COMPUTATION OF POTENTIAL AT SURFACE ABOVE AN ENERGIZED GRID OR OTHER ELECTRODE, ALLOWING FOR NON-UNIFORM CURRENT DISTRIBUTION

Robert J. Heppe, Senior Member, IEEE
Computer Sciences Corporation
6565 Arlington Boulevard
Falls Church, Virginia 22046

Abstract - Equations presented in most papers on the computation of voltages at the surface of the earth near a buried energized electrode grid of bare wires are based on the assumption that leakage current density is the same at all points along the wire grid. This paper presents a method for allowing for the effects of variation of leakage current density caused by the proximity of parallel conductors, cross conductors, angled conductors, and end effects. Examples are presented showing the effect of the non-uniform leakage in some cases. The method given herein also can be used to compute surface voltages in the vicinity of electrodes composed of wires oriented at arbitrary angles, such as combinations of stars and polygons, and the resistance to ground of such electrodes.

INTRODUCTION

The grid of wires under a substation rises to a high voltage relative to distant earth during fault conditions. Although the wires are usually at nearly the same potential throughout the grid, the leakage current density from the wires to the soil varies because of the proximity of nearby parallel and/or cross conductors and because of end effects. In a uniformly spaced grid, the leakage current density is smallest near the center of the grid and highest at the edges and corners. Nevertheless, equations and methods presented in the literature for computing voltages and voltage gradients at the surface of the earth in the vicinty of such a grid often are based on the simplifying approximation that leakage current density is the same at all points of all the wires of the grid. Appendix 1 of IEEE Std 80, for example, gives a mathematical analysis of the surface voltage gradient problem using the assumptions that the leakage currents are the same in conductors near the edge of the grid as at the center, that leakage current is the same at all points on each conductor, and that cross conductors have a negligible effect [1]. In the body of [1] there is an attempt to allow for the effects of irregularities, but the method given therein can yield physically impossible negative mesh voltages for certain design parameters. Some papers have been published giving methods of computer analysis allowing for varying leakage current, but these generally have not been as simple to understand and apply as one might wish [2, 3]. The method given herein is relatively simple, straightforward, and easy to program. Although it is based on the matrix method, computer time and core requirements are relatively modest. For example, solving for the leakage currents in the 112 wire segments of a square 8x8 grid, and using these 112 leakage currents to compute the resistance of the grid and the surface voltage at several points, required the solution of only 16 simultaneous equations in 16 unknowns, and took only 4 seconds of CPU time. Modifying the program to handle a 16x16 grid required no additional lines of code in the 62 line program or its subroutines. The computer time and the computer program required to handle grids with varied spacing is exactly the same as for grids with uniform spacing, if the spacing is symmetrical about the center of the grid in both directions.

Finally, this paper contains equations permitting one to compute the mutual resistance of orthogonal and angled conductors. These equations are not easy to locate in the literature, nor to derive.

METHOD

The problem divides naturally into two parts: Finding the values of leakage currents in segments of the conductors, and using these leakage current values to compute the voltage at any desired point on the surface. Most of this paper will focus on the first of these two parts, because the second part is simple and straightforward.

Finding the Current Distribution

In order to find the leakage current distribution, one conceptually breaks up the electrode into many segments, each segment consisting of a single piece of straight wire. Within each segment, leakage current density will be assumed to be constant, but will be allowed to be different from segment to segment. (More complicated approximations could alternatively be used such as assuming that leakage current density varies linearly from one end of a segment to the other, but these refinements complicate the equations without necessarily improving the accuracy of the solution [4].)

If the electrode is in the form of a rectangular grid (Figure 1), a natural choice for the segments would be the pieces of wire between the crossings. If one wants even better accuracy, one can further subdivide these pieces of wire into smaller segments. An example later in this paper demonstrates how this is done and evaluates the effect this refinement has on the results of the example.

After conceptually dividing the grid into segments, the next step is to number the segments, using consecutive numbers for segments which can be seen by symmetry to have the same leakage current. Figure 1, for example, shows a square grid in earth of uniform resistivity with lines spaced closer together near the edges than near the center, but with the same spacing for the north-south lines as the east-west lines. Furthermore, the north-south lines are spaced symmetrically about a north-south axis through the center of the grid, and the east-west lines are spaced symmetrically about an east-west axis through the center of the grid. Inspection reveals that the line segments numbers 1 through 8 must, by symmetry, all have the same leakage current, because they are all of the same length and are symmetrically placed as end segments of an outside line. These will be called type 1 segments. Similarly, segments 9 through 16 have the same leakage current, being end segments of a

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second-from-outside line, and will be called type 2. Segments 17 through 20 are center segments of an outside line and will be called type 3. Segments 21 through 24 are center segments of second lines, herein called type 4. There are thus only 4 different current values to solve for, although there are 24 conductor segments in this example. If the north-south spacing were different from the east-west there would be eight values of current to solve for.

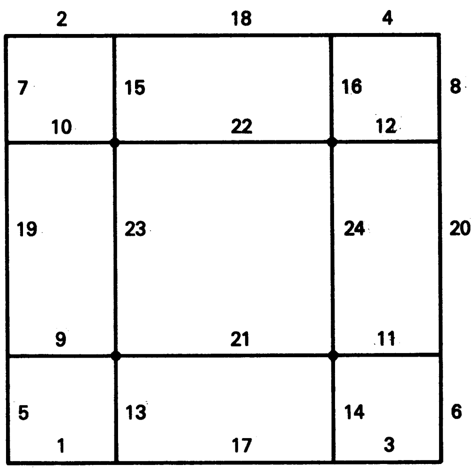

Fig. 1. 4x4 grid.

As will be seen, the consecutive numbering of similar segments is helpful. The order in which segments of the same type are numbered is arbitrary and immaterial, but the computer program is simplified if numbering is done so that the types of segments that have eight members are numbered first, then the types that have four members. This is easy to do. In a large grid, the segements can be numbered and their coordinates generated with a few lines of computer code, if the grid is symmetrical.

The four values of leakage current can be found by solving the following set of four simultaneous linear equations:

$$\begin{aligned} r_{11}i_1 + r_{12}i_2 + r_{13}i_3 + r_{14}i_4 &= v_1 \\ r_{21}i_1 + r_{22}i_2 + r_{23}i_3 + r_{24}i_4 &= v_2 \\ r_{31}i_1 + r_{32}i_2 + r_{33}i_3 + r_{33}i_4 &= v_3 \\ r_{41}i_1 + r_{42}i_2 + r_{43}i_3 + r_{44}i_4 &= v_4 \end{aligned} \qquad (1)$$

In these equations r_{jj} is the sum of the self resistance of a type j segment and the mutual resistances between the segment and other segments of type j, and r_{jk} is the sum of the mutual resistance between a segment of type j and segments of type k. The voltage with respect to far distant ground on the segments if v_j. Currents i_j are the values of leakage current in the four types of segments. The equations state that the sum of the voltages induced into a segment by its own leakage and the leakage from all other segments must be equal to the applied voltage. Expressed more compactly, in a grid with n types of segments we have

$$\sum_{k=1}^{n} r_{jk}i_k = v_j \qquad j = 1,2,3,\ldots n$$

or, still more compactly,

$$RI = V$$

where R is the matrix of self and mutual resistance values, I is the column vector (list) of leakage currents in the n types of segments, and V is the list of voltages on the n segments. It will be assumed herein that all of the segments are at the same voltage. This is usually permissible at power frequencies because the resistance and inductive reactance of the wires are small compared to the resistance between the segments and the earth. If one is interested in the response of the grid to brief transients, such as those associated with lightning surges, inductive effects become important. This may also be the case at power frequencies if the conductors are exceptionally long, approaching or exceeding a skin depth in the earth. The skin depth, δ, is given by

$$\delta = \frac{1}{2\pi}\sqrt{\rho \cdot 10^7/f}$$

where

ρ = resistivity in ohm - m
f = frequency in Hz

In such situations it is necessary to express each v_j in terms of the applied voltage and the voltage drops caused by leakage currents flowing through the resistances and inductances and mutual inductances of the segments. This paper will confine itself to the case where all v_j's are equal to v, the voltage applied to the grid.

Once the r coefficients have been computed, all that is necessary (on most computers) is to call a simultaneous equation solver routine from the library to find the currents. As will be seen, generating the coefficients, even in a large grid, requires only a few lines of computer code. In order to generate the coefficients, we must relate the r terms in (1) to the mutual resistances between the segments as numbered in Figure 1. These actual mutual resistances will be designated as r'. With the numbering system we have adopted in Figure 1 the relationship is:

$$r_{ij} = \sum_{k=1}^{8} r'_{8(i-1)+1,\ 8(j-1)+k} \qquad i = 1,2;\ j = 1,2$$

$$r_{ij} = \sum_{k=1}^{8} r'_{4(i-1)+9,\ 8(j-1)+k} \qquad i = 3,4;\ j = 1,2$$

$$r_{ij} = \sum_{k=17}^{20} r'_{8(i-1)+1,\ 4(j-3)+k} \qquad i = 1,2;\ j = 3,4$$

$$r_{ij} = \sum_{k=17}^{20} r'_{4(i-1)+9,\ 4(j-3)+k} \qquad i = 3,4;\ j = 3,4$$

The rationale for these equations is simple. For example, r_{44} is the voltage induced in a type 4 segment by leakage currents from type 4 segments. Specifically, the voltage induced in segment 21 (which is a type 4 segment) by unit currents leaking from segments of type 4 is $r'_{21,21} + r'_{21,22} + r'_{21,23} + r'_{21,24}$. (The self resistence of segment 21 is $r'_{21,22}$, which was arbitrarily selected as the one segment of its type whose current would be solved for).

Each of the above four relationships can be programmed in five lines of FORTRAN code. For example, the fourth relationship can be programmed as follows:

```
DO 10 I=3,4
DO 10 J=3,4
```

```
    DO 10 K=17,20
    CALL RMUT(4*(I-1)+9,4*(J-3)+K,RPRIME)
 10 R(I,J)=R(I,J) + RPRIME)
```

In the above, RMUT is a subroutine which accepts the subscript numbers, obtains the coordinates and orientation of the segments having those numbers, and computes their mutual resistance which it returns as the variable RPRIME. The same subroutine, and a similar set of four relationships, can handle much larger grids. Thus, generating the r coefficients takes no more than 20 lines of FORTRAN (exclusive of the subroutine) if the grid is symmetrical.

It should also be noted that the $24^2 = 576$ possible self and mutual resistances in the 4x4 grid did not have to be calculated or stored. Instead, only 96 mutual resistance values were computed, and these were summed as computed into only 16 coefficients which had to be stored in core. Computer time and memory requirements are, therefore, more modest than might be expected with the matrix method. CPU time required on the Computer Sciences Infonet system was about 0.4 seconds for a 4x4 grid, 4 seconds for an 8x8 grid, and 64 seconds for a 16x16 grid. This included reading in the geometrical data, computing the coefficients, solving the simultaneous equations for the currents and printing them out, computing and printing out the grounding resistance of the grid, and computing and printing out the surface voltages at several points. No lines had to be added to the program or the subroutines to handle the larger grids. Of course, it was necessary to change the numbers in some of the lines of the program.

A further reduction in computing time can be achieved by taking advantage of the fact that in the first and fourth of the above equations $r_{ij} = r_{ji}$. (In all situations $r'_{ij} = r'_{ji}$. The r coefficients do not form a symmetric matrix because some are the sum of four mutual resistances and some are the sum of eight.)

The final requirement is the equations to compute the mutual resistance of two segments as a function of their angle and separation. These equations are given in the Appendix for parallel, perpendicular, and angled segments (coplanar or skew), in homogeneous or two-layer soil. The self-resistance of a segment is simply the mutual resistance of two parallel segments separated by distance a, where a is the radius of the wire. The equation for this special case of parallel segments is also given in the Appendix.

The Number of Equations

If in horizontally uniform earth there are n parallel lines symmetrically spaced about a parallel axis through the center, the currents in the two outside lines will be the same, the currents in the next two lines will be the same, etc. There are, therefore, n/2 different kinds of lines if n is even, and (n+1)/2 different kinds of lines if n is odd.

If the n lines are crossed by m orthogonal symmetrically-spaced lines, each of the n lines is divided into m-1 segments. The outermost segments in a line will have equal currents, the next ones will have equal currents, etc. There are thus (m-1)/2 kinds of segments per line if m is odd, and m/2 kinds if m is even. Similarly, the m lines will be divided into n-1 segments, with (n-1)/2 different currents per line if n is odd and n/2 if n is even.

If the lines are orthogonal, with the same spacing and number of lines in both directions, the currents in corresponding segments of the orthogonal sets of lines will be equal, and the total number of different currents to be solved for is $n^2/4$ if n is even and $(n^2-1)/4$ if n is odd.

Finding the Surface Voltage

Once the leakage currents in the various types of segments have been found, the surface voltage at any desired point can be found by superposition, by adding up the contributions of all the segments. The equation to use for the contribution of a leakage current of I amps from a segment of length L is:

$$V = \frac{I\rho}{2\pi L} \ln \frac{\sqrt{x^2+y^2+D^2} + x}{\sqrt{(x-L)^2+y^2+D^2} + x-L} \tag{2}$$

where ρ is the resistivity of the soil (assumed homogeneous) and D is the depth. A coordinate system is assumed in which the x axis lies on the surface above and parallel to the segment, with the origin over the left end of the segment, and the y axis is horizontal and perpendicular to the x axis. Quantities x and y are the coordinates of the surface point in this system.

If the soil has a surface layer of thickness H with resistivity ρ_1 and a basement layer with resistivity ρ_2, equation 3 should be used if the segment is in the top layer and equation 4 if the segment is in the bottom layer:

$$V = \frac{I\rho_1}{2\pi L}\left\{\ln\left(\frac{\sqrt{x^2+y^2+D^2} + x}{\sqrt{(x-L)^2+y^2+D^2}+x-L}\right) + \sum_{n=1}^{\infty} K^n \left[\ln\left(\frac{\sqrt{x^2+y^2+(2nH+D)^2} + x}{\sqrt{(X-L)^2+y^2+(2nH+D)^2}+x-L}\right) + \ln\left(\frac{\sqrt{x^2+y^2+(2nH-D)^2} + x}{\sqrt{(x-L)^2+y^2+(2nH-D)^2}+x-L}\right)\right]\right\} \tag{3}$$

$$V = \frac{I\rho_1(1+K)}{2\pi L}\left[\ln\left(\frac{\sqrt{x^2+y^2+D^2} + x}{\sqrt{(x-L)^2+y^2+D^2}+x-L}\right) + \sum_{n=1}^{\infty} K^n \ln\left(\frac{\sqrt{x^2+y^2+(2nH+D)^2} + x}{\sqrt{(x-L)^2+(2nH+D)^2}+x-L}\right)\right] \tag{4}$$

where $K = \frac{\rho_2-\rho_1}{\rho_2+\rho_1}$. Equations (2), (3), and (4) are based on Reference [8]. The end of Appendix I tells how to determine where to break off the infinite summation.

ILLUSTRATIVE EXAMPLES

1. As a first illustrative example, consider the square formed by two N-S and 2 E-W wires as shown in Figure 2.

For purposes of this example the following parameters will be assumed:

- s = N-S spacing = E-W spacing = 8 m
- D = depth of burial = 0.5 m
- ρ = soil conductivity = 1000 ohm-m (homogeneous soil)
- a = wire radius = 0.007 m (4/0 wire)
- V = 15,000 volts

By symmetry, the currents in all four segments must be the same, so we get just one equation in one unknown, namely:

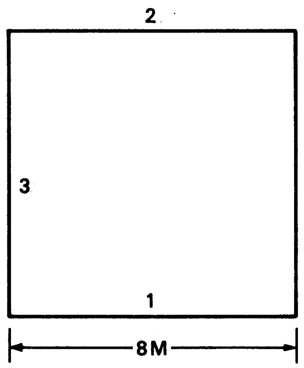

Fig. 2. 2x2 grid.

$$(r'_{11} + r'_{12} + r'_{13} + r'_{14})i = 15{,}000$$

Using equations (A9), (A6), (A5), and (A11), and remembering to include the effects of the images when using (A6) and (A11), we obtain

$$r'_{11} = 171.668 \text{ ohms}$$

$$r'_{12} = 18.524 \text{ ohms}$$

$$r'_{13} = r'_{14} = 33.225 \text{ ohms}$$

Substituting, we obtain i = 15,000/256.64 = 58.447 amps/segment. Total current is thus 4 x 58.447 = 233.79 amps and grounding resistance R_g = 15,000/223.79 = 64.16 ohms.

Applying (2), we find the surface voltage at the center of the cell is 2036.9 volts due to each of the 4 wires. By superposition, the total voltage at the center is 4 x 2036.9 = 8148 volts. The mesh voltage is 15,000 - 8184 = 6852 volts.

2. To improve the accuracy of the above analysis, we could conceptually divide each of the 4 segments into, say, eight parts, as shown in Figure 3. By symmetry, there are only four different values of leakage current - that in a segment adjacent to a corner, once removed from a corner, twice removed, or thrice removed. Thus, we will get only 4 simultaneous equations, rather than 32.

The segments can be equal or unequal in length. In either case there will be four equations. In this example, we will arbitrarily divide each 8 m side into 8 equal pieces, each 1 m long.

The four simultaneous equations are identical with (1), with v = 15,000. Here we have

$$r_{i,j} = \sum_{k=8(j-1)+1}^{8j} r'_{8(i-8)+1,k} \qquad \begin{matrix} i = 1,2,3,4 \\ j = 1,2,3,4 \end{matrix}$$

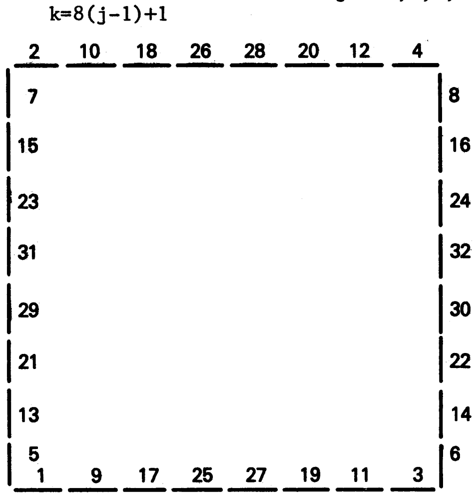

Fig. 3. Subdivided 2 x 2 grid.

Solving for the currents, we get

i_1 = 7.46 amps $\qquad$ i_3 = 7.25 amps

i_2 = 7.27 amps $\qquad$ i_4 = 7.25 amps

Total current is equal to 8 $(i_1+i_2+i_3+i_4)$ = 233.84 amps. R_g = 15,000/233.84 = 64.16 ohms, the same as in the previous example. Voltage at the center of the mesh is computed to be 8139 volts, which differs from the previous example by only 0.1 percent.

Since dividing the sides of a mesh into smaller pieces produced such a small change in the results in this example, further examples in this paper use segment length equal to mesh size. Of course, if the sides of the mesh were much larger it might be desirable to subdivide them.

3. Consider a square 4x4 grid, as shown in Figure 4, with adjacent lines spaced 8 m apart, with other parameters as in the previous examples. Using the approach outlined earlier the four different current values were found to be as shown in Figure 4. We obtain R_g = 20.6 ohms. The voltage at the center of the center square is 11,797 volts, and in the corner square is 10,836 volts. Subtracting these values from 15,000 to obtain mesh voltages at these points, we obtain 3203 and 4164 volts, respectively.

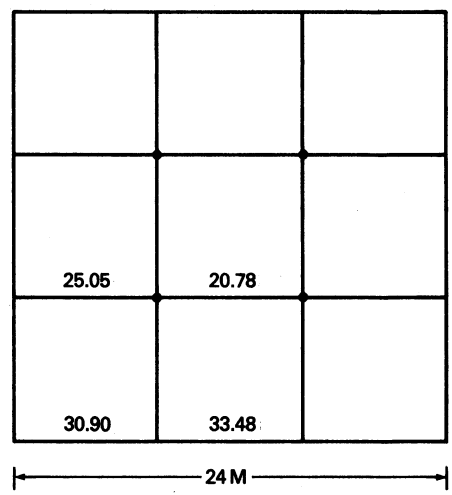

Fig. 4. 4x4 grid, showing currents.

4. Increasing the grid to 8x8 lines, still with 8 m spacing, we get the same configuration that was used in an illustrative example in Appendix 1 of [1]. Leakage currents in the 16 different types of segments are as shown in Figure 5.

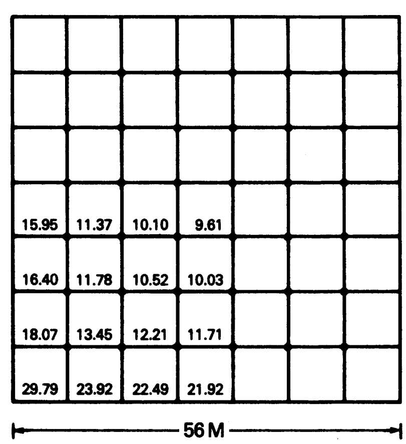

Fig. 5. 8x8 grid, showing currents.

Total current is 1781 amps, making the average leakage current density 1781/896 = 1.988 A/m. R_g is 8.42 ohms. Computed mesh voltage in the center of the corner square is 3101 volts. Maximum mesh voltage within the grid is 3254 volts, and is found on the main diagonals of the grid 2.7 m in from the outer conductors of the grid. Figure 6 depicts a three-dimensional plot of the voltage on the surface of the earth as a function of x and y. The highest voltage shown is 14,725 volts, the lowest is 4872 volts, at the bottom corner of the area shown. The area shown covers 32 m x 32 m, with the center of the plotted area at one corner of the 8x8 grid. The mesh voltage is the distance between the surface shown and the ceiling of 15,000 volts.

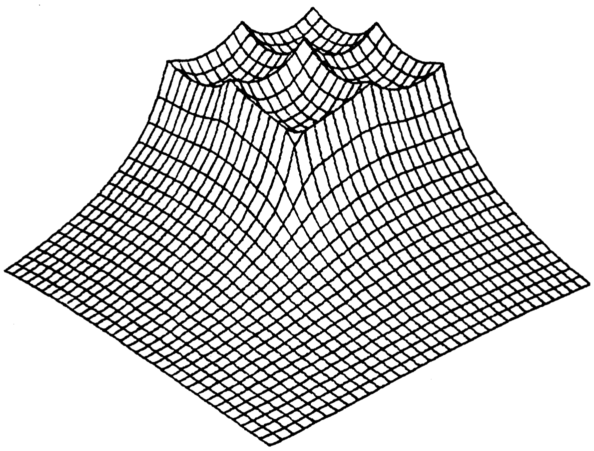

Fig. 6. Surface voltage near corner of 8x8 grid.

Step voltage was computed by finding the voltage differences beteween points 1 m apart in the x and y directions. This was done at 1 m intervals. These differences were called dv/dx and dv/dy. Step voltage was calculated as $\sqrt{(dv/dx)^2 + (dv/dy)^2}$. The maximum step voltage was 1622 V/m, located at the edge of the grid, 8 m from a corner.

These results may be compared with the results which would be obtained using the methods described in [1]. Applying equation (20) of reference [1], one obtains a value of 9.03 ohms for the grounding resistance of the mesh. With 15,000 volts applied, the computed current is 1661 amps, which yields 1.854 A/m. Equations (70) and (71) yield mesh voltages of 1359 volts in the center of the mesh and 1489 volts at a point 2.4 m from the outer conductors. Applying the irregularity factor K = 2.026 as described on pages 20 and 23 of [1], one obtains mesh voltages of 2753 and 3017 volts at these two points. Maximum mesh voltage computed by the method of [1] is about 7 percent low, which may be acceptable. However, as will be seen in the next example, in denser grids, the error in the method of reference [1] is much larger.

According to (73) of [1], the maximum step voltage in this example is 775 V/m. Applying the irregularity factor K = 2.026 yields an adjusted value of 1570 V/m as the maximum step voltage. Maximum step voltage as computed by [1] is thus 3 percent low in this example. Again, the next example shows larger errors in a denser grid.

5. Increasing the grid size to 16x16 lines and decreasing the conductor spacing to 3 m produces a denser grid with a grounding resistance R = 9.8 ohms.

Leakage currents range from 1.71 amps/segment in the center of the grid to 10.17 amps/segment at the corners. Mesh voltage at the center of the corner mesh is 1876 volts. Maximum mesh voltage within the grid is 2720 volts, and occurs at the corner of the grid. Figure 7 shows the voltages in a 12x12 m square, centered on a corner of the grid. In this plot, the highest voltage is 14,590 volts and the lowest is 6894 volts. Step voltage was computed at 0.5 m intervals in both directions. Maximum step voltage found was 1731 V/m, located 0.5 m outside the grid, near a corner.

Once the currents in the 64 types of segments were found, it required 89 seconds of CPU time on Infonet to calculate the surface voltage at 625 points, find the maximum step voltage, and produce Figure 7.

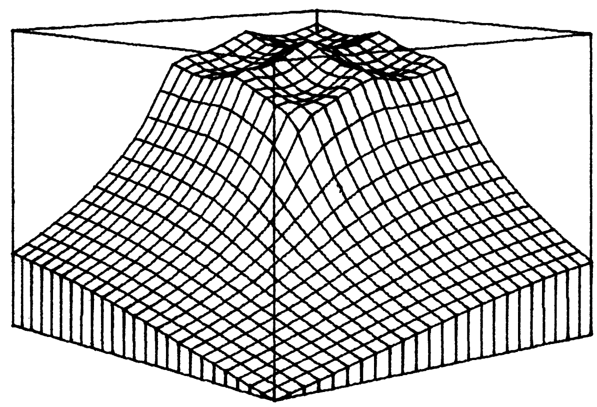

Fig. 7. Surface voltage near corner of 16x16 grid.

Using the methods of [1], R_g = 10.54 ohms, and leakage current density would be 0.988 A/m. Using equation (66) of [1], which is stated to be more accurate than the others on that page, yields a computed mesh voltage of 339 volts at the center of the corner mesh and a maximum mesh voltage of 562 volts at the outer conductor. Applying K_i = 3.402, one obtains an adjusted mesh voltage of 1153 volts in the center of the corner mesh and 1912 volts over the outer conductors. In this example the mesh voltages computed by the methods of [1] are about 30 to 40 percent lower than those computed by the method given herein. With still denser grids the difference would be even larger.

Step voltage computed by equations (21) and (50) of [1], again using K_i = 3.402, comes out to 2170 V/m. In this case the step voltage computed by the matrix method was 20 percent smaller than that computed by [1].

CONCLUSIONS

1. The equations given herein permit the grounding resistance of horizontal wire electrodes of complicated shape to be computed. Wire segments may have any desired angular orientation or displacement from each other.

2. Leakage currents are computed allowing for the effects of all segments on each other. From these leakage currents the surface voltages may be computed. In some cases the differences between the results obtained with this method and those obtained with simpler but less accurate methods can be large.

3. The method uses modest amounts of programming time, computer time, and core memory even for rather large grids, if they are symmetrical.

ACKNOWLEDGMENT

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APPPENDIX

In [5], Sunde gives an equation for the voltage produced at a point P in the earth by a point source of I amps. He states that the voltage produced is

$$V = \frac{I\rho}{4\pi}\left(\frac{1}{r} + \frac{1}{r'}\right)$$

where ρ is the resistivity of the soil, r is the distance between P and the source of current, and r' is the distance between P and the image of the source of current. The image is an imaginary second source of current, located directly over the current source as far above the surface as the current source is below the surface. (The r and r' used here have nothing to do with the r and r' in the body of the paper.) Choosing a coordinate system in which the x and y axes lie on the surface, with origin above the current source, one obtains the following equation for V:

$$V = \frac{I\rho}{4\pi}\left(\sqrt{x^2+y^2+(D-z)^2} + \sqrt{x^2+y^2+(D+z)^2}\right)$$

where x, y, z are the coordinates of point P and D is the depth of the current source. Note that voltage depends on the absolute value of the x, y, z components of distance. At the surface z=0, yielding

$$V = \frac{I\rho}{2\pi}\sqrt{x^2+y^2+D^2}$$

The voltage produced at point P by a line source L_1 meters long having constant leakage current density along its length is given by

$$V = \frac{I\rho}{4\pi L_1}\left[\int\frac{ds}{r} + \int\frac{ds}{r'}\right]$$

Again, the first term in the bracket is caused by the source itself, and the second term is caused by the image. It is apparent that our task will be simplified if we derive a general formula for the voltage produced by a line source, and always remember to apply it twice, once with the z coordinate of the source and once with the z coordinate of the image. That will be done in the remainder of this Appendix.

In order to find the average voltage produced in a line segment of length L_2 by a line source of length L_1, one integrates over the length of L_2 and divides by L_2 to obtain the average value, obtaining

$$V = \frac{I\rho M}{4\pi L_1 L_2}, \text{ where } M = \iint\frac{dSds}{r} \tag{A-1}$$

The mutual resistance between segments 1 and 2 is the voltage produced in segment 2 by one ampere leaking from segment 1. The symmetry of (A1) shows that this will also be the voltage produced in segment 1 by one ampere leaking from segment 2. Therefore, $R_{jk} = R_{kj}$, where R is mutual resistance.

In (A1) r is the distance between a source element of length ds and a receiving element of length dS. One is therefore faced with the problem of evaluating the above integrals for line segments which may be parallel, coplanar but angled, or skew. The parallel case is easy to integrate. The others are more difficult.

In reference [6] Campbell, in the process of computing mutual inductances between wire segments, gives a way of evaluting the Neumann integral:

$$N = \cos\theta\iint\frac{dSds}{r}$$

where θ is the angle between the two line segments and S and s are measured along the segments from their intersection with their common perpendicular. Modifying his result (and notation) to remove the cos θ term and to avoid use of mixed lower case and upper case letters (which are difficult to program), one obtains, for $0 \leq \theta < \pi$,

$$M = \iint\frac{dSds}{r} = \underline{CB}\ln\frac{BF+\underline{B'F}}{BE+\underline{B'E}} - \underline{CA}\ln\frac{AF+\underline{A'F}}{AE+\underline{A'E}} + \underline{GF}\ln\frac{BF+\underline{F'B}}{AF+\underline{F'A}} - \underline{GE}\ln\frac{BE+\underline{E'B}}{AE+\underline{E'A}} - \frac{CG\Omega}{\sin\theta} \tag{A-2}$$

$$\begin{aligned} \text{where } \Omega = &\tan^{-1}\left(\frac{CG}{BF\tan\theta} + \frac{\underline{CB}}{\underline{CG}}\cdot\frac{\underline{GF}\sin\theta}{BF}\right) \\ &-\tan^{-1}\left(\frac{CG}{BE\tan\theta} + \frac{\underline{CB}}{\underline{CG}}\cdot\frac{\underline{GE}\sin\theta}{BE}\right) \\ &-\tan^{-1}\left(\frac{CG}{AF\tan\theta} + \frac{\underline{CA}}{\underline{CG}}\cdot\frac{\underline{GF}\sin\theta}{AF}\right) \\ &+\tan^{-1}\left(\frac{CG}{AE\tan\theta} + \frac{\underline{CA}}{\underline{CG}}\cdot\frac{\underline{GF}\sin\theta}{AE}\right) \end{aligned} \tag{A-3}$$

Figure A-1 is the diagram corresponding to these equations. Line segments AB and EF are shown projected in planes perpendicular to their common

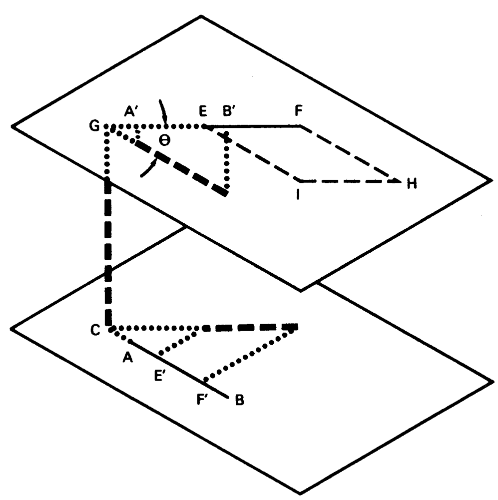

Fig. A-1. Skew line segments.

perpendicular CG. E' and F' are the projections of E and F on AB. A' and B' are the projections of A and B on EF. CG and all other distances are taken as positive except those (underscored) measured along AB and EF which are taken as algebraically positive in the directions AB and EF, respectively. Ω is the (positive) solid angle subtended at B by a parallelogram EFHI constructed on EF with FH parallel and equal to AB.

In [6] none of the terms in the equation for Ω were underscored. I believe this was an oversight, because otherwise Ω would not vary smoothly when the terms shown underscored in (A3) change sign.

The above equations become greatly simplified in some frequently-encountered cases, such as coplanar line segments, parallel lines, and perpendicular lines.

1. Coplanar non-parallel line segments

Since $\tan^{-1}(y/x) = \pi/2 - \tan^{-1}(x/y)$, we can write (A3) as

$$\begin{aligned}\Omega = {} & \tan^{-1}\left(\frac{BE\cdot CG\tan\theta}{(CG)^2+\underline{CB}\cdot\underline{GE}\sin\theta\tan\theta}\right) \\ & + \tan^{-1}\left(\frac{AF\cdot CG\tan\theta}{(CG)^2+\underline{CA}\cdot\underline{GF}\sin\theta\tan\theta}\right) \\ & - \tan^{-1}\left(\frac{BF\cdot CG\tan\theta}{(CG)^2+\underline{CB}\cdot\underline{GF}\sin\theta\tan\theta}\right) \\ & - \tan^{-1}\left(\frac{AE\cdot CG\tan\theta}{(CG)^2+\underline{CA}\cdot\underline{GE}\sin\theta\tan\theta}\right)\end{aligned} \qquad \text{(A-4)}$$

With $\Theta \neq 0$, as CG is reduced towards zero, making the segments coplanar, Ω approaches zero, and $CG\,\Omega/\sin\Theta$ approaches zero.

2. Angled line segments, coplanar or skew

If the line segments are neither parallel nor perpendicular, the following procedure may be used to obtain the required quantities in (A2) and (A3). (As noted above, (A3) does not have to be evaluated if the segments are coplanar.)

Let the x axis lie along the segment having length L_1, extending to the right from the origin at point A. (See Figure A-2). Let end E of the segment having length L_1 be at coordinates x, y, z in this system, and let the angle between the two segments be Θ, where $0 \le \Theta \le \pi$. The mutual resistance will be computed from the values of x, y, z, Θ, L_1, and L_2.

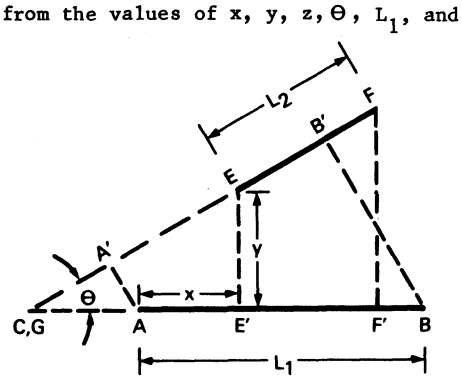

Fig. A-2. Angled segments.

To obtain the quantities needed to compute M in (A2), proceed as follows: If z=0 and y=0, set y = a small number, such as the wire radius. This will avoid the possibility of attempting to take the logarithm of zero. It will not change the result appreciably.

$W^2 = y^2+z^2$

$X_F = x+L_2\cos\theta$

$Y_F = y+L_2\sin\theta$

$\underline{GE} = y/\sin\theta$

$\underline{GF} = y_F/\sin\theta$

$CE' = y/\tan\theta$

$CF' = y_F/\tan\theta$

$\underline{CA} = CE'-X$

$\underline{CB} = CA+L_1$

$GA' = CA\cos\theta$

$GB' = CB\cos\theta$

$BE = \sqrt{(x-L_1)^2+W^2}$

$BF = \sqrt{(X_F-L_1)^2+Y_F^2+z^2}$

$\underline{B'E} = GE-GB'$

$\underline{B'F} = GF-GB'$

$AE = \sqrt{x^2+W^2}$

$AF = \sqrt{X_F^2+Y_F^2+z^2}$

$\underline{A'E} = GE-GA'$

$\underline{A'F} = GF-GA'$

$\underline{E'A} = CA-CE'$

$\underline{F'A} = CA-CF'$

$\underline{E'B} = CB-CE'$

$F'B = CB-CF'$

$CG = z$

M can be computed from the above constants, using (A2). The mutual resistance R is computed from M from (A5):

$$R = M\rho/\left(4\pi L_1L_2\right) \qquad \text{(A-5)}$$

Remember that two values of R must be computed and added to allow for the effect of the surface: The value for the two wires themselves, and the mutual resistance between one wire and the image of the other. This can conveniently be done by calling the mutual resistance subroutine twice, once with z=0, and once with z=2D (for horizontal coplanar wires).

3. Parallel line segments, non-colinear

If AB and EF are parallel but non-colinear the equations get much simpler. We have $\Theta = 0$ and $CG \neq 0$. As Θ decreases towards 0, equation (A4) approaches $\Omega = \Theta\,(BE+AF-BF-AE)/CG$, and $CG\,\Omega/\sin\Theta$ approaches BE+AF-BF-AE.

Again letting the x axis lie along segment AB with the origin at point A, and letting x, y, z be the coordinates of point E, we get the geometry shown in Figure A-3. Here CG = y, and equations (A2) and (A3) simplify to (A6):

$$\begin{aligned}M = {} & L_1\ln\frac{\sqrt{(x+L_2-L_1)^2+W^2}+x+L_2-L_1}{\sqrt{(x-L_1)^2+W^2}+x-L_1} \\ & + (x+L_2)\ln\frac{\sqrt{(x+L_2-L_1)^2+W^2}-(x+L_2-L_1)}{\sqrt{(x+L_2)^2+W^2}-(x+L_2)} \\ & - x\ln\frac{\sqrt{(x-L_1)^2+W^2}-(x-L_1)}{\sqrt{x^2+W^2}-x} \\ & - \sqrt{(x-L_1)^2+W^2} - \sqrt{(X+L_2)^2+W^2} \\ & + \sqrt{(x+L_2-L_1)^2+W^2} + \sqrt{x^2+W^2}\end{aligned} \qquad \text{(A-6)}$$

where $W^2 = y^2 + z^2$

As before, mutual resistance is obtained from M by use of (A5). Again, two values of M must be computed with different values of z (one with z=0 and one with z=2D), if the segments are coplanar and horizontal.

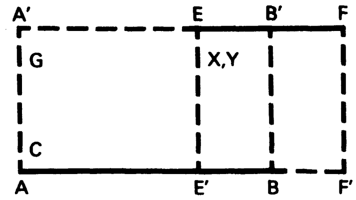

Fig. A-3. Parallel lines.

4. Parallel Line Segments, Colinear

Although (A6) was derived on the basis that $CG \neq 0$, it can be used for colinear segments by setting y equal to the radius of the wire. This prevents some of the expressions from becoming indeterminate while not changing the physical situation enough to be significant. For example, if y and z are both zero W=0, and the third expression in (A6) becomes $0 \cdot \ln(0/0)$. Alternatively, a test for this condition can be used, and the expression can be set equal to zero if it occurs, since that is the limit it approaches as W approaches zero.

Roundoff errors can be very significant in expressions like $\sqrt{x^2+W^2}-x$ when $x >> W$. A later paragraph tells how to cope with this problem.

5. Self resistance

The self resistance of a segment is the ratio of the voltage on a segment to the current that will flow out of it, neglecting the effects of other segments. Alternatively, it can be thought of as the voltage produced in the segment as a result of the current flowing out of it, just as the mutual resistance is the voltage produced in it by current flowing out of another segment. To compute the self resistance of a bare wire of length L, use equation (A6) with $L_1=L_2=L$, $x=0$, $y=a$, where a is the radius of the wire. Doing this, we find that (A6) simplifies to (A7) and (A8) for the M of the wire itself and the M of the wire with its image, respectively:

$$M = 2L \ln\left(\frac{a}{\sqrt{L^2+a^2}-L}\right) + 2\left(a - \sqrt{L^2+a^2}\right) \qquad \text{(A-7)}$$

$$M = 2L \ln\left(\frac{\sqrt{a^2+4D^2}}{\sqrt{L^2+a^2+4D^2}-L}\right) + 2\left(\sqrt{a^2+4D^2} - \sqrt{L^2+a^2+4D^2}\right) \qquad \text{(A-8)}$$

Adding these two equations to get the total M including the effect of the image, substituting $\sqrt{L^2+a^2}-L = a^2/(\sqrt{L^2+a^2}+L)$, simplifying, and applying (A5), we obtain for the self resistance (including the effect of the image):

$$R_{jj} = \frac{\rho}{2\pi L^2}\left[L \ln\left(\frac{\sqrt{L^2+a^2}+L}{a} \cdot \frac{\sqrt{L^2+a^2+4D^2}+L}{\sqrt{a^2+4D^2}}\right) + a + \sqrt{a^2+4D^2} - \sqrt{L^2+a^2} - \sqrt{L^2+a^2+4D^2}\right] \qquad \text{(A-9)}$$

This result could have been obtained more simply by direct integration of (A1), but it is gratifying to see it emerging as a special case of the procedure used herein.

If $L >> D >> a$, we get $R_{jj} = \frac{\rho}{\pi L}\left[\ln\left(\frac{2L}{\sqrt{2aD}}\right) - 1\right]$ (A-10)

This is Sunde's equation 3.36 in reference [5].

6. Perpendicular Segments

If the line segments are horizontal but perpendicular (coplanar or skew), the situation is as shown in Figure A-4. Again we will assume that the x axis lies along segment AB with the origin at A, and the coordinates of point E are x, y, z. Equation (A2) becomes:

$$\begin{aligned} M = {} & (L_1-x) \ln\left(\frac{\sqrt{(L_1-x)^2+(L_2+y)^2+z^2}+L_2+y}{\sqrt{(L_1-x)^2+y^2+z^2}+y}\right) \\ & + x \ln\left(\frac{\sqrt{x^2+(L_2+y)^2+z^2}+L_2+y}{\sqrt{x^2+y^2+z^2}+y}\right) \\ & + (L_2+y) \ln\left(\frac{\sqrt{(L_1-x)^2+(L_2+y)^2+z^2}+L_1-x}{\sqrt{x^2+(L_2+y)^2+z^2}-x}\right) \\ & - y \ln\left(\frac{\sqrt{(L_1-x)^2+y^2+z^2}+L_1-x}{\sqrt{x^2+y^2+z^2}-x}\right) - |U| , \end{aligned} \qquad \text{(A-11)}$$

where U=0 if z=0. If $z \neq 0$ we get

$$\begin{aligned} U = z \Bigg[& \tan^{-1}\left(\frac{(L_1-x)(L_2+y)}{z\sqrt{(L_1-x)^2+(L_2+y)^2+z^2}}\right) \\ & - \tan^{-1}\left(\frac{(L_1-x)y}{z\sqrt{(L_1-x)^2+y^2+z^2}}\right) - \tan^{-1}\left(\frac{-x(L_2+y)}{z\sqrt{x^2+(L_2+y)^2+z^2}}\right) \\ & + \tan^{-1}\left(\frac{-xy}{z\sqrt{x^2+y^2+z^2}}\right) \Bigg] \end{aligned}$$

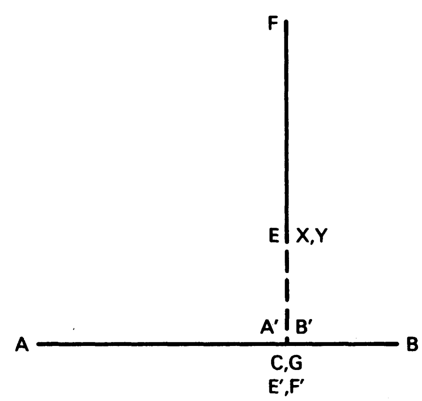

Fig. A-4. Perpendicular lines.

Roundoff Error

Many of the expressions used herein are of the form $\sqrt{b^2+c^2}-b$ or $\sqrt{b^2+c^2}+b$. The former is subject to severe roundoff error if b is positive and much larger than c. The latter is subject to severe roundoff error if b is negative and has an absolute value much larger than c. In either case, the expression is the difference of two numbers which are very nearly equal, and roundoff error can cause the difference to be computed as zero. This is catastrophic if the expression is in a denominator, or if one wants to take a logarithm of it. The following exact substitutions should therefore always be made. They will eliminate the problem.

In evaluating $\sqrt{b^2+c^2}-b$, if b is positive, always substitute $\sqrt{b^2+c^2}-b = c^2/(\sqrt{b^2+c^2}+b)$.

In evaluating $\sqrt{b^2+c^2}+b$, if b is negative always substitute $\sqrt{b^2+c^2}+b = c^2/(\sqrt{b^2+c^2}-b)$.

LAYERED EARTH

The idea of adding the effects of a line source and its image, as described above, is appropriate if the earth is homogeneous to a great depth. If the soil consists of two layers, the top layer having resistivity ρ_1 and thickness H, the botton layer having resistivity ρ_2 and extending to great depth, there are an infinite number of images of decreasing magnitude. Instead of calling the mutual resistance subroutine twice, it is necessary to call it several times, with increasing values of z, until the magnitude of the images is small enough to be disregarded.

Let the reflection factor $K = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$.

Both Segments in Top Layer

The following analysis is based upon reference [7]. Let z be positive going downward from the surface. Consider horizontal segments i and j at depths z_i and z_j below the surface, both being in the top layer of soil. If segment j inserts I_j amps, segment i will see not only segment j itself at vertical coordinate z_j, but also an image reflected in the surface of the earth at coordinate $-z_j$. This image will also appear to insert I_j amps. The vertical distances of these two sources from segment i are $z_j - z_i$ and $z_j + z_i$.

Segment j and the image will both be reflected in the interface between the two layers, appearing as segments injecting currents of KI_j amps. These reflections are at coordinates $2H-z_j$ and $2H+z_j$, respectively. These two images will be reflected with equal intensity by the top surface of the earth, appearing at coordinates $-2H+z_j$ and $-2H-z_j$, respectively. The vertical distances of these four images from segment i are $2H-z_j-z_i$, $2H+z_j-z_i$, $2H-z_j+z_i$, and $2H+z_j+z_i$. Continuation of this process leads to the following equation for the mutual resistance of segment j as seen by segment i, that is, the voltage produced in segment i per ampere leaking from segment j:

$$R_m = \frac{\rho_1}{4\pi L_1 L_2}\left\{\sum_{n=0}^{\infty} K^n\left[M(2nH+z_j+z_i) + M(2nH+z_j-z_i)\right] + \sum_{n=1}^{\infty} K^n\left[M(2nH-z_j+z_i) + M(2nH-z_j-z_i)\right]\right\}$$

where M(z) is the function of z defined previously. M is, of course, also a function of the x and y displacement of one segment relative to the other, the lengths, and the angle Θ, but these are the same for all of the images of a given pair of segments.

In homogeneous soil K=0. All of the above terms drop out except for n=0. Remembering that 0^0 is defined as 1, in homogeneous soil the above equation reduces to

$$R_m = \frac{\rho_1}{4\pi L_1 L_2}\left[M(z_i+z_j) + M(z_i-z_j)\right]$$

That is, of course, just the mutual resistance of the two segments themselves plus the mutual resistance between one segment and the image of the other.

As usual, the equations are symmetrical, so that the above equations also give the mutual resistance of segment i as seen by segment j.

Both Segments in Bottom Layer

Using the same geometry as above except with both segments in the bottom layer, and again basing our analysis on [7], segment i sees segment j injecting I_j amps at vertical coordinate z_j. This is at a vertical distance from segment i of z_j-z_i.

Segment i sees a reflection of segment j in the interface between the two soil layers. This reflection appears to be injecting $-KI_j$ amps, and is located at vertical coordiate $2H-z_j$, and thus at a distance from i of $2H-z_j+z_i$. Call this image j'.

Segment j is reflected by the top surface of the earth. This image is at vertical coordinate $-z_j$, and appear to inject $(1-K^2)\ I_j$ amps. Call this image j". Its distance from i is z_j+z_i.

Image j" is reflected in the interface between the two layers, and is re-reflected by the top surface, appearing at successively higher altitudes going up in steps of 2H. There is no reflecting surface below segment i, so all the reflections appear to i to be above it. Each successive reflection of j" in the soil interface is multiplied in amplitude by K.

Image j' is not re-reflected. It is caused by energy leaving j which strikes the botton of the interface between the two layers, and bounces downward off the interface towards i. Such energy does not hit any additional reflecting surfaces. Energy which penetrates into the top layer, by contrast, can keep bouncing back and forth, some leaking out each time it hits the boundary between the two layers.

The equation for the mutual resistance of segment j as seen by segment i, or segment i as seen by segment j is therefore:

$$R_m = \frac{\rho_2}{4\pi L_1 L_2}\left[M(z_j-z_i) - KM(2H-z_j+z_i) + (1-K^2)\sum_{n=0}^{\infty} K^n M(2nH+z_j+z_i)\right]$$

One Segment in Top Layer, One in Bottom

Proceeding similarly, one obtains the following equation for the mutual resistance of segment j in the basement layer as seen by segment i in the top layer:

$$R_m = \frac{\rho_{2}(1-K)}{4\pi L_1 L_2}\left[M(z_j-z_i) + \sum_{n=0}^{\infty} K^n\ M(2nH+z_j+z_i) + \sum_{n=1}^{\infty} K^n\ (2nH+z_j-z_i)\right]$$

The first term is the mutual resistance of the two segments themselves. The first infinite sum is caused by the images point i sees looking up towards the surface. The second infinite sum is caused by the reflections of these images segment i sees when looking down toward the interface between the two layers.

The mutual resistance of segment i as seen by segment j has $\rho_1\ (1+K)$ in place of $\rho_2\ (1-K)$. These two expressions are equal, again making $r'_{ij} = r'_{ji}$.

Evaluating the Infinite Series

The equations for surface voltage and mutual resistance in the two-layer earth case contain summations to infinity. To evaluate them one must break off the summation someplace. One can decide on how many terms to retain by noting that the terms of the series consist of the product of two factors, both of which decrease with increasing values of n. The first factor is K , which decreases very rapidly with n if K is near zero, a condition which occurs if the conductivities of the two layers are nearly equal. The second term also decreases with n, representing the mutual resistance of two segments that are getting farther apart in the vertical direction. The rate of decrease can be small if H is small. The way in which the second factor decreases with n is difficult to estimate, but, a bound on the error can be obtained by assuming that the second factor does not decrease at all with n. The series starting at any value of n is then just a simple geometric series, and one can compute the sum of the remaining terms from n to infinity. If this calculation shows that the sum of the remaining terms to infinity cannot exceed, say 1 percent of the summation of the terms already evaluated, the summation can be broken off. Because the second term is also decreasing, the error will be smaller than the percentage chosen.

Discussion

P. Kouteynikoff (Electricite de France, Clamart): The author must be complimented for his detailed presentation. It is assumed that, within each segment, the leakage current density is constant. This is a commonly used simplification, though the validity of which is not always clearly proved. It seems this assumption was checked (ref [4] in discussed paper). Could the author give some details on this point, particularly in the case of vertical segments close to a boundary surface (air-top layer, or top layer-bottom layer)?

In the case of a stratified earth, infinite series have to be computed. This may raise some difficulties, as pointed out by the author. However, it exists some very efficient methods for accelerating the convergence of series [1], [2]. Did the author try to use similar methods?

Thanks to symmetry considerations, the author showed it was possible to reduce significantly the amount of calculations. However, I believe these simpliciations are rarely applicable in practical cases.

Otherwise, it would be interesting to have some details about the choice of n, number of segments which a given electrode is broken into. The value of n can be either calculated in an automatic routine (inside the program) or choosen by the program's user and then introduced as a data. Which solution has been choosen by the author, and which criterion is used to determine n?

Finally, I would like to raise a more general problem, concerning the conception and use of computer programs for earth resistance determination. In my opinion, such a program should be as general as possible and usable by people who are not familiar with grounding computations. Of course, such objectives lead to rather complex programs, with no possibility to use symmetry considerations and with a need for an automatic choice of the number of segments which the earth electrode is divided into. However, the increase of programming difficulties would seem negligible when compared to the much greater readiness of use and versatility that would be achieved.

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[2] D. Shanks, "Nonlinear transformations of divergent and slowly convergent sequences." J. Math. and Phys. 24/1955.

Manuscript received March 5,1979.

F. Dawalibi (Safe Electrical System dm Ltd. Montreal, Quebec, Canada): I would like to compliment the author for a timely and well written paper. The author's work is of great value to Engineers involved in grounding studies because it explains in detail the Matrix method [1, 2]. I have the following comments:

1—When the author claims that the matrix method requires modest computer time and core, he refers to symmetrical grids buried in uniform soils. However in practice the grid is seldom symmetrical and is usually buried in nonuniform soils. In such cases, computer time and core requirements may be substantial. A computer program which is designed based on symmetrical or quasi-symmetrical grids is not practical to use for real life installations, where economical grids are required. For this reason our program MALT [3.4] was designed to study symmetrical as well as irregular grids.

2—The author uses several different equations to describe the performance of various conductors. In MALT program only one general equation is used for all types of conductors ([5] and reference [3] of author's paper). Is there a reason for author's approach?

3—The equations developed by the author are valid for horizontal conductors. When oblique or vertical conductors are considered, the terms of the infinite series (two-layer soils) when expressed as inverse hyperbolic sines, have a numerator which is also a function of n. Hence this term may not decrease with n [5], and therefore, it is not possible to obtain a bound on the error by proceeding as described in page 9 of author's paper.

I will appreciate the author comments on the above discussion and I congratulate again the author for his excellent paper.

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[3] F. Dawalibi, D. Mukhedkar, "Parametric analysis of grounding grids". IEEE Transactions paper F79243-7, N.Y. 1979.

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[5] F. Dawalibi, D. Mukhedkar, "Resistance measurement of large grounding systems", Paper submitted for presentation at IEEE PES summer meeting, Vancouver, 1979.

Manuscript received March 2, 1979.

G. B. Niles (Baltimore Gas and Electric Company, Baltimore, MD): The author has shown some interesting effects of nonuniform leakage in ground grids. It appears from the examples given that as the conductor spacing decreases, the step voltage decreases from that computed by IEEE Guide 80. Is this a general trend and can the author give an idea of the reduction by his method vs. IEEE Guide 80 relative to the conductor spacing?

It is noted these differences are given for the maximum step voltage, is it also true for all step voltages?

In IEEE Guide 80, ground rods at each interconnection are shown. However, the guide states these rods have "negligible effect on safety". Did the author use ground rods in his grid network and if so does he agree with the guide on the safety aspect? It would appear to me that there is greater chance for current density nonuniformity with these rods in the grid. However, coupled with nonuniform soil this discusser realizes there could be difficult computations involved to resolve this interaction. Has the author attempted any computations of this kind?

It also appears to me that rods at the perimeter where maximum step potentials probably occur would reduce this maximum step potential. Would the author comment on this?

Manuscript received February 23, 1979.

D. L. Garrett (Southern Company Services, Inc., Birmingham, AL): The author is to be congratulated for his development of a computer oriented method of modeling the grounding system of a power substation. The technique and equations presented in the paper allow the engineer to avoid many assumptions and approximations that are necessary in analyzing a grounding system using previous methods [1]. These assumptions and approximations have tended to make previous methods inaccurate when modeling irregular shaped or very large substations. However, when each segment of the grounding system is modeled separately, as proposed by the author, then the validity of the results obtained should not be affected appreciably by the size or configuration of the grounding system.

There is, however, a major drawback to the method proposed by the author. This is the enormous amount of computer storage and computation time that may be required to model the grounding system. The author emphasizes the fact that for symmetrical grounding systems (i.e., systems that are symmetrical in overall shape, grid conductor spacings, and ground rod locations), only a part of the grounding system needs to be studied. However, it has been this discussor's experience to more frequently encounter grounding systems that are unsymmetrical, such as Figure 1, below. In a grounding system such as this, each branch of each mesh would be classified as a distinct "type." Since the author did not give examples of analyzing unsymmetrical grids in the paper, I would like to have his comments on the amount of computer storage and time required to model such grounding systems as compared to the simpler symmetrical systems.

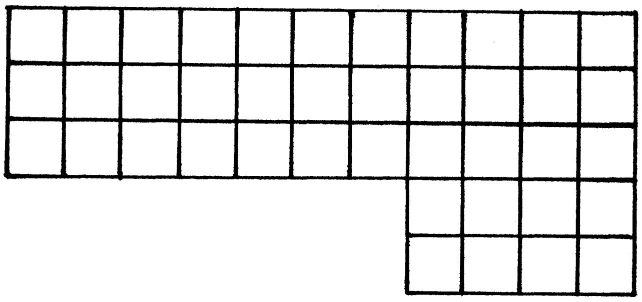

Figure 1
Example of a Typical Unsymmetrical Grounding System

The discussor also wishes for the author to elaborate on how many points need be evaluated in determining the maximum step and touch potentials within the grounding system. While an unsymmetrical grid will increase the number of self and mutual resistances to be determined, it will increase even more the number of points at which the potential must be calculated. This is because the maximum step and touch potentials have been found to occur in the corner meshes of uniformly spaced symmetrical grounding systems, but for an unsymmetrical grid such as Figure 1, it is not obvious in which mesh these maximum potentials will occur. Therefore, it may be necessary to evaluate the potentials at many more points for an unsymmetrical grid than for a symmetrical grid in order to find the maximum step and touch potentials.

In developing equations for the mutual resistance between conductor segments in two layer earth, the author evaluates two cases of both conductors in either the top or bottom layer, and one conductor in the top layer with the second conductor in the bottom layer. However, the author does not include the possibility of conductor segments which extend from one layer into the second layer, as would often be the case for deep driven ground rods. It appears to this discussor that a valid model of such a conductor segment would be to model the segment as two separate segments, one in each of the layers. Would the author care to comment on this assumption or the relative need for this additional detail in the model which is sure to add significantly to the number of distinct "types" of conductor segments in a grounding system?

In closing, it should be noted that the accuracy of the results obtained using the technique presented by Mr. Heppe should depend mainly on the proper choice of equations for determining the self and mutual resistances of the conductor segments (i.e., homogeneous, two layer, or multi-layer earth). It is the opinion of this discussor that until more work is done in validating the equations for two layer or multilayer earth, and until better means are developed for determining the values of ϱ_1, ϱ_2 and H from field test data, the system protection engineer should avoid basing his design of a safe grounding system on these layered earth models.

REFERENCE

[1] "Guide for Safety in Alternating-Current Substation Grounding," *IEEE Standard No. 80,* New York, N.Y.: The Institute of Electrical and Engineers, Inc. 1976.

Manuscript received February 23, 1979.

Robert J. Heppe: I thank the discussors for their thoughtful comments, kind words, and the references they have brought to my attention.

Most of the discussors note that the method presented applies only to symmetrical grids. This is, unfortunately, true, although if an unsymmetrical grid is as small as that shown in Mr. Garrett's discussion (98 segments) it can be handled within reasonable computer time and memory constraints by treating each of the segments as being of a different "type". This is, of course, a straight matrix solution of the problem, and while the computer program is a little simpler than the one discussed in the body of the paper, it has no special shortcuts.

There are several approaches to handling unsymmetrical grids. If one examines a grid of the type shown by Mr. Garrett, one may note that there are large numbers of pairs of segments which have the same relative spacing. As a matter of fact, there are only a total of 10 different mutual resistance values in the east-west direction and 5 in the north-south direction, so that the $98 \times 97 = 9506$ mutual resistance values in this grid all consist of the same 50 numbers. It turns out that even if the spacing of the conductors is non-uniform the number of different mutual resistance values which must be calculated is far smaller than the number of pairs of segments.

A second technique which may be exploited is that it is not necessary to calculate individually all values of mutual resistance between two segments, or between a segment and a surface point, even when they are different. A human being faced with the problem would calculate a few points covering the range of interest, draw a graph, and use it to interpolate to find intermediate values. A computer can, in effect, do the same thing. Both of these techniques, but especially the second, can be enormously helpful in calculating the surface voltage at a grid of points over the buried grid of wires. As Mr. Garrett points out, in an unsymmetrical grid it is not easy to tell where the maximum step and touch voltages occur, so the potential at many points must be computed. By doing a simple interpolation into tables of mutual resistance between a buried conductor of a given length at various spacings in the x and y directions from a given point, the contribution of each segment to the surface voltage at the point can be obtained much faster than if a long, slowly-converging series had to be evaluated.

Mr. Sverak and I hope to prepare a paper for presentation to the Power Engineering Society which will report on a computer program using these techniques to find the current distribution and surface voltage distribution for unsymmetrical grids.

To answer Mr. Garrett's other question, recent results by Mssrs. Dawalibi and Mukhedkar which appear in the fourth reference cited in Mr. Dawalibi's comments indicate that ground rods can be modelled as two vertical segments, one in the top layer and one in the basement layer. If one layer has a resistivity which is very different from the other (which causes the series to converge slowly) it may sometimes be possible to neglect the effect of the segment in the higher resistivity layer, if I understand his results correctly. The grounding rods will add to the number of distinct "types" of conductor segments in a grounding system.

In answer to Mr. Niles' question, I did not use ground rods in the network I investigated, and am thus unable to comment on his questions concerning them. Reference 4 of Mr. Dawalibi's discussion provides a great deal of information on the effects of grounding rods.

I did not run any examples to verify whether decreasing the conductor spacing always decreases the step voltage as compared to what IEEE Guide 80 would predict.

I am indebted to Mr. Dawalibi for his remarks on the convergence of the series for vertical conductors such as ground rods. This is an area I had not explored. In answer to his other question, several different equations were presented for computing the mutual resistance of conductor segments, depending on whether they are parallel, perpendicular, etc., because the special cases cause the general equation to become simpler to compute, and are also less subject to roundoff error, I believe.

I thank M. Kouteynikoff for his references on evaluation of infinite series. I have not yet tried to apply them to this problem. The question as to what size the segments should be to produce negligible error from the approximation that leakage current density is constant within a segment is a difficult one. The answer depends on exactly what one is trying to compute. For example, relatively long segments can be used to compute the grounding resistance of a grid, or to compute the effects of a distant conductor on the voltage of a segment or a surface point. Use of long segments when evaluating the effects of nearby conductors would introduce larger errors. Varying the segment length depending on the location of the conductor relative to a point of interest can be used to reduce the computing time required, but it requires use of a more complicated computer program.

As M. Kouteynikoff says, it would be desirable to develop a computer program which would apply the various shortcuts when they are applicable, and which would be usable by those who are unfamiliar with grounding equations. Progress is being made towards this goal. But as Mr. Garrett points out, the accuracy one can expect to achieve depends ultimately on the accuracy with which the resistance distribution in the earth is known and modelled.

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