# Supplementary materials for "Reevaluating Exponential Crossover in Differential Evolution"
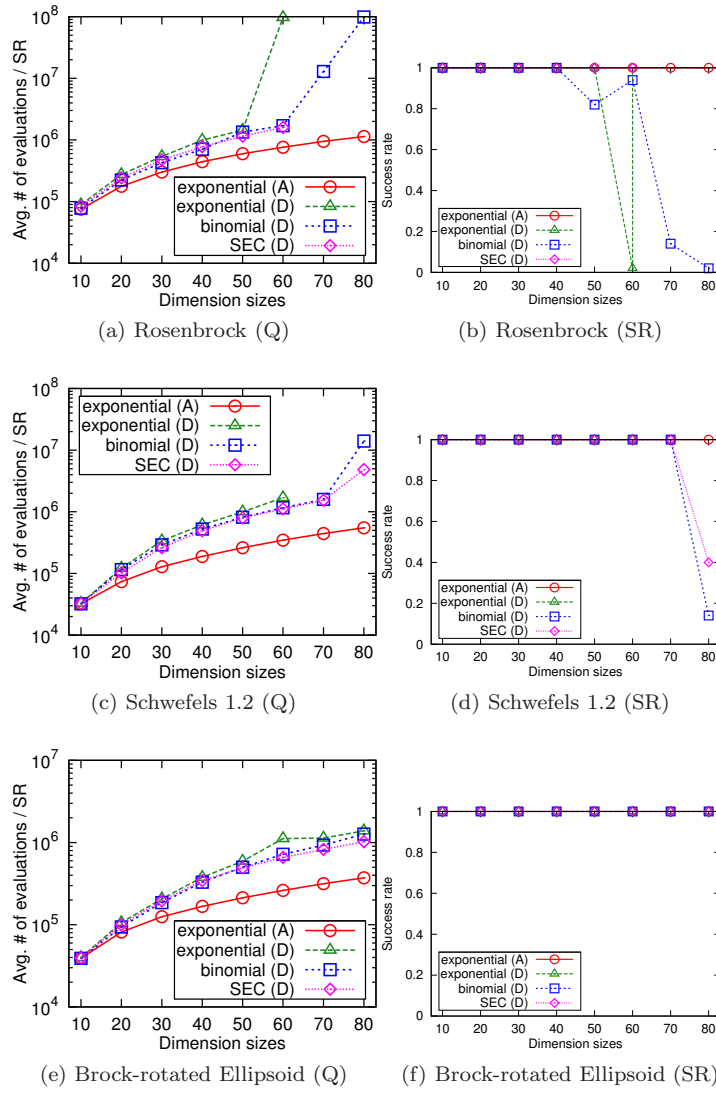
Ryoji Tanabe and Alex Fukunaga

31/Mar/2014

Figure 1: Evaluation of exponential crossover (**standard DE**). The horizontal axis represents the dimensionality $D$, and the vertical axis represents: (**left**) the mean fitness evaluations (for successful runs) divided by success rate, (**right**) the success rate which is the number of "successes" divided by 50.

Figure 2: Evaluation of exponential crossover (**jDE**). The horizontal axis represents the dimensionality $D$, and the vertical axis represents: (**left**) the mean fitness evaluations (for successful runs) divided by success rate, (**right**) the success rate which is the number of "successes" divided by 50.
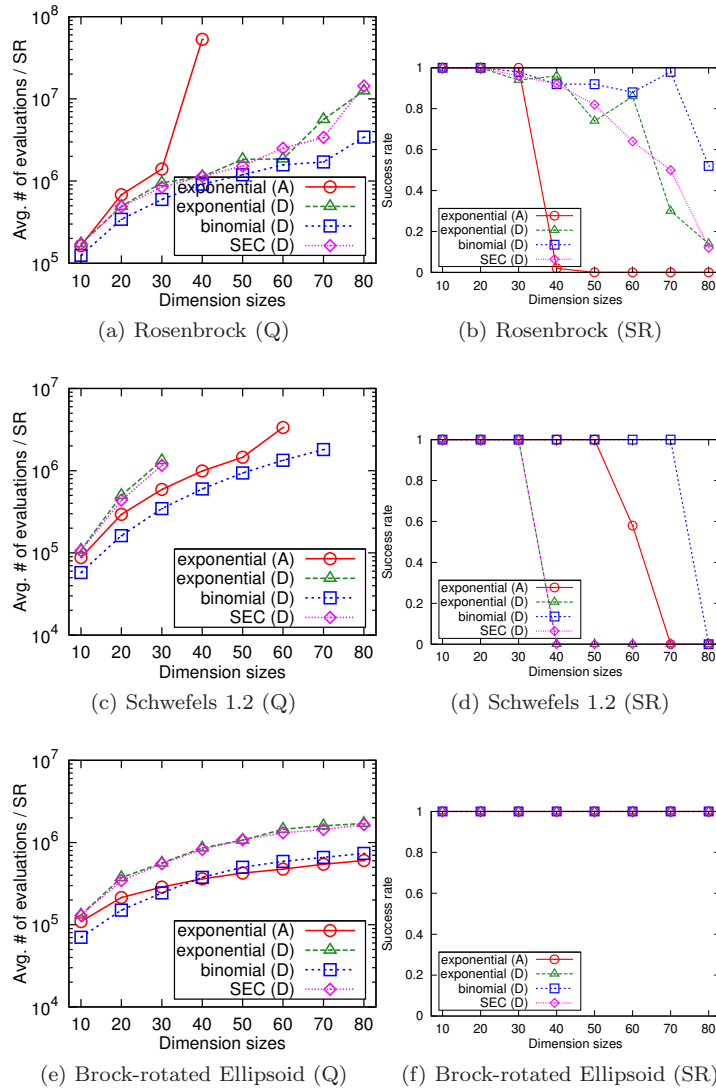
Figure 3: Evaluation of exponential crossover (**JADE**). The horizontal axis represents the dimensionality $D$, and the vertical axis represents: (**left**) the mean fitness evaluations (for successful runs) divided by success rate, (**right**) the success rate which is the number of "successes" divided by 50.
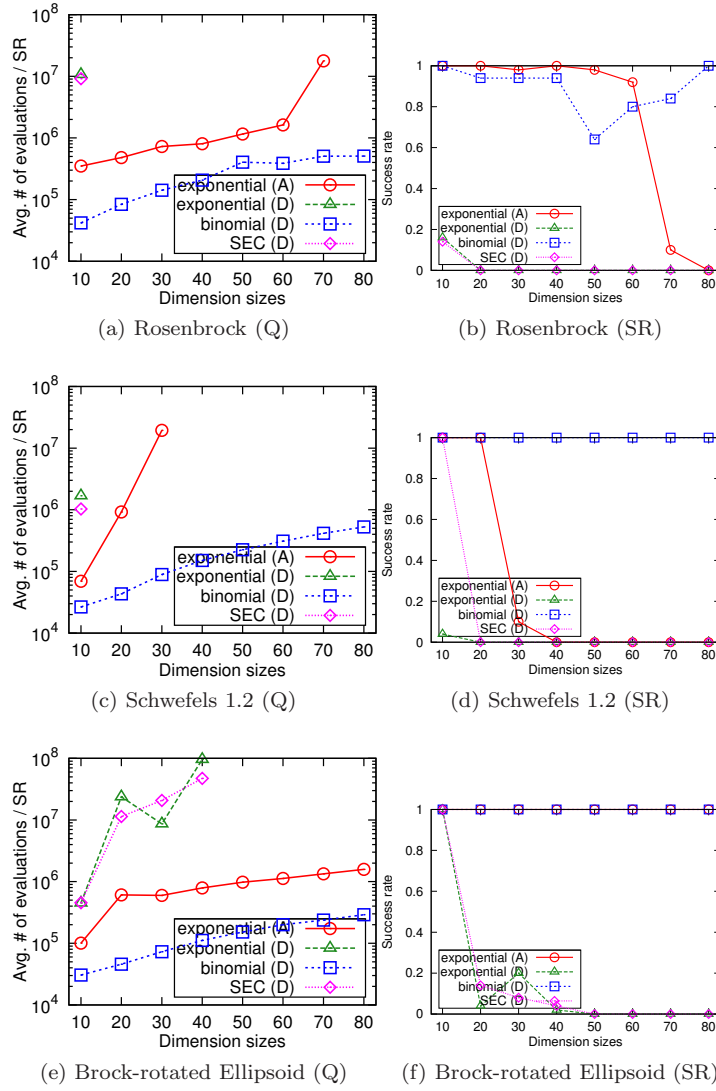
(a) Rosenbrock (Q)

(b) Rosenbrock (SR)

(c) Schwefels 1.2 (Q)

(d) Schwefels 1.2 (SR)

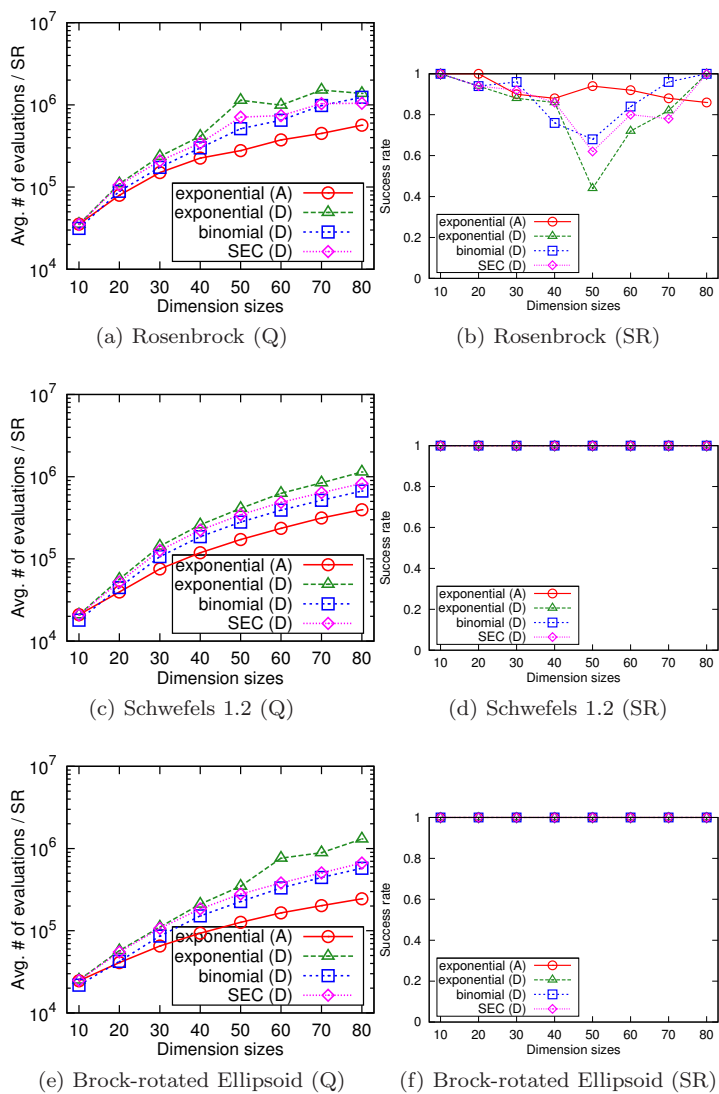(e) Brock-rotated Ellipsoid (Q)

(f) Brock-rotated Ellipsoid (SR)

Figure 4: Evaluation of exponential crossover (**SHADE**). The horizontal axis represents the dimensionality $D$, and the vertical axis represents: (**left**) the mean fitness evaluations (for successful runs) divided by success rate, (**right**) the success rate which is the number of "successes" divided by 50.

Table 1: Comparison of shuffle exponential crossover with binomial and exponential crossover in **standard DE** on the CEC2014 benchmark functions [1] (10 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 100,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_2$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_3$ | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_4$ | **6.48e+00±1.25e+01** | 9.46e+00±1.43e+01≈ | 7.58e+00±1.37e+01≈ |
| $F_5$ | 1.97e+01±1.16e+00 | 1.96e+01±3.00e+00+ | **1.91e+01±4.68e+00**+ |
| $F_6$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_7$ | 3.75e-02±2.64e-02 | 3.70e-02±4.61e-02≈ | **2.91e-02±2.14e-02**≈ |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | 1.14e+01±7.39e+00− |
| $F_9$ | **1.18e+01±2.25e+00** | 1.18e+01±2.19e+00≈ | 1.42e+01±8.25e+00≈ |
| $F_{10}$ | **4.19e+00±2.19e+00** | 4.58e+00±2.23e+00≈ | 5.19e+02±3.48e+02− |
| $F_{11}$ | **5.54e+02±1.30e+02** | 5.98e+02±1.12e+02≈ | 8.66e+02±3.03e+02− |
| $F_{12}$ | 4.71e-01±9.08e-02 | **4.67e-01±8.00e-02**≈ | 6.55e-01±2.12e-01− |
| $F_{13}$ | 1.25e-01±3.07e-02 | 1.26e-01±3.26e-02≈ | **1.22e-01±2.64e-02**≈ |
| $F_{14}$ | 1.43e-01±3.17e-02 | **1.34e-01±3.54e-02**≈ | 1.51e-01±3.02e-02≈ |
| $F_{15}$ | 1.36e+00±2.12e-01 | **1.32e+00±1.79e-01**≈ | 1.77e+00±4.66e-01− |
| $F_{16}$ | **2.31e+00±2.12e-01** | 2.33e+00±2.22e-01≈ | 2.51e+00±3.02e-01− |
| $F_{17}$ | 3.86e-01±2.49e-01 | **3.58e-01±2.24e-01**+ | 4.88e-01±1.53e+00− |
| $F_{18}$ | 1.56e-01±1.47e-01 | 1.62e-01±2.92e-01− | **1.43e-01±1.30e-01**≈ |
| $F_{19}$ | **2.94e-01±1.03e-01** | 3.21e-01±9.55e-02≈ | 3.26e-01±9.03e-02− |
| $F_{20}$ | **4.77e-02±9.93e-02** | 7.42e-02±1.12e-01− | 8.72e-02±1.44e-01≈ |
| $F_{21}$ | **2.27e-01±2.55e-01** | 3.47e-01±2.69e-01− | 3.09e-01±2.55e-01≈ |
| $F_{22}$ | **1.16e-01±1.11e-01** | 1.97e-01±1.77e-01− | 2.34e-01±2.11e-01− |
| $F_{23}$ | **3.23e+02±4.61e+01** | 3.29e+02±0.00e+00≈ | 3.29e+02±0.00e+00≈ |
| $F_{24}$ | 1.15e+02±6.08e+00 | 1.17e+02±6.54e+00− | **1.13e+02±7.86e+00**≈ |
| $F_{25}$ | 1.47e+02±4.11e+01 | 1.44e+02±3.42e+01≈ | **1.42e+02±4.17e+01**≈ |
| $F_{26}$ | 1.00e+02±2.25e-02 | 1.00e+02±3.57e-02≈ | **1.00e+02±2.57e-02**≈ |
| $F_{27}$ | 6.06e+01±1.38e+02 | 5.07e+01±1.25e+02≈ | **4.68e+01±1.25e+02**+ |
| $F_{28}$ | 3.66e+02±1.72e+01 | **3.63e+02±5.93e+00**≈ | 3.71e+02±2.83e+01≈ |
| $F_{29}$ | **2.03e+02±4.00e+01** | 2.04e+02±3.55e+01≈ | 2.11e+02±3.11e+01− |
| $F_{30}$ | **4.62e+02±1.28e+00** | 4.63e+02±7.70e-01− | 4.64e+02±5.22e+00− |
| vs. shuffle exponential     + (better) | | 2 | 2 |
| (Wilcoxon rank-sum)   − (worse) | | 6 | 11 |
| $p < 0.05$    ≈ (no sig.) | | 22 | 17 |

Table 2: Comparison of shuffle exponential crossover with binomial and exponential crossover in **standard DE** on the CEC2014 benchmark functions [1] (30 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 300,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 6.74e+04±6.78e+04 | 1.05e+05±8.05e+04− | **3.29e+04±2.47e+04+** |
| $F_2$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_3$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_4$ | **9.38e-02±1.01e-01** | 4.79e-01±3.63e-01− | 2.06e-01±1.35e-01− |
| $F_5$ | 2.05e+01±3.99e-02 | **2.05e+01±3.85e-02≈** | 2.07e+01±1.87e-01− |
| $F_6$ | 5.36e-01±7.07e-01 | 3.85e-01±7.94e-01≈ | **3.74e-01±5.56e-01+** |
| $F_7$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | 1.88e+01±5.59e+00− |
| $F_9$ | 8.81e+01±9.61e+00 | 9.14e+01±9.01e+00≈ | **3.60e+01±2.29e+01+** |
| $F_{10}$ | **2.30e+01±4.92e+00** | 2.66e+01±4.31e+00− | 1.16e+03±1.43e+03− |
| $F_{11}$ | **3.83e+03±2.45e+02** | 3.83e+03±2.11e+02≈ | 3.89e+03±1.70e+03− |
| $F_{12}$ | **7.04e-01±1.02e-01** | 7.35e-01±7.55e-02≈ | 7.54e-01±6.35e-01− |
| $F_{13}$ | 2.89e-01±8.40e-02 | 3.15e-01±7.03e-02≈ | **2.77e-01±7.15e-02≈** |
| $F_{14}$ | **2.54e-01±3.20e-02** | 2.54e-01±2.63e-02≈ | 2.59e-01±3.63e-02≈ |
| $F_{15}$ | 9.73e+00±8.17e-01 | 9.60e+00±8.70e-01≈ | **6.64e+00±3.33e+00+** |
| $F_{16}$ | **1.09e+01±3.29e-01** | 1.09e+01±2.66e-01≈ | 1.09e+01±1.34e+00≈ |
| $F_{17}$ | 1.86e+02±1.71e+02 | 1.83e+02±1.21e+02≈ | **1.25e+02±1.08e+02≈** |
| $F_{18}$ | 1.00e+01±4.63e+00 | 1.12e+01±4.06e+00≈ | **7.78e+00±3.52e+00+** |
| $F_{19}$ | 3.32e+00±1.09e+00 | 3.40e+00±7.08e-01≈ | **3.08e+00±7.66e-01≈** |
| $F_{20}$ | 9.98e+00±6.40e+00 | 9.39e+00±3.80e+00≈ | **7.58e+00±4.44e+00+** |
| $F_{21}$ | 9.29e+01±1.02e+02 | 7.98e+01±9.69e+01≈ | **6.33e+01±8.86e+01+** |
| $F_{22}$ | 3.32e+01±2.42e+01 | 3.65e+01±3.04e+01≈ | **2.81e+01±5.03e+00+** |
| $F_{23}$ | **3.15e+02±0.00e+00** | **3.15e+02±0.00e+00≈** | 3.15e+02±0.00e+00≈ |
| $F_{24}$ | 2.16e+02±9.54e+00 | **2.15e+02±1.00e+01≈** | 2.18e+02±8.97e+00≈ |
| $F_{25}$ | 2.03e+02±1.38e-01 | 2.03e+02±2.27e-01− | **2.03e+02±1.51e-01≈** |
| $F_{26}$ | 1.00e+02±7.29e-02 | 1.00e+02±7.76e-02≈ | **1.00e+02±7.07e-02≈** |
| $F_{27}$ | 3.64e+02±4.85e+01 | 3.64e+02±4.59e+01≈ | **3.47e+02±4.78e+01+** |
| $F_{28}$ | 7.90e+02±2.06e+01 | **7.85e+02±3.20e+01+** | 8.00e+02±2.34e+01− |
| $F_{29}$ | 6.52e+02±1.56e+02 | 6.63e+02±1.65e+02≈ | **5.72e+02±2.45e+02+** |
| $F_{30}$ | 5.92e+02±1.72e+02 | 6.27e+02±2.28e+02≈ | **5.25e+02±1.78e+02+** |
| vs. shuffle exponential   + (better) | | 1 | 11 |
| (Wilcoxon rank-sum)   − (worse) | | 4 | 7 |
| $p < 0.05$   ≈ (no sig.) | | 25 | 12 |

Table 3: Comparison of shuffle exponential crossover with binomial and exponential crossover in **standard DE** on the CEC2014 benchmark functions [1] (50 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 500,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 5.62e+05±2.49e+05 | 6.57e+05±3.11e+05≈ | **3.85e+05±1.78e+05+** |
| $F_2$ | **9.65e+00±1.60e+01** | 6.73e+02±8.56e+02− | 2.08e+02±6.43e+02≈ |
| $F_3$ | **1.30e-05±1.14e-05** | 3.12e-04±2.65e-04− | 1.93e-01±3.55e-01− |
| $F_4$ | 7.50e+01±3.26e+01 | 8.40e+01±2.52e+01≈ | **7.06e+01±2.99e+01≈** |
| $F_5$ | **2.06e+01±3.03e-02** | 2.06e+01±3.01e-02≈ | 2.08e+01±1.27e-01− |
| $F_6$ | 2.40e+00±1.93e+00 | 3.05e+00±2.26e+00≈ | **1.50e+00±1.67e+00+** |
| $F_7$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | 1.45e-04±1.04e-03≈ |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | 3.46e+01±7.07e+00− |
| $F_9$ | 1.97e+02±1.33e+01 | 2.00e+02±1.14e+01≈ | **5.85e+01±1.45e+01+** |
| $F_{10}$ | **3.49e+01±5.71e+00** | 4.28e+01±6.64e+00− | 1.60e+03±1.22e+03− |
| $F_{11}$ | 7.61e+03±2.96e+02 | 7.69e+03±2.68e+02≈ | **6.98e+03±3.04e+03+** |
| $F_{12}$ | 8.13e-01±8.02e-02 | 8.10e-01±7.04e-02≈ | **5.46e-01±4.20e-01+** |
| $F_{13}$ | 4.30e-01±8.04e-02 | 4.42e-01±7.68e-02≈ | **3.96e-01±7.35e-02+** |
| $F_{14}$ | **2.75e-01±3.65e-02** | 2.81e-01±3.34e-02≈ | 3.17e-01±6.58e-02− |
| $F_{15}$ | 2.14e+01±1.37e+00 | 2.10e+01±1.51e+00≈ | **9.11e+00±4.66e+00+** |
| $F_{16}$ | 1.96e+01±3.18e-01 | 1.97e+01±3.00e-01≈ | **1.93e+01±1.12e+00+** |
| $F_{17}$ | 1.46e+04±9.23e+03 | 1.65e+04±1.51e+04≈ | **1.25e+04±6.44e+03≈** |
| $F_{18}$ | 4.40e+01±1.87e+01 | 4.93e+01±1.80e+01≈ | **2.35e+01±8.63e+00+** |
| $F_{19}$ | **8.15e+00±2.06e+00** | 8.61e+00±1.97e+00≈ | 9.12e+00±2.35e+00− |
| $F_{20}$ | **4.55e+01±2.89e+01** | 5.83e+01±2.43e+01− | 4.90e+01±3.37e+01≈ |
| $F_{21}$ | 1.02e+03±8.02e+02 | 1.18e+03±1.19e+03≈ | **7.16e+02±4.44e+02+** |
| $F_{22}$ | **5.09e+02±2.65e+02** | 5.64e+02±2.10e+02≈ | 5.91e+02±3.61e+02≈ |
| $F_{23}$ | **3.44e+02±0.00e+00** | **3.44e+02±0.00e+00≈** | **3.44e+02±0.00e+00≈** |
| $F_{24}$ | **2.62e+02±3.65e+00** | 2.63e+02±3.76e+00≈ | 2.70e+02±2.36e+00− |
| $F_{25}$ | 2.06e+02±6.71e-01 | 2.06e+02±5.72e-01≈ | **2.05e+02±4.10e-01+** |
| $F_{26}$ | 1.00e+02±4.37e-02 | 1.00e+02±1.01e-01+ | **1.00e+02±7.98e-02+** |
| $F_{27}$ | **3.56e+02±3.49e+01** | 3.74e+02±2.75e+01− | 3.69e+02±3.80e+01− |
| $F_{28}$ | **1.08e+03±5.54e+01** | 1.10e+03±4.72e+01− | 1.08e+03±3.65e+01≈ |
| $F_{29}$ | **8.87e+02±6.75e+01** | 1.15e+03±2.49e+02− | 9.13e+02±2.15e+02≈ |
| $F_{30}$ | **7.93e+03±9.66e+01** | 7.98e+03±1.27e+02− | 8.27e+03±3.22e+02− |
| vs. shuffle exponential    + (better) | | 1 | 12 |
| (Wilcoxon rank-sum)    − (worse) | | 8 | 9 |
| $p < 0.05$    ≈ (no sig.) | | 21 | 9 |

Table 4: Comparison of shuffle exponential crossover with binomial and exponential crossover in **jDE** on the CEC2014 benchmark functions [1] (10 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 100,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 1.45e-05±2.46e-05 | 1.09e-03±4.60e-03≈ | **0.00e+00±0.00e+00+** |
| $F_2$ | 9.28e-07±1.99e-06 | 3.21e-05±5.32e-05− | **0.00e+00±0.00e+00+** |
| $F_3$ | 1.70e-07±5.00e-07 | 2.54e-09±7.04e-09+ | **0.00e+00±0.00e+00+** |
| $F_4$ | 3.93e+00±1.04e+01 | **1.29e+00±5.01e+00≈** | 1.42e+01±1.67e+01≈ |
| $F_5$ | 1.86e+01±3.49e+00 | **1.81e+01±3.60e+00≈** | 1.98e+01±1.30e+00− |
| $F_6$ | 1.52e-01±1.74e-01 | 1.07e-02±2.69e-02+ | **3.25e-06±1.70e-05+** |
| $F_7$ | 2.16e-02±1.11e-02 | 1.90e-02±1.09e-02≈ | **1.89e-02±1.38e-02≈** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 5.67e+00±1.21e+00 | **5.62e+00±1.23e+00≈** | 5.99e+00±1.25e+00≈ |
| $F_{10}$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_{11}$ | 2.67e+02±8.70e+01 | **2.48e+02±1.09e+02≈** | 2.96e+02±9.12e+01≈ |
| $F_{12}$ | **3.31e-01±6.11e-02** | 3.37e-01±6.28e-02≈ | 3.87e-01±6.99e-02− |
| $F_{13}$ | 1.64e-01±2.62e-02 | 1.61e-01±2.96e-02≈ | **1.42e-01±2.83e-02+** |
| $F_{14}$ | 1.60e-01±3.35e-02 | 1.66e-01±3.55e-02≈ | **1.56e-01±4.28e-02≈** |
| $F_{15}$ | 9.40e-01±1.66e-01 | **8.92e-01±1.52e-01≈** | 9.94e-01±1.78e-01≈ |
| $F_{16}$ | **2.08e+00±2.70e-01** | 2.08e+00±2.50e-01≈ | 2.12e+00±2.66e-01≈ |
| $F_{17}$ | 4.81e+01±2.31e+01 | 4.87e+01±2.80e+01≈ | **1.15e+01±1.03e+01+** |
| $F_{18}$ | 3.22e+00±1.04e+00 | 4.16e+00±1.32e+00− | **1.37e+00±6.87e-01+** |
| $F_{19}$ | 3.49e-01±1.09e-01 | 3.78e-01±1.18e-01≈ | **3.32e-01±1.05e-01≈** |
| $F_{20}$ | 4.80e-01±1.92e-01 | 1.07e+00±4.40e-01− | **2.06e-01±1.07e-01+** |
| $F_{21}$ | 3.88e+00±2.69e+00 | 1.04e+01±7.82e+00− | **4.99e-01±3.93e-01+** |
| $F_{22}$ | 3.58e-01±4.19e-01 | 7.02e-01±5.70e-01− | **1.72e-01±9.86e-02+** |
| $F_{23}$ | **3.29e+02±0.00e+00** | **3.29e+02±0.00e+00≈** | 3.29e+02±0.00e+00≈ |
| $F_{24}$ | 1.13e+02±2.19e+00 | 1.13e+02±1.89e+00≈ | 1.13e+02±2.29e+00≈ |
| $F_{25}$ | **1.27e+02±1.44e+01** | 1.31e+02±2.18e+01≈ | 1.27e+02±2.82e+01− |
| $F_{26}$ | 1.00e+02±2.62e-02 | 1.00e+02±2.81e-02≈ | **1.00e+02±3.17e-02+** |
| $F_{27}$ | **4.68e+01±1.12e+02** | 6.18e+01±1.21e+02≈ | 7.94e+01±1.42e+02− |
| $F_{28}$ | **3.57e+02±1.76e+00** | 3.58e+02±4.62e+00− | 3.61e+02±6.03e+00≈ |
| $F_{29}$ | 2.23e+02±7.81e-01 | 2.24e+02±6.59e-01≈ | **2.22e+02±5.15e-01+** |
| $F_{30}$ | 4.73e+02±1.17e+01 | 4.77e+02±1.39e+01− | **4.66e+02±9.19e+00+** |
| vs. shuffle exponential     + (better) | | 2 | 13 |
| (Wilcoxon rank-sum)   − (worse) | | 7 | 4 |
| $p < 0.05$   ≈ (no sig.) | | 21 | 13 |

Table 5: Comparison of shuffle exponential crossover with binomial and exponential crossover in **jDE** on the CEC2014 benchmark functions [1] (30 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 300,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 3.17e+05±1.91e+05 | 3.75e+05±2.30e+05≈ | **6.54e+04±4.95e+04+** |
| $F_2$ | **0.00e+00±0.00e+00** | 3.38e-09±1.58e-08− | **0.00e+00±0.00e+00≈** |
| $F_3$ | 7.75e-06±1.25e-05 | 7.25e-05±1.04e-04− | **0.00e+00±0.00e+00+** |
| $F_4$ | 5.74e+01±2.87e+01 | 5.35e+01±2.93e+01≈ | **5.08e+00±1.60e+01+** |
| $F_5$ | 2.03e+01±3.00e-02 | **2.03e+01±3.42e-02≈** | 2.03e+01±4.29e-02− |
| $F_6$ | 1.36e+01±9.86e-01 | 1.31e+01±1.11e+00+ | **3.84e+00±4.37e+00+** |
| $F_7$ | **0.00e+00±0.00e+00** | 3.67e-10±2.62e-09≈ | **0.00e+00±0.00e+00≈** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 5.05e+01±6.47e+00 | 5.09e+01±6.67e+00≈ | **4.25e+01±6.64e+00+** |
| $F_{10}$ | **4.08e-04±2.92e-03** | 8.16e-04±4.08e-03≈ | 2.04e-03±6.25e-03≈ |
| $F_{11}$ | **2.25e+03±2.36e+02** | 2.26e+03±2.71e+02≈ | 2.43e+03±2.68e+02− |
| $F_{12}$ | **3.87e-01±4.13e-02** | 3.91e-01±4.20e-02≈ | 4.63e-01±5.13e-02− |
| $F_{13}$ | 3.46e-01±3.25e-02 | 3.47e-01±3.93e-02≈ | **2.97e-01±3.60e-02+** |
| $F_{14}$ | **2.64e-01±2.81e-02** | 2.66e-01±3.00e-02≈ | 2.76e-01±3.22e-02− |
| $F_{15}$ | 6.97e+00±5.98e-01 | 6.77e+00±8.12e-01≈ | **5.66e+00±7.18e-01+** |
| $F_{16}$ | **9.75e+00±3.88e-01** | 9.79e+00±2.69e-01≈ | 9.89e+00±3.65e-01≈ |
| $F_{17}$ | 5.30e+04±5.19e+04 | 5.65e+04±4.53e+04≈ | **1.61e+03±1.46e+03+** |
| $F_{18}$ | 6.66e+02±6.64e+02 | 7.03e+02±6.86e+02≈ | **1.78e+01±1.19e+01+** |
| $F_{19}$ | 6.94e+00±5.47e-01 | 6.97e+00±6.14e-01≈ | **4.49e+00±6.49e-01+** |
| $F_{20}$ | 5.40e+01±1.40e+01 | 5.12e+01±1.31e+01≈ | **1.16e+01±3.15e+00+** |
| $F_{21}$ | 3.69e+03±2.02e+03 | 3.20e+03±1.71e+03≈ | **3.03e+02±1.36e+02+** |
| $F_{22}$ | 2.31e+02±6.96e+01 | 2.13e+02±8.10e+01≈ | **9.91e+01±5.98e+01+** |
| $F_{23}$ | **3.15e+02±0.00e+00** | **3.15e+02±0.00e+00≈** | 3.15e+02±0.00e+00≈ |
| $F_{24}$ | 2.26e+02±6.35e-01 | 2.26e+02±6.79e-01≈ | **2.24e+02±1.46e+00+** |
| $F_{25}$ | 2.04e+02±5.70e-01 | 2.04e+02±6.14e-01≈ | **2.03e+02±6.34e-01+** |
| $F_{26}$ | 1.00e+02±3.90e-02 | 1.00e+02±4.51e-02≈ | **1.00e+02±3.67e-02+** |
| $F_{27}$ | 4.08e+02±5.20e+00 | 4.11e+02±8.88e+00− | **3.56e+02±4.86e+01+** |
| $F_{28}$ | 8.53e+02±1.67e+01 | 8.53e+02±1.99e+01≈ | **7.88e+02±3.10e+01+** |
| $F_{29}$ | 1.30e+03±9.66e+01 | 1.30e+03±1.06e+02≈ | **8.07e+02±7.49e+01+** |
| $F_{30}$ | 1.78e+03±4.68e+02 | 1.61e+03±4.34e+02+ | **1.49e+03±6.80e+02+** |
| vs. shuffle exponential + (better) | | 2 | 20 |
| (Wilcoxon rank-sum) − (worse) | | 3 | 4 |
| $p < 0.05$ ≈ (no sig.) | | 25 | 6 |

Table 6: Comparison of shuffle exponential crossover with binomial and exponential crossover in **jDE** on the CEC2014 benchmark functions [1] (50 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 500,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 1.34e+06±6.59e+05 | 1.38e+06±6.40e+05≈ | **4.91e+05±2.37e+05+** |
| $F_2$ | 1.49e+03±9.54e+02 | 1.63e+03±1.40e+03≈ | **8.17e-09±3.61e-08+** |
| $F_3$ | 5.28e+01±4.38e+01 | 3.94e+01±3.44e+01≈ | **7.18e-09±2.27e-08+** |
| $F_4$ | 9.48e+01±4.31e+00 | 9.12e+01±1.68e+01≈ | **8.91e+01±1.26e+01≈** |
| $F_5$ | **2.04e+01±2.90e-02** | 2.04e+01±2.76e-02≈ | 2.04e+01±2.99e-02− |
| $F_6$ | 2.87e+01±1.83e+00 | 2.90e+01±1.67e+00≈ | **8.33e+00±6.88e+00+** |
| $F_7$ | 1.82e-05±4.91e-05 | 8.07e-05±1.53e-04− | **0.00e+00±0.00e+00+** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 1.22e+02±9.66e+00 | 1.24e+02±1.21e+01≈ | **9.34e+01±9.11e+00+** |
| $F_{10}$ | **9.80e-04±3.39e-03** | 1.22e-03±3.75e-03≈ | 3.43e-03±6.65e-03− |
| $F_{11}$ | 4.95e+03±3.04e+02 | **4.90e+03±3.01e+02≈** | 5.21e+03±3.70e+02− |
| $F_{12}$ | 4.17e-01±4.30e-02 | **4.09e-01±5.17e-02≈** | 4.94e-01±5.80e-02− |
| $F_{13}$ | 4.05e-01±4.22e-02 | 3.99e-01±3.77e-02≈ | **3.90e-01±4.56e-02≈** |
| $F_{14}$ | 3.14e-01±1.83e-02 | **3.13e-01±2.23e-02≈** | 3.40e-01±9.84e-02≈ |
| $F_{15}$ | 1.56e+01±1.51e+00 | 1.56e+01±1.67e+00≈ | **1.21e+01±1.15e+00+** |
| $F_{16}$ | 1.82e+01±3.54e-01 | **1.82e+01±3.30e-01≈** | 1.82e+01±4.21e-01≈ |
| $F_{17}$ | 7.26e+05±5.12e+05 | 6.73e+05±4.09e+05≈ | **2.15e+04±1.43e+04+** |
| $F_{18}$ | 9.80e+02±5.90e+02 | 1.08e+02±6.24e+02≈ | **4.56e+02±5.59e+02+** |
| $F_{19}$ | 1.78e+01±2.02e+00 | 1.78e+01±2.64e+00≈ | **1.43e+01±6.16e+00+** |
| $F_{20}$ | 6.85e+02±3.46e+02 | 9.12e+02±4.63e+02− | **5.17e+01±2.18e+01+** |
| $F_{21}$ | 1.28e+05±6.78e+04 | 1.28e+05±6.10e+04≈ | **8.36e+03±7.71e+03+** |
| $F_{22}$ | 7.53e+02±1.33e+02 | 7.29e+02±1.63e+02≈ | **5.46e+02±1.52e+02+** |
| $F_{23}$ | **3.44e+02±0.00e+00** | **3.44e+02±0.00e+00≈** | 3.44e+02±0.00e+00≈ |
| $F_{24}$ | **2.61e+02±3.44e+00** | 2.62e+02±3.64e+00≈ | 2.68e+02±2.07e+00− |
| $F_{25}$ | 2.11e+02±1.42e+00 | 2.12e+02±1.49e+00≈ | **2.07e+02±1.97e+00+** |
| $F_{26}$ | 1.00e+02±3.89e-02 | 1.00e+02±4.59e-02≈ | **1.00e+02±3.52e-02+** |
| $F_{27}$ | 9.69e+02±1.84e+02 | 1.03e+03±9.84e+01≈ | **4.33e+02±5.33e+01+** |
| $F_{28}$ | 1.29e+03±3.44e+01 | 1.29e+03±3.67e+01≈ | **1.11e+03±3.62e+01+** |
| $F_{29}$ | 1.66e+03±2.05e+02 | 1.70e+03±2.13e+02≈ | **1.08e+03±1.95e+02+** |
| $F_{30}$ | **8.64e+03±2.51e+02** | 8.66e+03±2.57e+02≈ | 8.65e+03±4.68e+02≈ |
| vs. shuffle exponential + (better) | | 0 | 18 |
| (Wilcoxon rank-sum) − (worse) | | 2 | 5 |
| $p < 0.05$ ≈ (no sig.) | | 28 | 7 |

Table 7: Comparison of shuffle exponential crossover with binomial and exponential crossover in **JADE** on the CEC2014 benchmark functions [1] (10 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 100,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 2.78e+04±1.96e+04 | 7.13e+04±3.24e+04− | **0.00e+00±0.00e+00+** |
| $F_2$ | 5.94e+01±5.06e+01 | 9.72e+01±9.41e+01≈ | **0.00e+00±0.00e+00+** |
| $F_3$ | 3.71e+01±5.31e+01 | 2.53e+01±2.38e+01≈ | **3.52e-05±2.07e-04+** |
| $F_4$ | 2.32e+00±6.57e+00 | **1.32e+00±3.39e+00≈** | 2.34e+01±1.62e+01− |
| $F_5$ | **1.74e+01±4.82e+00** | 1.77e+01±4.45e+00≈ | 1.77e+01±4.42e+00≈ |
| $F_6$ | 1.15e+00±3.25e-01 | 4.86e-01±1.61e-01+ | **9.92e-02±1.34e-01+** |
| $F_7$ | 1.87e-02±6.88e-03 | 2.01e-02±8.58e-03≈ | **1.05e-02±5.37e-03+** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 4.13e+00±9.57e-01 | 4.21e+00±8.75e-01≈ | **3.29e+00±8.73e-01+** |
| $F_{10}$ | **2.45e-03±1.22e-02** | 4.90e-03±1.70e-02≈ | 9.80e-03±2.29e-02− |
| $F_{11}$ | 1.17e+02±5.99e+01 | 1.34e+02±8.31e+01≈ | **8.83e+01±5.47e+01+** |
| $F_{12}$ | **2.34e-01±4.96e-02** | 2.38e-01±5.18e-02≈ | 2.55e-01±5.56e-02≈ |
| $F_{13}$ | 1.33e-01±2.63e-02 | 1.29e-01±2.47e-02≈ | **8.16e-02±1.98e-02+** |
| $F_{14}$ | 1.13e-01±3.19e-02 | 1.14e-01±3.60e-02≈ | **1.02e-01±3.64e-02≈** |
| $F_{15}$ | 6.71e-01±1.36e-01 | 6.38e-01±1.26e-01≈ | **5.54e-01±9.35e-02+** |
| $F_{16}$ | 1.82e+00±3.25e-01 | 1.85e+00±1.99e-01≈ | **1.63e+00±2.93e-01+** |
| $F_{17}$ | 7.37e+03±6.39e+03 | 1.10e+04±1.03e+04≈ | **3.14e+00±1.71e+01+** |
| $F_{18}$ | 1.92e+02±2.47e+02 | 5.38e+02±4.43e+02− | **4.00e-01±5.10e-01+** |
| $F_{19}$ | **2.69e-01±7.46e-02** | 3.05e-01±9.07e-02− | 2.71e-01±1.72e-01≈ |
| $F_{20}$ | 9.93e-01±6.15e-01 | 8.75e+01±1.04e+02− | **3.25e-01±8.75e-02+** |
| $F_{21}$ | 7.03e+02±6.99e+02 | 8.54e+02±8.34e+02≈ | **8.45e-01±1.53e+00+** |
| $F_{22}$ | 1.93e-01±1.03e-01 | 2.35e-01±8.86e-02− | **1.87e-01±8.77e-02≈** |
| $F_{23}$ | **3.04e+02±8.78e+01** | 3.17e+02±6.17e+01− | 3.29e+02±0.00e+00− |
| $F_{24}$ | 1.11e+02±2.03e+00 | 1.13e+02±1.89e+00− | **1.09e+02±1.71e+00+** |
| $F_{25}$ | 1.26e+02±5.55e+00 | 1.28e+02±1.11e+01≈ | **1.25e+02±2.30e+01+** |
| $F_{26}$ | 1.00e+02±3.07e-02 | 1.00e+02±3.17e-02≈ | **1.00e+02±1.45e-02+** |
| $F_{27}$ | 4.94e+01±1.13e+02 | **4.28e+01±1.06e+02≈** | 6.55e+01±1.39e+02− |
| $F_{28}$ | **3.68e+02±2.76e+01** | 3.74e+02±3.46e+01− | 4.03e+02±5.10e+01− |
| $F_{29}$ | 3.12e+02±3.49e+01 | 3.19e+02±3.91e+01≈ | **2.33e+02±2.83e+01+** |
| $F_{30}$ | 5.43e+02±3.44e+01 | 5.58e+02±4.32e+01≈ | **4.76e+02±2.13e+01+** |
| vs. shuffle exponential    + (better) | | 1 | 19 |
| (Wilcoxon rank-sum)   − (worse) | | 8 | 5 |
| $p < 0.05$    ≈ (no sig.) | | 21 | 6 |

Table 8: Comparison of shuffle exponential crossover with binomial and exponential crossover in **JADE** on the CEC2014 benchmark functions [1] (30 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 300,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential<br>Mean±Std Dev | exponential<br>Mean±Std Dev | binomial<br>Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 1.68e+07±5.71e+06 | 1.51e+07±4.72e+06≈ | **1.81e+03±2.42e+03**+ |
| $F_2$ | 5.37e+01±5.32e+01 | 2.43e+02±2.61e+02− | **0.00e+00±0.00e+00**+ |
| $F_3$ | 4.28e+02±5.30e+02 | 6.94e+02±8.97e+02≈ | **8.40e-05±5.03e-04**+ |
| $F_4$ | 5.99e+01±2.17e+01 | 5.10e+01±2.30e+01+ | **7.82e-02±5.58e-01**+ |
| $F_5$ | 2.02e+01±2.92e-02 | **2.02e+01±3.83e-02**≈ | 2.03e+01±3.10e-02− |
| $F_6$ | 1.33e+01±1.11e+00 | 1.30e+01±1.08e+00≈ | **9.01e+00±2.71e+00**+ |
| $F_7$ | **1.53e-04±1.07e-04** | 2.24e-04±1.86e-04− | 5.32e-04±2.22e-03− |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_9$ | 4.29e+01±5.73e+00 | 4.41e+01±5.70e+00≈ | **2.56e+01±4.47e+00**+ |
| $F_{10}$ | **2.45e-03±6.77e-03** | **2.45e-03±6.77e-03**≈ | 6.94e-03±1.15e-02− |
| $F_{11}$ | **1.65e+03±2.05e+02** | 1.70e+03±2.20e+02≈ | 1.67e+03±2.16e+02≈ |
| $F_{12}$ | 2.27e-01±2.96e-02 | **2.19e-01±3.50e-02**≈ | 2.52e-01±3.60e-02− |
| $F_{13}$ | 2.94e-01±3.83e-02 | 3.00e-01±4.07e-02≈ | **2.05e-01±2.90e-02**+ |
| $F_{14}$ | **2.16e-01±2.26e-02** | 2.20e-01±2.09e-02≈ | 2.24e-01±3.48e-02≈ |
| $F_{15}$ | 5.29e+00±5.84e-01 | 5.14e+00±6.60e-01≈ | **3.12e+00±3.91e-01**+ |
| $F_{16}$ | 9.51e+00±3.28e-01 | 9.53e+00±3.79e-01≈ | **9.39e+00±3.07e-01**≈ |
| $F_{17}$ | 2.54e+06±1.18e+06 | 2.83e+06±1.35e+06≈ | **1.12e+03±3.69e+02**+ |
| $F_{18}$ | 1.62e+04±9.81e+03 | 1.69e+04±9.15e+03≈ | **9.06e+01±1.17e+02**+ |
| $F_{19}$ | 7.21e+00±7.23e-01 | 7.20e+00±8.24e-01≈ | **4.47e+00±7.11e-01**+ |
| $F_{20}$ | 5.28e+03±2.16e+03 | 5.44e+03±2.31e+03≈ | **2.38e+03±2.28e+03**+ |
| $F_{21}$ | 4.33e+05±2.30e+05 | 5.03e+05±1.90e+05− | **1.07e+04±4.42e+04**+ |
| $F_{22}$ | 2.07e+02±8.65e+01 | 2.09e+02±6.98e+01≈ | **1.60e+02±6.56e+01**+ |
| $F_{23}$ | 3.15e+02±4.42e-02 | 3.15e+02±6.44e-02≈ | **3.15e+02±0.00e+00**+ |
| $F_{24}$ | 2.28e+02±7.51e-01 | 2.28e+02±6.55e-01≈ | **2.26e+02±3.90e+00**+ |
| $F_{25}$ | 2.08e+02±8.08e-01 | 2.08e+02±1.06e+00− | **2.05e+02±2.31e+00**+ |
| $F_{26}$ | **1.00e+02±5.38e-02** | 1.00e+02±4.77e-02≈ | 1.02e+02±1.40e+01− |
| $F_{27}$ | 4.04e+02±5.69e+01 | 4.13e+02±4.06e+00≈ | **3.42e+02±4.97e+01**+ |
| $F_{28}$ | 8.71e+02±3.48e+01 | 8.68e+02±2.82e+01≈ | **7.93e+02±4.17e+01**+ |
| $F_{29}$ | 1.71e+03±1.32e+02 | 1.67e+03±1.45e+02≈ | **7.28e+02±1.50e+01**+ |
| $F_{30}$ | 4.10e+03±8.39e+02 | 4.13e+03±1.11e+03≈ | **1.48e+03±4.78e+02**+ |
| vs. shuffle exponential     + (better) | | 1 | 21 |
| (Wilcoxon rank-sum)   − (worse) | | 4 | 5 |
| $p < 0.05$   ≈ (no sig.) | | 25 | 4 |

Table 9: Comparison of shuffle exponential crossover with binomial and exponential crossover in **JADE** on the CEC2014 benchmark functions [1] (50 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 500,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 1.60e+07±3.46e+06 | 1.73e+07±3.95e+06≈ | **1.86e+04±1.45e+04+** |
| $F_2$ | 4.19e+04±1.70e+04 | 4.26e+04±2.45e+04≈ | **0.00e+00±0.00e+00+** |
| $F_3$ | 5.81e+03±2.99e+03 | 6.68e+03±2.47e+03≈ | **3.66e+03±2.17e+03+** |
| $F_4$ | 8.53e+01±1.59e+01 | 8.69e+01±1.43e+01≈ | **5.47e+00±2.06e+01+** |
| $F_5$ | 2.03e+01±3.42e-02 | **2.03e+01±2.98e-02≈** | 2.04e+01±3.42e-02− |
| $F_6$ | 2.58e+01±1.88e+00 | 2.56e+01±1.87e+00≈ | **1.67e+01±6.39e+00+** |
| $F_7$ | 5.49e-03±2.20e-03 | 6.06e-03±2.81e-03≈ | **4.88e-03±5.94e-03≈** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 1.00e+02±9.96e+00 | 1.02e+02±1.06e+01≈ | **5.41e+01±7.49e+00+** |
| $F_{10}$ | **3.43e-03±6.65e-03** | 6.61e-03±9.13e-03≈ | 8.57e-03±8.47e-03− |
| $F_{11}$ | 3.82e+03±2.88e+02 | 3.83e+03±3.18e+02≈ | **3.82e+03±3.11e+02≈** |
| $F_{12}$ | 2.07e-01±2.34e-02 | **2.05e-01±2.33e-02≈** | 2.61e-01±2.75e-02− |
| $F_{13}$ | 3.71e-01±4.09e-02 | 3.73e-01±3.61e-02≈ | **3.17e-01±4.59e-02+** |
| $F_{14}$ | **2.72e-01±2.04e-02** | 2.72e-01±2.45e-02≈ | 2.95e-01±6.33e-02− |
| $F_{15}$ | 1.25e+01±1.23e+00 | 1.25e+01±1.13e+00≈ | **7.48e+00±9.07e-01+** |
| $F_{16}$ | 1.78e+01±3.44e-01 | 1.79e+01±3.09e-01≈ | **1.77e+01±4.16e-01≈** |
| $F_{17}$ | 6.64e+06±1.79e+06 | 6.55e+06±1.69e+06≈ | **2.47e+03±6.84e+02+** |
| $F_{18}$ | 1.41e+04±5.86e+03 | 1.41e+04±6.93e+03≈ | **1.63e+02±4.75e+01+** |
| $F_{19}$ | 1.80e+01±2.30e+00 | 1.86e+01±2.94e+00≈ | **1.55e+01±7.24e+00+** |
| $F_{20}$ | 2.16e+04±5.63e+03 | 2.16e+04±6.10e+03≈ | **6.23e+03±6.29e+03+** |
| $F_{21}$ | 3.38e+06±1.44e+06 | 3.57e+06±1.32e+06≈ | **4.63e+04±2.30e+05+** |
| $F_{22}$ | 5.79e+02±1.72e+02 | 5.69e+02±1.41e+02≈ | **4.95e+02±1.53e+02+** |
| $F_{23}$ | 3.45e+02±8.14e-01 | 3.46e+02±1.48e+00− | **3.44e+02±0.00e+00+** |
| $F_{24}$ | 2.58e+02±7.57e-01 | **2.58e+02±5.80e-01≈** | 2.74e+02±2.59e+00− |
| $F_{25}$ | **2.16e+02±1.46e+00** | 2.16e+02±1.34e+00− | 2.22e+02±3.82e+00− |
| $F_{26}$ | **1.00e+02±4.12e-02** | 1.00e+02±4.93e-02− | 1.02e+02±1.40e+01− |
| $F_{27}$ | 8.77e+02±2.52e+02 | 8.59e+02±2.70e+02≈ | **4.55e+02±5.63e+01+** |
| $F_{28}$ | 1.42e+03±5.76e+01 | 1.43e+03±6.32e+01≈ | **1.14e+03±5.45e+01+** |
| $F_{29}$ | 2.66e+03±4.30e+02 | 2.61e+03±3.46e+02≈ | **9.04e+02±8.26e+01+** |
| $F_{30}$ | 1.07e+04±9.80e+02 | 1.05e+04±8.04e+02≈ | **1.00e+04±9.22e+02+** |
| vs. shuffle exponential    + (better) | | 0 | 19 |
| (Wilcoxon rank-sum)   − (worse) | | 3 | 7 |
| $p < 0.05$    ≈ (no sig.) | | 27 | 4 |

Table 10: Comparison of shuffle exponential crossover with binomial and exponential crossover in **SHADE** on the CEC2014 benchmark functions [1] (10 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 100,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_2$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_3$ | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_4$ | **1.34e+01±1.67e+01** | 1.67e+01±1.72e+01≈ | 2.87e+01±1.32e+01− |
| $F_5$ | 1.63e+01±6.64e+00 | 1.75e+01±5.46e+00≈ | **1.54e+01±7.08e+00**≈ |
| $F_6$ | 1.58e-01±3.07e-01 | 2.64e-03±6.68e-03+ | **0.00e+00±0.00e+00**+ |
| $F_7$ | 4.89e-03±5.31e-03 | **4.72e-03±6.25e-03**≈ | 7.75e-03±1.16e-02≈ |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00**≈ | **0.00e+00±0.00e+00**≈ |
| $F_9$ | 3.40e+00±1.15e+00 | 3.45e+00±1.16e+00≈ | **2.92e+00±8.18e-01**+ |
| $F_{10}$ | 2.45e-03±1.22e-02 | **1.22e-03±8.75e-03**≈ | 1.47e-02±2.95e-02− |
| $F_{11}$ | 9.26e+01±7.14e+01 | 9.62e+01±7.10e+01≈ | **7.57e+01±5.16e+01**≈ |
| $F_{12}$ | 1.54e-01±3.19e-02 | 1.56e-01±3.22e-02≈ | **1.42e-01±3.80e-02**≈ |
| $F_{13}$ | 1.20e-01±2.47e-02 | 1.21e-01±2.74e-02≈ | **7.15e-02±1.80e-02**+ |
| $F_{14}$ | 9.97e-02±2.56e-02 | 1.00e-01±2.49e-02≈ | **9.95e-02±3.09e-02**≈ |
| $F_{15}$ | 5.31e-01±9.03e-02 | 5.00e-01±9.39e-02≈ | **4.92e-01±8.10e-02**≈ |
| $F_{16}$ | 1.86e+00±2.35e-01 | 1.86e+00±2.36e-01≈ | **1.52e+00±2.75e-01**+ |
| $F_{17}$ | 1.34e+00±2.32e+00 | **1.26e+00±2.39e+00**≈ | 1.66e+00±3.18e+00≈ |
| $F_{18}$ | **1.79e-01±2.35e-01** | 3.71e-01±4.07e-01− | 2.67e-01±3.38e-01≈ |
| $F_{19}$ | **1.92e-01±4.92e-02** | 2.37e-01±1.44e-01≈ | 2.16e-01±2.44e-01− |
| $F_{20}$ | **1.91e-01±7.01e-02** | 4.03e-01±1.73e-01− | 2.04e-01±1.59e-01≈ |
| $F_{21}$ | 4.09e-01±2.74e-01 | 4.04e-01±2.69e-01≈ | **3.10e-01±2.48e-01**≈ |
| $F_{22}$ | **1.58e-01±6.70e-02** | 2.61e-01±1.28e-01− | 2.57e-01±8.25e-02− |
| $F_{23}$ | 3.29e+02±0.00e+00 | **3.23e+02±4.61e+01**≈ | 3.29e+02±0.00e+00≈ |
| $F_{24}$ | 1.10e+02±1.91e+00 | 1.11e+02±2.01e+00≈ | **1.09e+02±1.86e+00**+ |
| $F_{25}$ | **1.16e+02±2.23e+01** | 1.17e+02±1.77e+01≈ | 1.22e+02±2.74e+01− |
| $F_{26}$ | 1.00e+02±2.31e-02 | 1.00e+02±2.50e-02≈ | **1.00e+02±1.93e-02**+ |
| $F_{27}$ | **4.70e+01±1.25e+02** | 8.02e+01±1.60e+02≈ | 1.07e+02±1.60e+02− |
| $F_{28}$ | **3.72e+02±2.44e+01** | 3.79e+02±3.36e+01≈ | 3.97e+02±4.58e+01− |
| $F_{29}$ | 2.22e+02±6.04e-01 | 2.22e+02±5.73e-01≈ | **2.22e+02±5.07e-01**≈ |
| $F_{30}$ | 4.66e+02±1.35e+01 | **4.64e+02±6.82e+00**≈ | 4.71e+02±2.12e+01≈ |

| vs. shuffle exponential | + (better) | 1 | 6 |
|---|---|---|---|
| (Wilcoxon rank-sum) | − (worse) | 3 | 7 |
| $p < 0.05$ | ≈ (no sig.) | 26 | 17 |

Table 11: Comparison of shuffle exponential crossover with binomial and exponential crossover in **SHADE** on the CEC2014 benchmark functions [1] (30 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 300,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential Mean±Std Dev | exponential Mean±Std Dev | binomial Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 1.32e+03±2.15e+03 | 1.62e+03±2.34e+03≈ | **8.53e+02±1.85e+03+** |
| $F_2$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_3$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_4$ | 1.24e+00±8.88e+00 | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_5$ | 2.01e+01±2.44e-02 | 2.01e+01±1.91e-02≈ | **2.01e+01±1.39e-02+** |
| $F_6$ | 1.19e+01±1.22e+00 | 1.12e+01±1.15e+00+ | **5.82e-01±7.23e-01+** |
| $F_7$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | 1.45e-04±1.04e-03≈ |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 4.11e+01±5.39e+00 | 4.00e+01±5.15e+00≈ | **1.51e+01±2.54e+00+** |
| $F_{10}$ | 5.31e-03±1.01e-02 | **4.90e-03±9.84e-03≈** | 1.02e-02±1.58e-02− |
| $F_{11}$ | 1.70e+03±2.35e+02 | 1.75e+03±1.75e+02≈ | **1.44e+03±2.20e+02+** |
| $F_{12}$ | 1.79e-01±2.85e-02 | 1.83e-01±2.28e-02≈ | **1.61e-01±2.19e-02+** |
| $F_{13}$ | 3.02e-01±3.98e-02 | 3.05e-01±4.19e-02≈ | **2.16e-01±3.26e-02+** |
| $F_{14}$ | 2.13e-01±2.32e-02 | **2.10e-01±2.61e-02≈** | 2.39e-01±3.56e-02− |
| $F_{15}$ | 3.15e+00±3.55e-01 | 3.20e+00±4.11e-01≈ | **2.53e+00±2.74e-01+** |
| $F_{16}$ | 9.72e+00±3.72e-01 | 9.66e+00±3.44e-01≈ | **9.05e+00±4.26e-01+** |
| $F_{17}$ | **8.63e+02±2.91e+02** | 1.02e+03±3.33e+02− | 1.02e+03±3.40e+02− |
| $F_{18}$ | 3.14e+01±2.25e+01 | **2.85e+01±2.23e+01≈** | 5.54e+01±3.01e+01− |
| $F_{19}$ | **4.06e+00±5.09e-01** | 4.15e+00±5.99e-01≈ | 4.37e+00±6.91e-01− |
| $F_{20}$ | **1.32e+01±5.04e+00** | 1.36e+01±5.09e+00≈ | 1.36e+01±8.45e+00≈ |
| $F_{21}$ | **2.01e+02±9.83e+01** | 2.35e+02±9.67e+01≈ | 2.39e+02±1.28e+02≈ |
| $F_{22}$ | **1.06e+02±6.88e+01** | 1.28e+02±6.75e+01≈ | 1.08e+02±6.69e+01≈ |
| $F_{23}$ | **3.15e+02±0.00e+00** | **3.15e+02±0.00e+00≈** | **3.15e+02±0.00e+00≈** |
| $F_{24}$ | 2.25e+02±1.01e+00 | **2.24e+02±1.56e+00+** | 2.26e+02±3.38e+00≈ |
| $F_{25}$ | **2.03e+02±3.43e-01** | 2.03e+02±4.07e-01≈ | 2.04e+02±8.06e-01− |
| $F_{26}$ | 1.00e+02±4.11e-02 | **1.00e+02±5.21e-02≈** | 1.02e+02±1.40e+01− |
| $F_{27}$ | 4.04e+02±5.26e+00 | 4.06e+02±4.33e+00− | **3.23e+02±3.63e+01+** |
| $F_{28}$ | **8.14e+02±1.57e+01** | 8.21e+02±1.75e+01≈ | 8.28e+02±3.87e+01− |
| $F_{29}$ | 7.26e+02±1.91e+01 | 7.23e+02±5.29e+01≈ | **7.16e+02±3.92e+01≈** |
| $F_{30}$ | 1.26e+03±4.48e+02 | **1.17e+03±3.35e+02≈** | 1.56e+03±7.59e+02≈ |
| vs. shuffle exponential + (better) | | 2 | 10 |
| (Wilcoxon rank-sum) − (worse) | | 2 | 8 |
| $p < 0.05$ ≈ (no sig.) | | 26 | 12 |

Table 12: Comparison of shuffle exponential crossover with binomial and exponential crossover in **SHADE** on the CEC2014 benchmark functions [1] (50 dimensions). For all problems, the maximum number of objective function evaluations is $D \times 10,000 = 500,000$. All results are the means of 51 runs.

| $F$ | shuffle exponential<br>Mean±Std Dev | exponential<br>Mean±Std Dev | binomial<br>Mean±Std Dev |
|---|---|---|---|
| $F_1$ | 3.97e+04±2.14e+04 | 6.02e+04±2.63e+04− | **1.74e+04±1.00e+04+** |
| $F_2$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_3$ | **0.00e+00±0.00e+00** | 1.29e-06±9.12e-06− | **0.00e+00±0.00e+00≈** |
| $F_4$ | **1.55e+01±3.60e+01** | 2.52e+01±4.31e+01− | 2.28e+01±4.10e+01≈ |
| $F_5$ | 2.02e+01±2.26e-02 | 2.02e+01±1.80e-02≈ | **2.01e+01±1.93e-02+** |
| $F_6$ | 2.52e+01±1.70e+00 | 2.57e+01±1.67e+00≈ | **5.27e+00±2.05e+00+** |
| $F_7$ | 3.48e-03±6.36e-03 | 3.09e-03±5.09e-03≈ | **1.55e-03±3.45e-03≈** |
| $F_8$ | **0.00e+00±0.00e+00** | **0.00e+00±0.00e+00≈** | **0.00e+00±0.00e+00≈** |
| $F_9$ | 9.63e+01±8.72e+00 | 1.01e+02±9.22e+00− | **3.31e+01±4.45e+00+** |
| $F_{10}$ | 7.59e-03±1.09e-02 | **7.35e-03±8.35e-03≈** | 1.10e-02±1.21e-02≈ |
| $F_{11}$ | 3.76e+03±2.96e+02 | 3.81e+03±3.45e+02≈ | **3.38e+03±3.36e+02+** |
| $F_{12}$ | 1.77e-01±2.16e-02 | 1.71e-01±1.98e-02≈ | **1.60e-01±1.89e-02+** |
| $F_{13}$ | 3.87e-01±3.44e-02 | 3.77e-01±3.39e-02≈ | **3.12e-01±5.14e-02+** |
| $F_{14}$ | **2.41e-01±3.31e-02** | 2.46e-01±2.54e-02≈ | 2.88e-01±4.11e-02− |
| $F_{15}$ | 6.92e+00±8.56e-01 | 6.88e+00±6.65e-01≈ | **5.79e+00±6.53e-01+** |
| $F_{16}$ | 1.82e+01±4.73e-01 | 1.82e+01±3.86e-01≈ | **1.75e+01±4.16e-01+** |
| $F_{17}$ | 2.87e+03±8.34e+02 | 2.89e+03±9.35e+02≈ | **2.58e+03±7.50e+02≈** |
| $F_{18}$ | 1.25e+02±5.26e+01 | **1.14e+02±4.41e+01≈** | 1.49e+02±3.78e+01− |
| $F_{19}$ | 1.15e+01±9.13e-01 | 1.16e+01±1.22e+00≈ | **9.68e+00±2.29e+00+** |
| $F_{20}$ | **9.08e+01±3.34e+01** | 1.20e+02±4.48e+01− | 1.98e+02±6.83e+01− |
| $F_{21}$ | **1.23e+03±3.35e+02** | 1.35e+03±5.27e+02≈ | 1.27e+03±3.61e+02≈ |
| $F_{22}$ | 4.65e+02±1.14e+02 | 4.81e+02±1.55e+02≈ | **3.77e+02±1.64e+02+** |
| $F_{23}$ | **3.44e+02±0.00e+00** | **3.44e+02±0.00e+00≈** | **3.44e+02±0.00e+00≈** |
| $F_{24}$ | **2.59e+02±3.55e+00** | 2.60e+02±3.51e+00≈ | 2.74e+02±2.42e+00− |
| $F_{25}$ | **2.06e+02±1.00e+00** | 2.06e+02±1.07e+00≈ | 2.12e+02±7.08e+00− |
| $F_{26}$ | **1.00e+02±4.21e-02** | 1.00e+02±4.69e-02≈ | 1.02e+02±1.40e+01≈ |
| $F_{27}$ | 7.94e+02±2.51e+02 | 8.81e+02±2.22e+02≈ | **4.49e+02±5.15e+01+** |
| $F_{28}$ | 1.18e+03±2.72e+01 | 1.19e+03±3.77e+01≈ | **1.14e+03±6.05e+01+** |
| $F_{29}$ | 9.08e+02±7.61e+01 | 9.22e+02±9.16e+01≈ | **8.89e+02±5.30e+01≈** |
| $F_{30}$ | **8.59e+03±4.26e+02** | 8.60e+03±4.15e+02≈ | 9.43e+03±7.18e+02− |
| vs. shuffle exponential + (better) | | 0 | 13 |
| (Wilcoxon rank-sum) − (worse) | | 5 | 6 |
| $p < 0.05$ ≈ (no sig.) | | 25 | 11 |

# References

[1] J. J. Liang, B. Y. Qu, and P. N. Suganthan. Problem Definitions and Evaluation Criteria for the CEC 2014 Special Session and Competition on Single Objective Real-Parameter Numerical Optimization. Technical report, Zhengzhou University and Nanyang Technological University, 2013.