A Supplementary File for
"Review and Analysis of Three Components of the Differential Evolution Mutation Operator in MOEA/D-DE"

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Table S.1: Comparison of MOEA/D-DE with the three index selection methods on the 16 problems (the four DTLZ and nine WFG problems) with $M \in \{2, 3, 4, 5\}$. The current/1 mutation strategy and the replacement method were used in MOEA/D-DE. The means and standard deviations of the hypervolume indicator values of 51 runs are shown (higher is better).

(a) $M = 2$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	4.75e-01 (8.62e-02)	4.88e-01 (8.69e-02)	**4.94e-01 (8.70e-02)**
DTLZ2	4.22e-01 (7.10e-06)	4.22e-01 (6.15e-06)	**4.22e-01 (6.91e-06)**
DTLZ3	2.05e-01 (1.08e-01)	2.10e-01 (1.27e-01)	**2.25e-01 (9.89e-02)**
DTLZ4	4.22e-01 (1.49e-05)	**4.22e-01 (1.03e-05)**	4.22e-01 (1.19e-05)
DTLZ5	4.22e-01 (7.10e-06)	4.22e-01 (6.15e-06)	**4.22e-01 (6.91e-06)**
DTLZ6	4.16e-01 (5.85e-03)	**4.17e-01 (6.69e-03)**	4.16e-01 (9.67e-03)
DTLZ7	3.53e-01 (1.20e-01)	3.48e-01 (1.21e-01)	**3.67e-01 (1.18e-01)**
WFG1	3.69e-01 (7.11e-02)	3.89e-01 (7.31e-02)	**4.29e-01 (7.01e-02)**
WFG2	**7.64e-01 (5.69e-04)**	7.64e-01 (8.12e-04)	7.64e-01 (7.89e-04)
WFG3	7.05e-01 (3.88e-04)	7.05e-01 (4.58e-04)	**7.05e-01 (4.54e-04)**
WFG4	3.94e-01 (2.86e-03)	**3.99e-01 (2.68e-03)**	3.95e-01 (2.68e-03)
WFG5	3.74e-01 (4.25e-04)	**3.75e-01 (6.73e-04)**	3.75e-01 (3.58e-04)
WFG6	**3.52e-01 (2.00e-02)**	3.51e-01 (1.84e-02)	3.49e-01 (1.57e-02)
WFG7	4.21e-01 (1.47e-04)	4.21e-01 (1.30e-04)	**4.21e-01 (1.52e-04)**
WFG8	**3.69e-01 (2.93e-03)**	3.68e-01 (3.17e-03)	3.68e-01 (3.65e-03)
WFG9	3.46e-01 (1.87e-02)	**3.63e-01 (3.10e-02)**	3.53e-01 (2.64e-02)

(b) $M = 3$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	9.03e-01 (1.32e-01)	**9.06e-01 (1.19e-01)**	9.05e-01 (1.64e-01)
DTLZ2	7.38e-01 (1.66e-03)	**7.39e-01 (1.48e-03)**	7.39e-01 (1.55e-03)
DTLZ3	**3.13e-01 (2.37e-01)**	2.67e-01 (2.37e-01)	2.71e-01 (2.41e-01)
DTLZ4	**7.37e-01 (2.58e-02)**	7.27e-01 (3.84e-02)	7.32e-01 (3.32e-02)
DTLZ5	2.65e-01 (3.89e-05)	**2.65e-01 (2.10e-05)**	2.65e-01 (3.32e-05)
DTLZ6	2.55e-01 (5.93e-03)	**2.57e-01 (6.60e-03)**	2.54e-01 (7.59e-03)
DTLZ7	3.08e-01 (3.90e-02)	**3.18e-01 (4.26e-02)**	3.11e-01 (3.57e-02)
WFG1	4.31e-01 (5.02e-03)	**4.38e-01 (1.05e-02)**	4.34e-01 (7.82e-03)
WFG2	**1.21e+00 (5.69e-03)**	1.20e+00 (6.19e-03)	1.21e+00 (7.24e-03)
WFG3	8.41e-01 (3.98e-03)	8.41e-01 (6.36e-03)	**8.42e-01 (3.64e-03)**
WFG4	6.48e-01 (7.41e-03)	**6.61e-01 (7.31e-03)**	6.50e-01 (6.75e-03)
WFG5	6.64e-01 (2.49e-03)	**6.65e-01 (2.38e-03)**	6.65e-01 (2.96e-03)
WFG6	6.44e-01 (1.75e-02)	6.45e-01 (1.58e-02)	**6.47e-01 (1.74e-02)**
WFG7	7.04e-01 (4.96e-03)	**7.13e-01 (4.83e-03)**	7.06e-01 (5.99e-03)
WFG8	5.79e-01 (1.23e-02)	**5.84e-01 (1.20e-02)**	5.75e-01 (1.03e-02)
WFG9	6.19e-01 (7.39e-03)	6.17e-01 (7.11e-03)	**6.19e-01 (9.59e-03)**

(c) $M = 4$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	1.09e+00 (3.90e-01)	1.15e+00 (2.69e-01)	**1.15e-00 (3.17e-01)**
DTLZ2	**9.33e-01 (2.33e-03)**	9.32e-01 (3.01e-03)	9.32e-01 (3.04e-03)
DTLZ3	3.58e-01 (3.38e-01)	3.19e-01 (3.43e-01)	**4.61e-01 (3.75e-01)**
DTLZ4	9.27e-01 (2.55e-02)	9.27e-01 (2.72e-02)	**9.28e-01 (2.09e-02)**
DTLZ5	2.10e-01 (1.30e-04)	**2.10e-01 (1.03e-04)**	2.10e-01 (1.15e-04)
DTLZ6	**2.05e-01 (8.50e-03)**	2.03e-01 (7.53e-03)	2.03e-01 (1.29e-02)
DTLZ7	3.49e-01 (1.06e-02)	**3.50e-01 (1.32e-02)**	3.49e-01 (1.18e-02)
WFG1	4.80e-01 (2.71e-02)	4.86e-01 (2.38e-02)	**4.88e-01 (2.59e-02)**
WFG2	**1.34e+00 (1.55e-02)**	1.33e+00 (1.75e-02)	1.34e+00 (1.80e-02)
WFG3	**9.24e-01 (1.30e-02)**	9.18e-01 (1.58e-02)	9.19e-01 (1.39e-02)
WFG4	7.55e-01 (2.15e-02)	**7.65e-01 (1.69e-02)**	7.56e-01 (1.74e-02)
WFG5	7.82e-01 (1.47e-02)	7.83e-01 (1.35e-02)	**7.85e-01 (1.79e-02)**
WFG6	8.09e-01 (6.59e-03)	**8.13e-01 (1.09e-02)**	8.11e-01 (5.96e-03)
WFG7	8.16e-01 (2.15e-02)	**8.22e-01 (2.22e-02)**	8.11e-01 (2.18e-02)
WFG8	6.23e-01 (2.17e-02)	**6.25e-01 (2.19e-02)**	6.22e-01 (2.42e-02)
WFG9	**7.34e-01 (1.18e-02)**	7.30e-01 (9.96e-03)	7.31e-01 (1.04e-02)

(d) $M = 5$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	4.70e-01 (4.85e-01)	7.37e-01 (5.57e-01)	**7.38e-01 (5.86e-01)**
DTLZ2	1.14e+00 (2.25e-03)	**1.14e+00 (1.61e-03)**	1.14e+00 (1.89e-03)
DTLZ3	5.84e-02 (1.60e-01)	6.58e-02 (1.82e-01)	**8.93e-02 (2.14e-01)**
DTLZ4	1.08e+00 (2.85e-02)	**1.09e+00 (3.12e-02)**	1.08e+00 (3.28e-02)
DTLZ5	1.56e-01 (1.66e-04)	**1.56e-01 (4.15e-03)**	1.56e-01 (2.41e-04)
DTLZ6	1.48e-01 (3.81e-03)	**1.48e-01 (4.79e-03)**	1.48e-01 (3.96e-03)
DTLZ7	3.07e-01 (3.28e-02)	**3.23e-01 (2.78e-02)**	3.10e-01 (3.37e-02)
WFG1	4.81e-01 (5.33e-02)	**5.52e-01 (8.39e-02)**	5.23e-01 (7.45e-02)
WFG2	1.47e+00 (3.60e-02)	1.46e+00 (3.55e-02)	**1.47e+00 (2.57e-02)**
WFG3	**9.62e-01 (1.42e-02)**	9.59e-01 (1.69e-02)	9.57e-01 (1.63e-02)
WFG4	6.56e-01 (5.45e-02)	**6.69e-01 (5.16e-02)**	6.53e-01 (4.38e-02)
WFG5	7.07e-01 (4.78e-02)	7.01e-01 (4.43e-02)	**7.12e-01 (4.41e-02)**
WFG6	9.81e-01 (3.37e-02)	9.89e-01 (7.93e-03)	**9.89e-01 (1.97e-02)**
WFG7	**7.12e-01 (6.00e-02)**	6.95e-01 (5.40e-02)	6.99e-01 (5.10e-02)
WFG8	4.74e-01 (2.82e-02)	**4.80e-01 (2.97e-02)**	4.78e-01 (3.00e-02)
WFG9	**6.01e-01 (5.35e-02)**	5.99e-01 (5.07e-02)	5.89e-01 (5.05e-02)

Table S.2: Comparison of MOEA/D-DE with the three index selection methods on the 16 problems (the four DTLZ and nine WFG problems) with $M \in \{2,3,4,5\}$. The rand/1 mutation strategy and the replacement method were used in MOEA/D-DE. The means and standard deviation of the hypervolume indicator values of 51 runs are shown (higher is better).

(a) $M = 2$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	4.45e-01 (1.13e-01)	**4.66e-01 (1.36e-01)**	4.08e-01 (1.27e-01)
DTLZ2	4.22e-01 (8.97e-06)	**4.22e-01 (7.71e-06)**	4.22e-01 (8.44e-06)
DTLZ3	**2.04e-01 (1.01e-01)**	1.89e-01 (1.11e-01)	1.76e-01 (1.08e-01)
DTLZ4	4.22e-01 (1.70e-05)	**4.22e-01 (1.16e-05)**	4.22e-01 (1.74e-05)
DTLZ5	4.22e-01 (8.97e-06)	**4.22e-01 (7.71e-06)**	4.22e-01 (8.44e-06)
DTLZ6	**4.15e-01 (7.98e-03)**	4.12e-01 (7.84e-03)	4.14e-01 (9.87e-03)
DTLZ7	3.62e-01 (1.19e-01)	3.58e-01 (1.20e-01)	**3.90e-01 (1.10e-01)**
WFG1	3.38e-01 (6.43e-02)	**3.89e-01 (7.48e-02)**	3.55e-01 (6.09e-02)
WFG2	**7.64e-01 (6.32e-04)**	7.63e-01 (3.34e-03)	7.64e-01 (8.21e-04)
WFG3	7.04e-01 (4.13e-04)	**7.05e-01 (5.02e-04)**	7.04e-01 (4.67e-04)
WFG4	3.92e-01 (3.19e-03)	**3.98e-01 (2.91e-03)**	3.92e-01 (3.25e-03)
WFG5	**3.75e-01 (6.78e-04)**	3.74e-01 (5.44e-04)	3.74e-01 (4.72e-04)
WFG6	3.47e-01 (1.21e-02)	**3.58e-01 (2.51e-02)**	3.48e-01 (1.41e-02)
WFG7	4.21e-01 (1.68e-04)	**4.21e-01 (1.83e-04)**	4.21e-01 (1.62e-04)
WFG8	3.67e-01 (2.66e-03)	3.64e-01 (3.31e-03)	**3.68e-01 (2.72e-03)**
WFG9	3.44e-01 (1.67e-02)	**3.55e-01 (2.72e-02)**	3.52e-01 (2.43e-02)

(b) $M = 3$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	8.10e-01 (2.73e-01)	**8.96e-01 (1.26e-01)**	8.57e-01 (1.79e-01)
DTLZ2	7.39e-01 (1.68e-03)	**7.39e-01 (1.34e-03)**	7.39e-01 (1.39e-03)
DTLZ3	**2.73e-01 (2.19e-01)**	2.69e-01 (2.57e-01)	2.27e-01 (2.40e-01)
DTLZ4	**7.41e-01 (1.87e-02)**	7.32e-01 (3.33e-02)	7.37e-01 (2.57e-02)
DTLZ5	2.65e-01 (5.56e-05)	**2.65e-01 (2.62e-05)**	2.65e-01 (4.88e-05)
DTLZ6	2.57e-01 (6.18e-03)	**2.57e-01 (5.92e-03)**	2.56e-01 (7.13e-03)
DTLZ7	**2.95e-01 (3.16e-02)**	2.90e-01 (3.11e-02)	2.94e-01 (3.40e-02)
WFG1	4.28e-01 (3.13e-03)	**4.34e-01 (6.03e-03)**	4.28e-01 (3.71e-03)
WFG2	1.20e+00 (6.32e-03)	1.20e+00 (6.57e-03)	**1.20e+00 (5.95e-03)**
WFG3	8.39e-01 (3.96e-03)	**8.40e-01 (4.66e-03)**	8.40e-01 (3.48e-03)
WFG4	6.45e-01 (6.70e-03)	**6.55e-01 (7.76e-03)**	6.46e-01 (6.33e-03)
WFG5	**6.64e-01 (2.46e-03)**	6.63e-01 (2.32e-03)	6.64e-01 (2.80e-03)
WFG6	**6.46e-01 (1.76e-02)**	6.44e-01 (1.49e-02)	6.45e-01 (1.82e-02)
WFG7	6.98e-01 (6.47e-03)	**7.03e-01 (6.95e-03)**	6.95e-01 (5.79e-03)
WFG8	5.74e-01 (9.94e-03)	5.74e-01 (1.32e-02)	**5.76e-01 (1.01e-02)**
WFG9	6.18e-01 (8.41e-03)	**6.19e-01 (7.96e-03)**	6.18e-01 (6.45e-03)

(c) $M = 4$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	1.08e+00 (3.11e-01)	**1.21e+00 (2.02e-01)**	1.14e+00 (2.38e-01)
DTLZ2	**9.31e-01 (2.47e-03)**	9.31e-01 (2.60e-03)	9.31e-01 (2.59e-03)
DTLZ3	3.43e-01 (3.56e-01)	3.10e-01 (3.40e-01)	**4.32e-01 (3.68e-01)**
DTLZ4	**9.29e-01 (2.52e-02)**	9.27e-01 (2.71e-02)	9.29e-01 (2.46e-02)
DTLZ5	2.10e-01 (1.00e-04)	**2.10e-01 (6.93e-05)**	2.10e-01 (9.78e-05)
DTLZ6	2.05e-01 (5.70e-03)	2.05e-01 (6.26e-03)	**2.05e-01 (5.26e-03)**
DTLZ7	3.45e-01 (1.24e-02)	3.44e-01 (1.69e-02)	**3.45e-01 (1.48e-02)**
WFG1	4.60e-01 (1.27e-02)	**4.64e-01 (1.81e-02)**	4.58e-01 (1.34e-02)
WFG2	1.34e+00 (1.43e-02)	1.32e+00 (1.60e-02)	**1.34e+00 (1.41e-02)**
WFG3	9.15e-01 (1.29e-02)	**9.18e-01 (1.39e-02)**	9.14e-01 (1.67e-02)
WFG4	7.50e-01 (2.06e-02)	**7.54e-01 (1.75e-02)**	7.51e-01 (1.87e-02)
WFG5	**7.79e-01 (1.29e-02)**	7.73e-01 (1.34e-02)	7.79e-01 (1.57e-02)
WFG6	8.09e-01 (6.85e-03)	8.09e-01 (6.13e-03)	**8.10e-01 (5.27e-03)**
WFG7	7.94e-01 (2.08e-02)	**7.99e-01 (2.21e-02)**	7.94e-01 (2.29e-02)
WFG8	6.12e-01 (2.08e-02)	**6.16e-01 (2.06e-02)**	6.10e-01 (2.37e-02)
WFG9	7.30e-01 (1.15e-02)	7.27e-01 (9.65e-03)	**7.33e-01 (1.10e-02)**

(d) $M = 5$

	WOR Mean (Std.)	WR Mean (Std.)	WPR Mean (Std.)
DTLZ1	**6.89e-01 (4.79e-01)**	6.68e-01 (5.26e-01)	6.21e-01 (5.59e-01)
DTLZ2	1.14e+00 (1.99e-03)	**1.14e+00 (2.00e-03)**	1.14e+00 (2.26e-03)
DTLZ3	9.08e-02 (1.93e-01)	8.16e-02 (1.64e-01)	**9.77e-02 (2.10e-01)**
DTLZ4	**1.09e+00 (2.99e-02)**	1.08e+00 (2.92e-02)	1.08e+00 (3.65e-02)
DTLZ5	1.56e-01 (1.30e-04)	1.57e-01 (5.92e-03)	**1.57e-01 (5.94e-03)**
DTLZ6	**1.51e-01 (9.68e-03)**	1.48e-01 (6.38e-03)	1.49e-01 (8.62e-03)
DTLZ7	**3.24e-01 (3.26e-02)**	3.22e-01 (3.64e-02)	3.08e-01 (3.20e-02)
WFG1	4.39e-01 (5.28e-02)	**4.99e-01 (8.14e-02)**	4.33e-01 (4.89e-02)
WFG2	**1.47e+00 (3.09e-02)**	1.45e+00 (3.27e-02)	1.47e+00 (2.37e-02)
WFG3	9.63e-01 (1.47e-02)	9.62e-01 (1.40e-02)	**9.64e-01 (1.64e-02)**
WFG4	6.40e-01 (4.50e-02)	**6.45e-01 (4.11e-02)**	6.44e-01 (4.42e-02)
WFG5	6.90e-01 (4.22e-02)	**7.06e-01 (6.15e-02)**	7.03e-01 (4.61e-02)
WFG6	**9.87e-01 (2.00e-02)**	9.84e-01 (2.88e-02)	9.82e-01 (2.83e-02)
WFG7	**6.83e-01 (5.52e-02)**	6.76e-01 (5.07e-02)	6.82e-01 (4.83e-02)
WFG8	**4.95e-01 (2.33e-02)**	4.87e-01 (2.45e-02)	4.84e-01 (1.67e-02)
WFG9	5.92e-01 (4.65e-02)	5.89e-01 (4.83e-02)	**5.96e-01 (5.25e-02)**

Table S.3: Comparison of MOEA/D-DE with the two mutation strategies on the 16 problems (the four DTLZ and nine WFG problems) with $M \in \{2, 3, 4, 5\}$. The means and standard deviations of the hypervolume indicator values of 51 runs are shown (higher is better).

(a) $M = 2$

	current/1 Mean (Std.)	rand/1 Mean (Std.)
DTLZ1	**4.88e-01 (8.69e-02)**	4.45e-01 (1.13e-01)
DTLZ2	**4.22e-01 (6.15e-06)**	4.22e-01 (8.97e-06)
DTLZ3	**2.10e-01 (1.27e-01)**	2.04e-01 (1.01e-01)
DTLZ4	**4.22e-01 (1.03e-05)**	4.22e-01 (1.70e-05)
DTLZ5	**4.22e-01 (6.15e-06)**	4.22e-01 (8.97e-06)
DTLZ6	**4.17e-01 (6.69e-03)**	4.15e-01 (7.98e-03)
DTLZ7	3.48e-01 (1.21e-01)	**3.62e-01 (1.19e-01)**
WFG1	**3.89e-01 (7.31e-02)**	3.38e-01 (6.43e-02)
WFG2	7.64e-01 (8.12e-04)	**7.64e-01 (6.32e-04)**
WFG3	**7.05e-01 (4.58e-04)**	7.04e-01 (4.13e-04)
WFG4	**3.99e-01 (2.68e-03)**	3.92e-01 (3.19e-03)
WFG5	**3.75e-01 (6.73e-04)**	3.75e-01 (6.78e-04)
WFG6	**3.51e-01 (1.84e-02)**	3.47e-01 (1.21e-02)
WFG7	**4.21e-01 (1.30e-04)**	4.21e-01 (1.68e-04)
WFG8	**3.68e-01 (3.17e-03)**	3.67e-01 (2.66e-03)
WFG9	**3.63e-01 (3.10e-02)**	3.44e-01 (1.67e-02)

(b) $M = 3$

	current/1 Mean (Std.)	rand/1 Mean (Std.)
DTLZ1	**9.06e-01 (1.19e-01)**	8.10e-01 (2.73e-01)
DTLZ2	**7.39e-01 (1.48e-03)**	7.39e-01 (1.68e-03)
DTLZ3	2.67e-01 (2.37e-01)	**2.73e-01 (2.19e-01)**
DTLZ4	7.27e-01 (3.84e-02)	**7.41e-01 (1.87e-02)**
DTLZ5	**2.65e-01 (2.10e-05)**	2.65e-01 (5.56e-05)
DTLZ6	**2.57e-01 (6.60e-03)**	2.57e-01 (6.18e-03)
DTLZ7	**3.18e-01 (4.26e-02)**	2.95e-01 (3.16e-02)
WFG1	**4.38e-01 (1.05e-02)**	4.28e-01 (3.13e-03)
WFG2	**1.20e+00 (6.19e-03)**	1.20e+00 (6.32e-03)
WFG3	**8.41e-01 (6.36e-03)**	8.39e-01 (3.96e-03)
WFG4	**6.61e-01 (7.31e-03)**	6.45e-01 (6.70e-03)
WFG5	**6.65e-01 (2.38e-03)**	6.64e-01 (2.46e-03)
WFG6	6.45e-01 (1.58e-02)	**6.46e-01 (1.76e-02)**
WFG7	**7.13e-01 (4.83e-03)**	6.98e-01 (6.47e-03)
WFG8	**5.84e-01 (1.20e-02)**	5.74e-01 (9.94e-03)
WFG9	6.17e-01 (7.11e-03)	**6.18e-01 (8.41e-03)**

(c) $M = 4$

	current/1 Mean (Std.)	rand/1 Mean (Std.)
DTLZ1	**1.15e+00 (2.69e-01)**	1.08e+00 (3.11e-01)
DTLZ2	**9.32e-01 (3.01e-03)**	9.31e-01 (2.47e-03)
DTLZ3	3.19e-01 (3.43e-01)	**3.43e-01 (3.56e-01)**
DTLZ4	9.27e-01 (2.72e-02)	**9.29e-01 (2.52e-02)**
DTLZ5	**2.10e-01 (1.03e-04)**	2.10e-01 (1.00e-04)
DTLZ6	2.03e-01 (7.53e-03)	**2.05e-01 (5.70e-03)**
DTLZ7	**3.50e-01 (1.32e-02)**	3.45e-01 (1.24e-02)
WFG1	**4.86e-01 (2.38e-02)**	4.60e-01 (1.27e-02)
WFG2	1.33e+00 (1.75e-02)	**1.34e+00 (1.43e-02)**
WFG3	**9.18e-01 (1.58e-02)**	9.15e-01 (1.29e-02)
WFG4	**7.65e-01 (1.69e-02)**	7.50e-01 (2.06e-02)
WFG5	**7.83e-01 (1.35e-02)**	7.79e-01 (1.29e-02)
WFG6	**8.13e-01 (1.09e-02)**	8.09e-01 (6.85e-03)
WFG7	**8.22e-01 (2.22e-02)**	7.94e-01 (2.08e-02)
WFG8	**6.25e-01 (2.19e-02)**	6.12e-01 (2.08e-02)
WFG9	7.30e-01 (9.96e-03)	**7.30e-01 (1.15e-02)**

(d) $M = 5$

	current/1 Mean (Std.)	rand/1 Mean (Std.)
DTLZ1	**7.37e-01 (5.57e-01)**	6.89e-01 (4.79e-01)
DTLZ2	**1.14e+00 (1.61e-03)**	1.14e+00 (1.99e-03)
DTLZ3	6.58e-02 (1.82e-01)	**9.08e-02 (1.93e-01)**
DTLZ4	1.09e+00 (3.12e-02)	**1.09e+00 (2.99e-02)**
DTLZ5	**1.56e-01 (4.15e-03)**	1.56e-01 (1.30e-04)
DTLZ6	1.48e-01 (4.79e-03)	**1.51e-01 (9.68e-03)**
DTLZ7	3.23e-01 (2.78e-02)	**3.24e-01 (3.26e-02)**
WFG1	**5.52e-01 (8.39e-02)**	4.39e-01 (5.28e-02)
WFG2	1.46e+00 (3.55e-02)	**1.47e+00 (3.09e-02)**
WFG3	9.59e-01 (1.69e-02)	**9.63e-01 (1.47e-02)**
WFG4	**6.69e-01 (5.16e-02)**	6.40e-01 (4.50e-02)
WFG5	**7.01e-01 (4.43e-02)**	6.90e-01 (4.22e-02)
WFG6	**9.89e-01 (7.93e-03)**	9.87e-01 (2.00e-02)
WFG7	**6.95e-01 (5.40e-02)**	6.83e-01 (5.52e-02)
WFG8	4.80e-01 (2.97e-02)	**4.95e-01 (2.33e-02)**
WFG9	**5.99e-01 (5.07e-02)**	5.92e-01 (4.65e-02)

Table S.4: Comparison of MOEA/D-DE with the five bound handling methods on the 16 problems (the four DTLZ and nine WFG problems) with $M \in \{2, 3, 4, 5\}$. The means and standard deviations of the hypervolume indicator values of 51 runs are shown (higher is better).

(a) $M = 2$

	replacement Mean (Std.)	reinitialization Mean (Std.)	reflection Mean (Std.)	r-reflection Mean (Std.)	resampling Mean (Std.)
DTLZ1	4.88e-01 (8.69e-02)	**5.90e-01 (1.14e-01)**	5.79e-01 (1.16e-01)	4.93e-01 (1.47e-01)	5.60e-01 (1.38e-01)
DTLZ2	**4.22e-01 (6.15e-06)**	4.22e-01 (5.34e-04)	4.22e-01 (1.26e-04)	4.22e-01 (3.89e-05)	4.22e-01 (2.05e-04)
DTLZ3	**2.10e-01 (1.27e-01)**	7.29e-02 (1.40e-01)	1.07e-01 (1.57e-01)	3.63e-02 (7.31e-02)	1.53e-01 (1.54e-01)
DTLZ4	**4.22e-01 (1.03e-05)**	4.22e-01 (1.27e-04)	4.22e-01 (2.02e-04)	4.22e-01 (4.30e-05)	3.91e-01 (9.37e-02)
DTLZ5	**4.22e-01 (6.15e-06)**	4.22e-01 (5.34e-04)	4.22e-01 (1.26e-04)	4.22e-01 (3.89e-05)	4.22e-01 (2.05e-04)
DTLZ6	**4.17e-01 (6.69e-03)**	3.01e-04 (1.87e-03)	2.64e-01 (4.84e-02)	2.81e-01 (4.91e-02)	3.71e-01 (2.93e-02)
DTLZ7	3.48e-01 (1.21e-01)	**4.58e-01 (1.12e-03)**	3.06e-01 (1.15e-01)	3.06e-01 (1.16e-01)	3.01e-01 (1.14e-01)
WFG1	3.89e-01 (7.31e-02)	3.51e-01 (5.10e-02)	3.87e-01 (6.60e-02)	3.71e-01 (6.87e-02)	**4.04e-01 (5.16e-02)**
WFG2	**7.64e-01 (8.12e-04)**	7.60e-01 (8.77e-03)	7.60e-01 (8.14e-03)	7.63e-01 (3.26e-03)	7.58e-01 (1.01e-02)
WFG3	**7.05e-01 (4.58e-04)**	7.04e-01 (4.13e-04)	7.04e-01 (4.16e-04)	7.04e-01 (4.39e-04)	7.04e-01 (4.68e-04)
WFG4	3.99e-01 (2.68e-03)	3.99e-01 (2.67e-03)	3.99e-01 (2.56e-03)	3.99e-01 (2.46e-03)	**4.00e-01 (2.14e-03)**
WFG5	**3.75e-01 (6.73e-04)**	3.68e-01 (1.23e-03)	3.74e-01 (1.60e-04)	3.74e-01 (5.24e-04)	3.74e-01 (3.42e-04)
WFG6	3.51e-01 (1.84e-02)	**4.08e-01 (3.48e-03)**	4.04e-01 (3.17e-03)	4.04e-01 (1.34e-02)	4.06e-01 (3.75e-03)
WFG7	**4.21e-01 (1.30e-04)**	4.20e-01 (3.66e-04)	4.20e-01 (2.33e-04)	4.20e-01 (2.57e-04)	4.20e-01 (1.67e-04)
WFG8	**3.68e-01 (3.17e-03)**	3.65e-01 (3.22e-03)	3.66e-01 (3.49e-03)	3.66e-01 (2.97e-03)	3.64e-01 (2.59e-03)
WFG9	3.63e-01 (3.10e-02)	4.03e-01 (1.41e-03)	4.03e-01 (9.51e-04)	4.03e-01 (1.06e-03)	**4.04e-01 (8.95e-04)**

(b) $M = 3$

	replacement Mean (Std.)	reinitialization Mean (Std.)	reflection Mean (Std.)	r-reflection Mean (Std.)	resampling Mean (Std.)
DTLZ1	9.06e-01 (1.19e-01)	9.08e-01 (1.22e-01)	8.26e-01 (2.14e-01)	7.92e-01 (2.09e-01)	**9.46e-01 (1.08e-01)**
DTLZ2	**7.39e-01 (1.48e-03)**	7.32e-01 (3.17e-03)	7.30e-01 (3.33e-03)	7.33e-01 (3.35e-03)	7.37e-01 (1.71e-03)
DTLZ3	**2.67e-01 (2.37e-01)**	3.73e-02 (1.09e-01)	2.78e-02 (9.73e-02)	5.53e-03 (3.95e-02)	8.54e-02 (1.81e-01)
DTLZ4	7.27e-01 (3.84e-02)	**7.41e-01 (1.44e-03)**	6.94e-01 (9.76e-02)	7.33e-01 (2.79e-02)	7.00e-01 (9.21e-02)
DTLZ5	2.65e-01 (2.10e-05)	2.65e-01 (5.66e-04)	2.65e-01 (1.16e-03)	2.65e-01 (5.85e-04)	**2.65e-01 (5.15e-04)**
DTLZ6	**2.57e-01 (6.60e-03)**	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	7.18e-05 (5.13e-04)	9.19e-06 (4.88e-05)
DTLZ7	3.18e-01 (4.26e-02)	**3.39e-01 (4.42e-03)**	2.86e-01 (2.93e-02)	2.80e-01 (2.45e-02)	2.72e-01 (1.68e-02)
WFG1	**4.38e-01 (1.05e-02)**	3.46e-01 (6.23e-02)	3.92e-01 (3.13e-02)	3.94e-01 (1.38e-02)	4.15e-01 (2.20e-02)
WFG2	**1.20e+00 (6.19e-03)**	1.20e+00 (6.33e-03)	1.19e+00 (7.55e-03)	1.20e+00 (6.29e-03)	1.17e+00 (5.90e-02)
WFG3	8.41e-01 (6.36e-03)	8.40e-01 (3.54e-03)	8.38e-01 (4.04e-03)	8.39e-01 (4.09e-03)	**8.42e-01 (3.60e-03)**
WFG4	**6.61e-01 (7.31e-03)**	6.46e-01 (5.75e-03)	6.40e-01 (7.33e-03)	6.46e-01 (7.40e-03)	6.61e-01 (6.16e-03)
WFG5	**6.65e-01 (2.38e-03)**	5.10e-01 (1.81e-02)	6.37e-01 (6.29e-03)	6.46e-01 (5.14e-03)	6.31e-01 (7.11e-03)
WFG6	6.45e-01 (1.58e-02)	6.59e-01 (8.19e-03)	6.27e-01 (9.73e-03)	6.19e-01 (1.21e-02)	**6.62e-01 (6.27e-03)**
WFG7	**7.13e-01 (4.83e-03)**	6.86e-01 (3.97e-03)	6.84e-01 (7.23e-03)	6.89e-01 (6.81e-03)	7.03e-01 (5.36e-03)
WFG8	**5.84e-01 (1.20e-02)**	5.65e-01 (7.77e-03)	5.72e-01 (1.06e-02)	5.72e-01 (1.03e-02)	5.82e-01 (7.91e-03)
WFG9	6.17e-01 (7.11e-03)	6.44e-01 (1.13e-02)	6.37e-01 (1.46e-02)	6.41e-01 (1.74e-02)	**6.45e-01 (1.64e-02)**

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(c) $M = 4$

	replacement Mean (Std.)	reinitialization Mean (Std.)	reflection Mean (Std.)	r-reflection Mean (Std.)	resampling Mean (Std.)
DTLZ1	**1.15e+00 (2.69e-01)**	3.91e-01 (3.99e-01)	2.58e-01 (3.80e-01)	3.85e-01 (3.79e-01)	6.32e-01 (3.88e-01)
DTLZ2	**9.32e-01 (3.01e-03)**	9.18e-01 (1.06e-02)	9.16e-01 (1.22e-02)	9.21e-01 (7.12e-03)	9.29e-01 (2.17e-03)
DTLZ3	**3.19e-01 (3.43e-01)**	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)
DTLZ4	**9.27e-01 (2.72e-02)**	9.07e-01 (1.81e-02)	9.05e-01 (3.53e-02)	9.07e-01 (2.98e-02)	8.64e-01 (8.26e-02)
DTLZ5	2.10e-01 (1.03e-04)	**2.11e-01 (1.44e-03)**	2.10e-01 (1.29e-03)	2.10e-01 (9.64e-04)	2.10e-01 (1.46e-04)
DTLZ6	**2.03e-01 (7.53e-03)**	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)
DTLZ7	**3.50e-01 (1.32e-02)**	2.41e-01 (2.14e-02)	3.14e-01 (1.39e-02)	3.32e-01 (1.41e-02)	3.12e-01 (1.64e-02)
WFG1	**4.86e-01 (2.38e-02)**	1.82e-01 (2.08e-02)	3.25e-01 (8.31e-02)	3.20e-01 (5.30e-02)	3.52e-01 (6.00e-02)
WFG2	1.33e+00 (1.75e-02)	**1.36e+00 (1.33e-02)**	1.33e+00 (2.43e-02)	1.33e+00 (1.75e-02)	1.32e+00 (5.99e-02)
WFG3	9.18e-01 (1.58e-02)	9.34e-01 (1.09e-02)	9.28e-01 (1.19e-02)	9.25e-01 (1.18e-02)	**9.38e-01 (9.98e-03)**
WFG4	**7.65e-01 (1.69e-02)**	7.24e-01 (1.45e-02)	7.01e-01 (1.62e-02)	7.10e-01 (1.43e-02)	7.52e-01 (1.70e-02)
WFG5	**7.83e-01 (1.35e-02)**	5.86e-01 (2.71e-02)	6.82e-01 (2.53e-02)	7.04e-01 (2.32e-02)	7.18e-01 (3.20e-02)
WFG6	**8.13e-01 (1.09e-02)**	7.48e-01 (2.16e-02)	6.64e-01 (2.23e-02)	6.67e-01 (1.94e-02)	7.25e-01 (1.26e-02)
WFG7	**8.22e-01 (2.22e-02)**	7.97e-01 (1.64e-02)	7.88e-01 (1.80e-02)	7.97e-01 (1.81e-02)	8.17e-01 (1.74e-02)
WFG8	6.25e-01 (2.19e-02)	6.17e-01 (1.04e-02)	6.14e-01 (1.33e-02)	6.11e-01 (1.71e-02)	**6.39e-01 (1.73e-02)**
WFG9	**7.30e-01 (9.96e-03)**	7.12e-01 (1.20e-02)	7.06e-01 (1.30e-02)	7.06e-01 (1.34e-02)	7.10e-01 (1.34e-02)

(d) $M = 5$

	replacement Mean (Std.)	reinitialization Mean (Std.)	reflection Mean (Std.)	r-reflection Mean (Std.)	resampling Mean (Std.)
DTLZ1	**7.37e-01 (5.57e-01)**	2.80e-01 (3.47e-01)	4.11e-02 (8.77e-02)	4.19e-02 (1.06e-01)	3.93e-01 (3.43e-01)
DTLZ2	**1.14e+00 (1.61e-03)**	8.97e-01 (5.07e-02)	8.75e-01 (5.26e-02)	9.62e-01 (6.65e-02)	1.13e+00 (7.81e-03)
DTLZ3	**6.58e-02 (1.82e-01)**	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	1.84e-03 (1.32e-02)
DTLZ4	**1.09e+00 (3.12e-02)**	1.01e+00 (4.24e-02)	9.80e-01 (5.10e-02)	1.01e+00 (3.90e-02)	9.49e-01 (8.54e-02)
DTLZ5	1.56e-01 (4.15e-03)	1.70e-01 (1.46e-02)	1.85e-01 (5.52e-03)	1.77e-01 (1.29e-02)	**1.88e-01 (4.31e-03)**
DTLZ6	**1.48e-01 (4.79e-03)**	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)
DTLZ7	**3.23e-01 (2.78e-02)**	2.81e-02 (2.03e-02)	9.87e-02 (4.95e-02)	1.54e-01 (5.11e-02)	2.92e-01 (4.01e-02)
WFG1	**5.52e-01 (8.39e-02)**	1.83e-01 (3.51e-02)	3.41e-01 (1.07e-01)	2.89e-01 (4.54e-02)	3.00e-01 (5.03e-02)
WFG2	1.46e+00 (3.55e-02)	**1.52e+00 (1.51e-02)**	1.46e+00 (3.77e-02)	1.46e+00 (3.06e-02)	1.42e+00 (9.84e-02)
WFG3	9.59e-01 (1.69e-02)	**1.04e+00 (8.92e-03)**	1.02e+00 (1.61e-02)	1.01e+00 (2.00e-02)	1.03e+00 (6.67e-03)
WFG4	6.69e-01 (5.16e-02)	6.58e-01 (2.34e-02)	5.90e-01 (2.77e-02)	5.92e-01 (3.34e-02)	**6.86e-01 (3.55e-02)**
WFG5	**7.01e-01 (4.43e-02)**	5.11e-01 (3.38e-02)	5.91e-01 (4.44e-02)	6.10e-01 (4.13e-02)	5.65e-01 (4.25e-02)
WFG6	**9.89e-01 (7.93e-03)**	6.83e-01 (3.94e-02)	5.30e-01 (4.05e-02)	5.28e-01 (4.27e-02)	6.20e-01 (4.00e-02)
WFG7	6.95e-01 (5.40e-02)	**7.52e-01 (5.16e-02)**	6.39e-01 (3.21e-02)	6.53e-01 (3.94e-02)	6.69e-01 (2.98e-02)
WFG8	4.80e-01 (2.97e-02)	5.08e-01 (1.28e-02)	4.86e-01 (2.33e-02)	4.81e-01 (1.87e-02)	**5.67e-01 (3.95e-02)**
WFG9	**5.99e-01 (5.07e-02)**	5.50e-01 (4.19e-02)	5.23e-01 (2.88e-02)	5.39e-01 (3.66e-02)	5.40e-01 (4.06e-02)