A Decomposition-based Evolutionary Algorithm for Multi-modal Multi-objective Optimization

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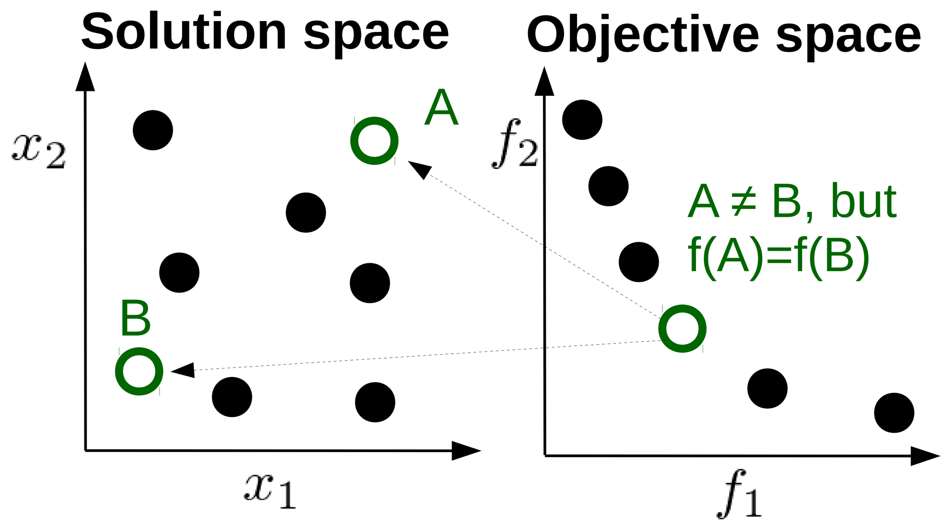
1. Multi-modal multi-objective optimization [Deb 08]

Locate as many as possible equivalent Pareto optimal solutions

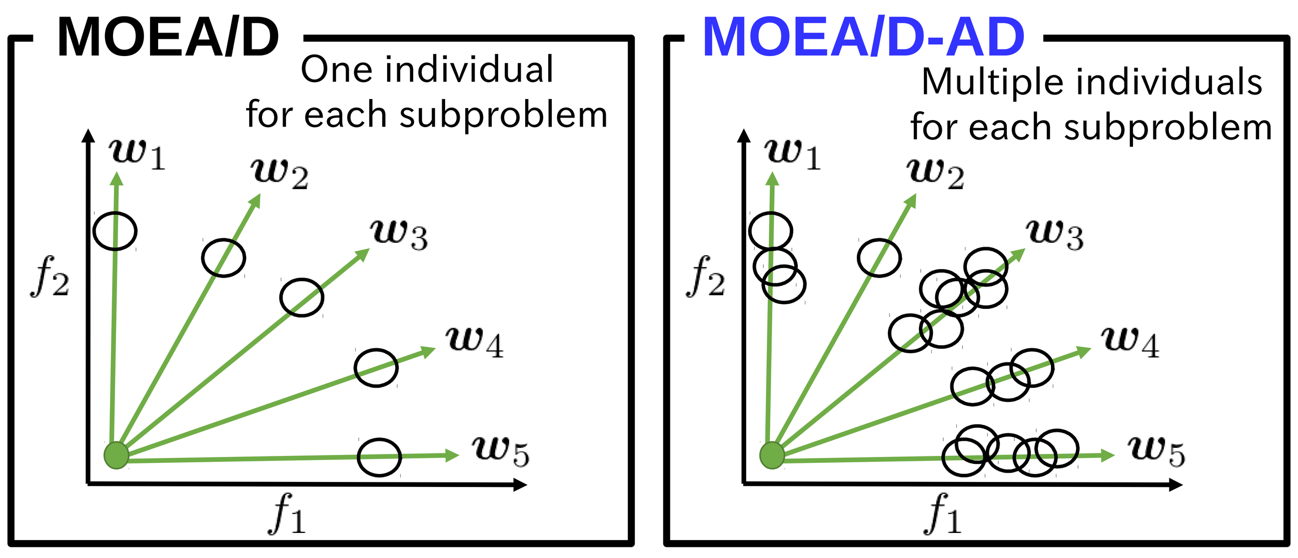


Multi-objective opt.
- Goal: approximate PF
- One of A and B is fine

Multi-modal multi-objective opt.
- Goal: approximate PS
- A and B are necessary

- E.g., Space mission design [Schütze 11] and rocket engine design [Kudo 11]
- Diverse nondominated solutions are helpful for decision-making [Hiroyasu 05]

2. Proposed: An MOEA/D with addition and deletion operators (MOEA/D-AD)

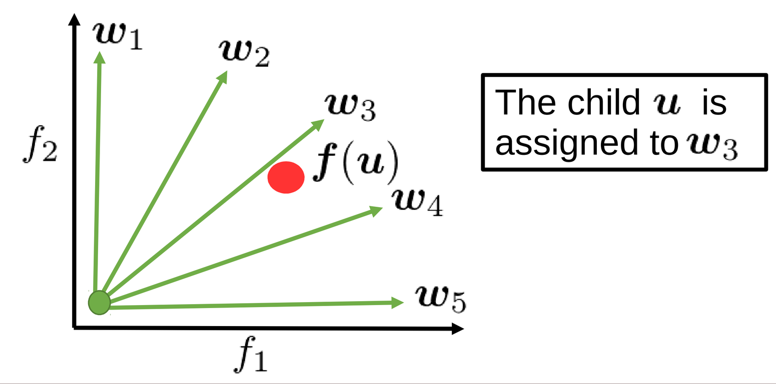


	Number of subproblems N	Population size μ
MOEA/D [Zhang 07]	constant	constant
MOEA/D-AD	constant	nonconstant

1 Initialize the population $\boldsymbol{P} = \{\boldsymbol{x}^1, ..., \boldsymbol{x}^\mu\}$ and weight vectors $\boldsymbol{W} = \{\boldsymbol{w}^1, ..., \boldsymbol{w}^N\}$;
2 **while** The termination criteria are not met **do**
3   Generate the child $\boldsymbol{u}$ by recombining two individuals randomly selected from $\boldsymbol{P}$;
4   Assign the child $\boldsymbol{u}$ to the closest subproblem in the objective space;
5   Perform the deletion operation;
6   Perform the addition operation;
7 **end**
8 **return** $\boldsymbol{P}$ with the size $N \leftarrow$ Apply a postprocessing solution-reducing procedure $(\boldsymbol{P})$;

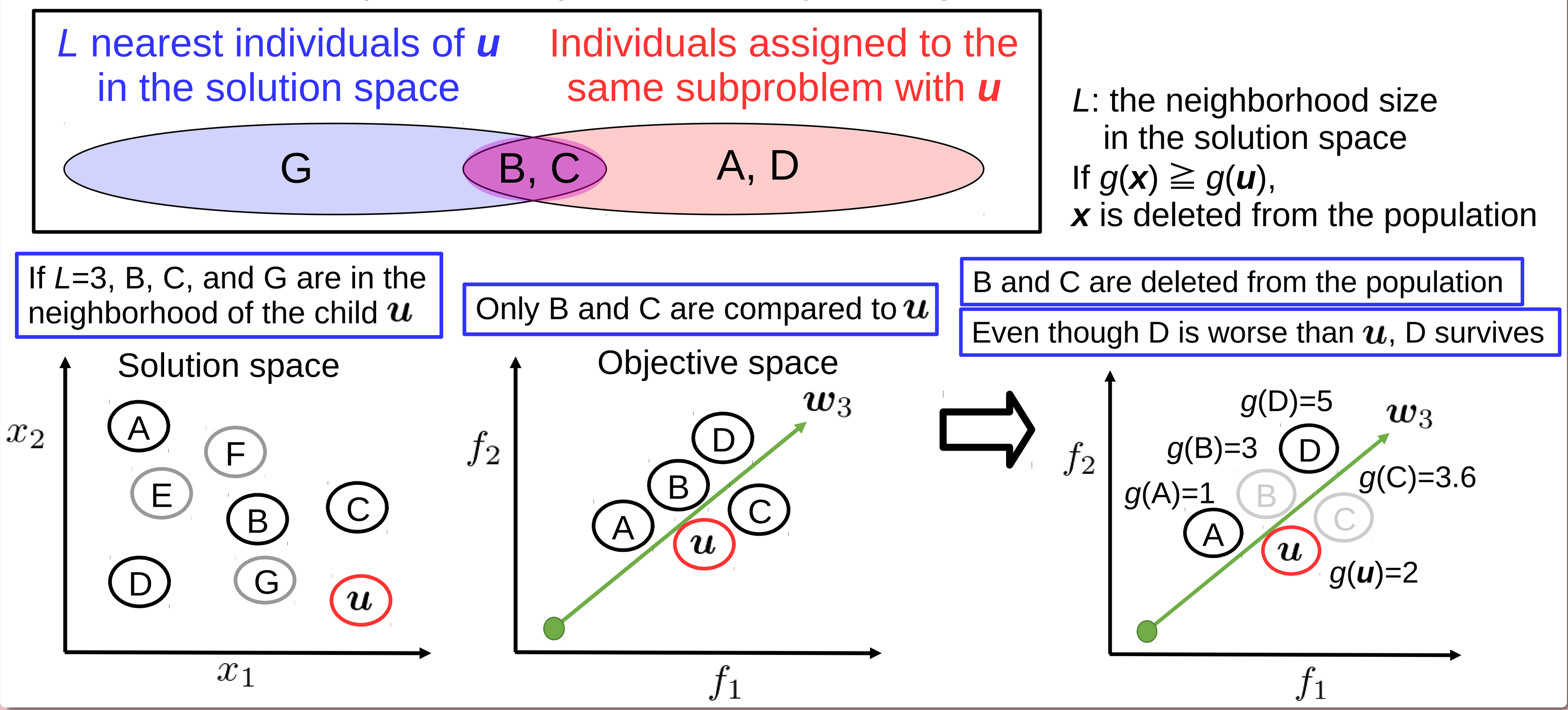
3. Assignment operation of MOEA/D-AD

- The same with MOEA/D-DU [Yuan 16]
- Calculate the perpendicular distance between $\boldsymbol{f}(\boldsymbol{u})$ and each weight $\boldsymbol{w}$
- Assign the child $\boldsymbol{u}$ to the subproblem having the minimum distance value



4. Deletion operation of MOEA/D-AD

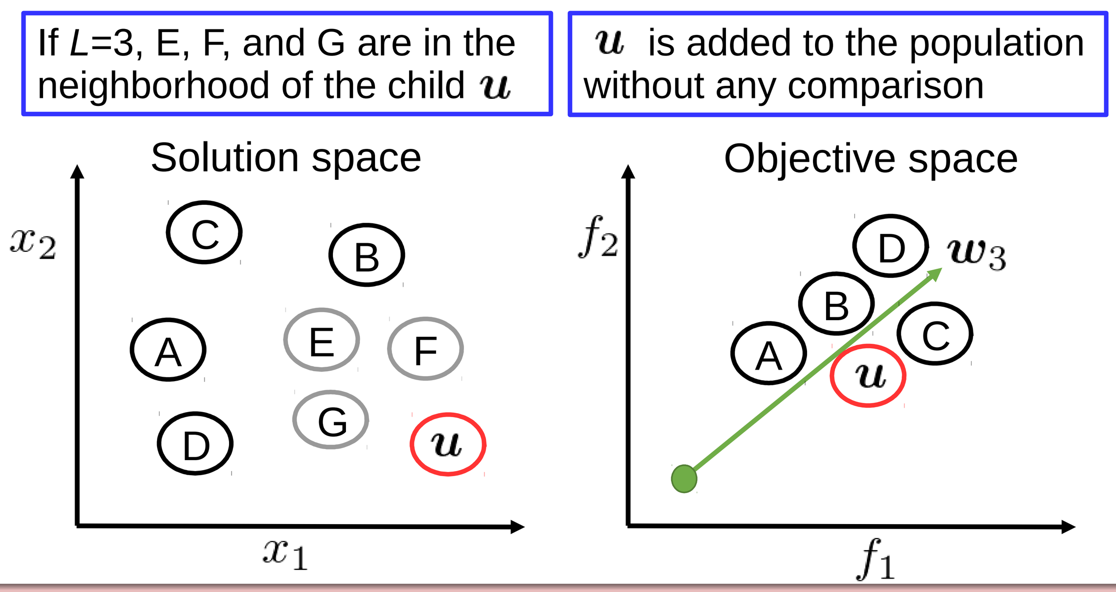
The deletion operation is performed only among...



L: the neighborhood size in the solution space
If $g(\boldsymbol{x}) \geqq g(\boldsymbol{u})$, $\boldsymbol{x}$ is deleted from the population

5. Addition operation of MOEA/D-AD

- Add the child $\boldsymbol{u}$ to the population $\boldsymbol{P}$
  1. If $\boldsymbol{u}$ is better than at least one individual in the deletion operation, or
  2. If $\boldsymbol{u}$ has no neighborhood individual in the solution space



6. Discussion: How to handle both objective and solution space diversity

- E.g., Omni-optimizer [Deb 08] uses the crowding distance in both the objective and solution spaces
- The objective space diversity is maintained using the uniformly distributed weight vectors
- The solution space diversity is maintained using the niching method with the relative distance

7. Experimental settings

- EMMO: Omni-optimizer [Deb 08], MO_Ring_PSO_SCD [Yue18]
- EMO: NSGA-II [Deb 02], MOEA/D [Zhang 07], MOEA/D-DU

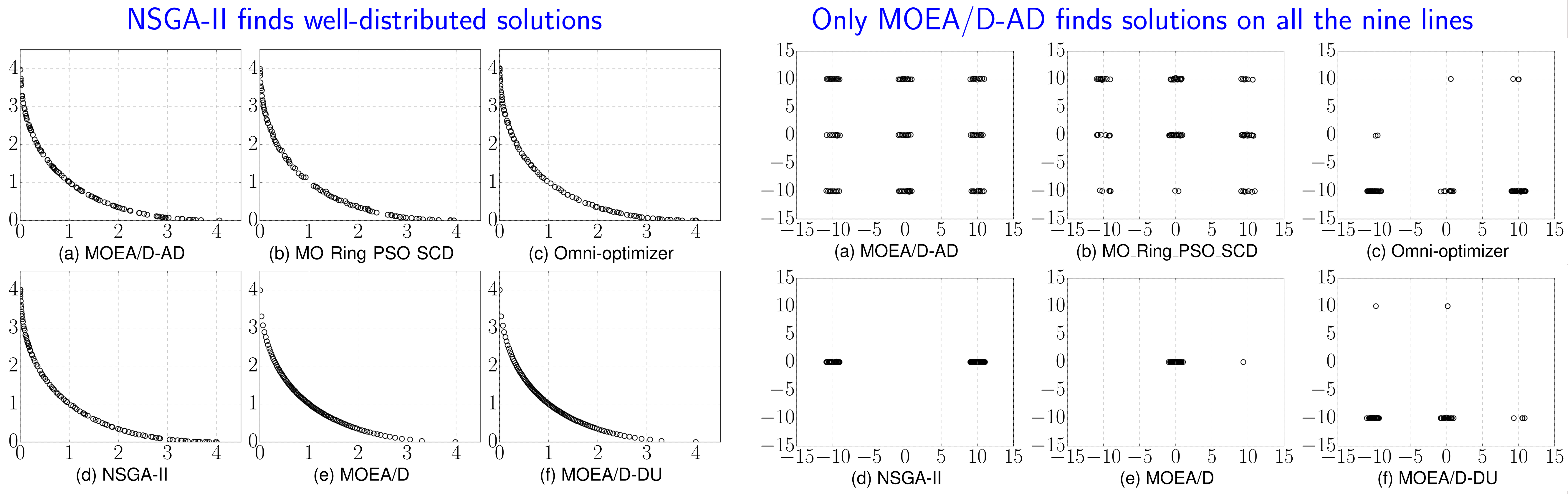
Test problems	# obj.	# var.	# PS sets	Disc. PS	PF
Two-On-One [Preuss 06]	2	2	2		convex
Omni-test [Deb 08]	2	5	360	✓	convex
SYM-PART1 [Rudolph 07]	2	2	9	✓	convex
SYM-PART2 [Rudolph 07]	2	2	9	✓	convex
SYM-PART3 [Rudolph 07]	2	2	9	✓	convex
SSUF1 [Liang 16]	2	2	2		convex
SSUF3 [Liang 16]	2	2	2	✓	convex

- Population size μ = Number of subproblems $N = 100$
  - Only 100 individuals selected by the postprocessing solution-reducing procedure were used for the performance evaluation of MOEA/D-AD
- SBX crossover and the polynomial mutation
- Tchebycheff scalarizing function [Zhang 07]
- Neighborhood size L in MOEA/D-AD: $L = \lfloor 0.1\mu \rfloor$
- Maximum number of evaluations $= 30\,000$
- Number of runs: $= 31$

8. Distribution of nondominated solutions on the SYM-PART1 problem

Objective space
NSGA-II finds well-distributed solutions

Solution space
Only MOEA/D-AD finds solutions on all the nine lines



Objective space: (a) MOEA/D-AD (b) MO_Ring_PSO_SCD (c) Omni-optimizer (d) NSGA-II (e) MOEA/D (f) MOEA/D-DU

Solution space: (a) MOEA/D-AD (b) MO_Ring_PSO_SCD (c) Omni-optimizer (d) NSGA-II (e) MOEA/D (f) MOEA/D-DU

9. Comparison on the seven two-objective test problems (the IGD and IGDX indicators)

Mean IGD values (objective space)
NSGA-II is the best multi-objective optimizer

	MOEA/D-AD	MO_Ring_PSO_SCD	Omni-optimizer	NSGA-II	MOEA/D	MOEA/D-DU
Two-On-One	0.0637 (5)	0.0606≈ (4)	*0.0489*+ (2)	0.0490+ (3)	**0.0450**+ (1)	0.0709− (6)
Omni-test	0.0755 (5)	0.1814− (6)	*0.0303*+ (2)	**0.0297**+ (1)	0.0517+ (4)	0.0458+ (3)
SYM-PART1	0.0302 (4)	0.0283+ (3)	*0.0236*+ (2)	**0.0210**+ (1)	0.0467− (5)	0.0478− (6)
SYM-PART2	0.0305 (3)	0.0312≈ (4)	*0.0284*+ (2)	**0.0229**+ (1)	0.0466− (5)	0.0474− (6)
SYM-PART3	*0.0307* (2)	0.0323− (3)	0.0343− (4)	**0.0228**+ (1)	0.0455− (5)	0.0470− (6)
SSUF1	0.0075 (6)	0.0065+ (5)	0.0060+ (4)	*0.0055*+ (2)	0.0055+ (3)	**0.0042**+ (1)
SSUF3	0.0190 (5)	0.0106+ (3)	0.0170+ (4)	**0.0073**+ (1)	0.0629− (6)	*0.0082*+ (2)

∗The IGD metric [Zitzler 03] evaluates how well the population approximates the Pareto front in the objective space

Mean IGDX values (solution space)
MOEA/D-AD is the best multi-modal multi-objective optimizer

	MOEA/D-AD	MO_Ring_PSO_SCD	Omni-optimizer	NSGA-II	MOEA/D	MOEA/D-DU
Two-On-One	**0.0353** (1)	*0.0369*− (2)	0.0383− (3)	0.1480− (4)	0.2805− (6)	0.2067− (5)
Omni-test	**1.3894** (1)	2.2227− (3)	*2.0337*− (2)	2.5664− (4)	4.3950− (6)	2.9251− (5)
SYM-PART1	**0.0686** (1)	*0.1482*− (2)	3.8027− (3)	7.9287− (5)	9.1551− (6)	5.0426− (4)
SYM-PART2	**0.0783** (1)	*0.1610*− (2)	1.0863− (3)	5.3711− (5)	9.4834− (6)	5.1610− (4)
SYM-PART3	**0.1480** (1)	*0.4909*− (2)	1.3620− (3)	5.8410− (5)	7.3969− (6)	4.6767− (4)
SSUF1	**0.0761** (1)	*0.0860*− (2)	0.0899− (3)	0.1323− (5)	0.2443− (6)	0.1143− (4)
SSUF3	*0.0302* (2)	**0.0198**+ (1)	0.0541− (3)	0.0710− (5)	0.3083− (6)	0.0599− (4)

∗The IGDX metric [Zhou 09] evaluates how well the population approximates the Pareto-optimal solution sets in the solution space

∗ Source code of MOEA/D-AD (jMetal) can be downloaded from the first author's website (https://ryojitanabe.github.io/)