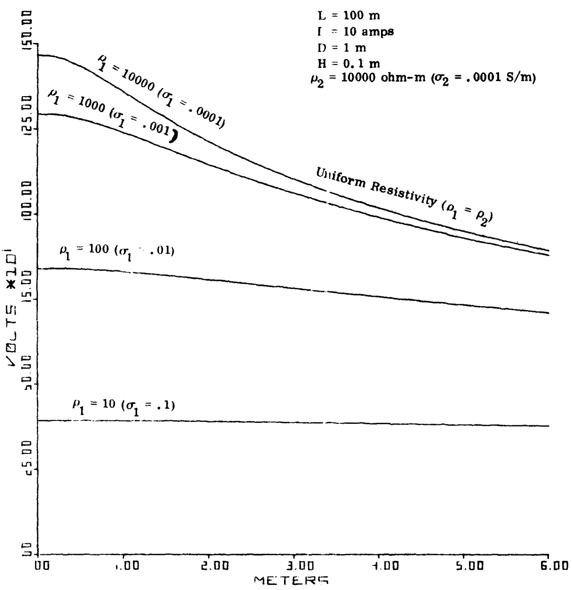
$$A = \ln\left(\frac{\sqrt{(x + L2)^2 + y^2 + D^2} + x + L2}{\sqrt{(x - L2)^2 + y^2 + D^2} + x - L2}\right)$$

$$B = \ln\left(\frac{\sqrt{(x + L2)^2 + y^2 + (2nH + D)^2} + x + L2}{\sqrt{(x - L2)^2 + y^2 + (2nH + D)^2} + x - L2}\right)$$

$$L2 = L/2$$

and the other variables are as previously defined. In the homogeneous earth case K=0, and the summation term is therefore zero.

Figure 1 shows the voltage on the surface of the earth near the center of the ground wire as a function of distance y from the wire, for four different values of surface conductivity. The top curve is the homogeneous earth case, the other three curves being for progressively more conducting top layers over the low conductivity basement in which the ground electrode lies. As can be seen, the voltage and maximum voltage gradient (as indicated by slope of curve) decrease as top layer conductivity increases. Figure 1 was drawn for x = 0, L = 100m, I = 10 amps, D = 1m, H = 0.1m, σ_2 = .0001 S/m. These parametric values are purely for illustrative purposes, and do not represent a proposed design.



y, Horizontal distance perpendicular to center of conductor

Fig. 1. Voltage at surface of earth near center of electrode buried 1 m below surface in 10000 ohm-m soil, for different surface layers.

If the ground wire is in the top layer of soil (2) should be used instead of (1):

$$V(x, y) = \frac{I \rho_1}{2\pi L}\left[A + \sum_{n=1}^{\infty} K^n (B + C)\right] \qquad (2)$$

where

$$C = \ln\left(\frac{\sqrt{(x + L2)^2 + y^2 + (2nH - D)^2} + x + L2}{\sqrt{(x - L2)^2 + y^2 + (2nH - D)^2} + x - L2}\right)$$

A derivation of (1) and (2) is given in the Appendix, and a different derivation is given in [7]. Equations (1) and (2) reduce to equation 3.55 of Sunde [8] when $\rho_1 = \rho_2$.

Voltage Gradient

The voltage gradient is, of course, the derivative of voltage with respect to distance. It is significant because the voltage between a person's feet is equal to the average voltage gradient between his feet multiplied by the distance between them. This voltage difference is called step voltage or step potential. For convenience, it will be assumed that a person has his feet one meter apart and that they are positioned in the direction of the maximum gradient, which near the center of the wire is along a line at right angles to the wire. The maximum step voltage will then be slightly less then the maximum voltage gradient, since the gradient does not remain at its maximum value over the entire distance between a person's feet.

Differentiating (1) and setting x = 0, we obtain the voltage gradient near the center of the wire, for the case in which the wire is in the bottom layer:

$$\frac{\partial V}{\partial x}(0, y) = 0$$

$$\frac{\partial V}{\partial y}(0, y) = -\frac{I \rho_1 (1 + K)\, y}{2\pi}\left(E + \sum_{n=1}^{\infty} K^n F\right) \qquad (3)$$

where

$$E = \frac{1}{(y^2 + D^2)\sqrt{L2^2 + y^2 + D^2}}$$

$$F = \frac{1}{[y^2 + (2nH + D)^2]\sqrt{y^2 + (2nH + D)^2 + L2^2}}$$

Figure 2 shows the magnitude of the voltage gradient on the surface of the earth near the center of the ground wire as a function of distance y from the wire, with the same parameter values as in Figure 1. The top curve is for the homogeneous earth case. As can be seen, the voltage gradient is very high, the maximum value occurring at the point at which y = D. If (1) is differentiated with respect to x instead of y, it will be found that in the homogeneous earth case the maximum gradient in line with the end of the of the ground wire occurs over the end of the wire, and is the same as that shown in Figure 2 near the center of the wire. When that conductivity of the top layer increases, Figure 2 shows that the maximum gradient becomes much smaller, but occurs farther from the wire, and the gradient does not fall off as rapidly with increasing distance.

Equation (4) gives the gradient for y = 0. The variable x_1 has been substituted for x, where $x_1 = x - L/2$. Thus x_1 is the distance beyond the end of the ground wire.

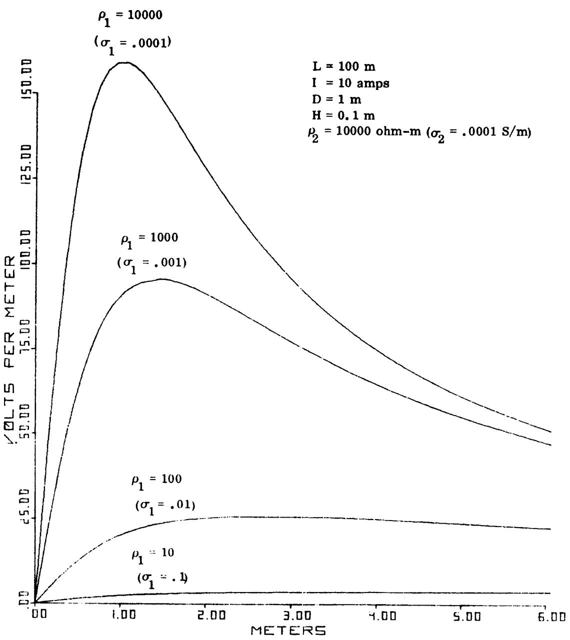
$$\frac{\partial V}{\partial x_1}(x_1, 0) = \frac{I \rho_1 (1 + K)}{2\pi L}\left(R + \sum_{n=1}^{\infty} K^n S\right) \qquad (4)$$

$$\frac{\partial V}{\partial y}(x_1, 0) = 0$$

where

$$R = \frac{1}{\sqrt{(x_1 + L)^2 + D^2}} - \frac{1}{\sqrt{x_1^2 + D^2}}$$

$$S = \frac{1}{\sqrt{(x_1 + L)^2 + (2nH + D)^2}} - \frac{1}{\sqrt{x_1^2 + (2nH + D)^2}}$$



y, Horizontal distance perpendicular to center of conductor

Fig. 2. Voltage gradient at surface of earth near center of electrode buried 1 m below surface in 10000 ohm-m soil, for different surface layers.

If the ground wire is in the top layer, the gradients are given by (5) and (6), rather than by (3) and (4):

$$\frac{\partial V}{\partial y}(0, y) = -\frac{I \rho_1 y}{2\pi}\left[E + \sum_{n=1}^{\infty} K^n (F + G)\right] \tag{5}$$

$$\frac{\partial V}{\partial x_1}(x_1, 0) = \frac{I \rho_1}{2\pi L}\left[R + \sum_{n=1}^{\infty} K^n (S + T)\right] \tag{6}$$

where

$$G = \frac{1}{[y^2 + (2nH - D)^2]\sqrt{y^2 + (2nH - D)^2 + L2^2}}$$

$$T = \frac{1}{\sqrt{(x_1 + L)^2 + (2nH - D)^2}} - \frac{1}{\sqrt{x_1^2 + (2nH - D)^2}}$$

It should be noted that the above gradients are based on the assumption that the conductivity of each layer is constant as one moves horizontally. If there are variations in conductivity as one moves horizontally, as well as vertically, gradients in the vicinity of the changes in conductivity will be different than the above equations would predict. Such effects are difficult to calculate, and are beyond the scope of this paper.

Contact Resistance and Body Current

Reference [9] gives the following equation for computing the contact resistance between a person's feet and the ground:

$$R_c = 6\rho \tag{7}$$

In this equation R_c is the contact resistance between the soil and the two feet in series, and ρ is the surface resistivity. The equation was derived by modeling each foot as a perfectly conducting disc having a radius of 8 cm. Mutual resistance effects between the two feet were not considered. These effects are small if the separation between the feet is large compared to the diameter of the foot.

The body current passing through a person's legs is given by:

$$i_B = V_s/(R_b + R_c) \tag{8}$$

where V_s is the step voltage, that is, the voltage between the person's feet. In the graphs herein, it will be assumed that the person's feet are one meter apart, that the voltage gradient is constant at its maximum value for the entire distance from one foot to the other, and that the person's feet are aligned in the direction of the maximum voltage gradient. All of these assumptions are conservative. R_b is the body resistance, and includes the resistance of the shoes, the skin on the feet, and the body itself. In accordance with the recommendation of Reference 9 it will be assumed herein that the person's shoes and skin are wet and have zero resistance, and that the resistance of the body is 1,000 ohms. Reference 9 based this figure on data from tests on subjects standing barefoot in pails of salt water.

Equation (7) was based on homogeneous soil. If the soil has two layers, and the top layer is thin, the contact resistance can be substantially affected by the conductivity of the bottom layer. The equation for contact resistance considering layering and mutual resistance effects is given by:

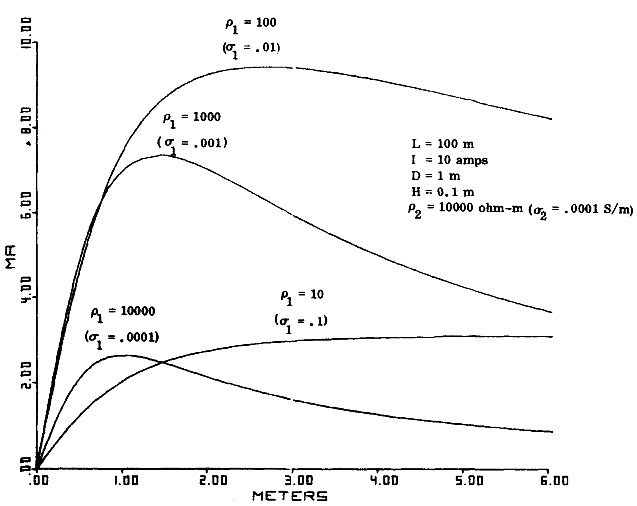
$$R_c = \frac{\rho_1}{2}\left[\frac{1}{b} + 2\sum_{n=1}^{\infty}\frac{K^n}{\sqrt{(2nH)^2 + b^2}}\right] - \frac{\rho_1}{\pi}\left[\frac{1}{s} + 2\sum_{n=1}^{\infty}\frac{K^n}{\sqrt{(2nH)^2 + s^2}}\right] \tag{9}$$

where b is the radius of the conducting disc representing the foot and s is the separation between the centers of the discs representing the two feet. The first expression gives the contact resistance of the two discs in series, and the second expression gives a correction term to allow for the effects of the mutual resistance between the two discs. The mutual resistance term tends to reduce the total contact resistance sightly when the feet are in series. Reference [7] gives the derivation of (9). Equation (9) is plotted in Figures 7-10 for typical values of the parameters.

Figure 3 shows the body current which would go through a person's legs when he walks near the center of the ground wire having the parameter values used previously, based on (3), (8), and (9) and b = 8 cm.

Although the maximum step voltage in Figure 2 went down from 159 v/m to 96 v/m when surface layer conductivity went from 0.0001 S/m to 0.001 S/m, the body current went up, going from 2.6 mA to 7.3 mA. The explanation is that the higher conductivity surface layer greatly decreased contact resistance. R_c dropped from 59.3 kohms to 12 kohms for the same change in surface conductivity. If surface conductivity is further increased to 0.01 S/m, maximum step voltage drops to 25.8 v/m, but R_c drops to 1743 ohms, increasing body current still further to 9.4 mA. But a further increase of surface conductivity to 0.1 S/m reduces body current instead of increasing it, because maximum step voltage drops to 3.7 v/m and R_c drops to 217 ohms. R_c is now small compared with the body resistance of 1,000 ohms, so body current decreases to 3.1 mA.

A key point to note is that if maximum body current were computed by simply dividing maximum step voltage of 159 v/m by minimum total resistance (body plus contact) of about 1,000 ohms, a computed value of 159 mA would have been obtained. This is about 17 times worse than what the body current could actually be, because the conditions that produce low contact resistance also drastically reduce step voltage.



y, Horizontal distance perpendicular to center of conductor

Fig. 3. Body current at surface of earth near center of electrode buried 1 m below surface in 10000 ohm-m soil, for different surface layers.

In order to find the worst combinations of surface layer depth, surface layer conductivity, and basement conductivity, several hundred combinations of values were evaluated. For each combination of parameter values the highest value of voltage gradient was computed by locating the value of y which maximized the gradient. This was done using the method of golden sections. Body current was computed for this value of V_s . Contours of constant magnitude of body current were then plotted as a function of surface layer conductivity and depth for three different values of basement conductivity, all rather low, since the low conductivity basement produces the worst situation with respect to body current. These plots are shown in Figure 4, 5, and 6. Parameter values used in Figures 4, 5, and 6 were different then those in Figures 1, 2, and 3. They were:

L = 4 km	I = 100 amps
D = 1.83 m (6 feet)	b = 10 cm

The results indicate that for a long ground electrode at this depth the worst value of surface layer depth is in the vicinity of 0.1 m and the worst value of surface conductivity is in the vicinity of 0.01 S/m. The crucial fact to note is that in no case did layering increase the body current by more than a factor of about four as compared with what would hae been experienced if the basement conductivity had continued all the way to the surface. Consequently, a ground electrode design which produces a relatively high step voltage when the ground is very dry or frozen all the way to the surface will not necessarily produce a lethal, or even a perceptible, shock when the surface is wet and conductive, because the conditions which lower contact resistance also lower step voltage. However, some additional safety factor must be allowed as compared with the homogeneous earth situation.

Additional runs (not shown) were made for a point electrode at a depth of 0.76 m (2.5 feet). In that case layering increased body current by a maximum of 50%. The worst case was σ_1 = .001 S/m, H = 0.2m.

It should be remembered that body currents can be greater than those produced by a one meter step under some conditions. For example, a person might be dragging an aluminum canoe or other long metal object across the ground, or two people might be carrying such an object. In that case the voltage could be developed over a distance of several meters, rather than one meter, and the feet would be in parallel rather than in series. And, as was mentioned previously, horizontal conductivity discontinuities can increase the fields in some places.

Finally, if the ground electrode is held at constant voltage, rather than constant current, gradients and body currents will be larger, as surface conductivity increases, because an increase in surface conductivity decreases the resistance between the ground electrode and the soil. This effect is discussed in the next section. The effect of σ_1 and H on R_c is shown in Figures 7-10. These contour plots are for b = 8 cm and σ_2 = 0.0001, 0.001, 0.01, and 0.1 S/m.

Effect of Layering on Grounding Resistance

Equations (10) and (11) give the resistance between the earth and a grounding electrode in the shape of a long wire for the

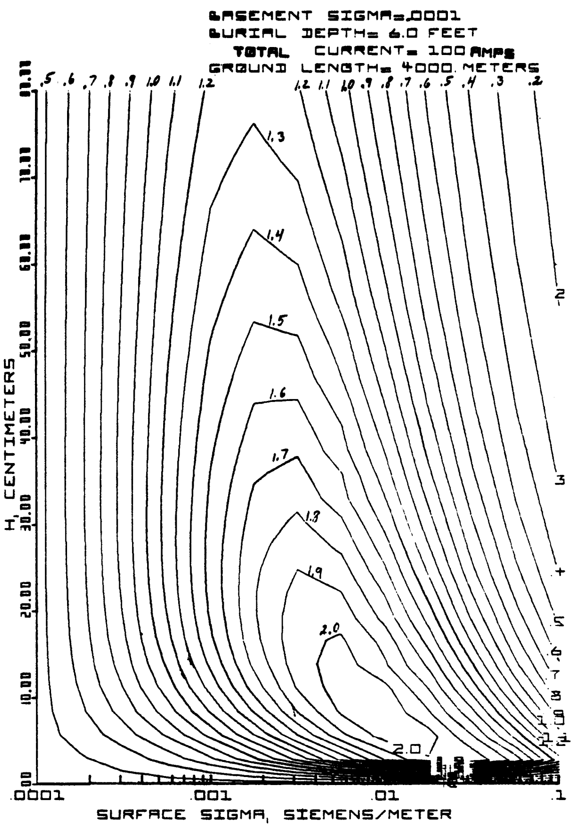


Fig. 4. Maximum body currents, mA., $\sigma_2 = 0.0001$ S/m.

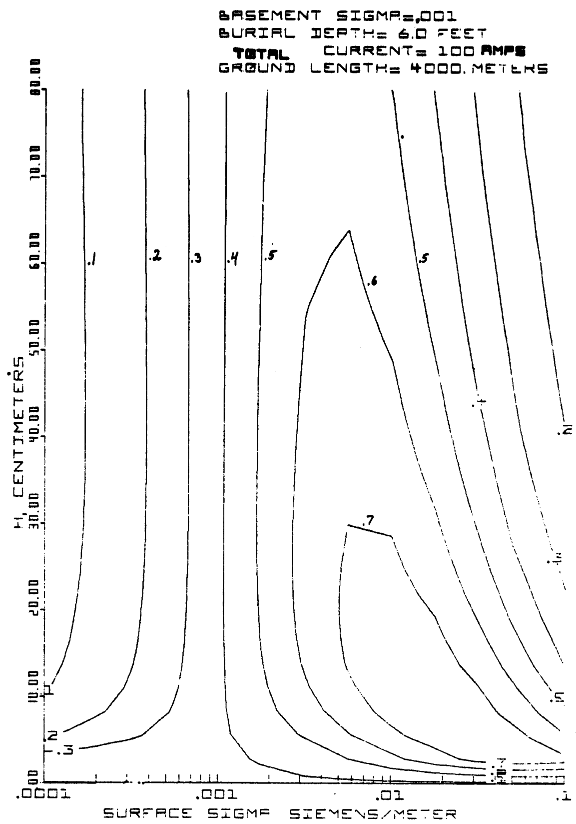


Fig. 6. Maximum body currents, mA., $\sigma_2 = 0.001$ S/m.

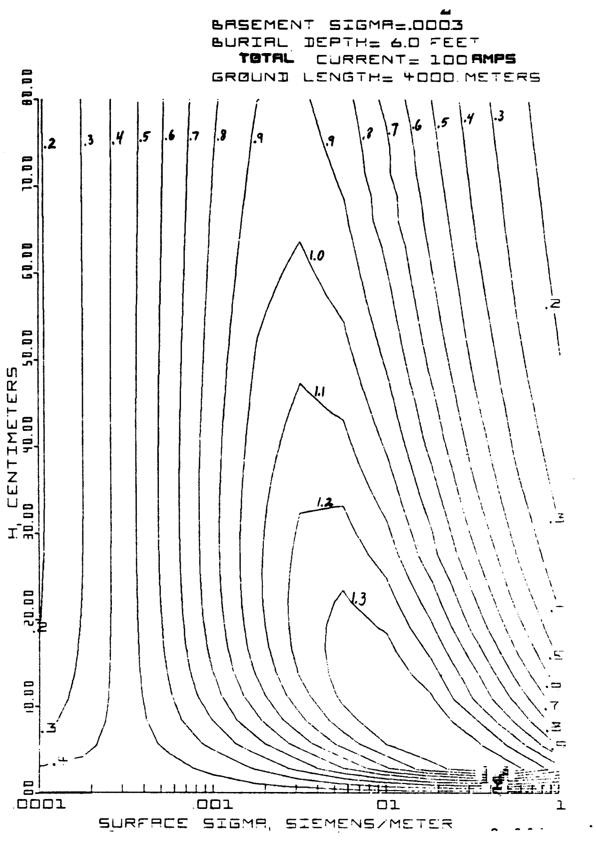


Fig. 5. Maximum body currents, mA., $\sigma_2 = 0.0003$ S/m.

cases where the electrode is in the top layer and the basement layer, respectively:

$$R_g = \frac{\rho_1}{2\pi L}\left[\ell n\left(\frac{2L}{a}\right) - 1 + U_1 + W_1\right] \; ; \quad D < H - \frac{a}{2} \qquad (10)$$

where

a = radius of ground wire in meters

$$U_1 = \ell n\left[\frac{1 + \sqrt{1 + 4\,D^2/L^2}}{2\,D/L}\right] + \frac{2D}{L} - \sqrt{1 + 4\,D^2/L^2}$$

$$W_1 = \sum_{n=1}^{\infty} K^n \;(U_2 + 2\,U_3 + U_4)$$

$$U_2 = \ell n\left[\frac{1 + \sqrt{1 + 4\,(nH - D)^2/L^2}}{2\,(nH - D)/L}\right] + \frac{2\,(nH - D)}{L} - \sqrt{1 + 4\,(nH - D)^2/L^2}$$

$$U_3 = \ell n\left[\frac{1 + \sqrt{1 + 4\,(nH)^2/L^2}}{2\,nH/L}\right] + \frac{2\,nH}{L} - \sqrt{1 + 4\,(nH)^2/L^2}$$

$$U_4 = \ell n\left[\frac{1 + \sqrt{1 + 4\,(nH + D)^2/L^2}}{2\,(nH + D)/L}\right] + \frac{2\,(nH + D)}{L} - \sqrt{1 + 4\,(nH + D)^2/L^2}$$

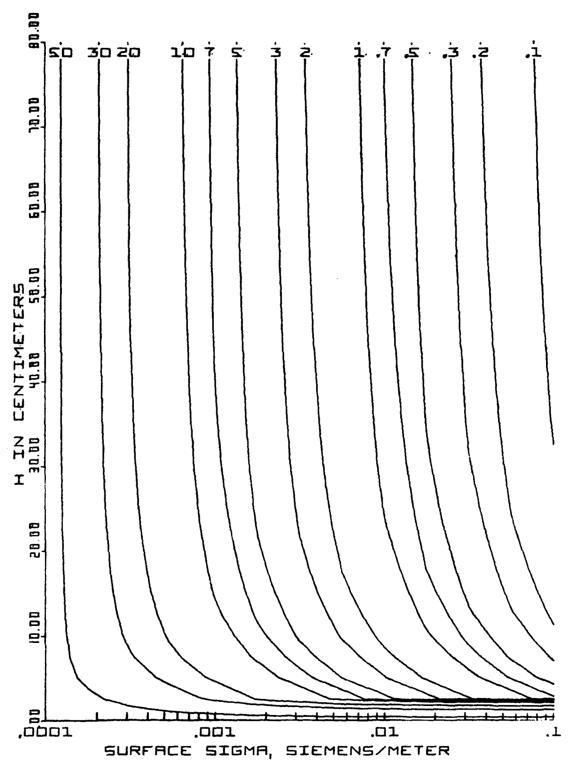


Fig. 7. Contact Resistance, Thousands of ohms
Basement Sigma = .0001 Siemens/meter

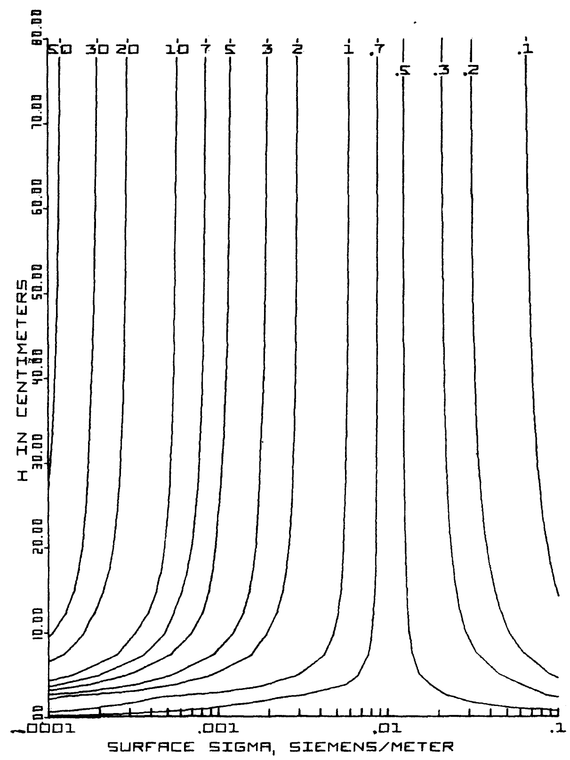


Fig. 9. Contact Resistance, Thousands of ohms
Basement Sigma = .01 Siemens/meter

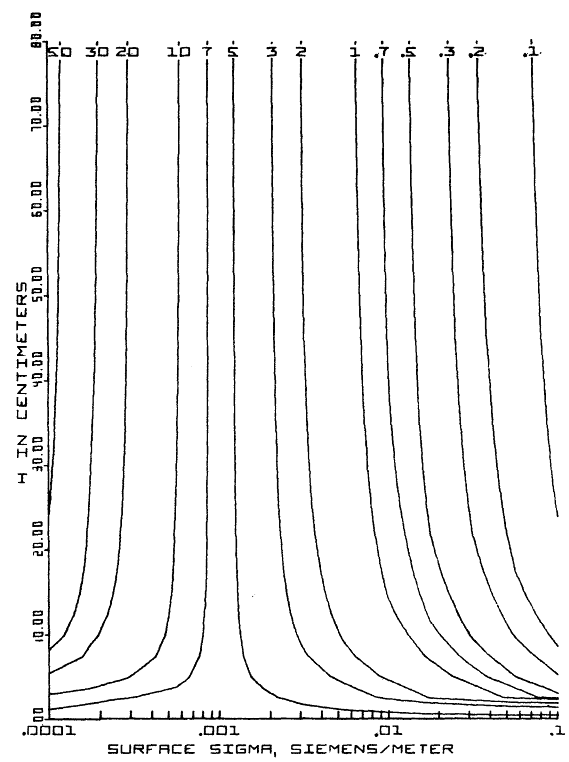


Fig. 8. Contact Resistance, Thousands of ohms
Basement Sigma = .001 Siemens/meter

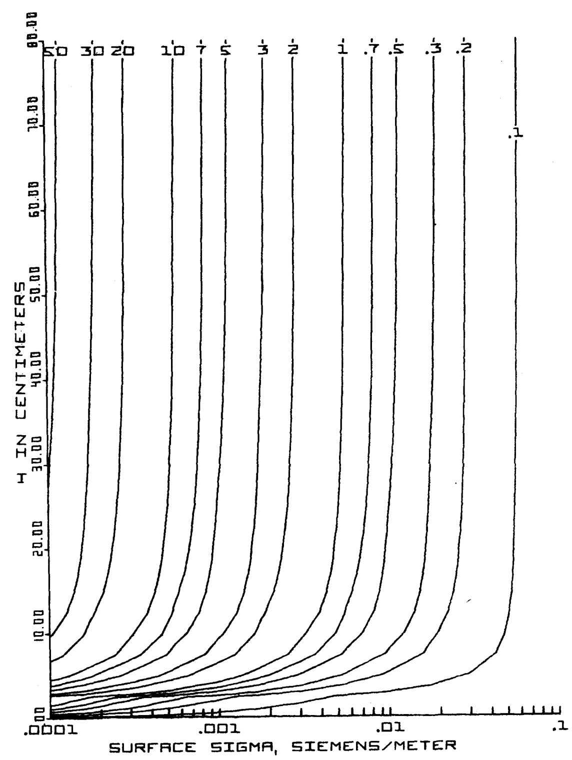


Fig. 10. Contact Resistance, Thousands of ohms
Basement Sigma = .1 Siemens/meter

$$R_g = \frac{\rho_2}{2\pi L}\left[\ell n\left(\frac{2L}{a}\right) - 1 - K\,U_5 + (1-K^2)\,(U_1 + W_2)\right]; \qquad D > H + \frac{a}{2} \tag{11}$$

where

$$W_2 = \sum_{n=1}^{\infty} K^n U_4$$

$$U_5 = \ell n\left[\frac{1 + \sqrt{1 + 4\,(D-H)^2/L^2}}{2\,(D-H)/L}\right] + \frac{2\,(D-H)}{L} - \sqrt{1 + 4\,(D-H)^2/L^2}$$

These equations were evaluated and plotted in a manner similar to that used for body current. As expected, it was found that the grounding resistance changes sharply if the thickness of the surface layer increases enough to put the ground wire in the top layer rather than in the basement layer. If the wire is in a basement layer of high conductivity, the conductivity of the top layer has little effect on the grounding resistance, but if the wire is in a basement layer of low conductivity, a highly conductive top layer of moderate thickness can strongly affect the value of R . For example, the parameter values used in Figures 1-3 produced the following values of R for a wire radius of 5.84 mm, corresponding to 4/0 wire:

ρ_2	ρ_1	σ_1	R_g (ohms)
10000	10000	0.0001	208
10000	1000	0.001	197
10000	100	0.01	163
10000	10	0.1	118

Effect of Length of the Ground Electrode

The above equations were derived under the assumption that the resistance of the ground conductor itself is negligible compared with R_g, and that inductive effects are negligible. This latter assumption is roughly equivalent to requiring that the length of the conductor be less than the skin depth of the earth at the frequency in question, where skin depth δ is given by (12):

$$\delta = 5000/(\pi\sqrt{10\,f\,\sigma_e}) \tag{12}$$

where f = frequency and σ_e is the conductivity of the earth. Since skin depth is always very large for any practical value of conductivity at power frequencies, the basement conductivity should be used for σ. At f = 60 Hz, skin depth ranges from 6.5 km for σ_e = 0.0001 S/m to 205 m for σ_e = 0.1 S/m.

If conductor length is large compared to skin depth, the portions of the conductor more than a skin depth away from a feed point will be at a relatively low voltage, and thus will be relatively ineffective. This effect can be reduced by feeding the ground conductor at the center, or, if necessary, at several points. For example, in Figure 6 σ_2 = 0.001 and L = 4 km, so the ground wire should be fed in the center, or, even better, consist of several separated segments or rays to make the equations be accurate.

To give a feel for the validity of the assumption that the resistance of the conductor should be small compared to R_g, the resistance of the 100 m ground conductor used in Figures 1-3 is about 0.017 ohms if it is made of 4/0 stranded copper wire. This is several orders of magnitude less than the values of Rg given in the above table.

Within the broad range where the above assumptions are valid, if L is large compared to D, H, and y, the voltage gradient and body current varies approximately inversely with L, if I is held constant. The voltage required to produce a given value of I is, of course, IR_g. As a rough approximation, R_g varies inversely with L if L is large compared with D and H.

The equations given herein are for dc. At high frequency the summation terms would have to allow for phase differences between the images. At power frequencies the dc approach is believed to be a good approximation.

Note on Evaluation of Equations

The above equations contain summations from n = 1 to infinity. To evaluate them one must break off the summation someplace. The question is, how many terms is enough?

An answer to this question may be obtained by noting that the terms in the infinite series consist of the product of two factors, both of which decrease with increasing values of n. The first factor is K^n, which decreases very rapidly with n if K is near zero, a condition that occurs if the conductivities of the two layers are nearly equal. The second term also can be shown to decrease with n, although the decrease is slow if H is small and D is large. The way in which the second factor decreases with n is quite complicated to evaluate, but a bound on the error can be obtained by assuming that the second factor does not decrease at all with n. The series, starting at any value of n, is then just a simple geometric series, and one can compute the sum of the terms from n to infinity. In evaluating the summations for the results presented herein, such a calculation was performed from time to time while doing the summations, and the summation was terminated if this calculation showed that the sum of the remaining terms up to infinity could not exceed 1% of the summation of terms 1 through n. In most cases only a few terms had to be summed to reach this criterion, but in cases having a very thin top layer with a much higher conductivity than the bottom layer it was necessary to sum over 1600 terms to compute R_g. Computation of voltage gradients and R_c required far fewer terms.

A, B, and C in (1) and (2) are subject to roundoff error if L2 - x is large compared to the other quantities. In that case, if x = 0, use the following identity:

$$\ln\frac{\sqrt{L2^2 + m^2} + L2}{\sqrt{L2^2 + m^2} - L2} = 2\ln\left(\frac{L2}{m} + \sqrt{1 + (L2/m)^2}\right)$$

If $x \neq 0$ and $L2 - x \gg m$, use the following approximation for the denominators of A, B, and C:

$$\sqrt{(x-L2)^2 + m^2} + x - L2 \approx m^2/2(L2-x)$$

If x≠0 and $-(L2+x) >> m$, use the following approximation for the numerators of A, B, and C:

$$\sqrt{(x+L2)^2 + m^2} + x + L2 \approx -m^2/2(L2+x)$$

CONCLUSIONS

Body currents in a two-layer earth were found to be up to about four times larger than they would be in homogeneous earth, for the same conditions of electrode size, current, depth of burial, etc. If the ground electrode is operated at a given voltage, rather than at a given current, a moderately conductive surface layer over a less conductive basement layer containing the electrode can produce even larger increase in body current as compared with homogeneous earth. However, the body currents are much smaller than would be computed by using a voltage gradient level which could be produced by low conductivity soil extending all the way to the surface, combined with a low value of contact resistance such as could be produced by a fairly conductive top layer. Such a design process, which some regulatory agencies might advocate, is based on a self-contradictory situation, and would probably produce an excessively conservative and costly design. This conclusion, however, should be tempered by the following considerations:

1. Variations in conductivity in the horizontal direction, as might be caused by puddles and streams, might produce hot spots, increasing the body currents a person could experience. While it is clear that horizontal variations in conductivity can increase the voltage gradient in areas where it would otherwise be small, by putting such areas into closer contact with regions of higher gradients, it is not clear whether, and to what extent, horizontal variations in conductivity can increase the maximum body currents encountered anywhere in an area. In the absence of such information, a safety factor must be allowed.
2. Only a limited range of burial depths for the ground electrode was investigated herein, most of the runs being for a burial depth of 1.83 m (6 feet). Some runs were made at 0.76 m (2.5 feet) and 1 m.
3. A person dragging a long metal object, such as a canoe, or two people carrying such an object, could experience a larger current than a person would experience by merely walking on the surface near a ground electrode.

In spite of these cautions, use of the two layer model should give more realistic design data than the assumption of a homogeneous earth could give.

APPENDIX
DERIVATION OF EQUATIONS (1) AND (2)

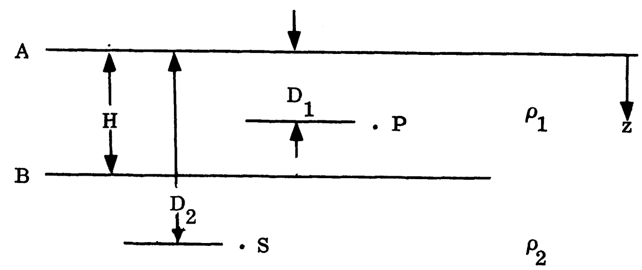
Effect of a Point Source in Bottom Layer on a Point in Top Layer

Consider a point source S in the bottom layer of soil, at coordinates x_s, y_s, D_2. Point P is in the top layer, at coordinates x_p, y_p, D_1. Let the z coordinate measure depth, downward from the top surface of the soil. Let H be the thickness of the top layer, σ_1 be the resistivity of the top layer, σ_2 be the resistivity of the bottom layer.

According to Maxwell's method of images [10], on which this derivation is based, the voltage at point P due directly to source S injecting current I will be:

$$E = \frac{I\rho_1\rho_2}{2\pi(\rho_1+\rho_2)} \cdot \frac{1}{\sqrt{(x_s-x_p)^2+(y_s-y_p)^2+(z_s-z_p)^2}} \qquad \text{(A-1)}$$

Here vertical distance $z_s - z_p$ is $D_2 - D_1$.



Point P will see an image of S, at distance D_2 above surface A. This image will also appear to inject current I, and will be at vertical distance $D_2 + D_1$ from point P. Its contribution to the voltage at point P will be the same as (1), except that the $z_s - z_p$ term is $D_2 + D_1$.

This image will have a reflection in surface B, of apparent magnitude KI, where $K = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$. This reflection will be at $z = D_2 + H$, so its vertical distance from P is $D_2 + 2H - D_1$. It, in turn, will have a reflection in A of magnitude KI, at vertical distance $D_2 + 2H + D_1$.

Similarly, there will be reflections of magnitude K^nI at distances $D_2 + 2nH - D_1$ and $D_2 + 2nH + D_1$, for n = 2, 3, 4, . . .

Noting that $2\rho_2/(\rho_1 + \rho_2) = 1 + K$, the total voltage at point P is given by:

$$E = \frac{I\rho_1}{4\pi}(1+K)\sum_{n=0}^{\infty}\left[\frac{K^n}{\sqrt{(x_s-x_p)^2+(y_s-y_p)^2+(2nH+D_2-D_1)^2}}\right.$$
$$\left.+\frac{K^n}{\sqrt{(x_s-x_p)^2+(y_s-y_p)^2+(2nH+D_2+D_1)^2}}\right]$$

If point P is on the surface, $D_1 = 0$, and we get:

$$E = \frac{I\rho_1}{2\pi}(1+K)\sum_{n=0}^{\infty}\frac{K^n}{\sqrt{(x_s-x_p)^2+(y_s-y_p)^2+(2nH+D_2)^2}} \qquad \text{(A-2)}$$

The effect of a line source, rather than a point source, is given in a separate section below.

Effect of a Point Source in the Top Layer on a Point in the Top Layer

Consider a point source S in the top layer, at coordinates x_s, y_s, D_2, and point P, also in the top layer, at x_p, y_p, D_1. In this case the voltage at P directly due to the source is given by:

$$E = \frac{I\rho_1}{4\pi} \cdot \frac{1}{\sqrt{(x_s - x_p)^2 + (y_s + y_p)^2 + (z_s - z_p)^2}}$$

For the source itself, the vertical distance $z_s = z_p$ is equal to $D_2 - D_1$.

There will be an image at $z = -D_2$, and thus at a vertical distance from P of $D_1 + D_2$. This image will inject a current of I amps.

The source and this image will have reflections in surface B, injecting currents of KI amps at vertical distances from P of $2H - D_2 - D_1$ and $2H + D_2 - D_1$. These will, in turn, be reflected in surface A, the reflections injecting currents of KI each, and being at vertical distances from P of $2H - D_2 + D_1$ and $2H + D_2 + D_1$.

Similarly, there will be reflections which appear as current sources of magnitude K^nI at vertical distances from P of $2nH - D_2 - D_1$, $2nH + D_2 - D_1$, $2nH - D_2 + D_1$, and $2nH + D_2 + D_1$.

If $D_1 = 0$, the total voltage at point P will be given by

$$E = \frac{I\rho_1}{2\pi} \left[\sum_{n=0}^{\infty} \frac{K^n}{\sqrt{(x_s - x_p)^2 + (y_s - y_p)^2 + (2nH + D_2)^2}} + \sum_{n=1}^{\infty} \frac{K^n}{\sqrt{(x_s - x_p)^2 + (y_s - y_p)^2 + (2nH - D_2)^2}} \right] \quad \text{(A-3)}$$

Voltage Produced at Point P by a Line Source

In (A-2) and (A-3), the voltage at point P produced by a point source consists of a sum of terms, each of the form

$$\frac{C}{\sqrt{(x_p - x_s)^2 + d^2}}$$

where C and d are constants.

If, instead of a point source, we have a line source of length L, extending in the x direction from $- L/2$ to $L/2$, each of these terms will change to

$$\frac{C}{L} \int_{-L/2}^{L/2} \frac{d\,x_s}{\sqrt{(x_p - x_s)^2 + d^2}}$$

Let $u = x_p - x_s$. The terms then become

$$-\frac{C}{L} \int_{x_p + L/2}^{x_p - L/2} \frac{du}{\sqrt{u^2 + d^2}}$$

Performing the integration, each of the terms becomes

$$\frac{C}{L} \ln \left(\frac{x_p + L/2 + \sqrt{(x_p + L/2)^2 + d^2}}{x_p - L/2 + \sqrt{(x_p - L/2)^2 + d^2}} \right) \quad \text{(A-4)}$$

Applying (A-4) to (A-2) and (A-3), we obtain equations (1) and (2).

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Robert J. Heppe (SM'78) was born in New York, NY on June 16, 1926. He received the BS EE degree (with honors) from Caltech in 1948, and the MS EE degree from Columbia University in 1955.

Mr. Heppe worked for ITT for 13 years, mostly on missile guidance and control systems. For the last 13 years he has been working for Computer Sciences Corp., doing a variety of computer simulation studies ranging from communication and surveillance satellite coverage studies to airport traffic and antisubmarine warfare. His most recent assignments have been a variety of studies connected with the Seafarer ELF communication system.

Mr. Heppe is a member of ORSA.

Discussion

R. McSweeny (Computer Sciences Corp., Falls Church, VA): Mr. Heppe's work is an excellent addition to the theory on which the design of large grounding electrodes is based. In many instances two-layer earth models are much better representations of real conditions than homogeneous earth models. As outlined below, the equations in Heppe's paper can be employed as approximations and the curves and insights in the paper can be used as guides in designing large grounding electrodes.

It should be remembered, however, that two-layer horizontal models often do not adequately represent the inhomogeneities at sites where electrodes are to be installed. More than two layers of different conductivities frequently exist and the layers have different thicknesses at different points and/or dips with respect to the surface. Inhomogeneities in the horizontal direction nearly always occur. In addition to variations in the soil and rock, buried metal pipes, open streams or puddles, underground lenses of clay or caliche and holes due to culverts and large drains may concentrate earth current with a resulting increase in voltage gradient along the surface near the discontinuity. Equations and associated computer programs that would accurately calculate the gradients under such conditions would be impractically complex. If such programs could be implemented, the necessary conductivity data input would require very large number of measurements at a site prior to designing the grounding electrode. Therefore it is generally not worth while to attempt modeling beyond two layers.

Specifications for maximum allowable body current in people walking above grounding electrodes customarily neglect the substantial insulation provided by dry shoes and skin or by overshoes usually worn in wet locations. Specifications for maximum allowable voltage gradient customarily neglect the very small probability that a person or creature will take a large step aligned with the gradient at the particular place and time when maximum occurs. It is interesting to note that if a similar disregard for probabilities of accidents were applied to automobile driving, cars would be forbidden.

Despite the statistically unrealistic requirements imposed thereby, grounding electrodes must sometimes conform to specifications promulgated by regulatory agencies for maximum voltage gradient along the surface or maximum body current. When this is the case, the equations and curves developed by Heppe can be used as follows:

a. Earth conductivity measurements are made along the center line of the proposed electrode and along lines parallel to the center line but spaced about ¼ the electrode length each side of the center line. Measuring technique and algorithms for reducing the measurements to an upper layer of various conductivities and thicknesses at different locations and a basement of another conductivity are similar to procedures used in electrical prospecting for ore bodies [1] but more emphasis is given to determining characteristics of the surface layer.

b. Heppe's equation (10) or (11) then can be used to design the electrode for the desired grounding resistance. Heppe's equations (3)–(6) can be used to estimate voltage gradients on the surface at locations where highest gradients are expected and Heppe's Figures 4–6 can be used to estimate maximum body currents at those locations. If any estimate exceeds the specification, the design is modified until it conforms.

c. Actual maximum voltage gradients along the surface and body currents should be measured after the electrode is installed. The measurements may be made at input currents less than the electrode is designed to handle and at a frequency close to the operating frequency but offset from 60 Hz to permit use of selective voltmeters to discriminate against interference from adjacent power system grounds and lines. Body current can be measured by using two electrodes spaced one meter driven into the ground to a depth that yields the same calculated contact resistance as a theoretical human foot (Heppe's Figures 7–10).

d. In case the measurements indicate that voltage gradient or body current at a few locations will exceed the specification, the offending portion of the electrode can be corrected by one or more of the following actions: (a) burying it deeper; (b) disconnecting it and bridging the gap in the electrode with insulated wire; (c) distributing the earth current over a larger area at the critical location by adding longer or paralleled grounding conductors or adding ground rods.

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Manuscript received January 23, 1978.

G. B. Niles (Baltimore Gas and Electric Company, Baltimore, MD): The author has done a commendable job of displaying the effects of two layer earth on the electrical parameters of a horizontal electrode such as voltage, voltage gradients and in particular maximum body currents. My specific questions are as follows:

1. The author mentions that "layering did not increase body currents by more than a factor of four" for the case of Figure 4, 5, and 6. However, for another case where the depth was 0.76 m (2.5 ft.) only a factor of 50% was encountered for layering. A common practice is to install horizontal electrodes at a depth of 0.46 m (1.5 ft.). Would the author comment on the effect layering could have on this depth? In addition, is there an expression to predict maximum effects of layering for different electrode depths?

2. Would the author clarify the parameters used in Figure 7 through 9. Also, why was a change made from a value of b = 10 cm (Figure 4, 5, 6) to a value of b = 8 cm?

3. Many utilities employ the use of ground rods with horizontal electrodes for grounding purposes. Has the author done any investigations into effects of layering on vertical electrodes?

Manuscript received February 17, 1978.

J. G. Sverak (Gibbs & Hill, Inc., New York*): This paper provides excellent insight into the often intricate matter of evaluating the performance of a given grounding system in terms of human safety, if the earth resistivity near the surface differs from that of underlying soil.

The Author has treated the results of his work with commendable honesty. Particularly valuable are his notes on the evaluation of equations, used in the described analytical model. Certainly, this convenient information is welcomed by anyone who has ever become aware of, or frustrated by some of the many subtle pitfalls, sometimes accompanying computerized modelling.

Since the topics, covered in this paper, are of considerable interest and complement the present efforts of Task Force on rewriting IEEE Standard No. 80, the following discussion is submitted with this circumstance in mind.

At present, all grounding calculations, currently accepted by the industry, are based on the following five assumptions.

1. Resistance of the human body is 1000 ohms.
2. Critical amount of electrical energy which a human body can safely absorb during electrical shock (and above which the danger of heart fibrillation becomes statistically significant), is characterized by an empirically derived energy constant of .0135 A^2s, which further resolves into a time-dependent limit for the maximum permissible rms current through the body,

$$I_B = 116\text{mA/second}^{1/2}. \qquad \text{(D-1)}$$

*The Discussor was with United Engineers & Constructors Inc., Philadelphia, PA, at the time of presentation.

3. Contact resistance of an unprotected foot, standing at the ground surface, is assumed to be equal to the ground resistance of an equivalent metal plate having approximately 15–16 cm diameter, imbedded in the surface of a homogeneous soil with uniform resistivity ϱ ohm-meters, calculated by Laurent's formula for a circular plate

$$R_F[\text{ohm}] = \varrho[\text{ohm-m}]/4r[\text{m}] \qquad \text{(D-2)}$$

which for $r = 0.75$ to 0.08m gives approximately

$$R_F = 3 \times [\varrho], \qquad \text{(D-2a)}$$

where $[\varrho]$ is the numerical value of earth resistivity, in ohms. Significantly, for a multi-layer soil, $[\varrho]$ is usually interpreted as resistivity of the upper soil layer, ϱ_1.

4. For a step contact, the combined resistance of two feet in series, positioned no more than 1m apart, is

$$R_{FS} = 2\,R_F = 6[\varrho], \qquad \text{(D-3)}$$

5. For a touch contact, the combined resistance of two feet in parallel is

$$R_{FP} = \tfrac{1}{2}\,R_F = 1.5[\varrho]. \qquad \text{(D-4)}$$

None of the above assumptions is beyond question.

The first assumption appears to be the least contravertial. Suffice to say, that 1000 ohms is a reasonably conservative approximation for the resistance of human body, generally accepted world-wide.

Referring to the second assumption, its emergence somehow resembles that of the national 55mph speed limit. Until recently [1, 2], a 165mA value was used as the limit of body current, which could be safely tolerated by 99.5% of all healthy men for 1 second. A 70kg average weight was anticipated. Following the 1976 revision of IEEE No. 80, this value has been reduced to 116mA—primarily in recognition of the fact that in a mixed population the average weight of a typical adult victim of electric shock is 50 kg. In the Discussor's opinion, the application of lower value is fully justified only for publicly accessible places, such as the areas under transmission lines, etc. [4]. For a remote substation, or a switchyard protected by a nonmetallic fence, one should be given a choice between using the predominatly male-, or the mixed-population oriented criterion. Also, it can be argued that the assumption of a smaller, 50kg body, requires to consider a proportional reduction of a foot size, resulting in an increase of the equivalent foot plate resistance.

As a corollary to the above, it is noted that the recent 1977 revision of a well-respected grounding standard DIN 57141, introduces a probability factor, which takes into account the fact that the calculation of critical current I_B is based on a multitude of the most unfavorable conditions. A 0.7 value is used for 110 kV facilities and above [5]. A similar standpoint has been indicated in an earlier Canadian paper [6]. Furthermore, although Germany, Austria and Switzerland use curves for the permissible body current, which for very short fault durations are similar to the curve based on Dalziel's investigations [7, 8]. Prof. Berger, Switzerland, recommends a permissible product of current and time as high as 70 mAS, i.e. within 0.1 and 0.2 second, from six times to three times the value per (D-1), respectively.

Finally, as what concerns the remaining assumptions 3, 4 and 5, the Author correctly suggests that, for instance, instead of calculating the combined resistance of two feet in series, assuming $R_{FS} = 6 \times [\varrho_1]$, for a step contact one should utilize Equation (9) in order to account for the presence of a bottom layer, for the finite depth of the upper layer, and for the mutual resistance of two feet, spaced only 1m apart.

Analyzing the effect of mutual resistance first, for a homogeneous soil the Equation (9) reduces into

$$R_{FS} = (1/2_b - 1/\pi_s) \times [r_1] \quad [\varrho_1] \qquad \text{(D-5)}$$

which for $b = 0.08$m and $s = 1$m, yields

$$R_{FS} = (6.25 - 0.318) \times [\varrho_1] = 5.93\,[\varrho_1] \qquad \text{(D-5a)}$$

Thus, about a 1.2% reduction of the combined foot resistance is being neglected, when using (D-3).

For two feet in parallel, the effect of mutual resistance is more pronounced, but luckily, the mutual resistance tends to increase the resultant resistance value:

$$R_{FP} = (1/8_b + 1/4\pi_s) \times [\varrho_1] = (1.563 + .0796/s) \times [\varrho_1] \qquad \text{(D-6)}$$

Estimating the average distance between person's feet to be between 0.5m and 1m, the resultant resistance of two feet in parallel, is approximately

$$R_{FP} = (1.642 \text{ to } 1.722) \times [\varrho_1] \qquad \text{(D-6a)}$$

Therefore, one can conclude that the present IEEE formula (D-4) contains about a 9.5% to 14.8% conservative margin with regard to the neglected effect of mutual resistance.

On the basis of experimental data given by the Author, the presence of a highly resistive bottom soil, underlying a relatively thin, more conductive surface layer (that is, thin in comparison with the skin depth of the bottom soil), can be roughly approximated by

$$R_F = R_{F*} \log_{10} (\varrho_2/\varrho_1); \ \varrho_2 \geqslant \varrho_1 \qquad \text{(D-7)}$$

where R_{F*} is the foot resistance value calculated under the assumption of a homogeneous soil with conductivity ϱ_1.

However, the Author states correctly that the worst case from a safety standpoint occurs when the ground electrode is in a low conductivity bottom layer, and a person stands on a moderately thin surface layer of moderately high conductivity. This is generally true both for the analyzed step contact, and for the touch contact, which has not been discussed.

As the most typical case of touch-contact, consider a man inside substation yard, standing on the top of a relatively thin layer of crushed stone, overlaying a grounding grid installed in much more conductive soil beneath.

By touching a grounded structure during a fault, he establishes himself an "accidental" ground system, which diverts some portion of the fault current to his body, discharging it through his feet into the crushed stone.

Accordingly to the established practice, the combined apparent resistance of his feet is to be calculated as a half of the ground resistance of the already mentioned, equivalent metal plate, imbedded in a homogeneous earth with the resistivity of 3,000 ohm-m—regardless of the actual thickness of the stone layer.

Thus, it would appear that in order to recognize the presence of a more conductive soil below the thin first layer, a certain reduction of the calculated foot resistance could be required, offsetting any relatively minor increase in the combined resistance of two feet, caused by the proximity effect.

Yet, as the earth becomes simultaneously saturated by the currents emanating from the grid, the interaction of the "accidental" and the "permanent" grounding systems also need to be taken into account.

Presently, the driving voltage E_t of a hand-body-two-feet-in-parallel, "accidental" circuit, is calculated as the difference between the full ground potential rise U_g, present both on the contacted structure and on the grid itself, and between the lowest "mesh" potential U_m, as it would be found along the plane of crushed stone-soil interface.

However, a grid, consisting of several conductors horizontally buried in a certain depth and spaced apart from each other, discharges currents which saturate the surrounding soil unevenly and the resulting potentials along the stone-soil boundary vary considerably from conductor to conductor.

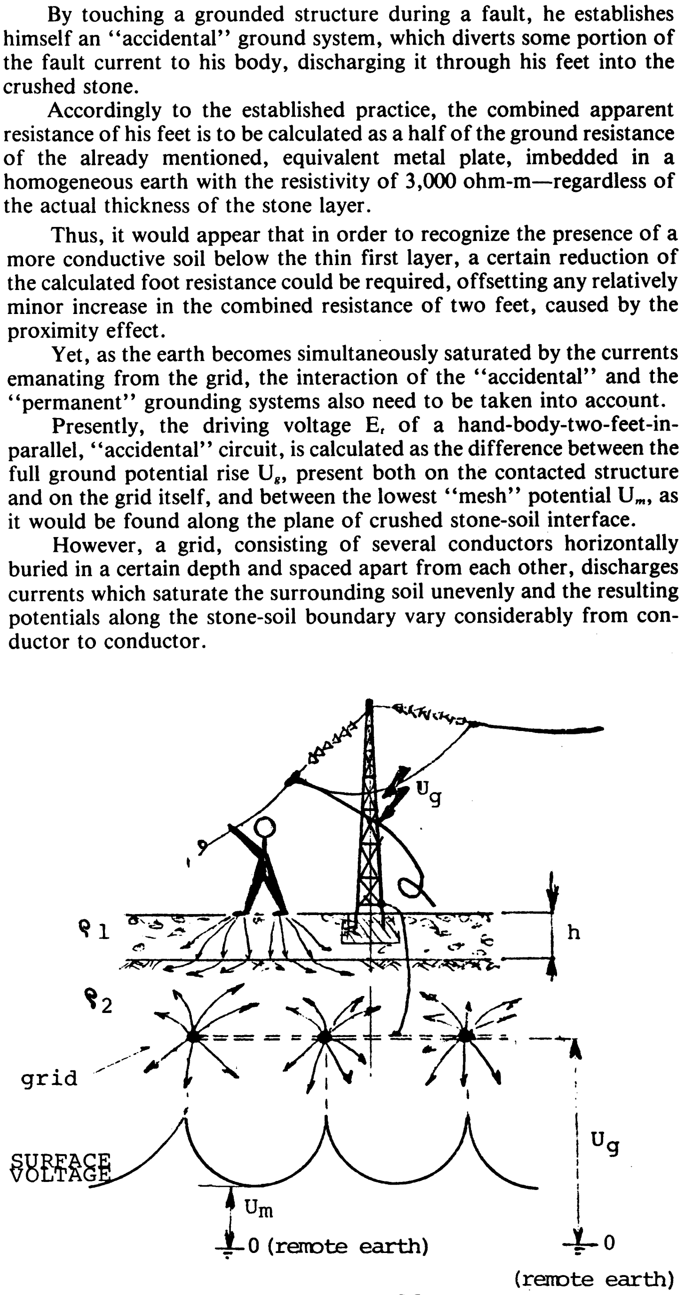

FIGURE 11

Another result of the uneven soil saturation is that for a certain minimum spacing distance between the grid conductors, the bottom soil may appear to have nearly infinite resistance at the depth of grid burial, if viewed from the point of foot contact. Consequently, most of the "accidental" current dissipated through the stone-soil boundary, can be imagined as being forced to spread wide, paralleling the boundary plane, instead of penetrating down into the earth. The net effect is nearly identical to having a bottom soil layer with its resistivity increasing to infinity downward.

Undoubtedly, with respect to the current IEEE method of calculating the permissible touch voltage E_t on the surface, as

$$E_t = (1000 + 1.5\,\varrho) \times 0.116/t^{1/2};\ [\text{V; ohm, second}] \qquad \text{(D-8)}$$

where $\varrho = [\varrho_1]$ is normally assumed to be a valid approximation, it would be of a great help to establish an auxiliary formula for determining the value of ϱ as a function of the actual resistivity of upper layer ϱ_1, its thickness h, and the resistivity of bottom soil ϱ_2, if $\varrho_1 \gg \varrho_2$ and the proximity of more conductive subsoil(s) can no longer be overlooked, i.e. $\varrho(h, \varrho_1, \varrho_2) \neq \varrho_1$.

Would the Author elaborate on a typical case of such a problem, say, for a 0.25m layer of crushed stone with a 5,000 ohm-m average resistivity, overlying a 250 ohm-m soil, in which are buried 2 counterpoise wires, each of a 0.014m diameter (4/0 size) installed in a 0.5m depth and spaced 10m apart, assuming further a potential rise of 15kV to remote earth and anticipating a man standing in central position with respect to the wires, and touching a grounded metal structure?

In closing, again I would like to compliment the Author on his timely contribution. Continued effort in research and development in this area is encouraged — looking forward to the next 1979 PES Winter Meeting, which will be devoted to the subject of grounding.

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Manuscript received May 25, 1978.

Robert J. Heppe: The author thanks the discussors for their thoughtful comments and kind words.

In response to Mr. Sverak's desire for a simple way of computing the contact resistance between the feet and the two-layer earth, the procedure below is recommended:

Equation (9) of the paper, for the contact resistance of the two feet in series, can be written as $R_c = 2(R_{self} - R_{mutual})$, where R_{self} is the contact resistance between the soil and the metal disc representing one foot, and R_{mutual} is the voltage produced on the metal disc by a current of one ampere injected into the disc representing the other foot, i.e., the mutual resistance. If one wants R_c', the contact resistance for the two feet in parallel, the equation becomes $R_c' = (R_{self} + R_{mutual})/2$. The equations below give R_{self} and R_{mutual}:

$$R_{self} = \frac{\rho_1}{4b}\,\alpha(b) \qquad\qquad R_{mutual} = \frac{\rho_1}{2\pi s}\,\alpha(s)$$

$$\text{where} \quad \alpha(r) = 1 + 2\sum_{n=1}^{\infty} \frac{K^n}{\sqrt{1 + (2nH/r)^2}}$$

Figure A-1 is a plot of α with r = 0.08 for several values of K ranging from −0.998 to 0.998, and H ranging from 0 to 0.5 meters. Values of α(b) for other values of K than those shown can be estimated by interpolation. (K, b, and s are defined in paper.) Alternatively, α can be evaluated directly with the aid of a programmable calculator or a computer. Unless the value of K is extreme, the number of terms which must be summed is not prohibitive. For example, in the situation which Mr. Sverak inquired about, with H = 0.25m, ϱ_1 = 5000 ohm-m, and ϱ_2 = 250 ohm-m, the value of K is −0.90476. In that situation summing 48 terms will yield α with less than 1% error. For those particular values, one obtain $\alpha(b) = \alpha(0.08) = 0.804$. Figures A-2 and A-3 are plots of α(s) vs H for s = 0.5 m and s = 1 m, respectively. For Mr. Sverak's parameter values one gets α(s) = 0.209 and 0.112 for these two cases. With s = 0.5 m, R_c' is 6447 ohms in this situation. Neglecting the effect of the bottom layer would have given R_c' = 8608 ohms. The effective resistance of the crushed stone is thus 75% of its nominal resistance, in this case.

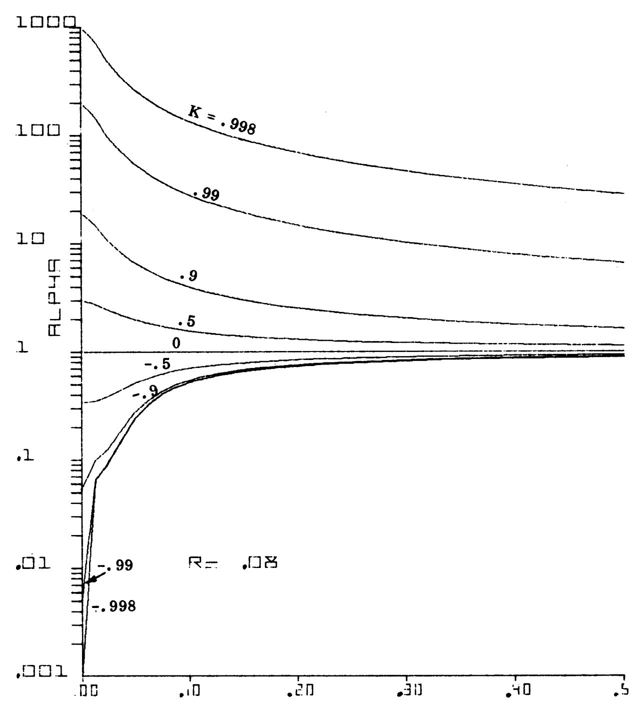

Fig. A-1. α(b)

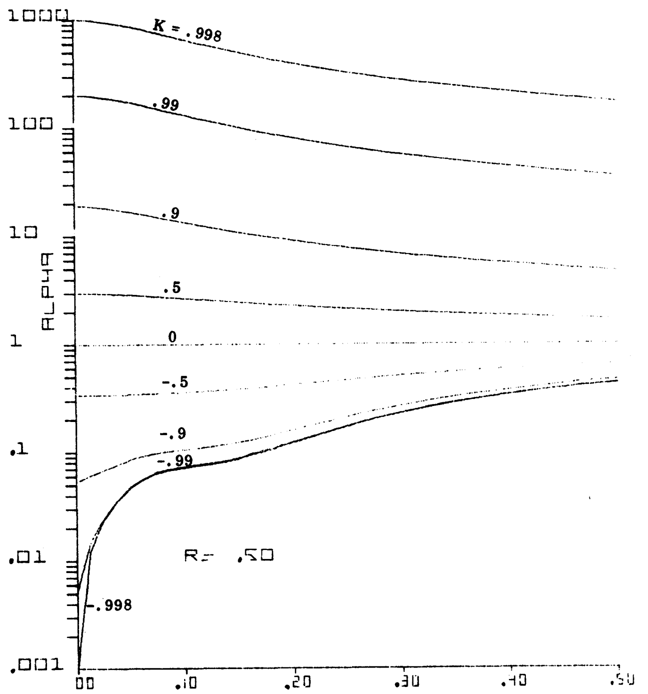

Fig. A-2. $\alpha(s)$ s = 0.5 m.

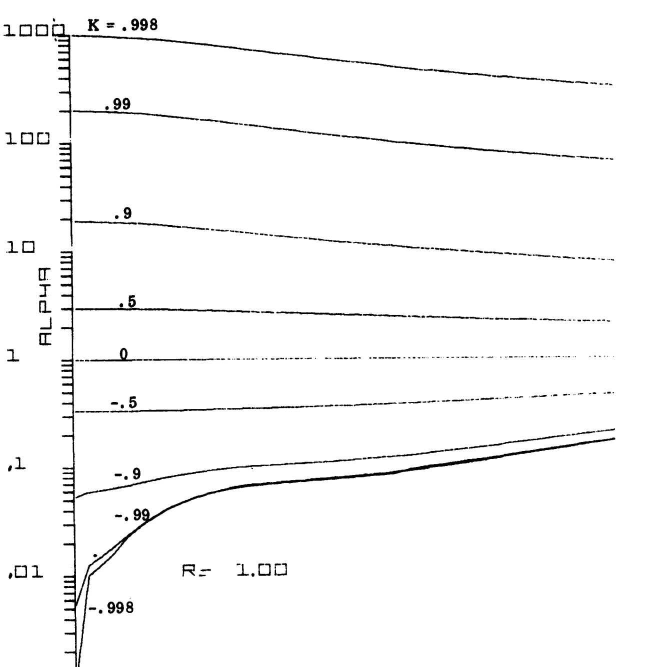

Fig. A-3. $\alpha(s)$ s = 1 m.

In order to answer Mr. Sverak's question about the touch or mesh voltage and current experienced by a person on the surface between two buried wires 10 m apart, if he touches a conductor at the same voltage as the buried wires, the problem was analyzed with six different techniques. This was done to provide an assessment of the errors produced by the approximations used in each technique.

The first, and crudest, approach was to treat each of the two wires as having a constant leakage current density of all points along its length, to neglect the mutual coupling between the two conductors, and to assume the voltage at the person's feet is the same as he would experience at the soil surface without the gravel. The presence of the gravel was taken into account only with respect to increasing the contact resistance at the person's feet, which was taken as 6447 ohms in each of the six approaches. In this approach a 13.4 mm diameter (4/0) wire was assumed to be buried at a depth of 0.5 m in 250 ohm-m soil. The wire was assumed to be 100 m long. With 15,000 volts on the wire, leakage current density was found to be an average of 27.69 amps/m, and voltage at a point on the surface 5 m to the side of the center of the wire was found to be 6596.8 volts. Using (incorrectly) the principle of superposition, the voltage at this point caused by two such wires, one on each side of the point, would be estimated as twice 6596.8, or 13193.6. These results are shown on the top line of the table below, but are only a crude approximation, because this use of superposition is improper. Adding the second wire reduces the current leaking from the first if both wires are held at 15 Kv. The surface voltage produced by two wires is thus less than twice the voltage produced by a single wire.

In the second approach, the effects of the two wires on each other was considered by setting up a pair of simultaneous equations to compute the average voltage produced in each wire by one amp from itself, and by one amp from the other wire. These equations were solved for the current in the two wires which would produce a total of 15,000 volts in each of them, averaged along the length of the wire. Results are shown in the second line of the table. As in all of the first five cases, body current was obtained by subtracting the voltage under the person's feet from 15,000 volts to obtain the mesh voltage, then dividing by 7447 ohms, which represents 1000 ohms body resistance and 6447 ohms contact resistance. Body current is found to be 2.7 times higher than the result of method 1, which shows that method 1 is highly inaccurate.

The next refinement was to divide each of the two wires, conceptually, into ten segments. Leakage current density was assumed to be constant in each segment, but could be different from one segment to another. The method for computing the current densities was similar to that used above, except that 20 equations, each with 20 terms, were used. Results are shown in line 3 of the table. Total leakage current went up slightly, but leakage current density at the center of the wires went down by about 8%, causing voltage under the person's feet to go down about 4%. This increased the voltage difference across the person, and thus the body current, by about 8.5%.

On line 4 the effect of dividing the wire into 40 segments, rather than 10 segments, is shown. Results are negligibly different from the ten segment case.

The fifth approach took the top layer of gravel into account in computing the leakage currents and the voltage under the person's feet. The equations were modified to include the terms for a two-layer earth. The wires were considered to be buried at a depth of 0.75 m, the top layer being 0.25 m thick and having a resistivity of 5000 ohm-m. (That would be unusually clean, dry gravel.) The gravel, by providing a poorly conducting shunt path, increased total current leakage from the wires from 4274.5 amps to 4280.8 amps, a change of only about 0.1%. The voltage at the top of the gravel, under the person's feet, became 9726.8 volts, rather than 9675.3 volts, a change of 0.5%. Body current went down by less than 1%.

Finally, the effect of the current going through the person on the current distribution in the ground wires was taken into account. This was done by adding an equation to the set of 80 equations previously used, and adding one term to each of the equations, so that the effects of all currents on each other was accounted for. As would be expected, the relatively small current entering the ground through the person's feet has little effect on the much larger currents leaking from the wires. Total leakage current from the wires went down by only 0.4 amps, from 4280.8 to 4280.4. Taking this interaction into account, the body current increased by only 0.04% as compared to the result obtained when neglecting it.

The mesh voltage can also be estimated by using equation (15) or (16) of IEEE80 [9]. Equation (15) of IEEE80 is $V_{mesh} = \varrho i$, where i is the leakage current density. Average leakage current density for two wires of length L is given by $i = V_{wire}/2LR_g$. Equation (3.38) of Sunde [8] gives R_g for two parallel wires. Taking D = 0.5 m, ϱ = 250 ohm-m, L = 100, and a = 6.7 mm, one obtains R_g = 3.499 ohms, yielding i = 21.43 amps/m and V_{mesh} = 5358 volts, not far from the correct value of 5273 volts. Equation (16) of IEEE80 would yield 5830 volts if K_i is taken as 1.

It can therefore be concluded that in a situation of this type (high resistance surface layer, wire in low resistance basement) the effect of the top layer can be neglected in computing the voltage under the person's feet, as long as both layers are taken into account in computing the contact resistance as described above. Interaction between the body current and the electrode current is also negligible.

In answer to Mr. Niles' questions, the author does not have an expression to predict maximum effects of layering for different electrode depths. However, the computer program which generated Figure 4 was re-run with the same parameters except that depth was changed to 0.46 m (1.5 ft.). The maximum body current was about 2.6 times higher than would have been experienced if the basement conductivity had continued all the way to the surface. The corresponding factor in Figure 4 was 4.9. It was less in Figures 5 and 6.

Figures 7 through 9 are plots of Equation 9. The only parameters used in (9) are ϱ_1, ϱ_2, b, s, and H. Surface conductivity, plotted along the x axis, is the reciprocal of ϱ_1. The captions of the figures give the basement conductivity, which is the reciprocal of ϱ_2. The values of H are given along the y axis. The values used for b and s were 0.08 m and 1 m respectively. The values of R_c are noted at the top of each curve. A change from b = 10 cm in Figure 4 through 6 to b = 8 cm in Figures 7 through 10 was made to conform to IEEE80. The larger 10 cm value had been used in some early work. The author has not investigated the effects of layering on vertical electrodes or on horizontal electrodes with ground rods, but some work has been done by others in the field. Sunde [8] briefly considered the subject (sec. 3.11). References [A1] and [A2] explore the subject more deeply. Reference [A2] states that equation 9.3 of [A1] is erroneous, and questions an assumption made in deriving equation 9.18 of [A1].

REFERENCES

A1. Tagg, G. F., Earth Resistances, Pitman Publishing Corp., N.Y., 1964.

A2. Project Sanguine, Special Topic Memorandum No. 19, Step Gradient Theory and Bravo Test Facility Phase I (BTF-1) Test Plans and Preliminary Measurements, Contract N00039-68-C-1518, RCA Communications Research Laboratory, Princeton, N.J., 17 April 1969 (Unclassified).

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Method	Segments per Line	Line Interaction Considered?	No. of Soil Layers	Total Ground Current amps	Leakage Current Density at Center amps/m	Voltage Under Feet	Mesh Voltage	Body Current ma
1	1	No	1	5539.0	27.69	13193.8	1806.4	242.5
2	1	Yes	1	4237.5	21.19	10093.6	4906.4	658.8
3	10	Yes	1	4267.1	19.54	9676.8	5323.2	714.8
4	40	Yes	1	4274.5	19.52	9675.3	5324.7	715.0
5	40	Yes	2	4280.8	19.53	9726.8	5273.2	708.1
6	40	Yes	2	4280.4	19.52	9724.5	5275.5	708.4