

On Calibration of Modern Neural Networks

Chuan Guo

Geoff Pleiss

Yu Sun

Kilian Q. Weinberger

Presented by Lukas Fluri

Introduction

VERY DEEP CONVOLUTIONAL

Deep Networks with Stochastic Depth

Deep Residual Learning

Densely Connected Convolutional Networks

**New state of the art results
For CIFAR 10/10+/100/100+**

Gao H.

C.

Zhuang Liu*
Tsinghua University
liang13@mails.tsinghua.edu.cn

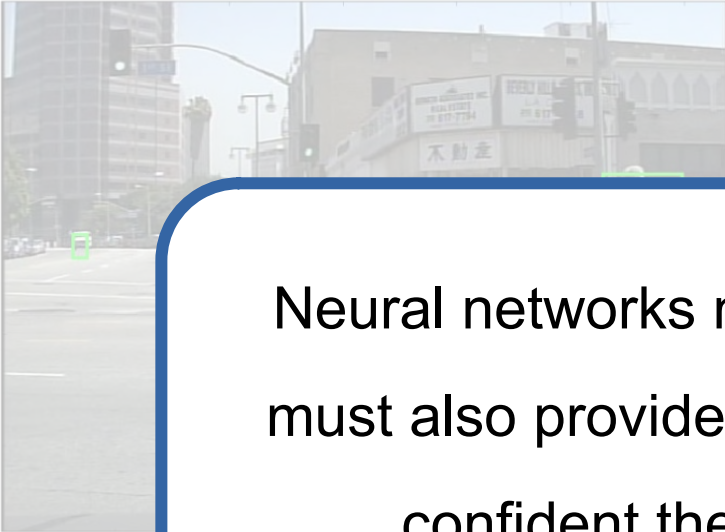
Kilian Q. Weinberger
Cornell University
kqw4@cornell.edu

Laurens van der Maaten
Facebook AI Research
lvdmaaten@fb.com

* Under certain assumptions

** As of 2020

Introduction

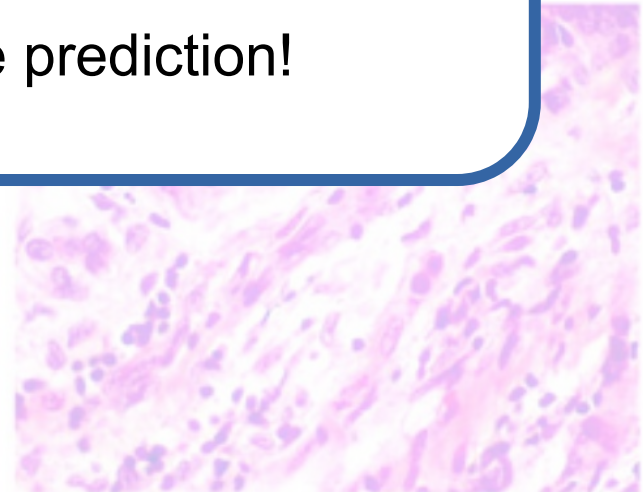


Pedestrian detection

Neural networks must not only be accurate, they must also provide a reliable estimation about how confident they are about the prediction!

Cancer detection

Survey: [Daoud Artif. Intell. Med. 2019]

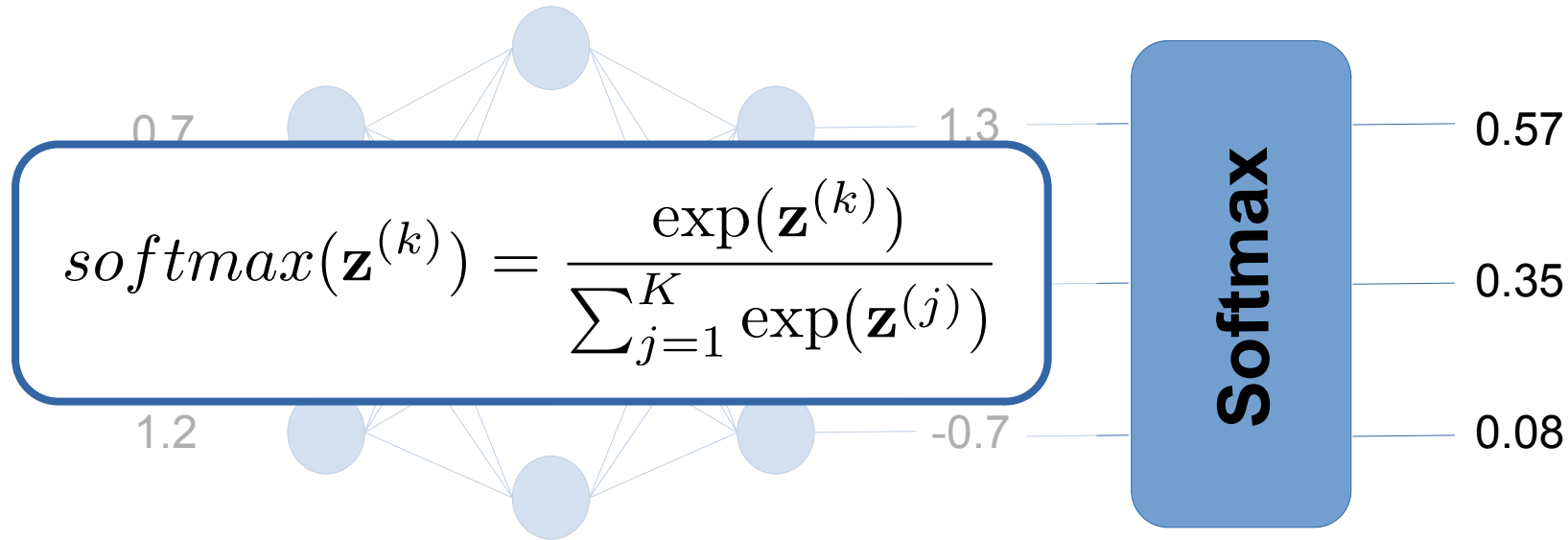


Source: [Spanhol et al. IEEE Trans Biomed 2016]

Overview

- **Introduction:** It's important for neural networks to be well-calibrated.
- **Definition:** How to measure model calibration?
- **Problem:** Modern neural networks are no longer calibrated!
- **Analysis:** Which factors might influence model calibration?
- **Mitigation:** How to calibrate neural networks?
- **Experiments:** Which calibration methods perform best?

How to create confidence estimates



Input:

\mathbf{x}

Output:

$\mathbf{z} = NN_{\Theta}(\mathbf{x})$

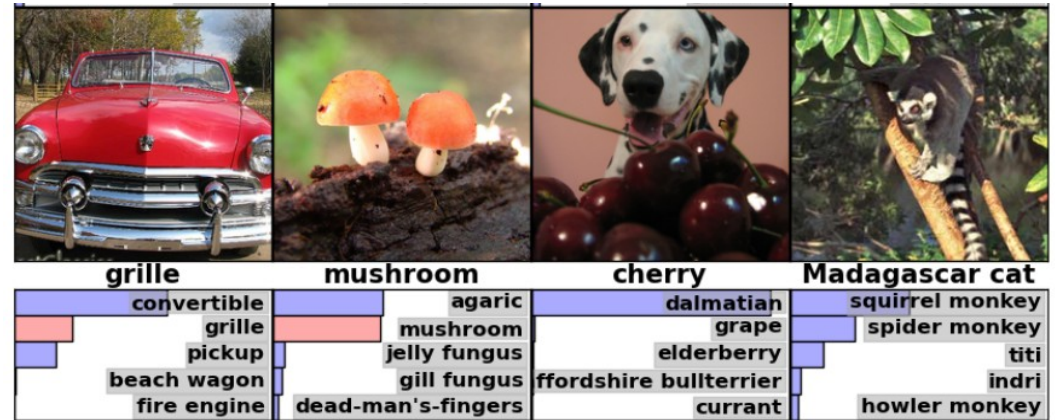
Confidence:

$\hat{\mathbf{p}} = \text{softmax}(\mathbf{z})$

How to interpret calibration

Input		Pred.	Conf.	
x_1	→	CAT	70%	✓
x_2	→	DOG	70%	✗
x_3	→	CAT	70%	✓
x_4	→	CAT	70%	✓
x_5	→	DOG	70%	✓
x_6	→	CAT	70%	✗
x_7	→	CAT	70%	✓
x_8	→	DOG	70%	✗
x_9	→	CAT	70%	✓
x_{10}	→	DOG	70%	✓

Different sources of error



Technically correct

Very close

Not able to recognize

How to define model-calibration

Input		Pred.	Conf.	True	
x_1	→	CAT	70%	CAT	✓
x_2	→	DOG	70%	CAT	✗
x_3	→	CAT	70%	CAT	✓
x_4	→	CAT	70%	CAT	✓
x_5	→	DOG	70%	DOG	✓
x_6	→	CAT	70%	DOG	✗
x_7	→	CAT	70%	CAT	✓
...			...		
		\hat{Y}	\hat{P}	Y	

Perfect calibration

A neural network has *perfect calibration* if for all $p \in [0, 1]$:

$$\mathbb{P} \left(\hat{Y} = Y | \hat{P} = p \right) = p$$

How to define model-calibration

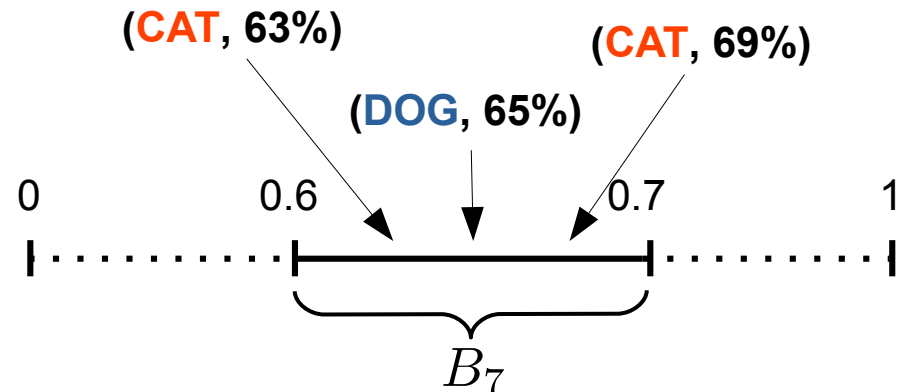
- Perfect calibration: $\mathbb{P} \left(\hat{Y} = Y | \hat{P} = p \right) = p \quad \forall p \in [0, 1]$
- Model calibration: $\mathbb{E} \left[\left| \mathbb{P} \left(\hat{Y} = Y | \hat{P} = p \right) - p \right| \right]$

Problem: In practice we only have finite data!
We need to approximate the model calibration

How to define model-calibration

- Expected Calibration Error (ECE):
 1. Train neural network on training data
 2. Create predictions and confidence estimates using the test data
 3. Group the predictions into M bins. Define bin B_m to be the set of all predictions (\hat{y}_i, \hat{p}_i) for which it holds that

$$\hat{p}_i \in \left(\frac{m-1}{M}, \frac{m}{M} \right]$$



How to define model-calibration

- Expected Calibration Error (ECE):

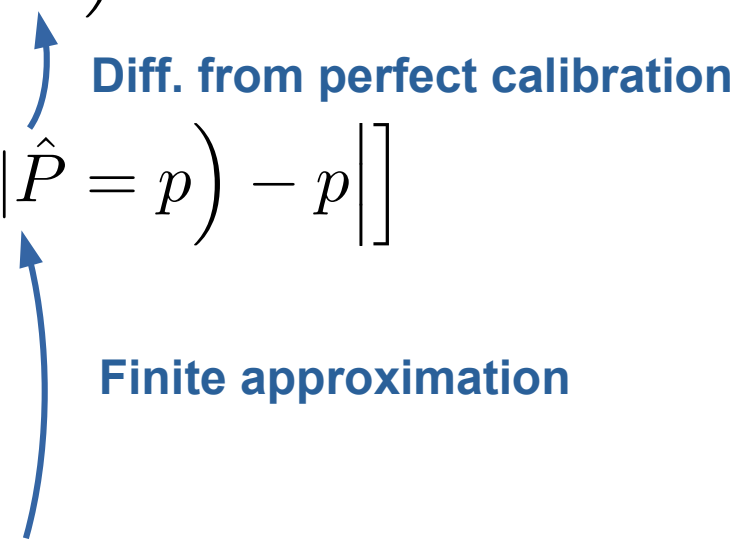
4. Compute the accuracy and confidence of bin B_m as:

$$acc(B_m) = \frac{1}{|B_m|} \sum_{i \in B_m} \mathbf{1}(\hat{y}_i = y_i) \qquad conf(B_m) = \frac{1}{|B_m|} \sum_{i \in B_m} \hat{p}_i$$

5. Compute the expected calibration error as:

$$ECE = \sum_{m=1}^M \frac{|B_m|}{n} |acc(B_m) - conf(B_m)|$$

How to define model-calibration

- Perfect calibration: $\mathbb{P}(\hat{Y} = Y | \hat{P} = p) = p \quad \forall p \in [0, 1]$
 - Model calibration: $\mathbb{E} \left[\left| \mathbb{P}(\hat{Y} = Y | \hat{P} = p) - p \right| \right]$
 - Expected Calibration Error:
- 
- Diff. from perfect calibration**
- Finite approximation**

$$ECE = \sum_{m=1}^M \frac{|B_m|}{n} |acc(B_m) - conf(B_m)|$$

How to define model-calibration

- Expected Calibration Error:

$$ECE = \sum_{m=1}^M \frac{|B_m|}{n} |acc(B_m) - conf(B_m)|$$

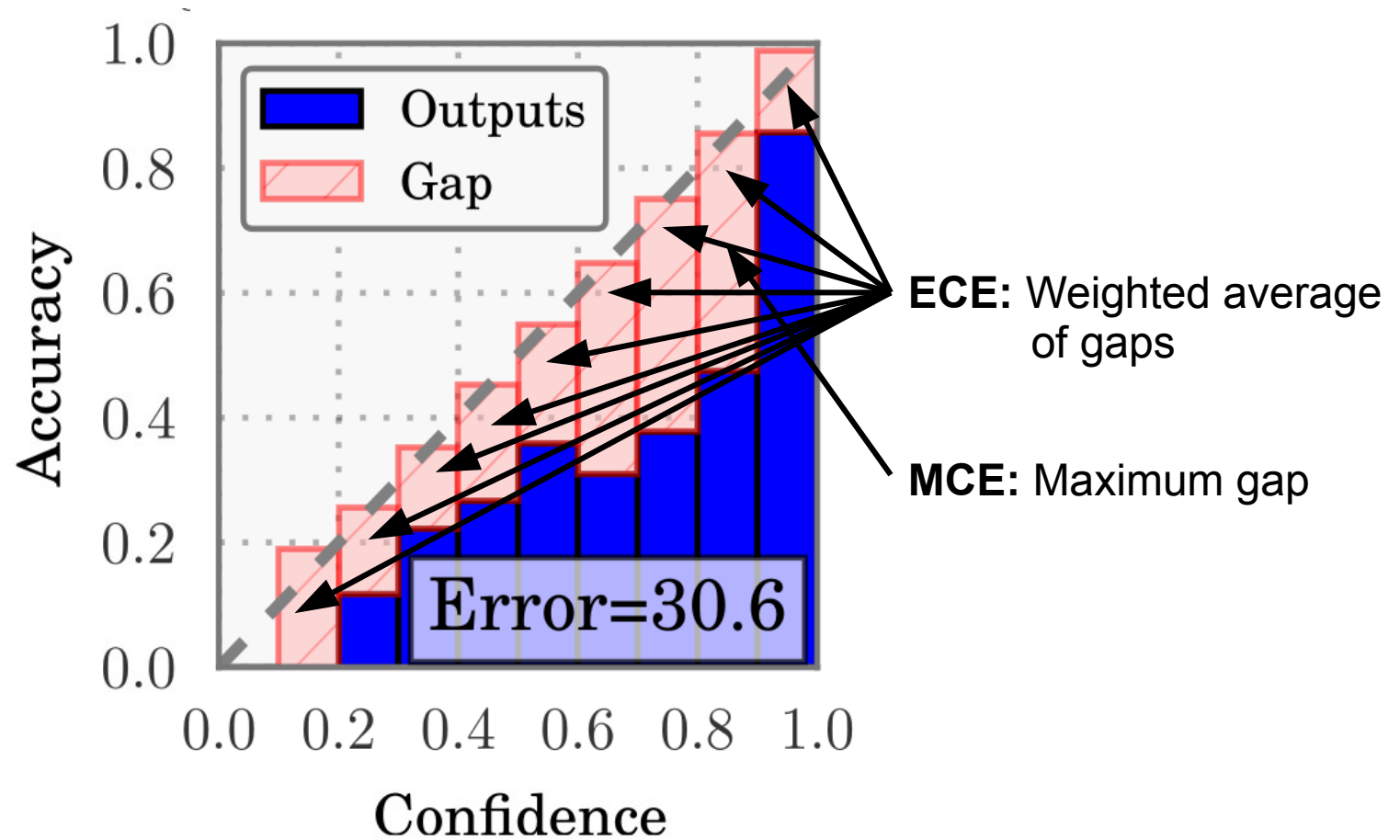
Computes weighted average of mis-calibration

- Maximum Calibration Error: Useful for high risk applications

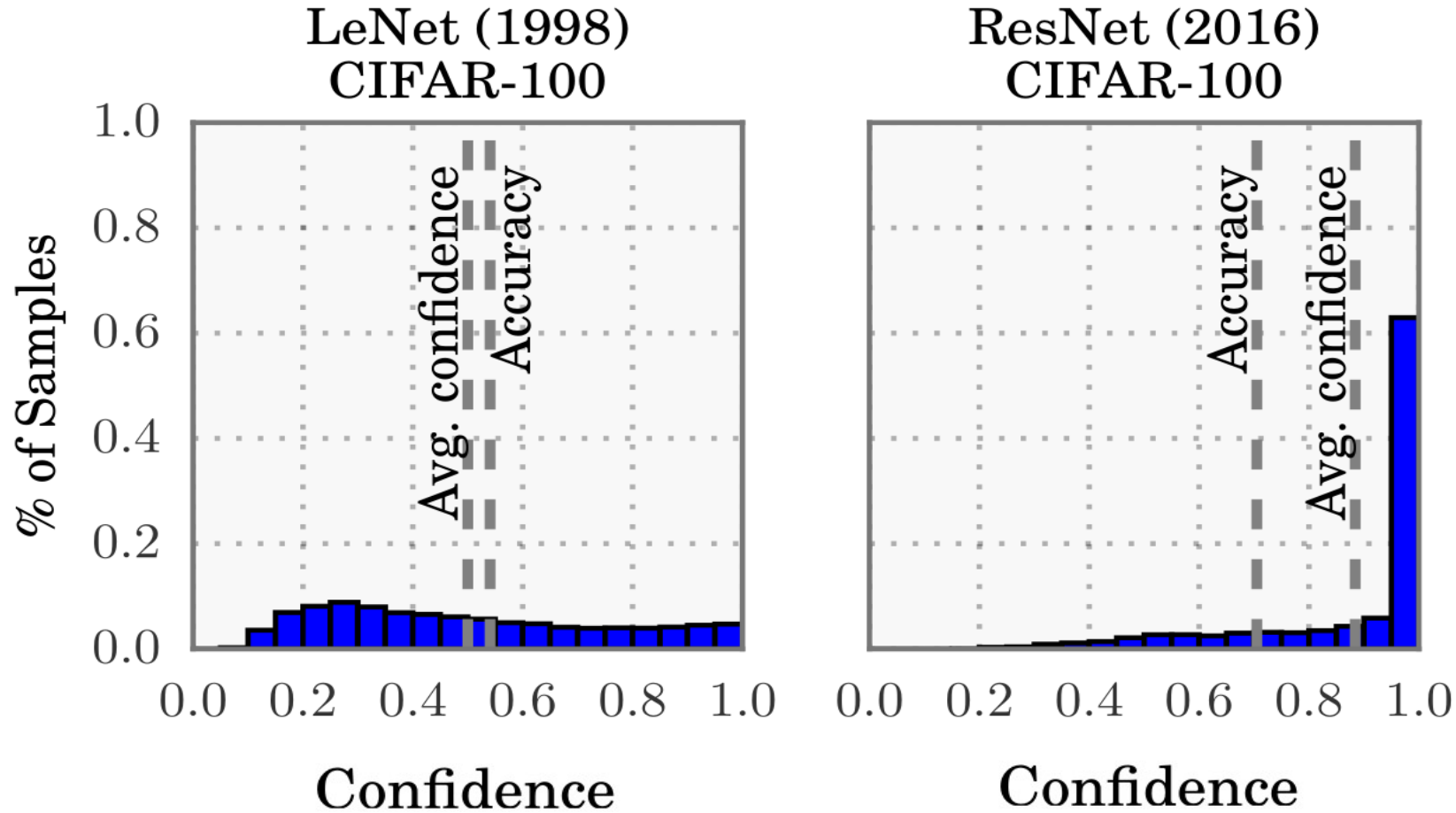
$$MCE = \max_{m \in \{1, \dots, M\}} |acc(B_m) - conf(B_m)|$$

Computes maximum mis-calibration

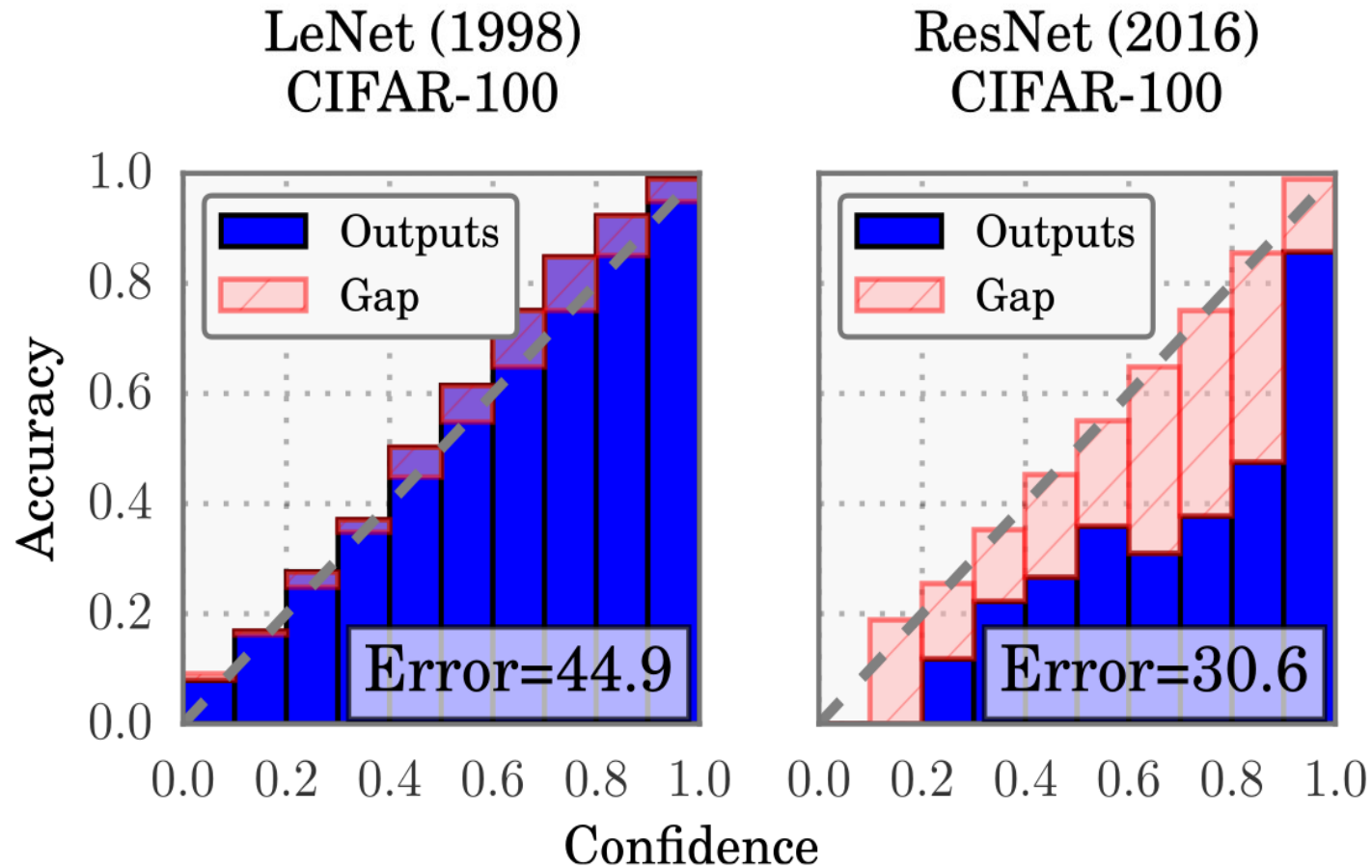
Reliability Diagram



Problem



Problem



Goal

- 1) Understand why neural networks have become miscalibrated
- 2) Identify and compare methods to alleviate this problem

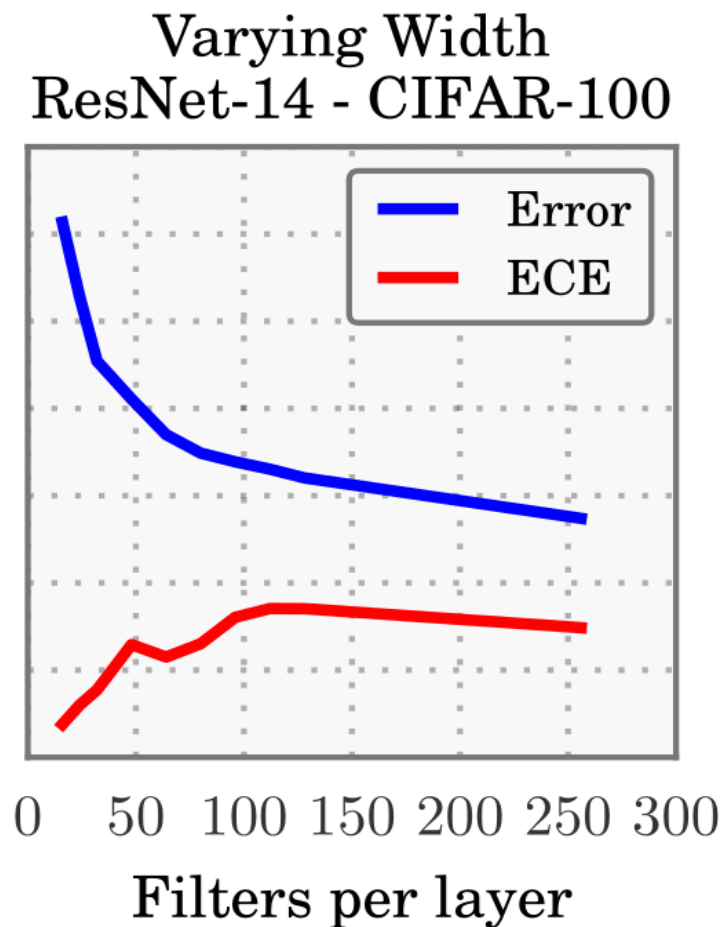
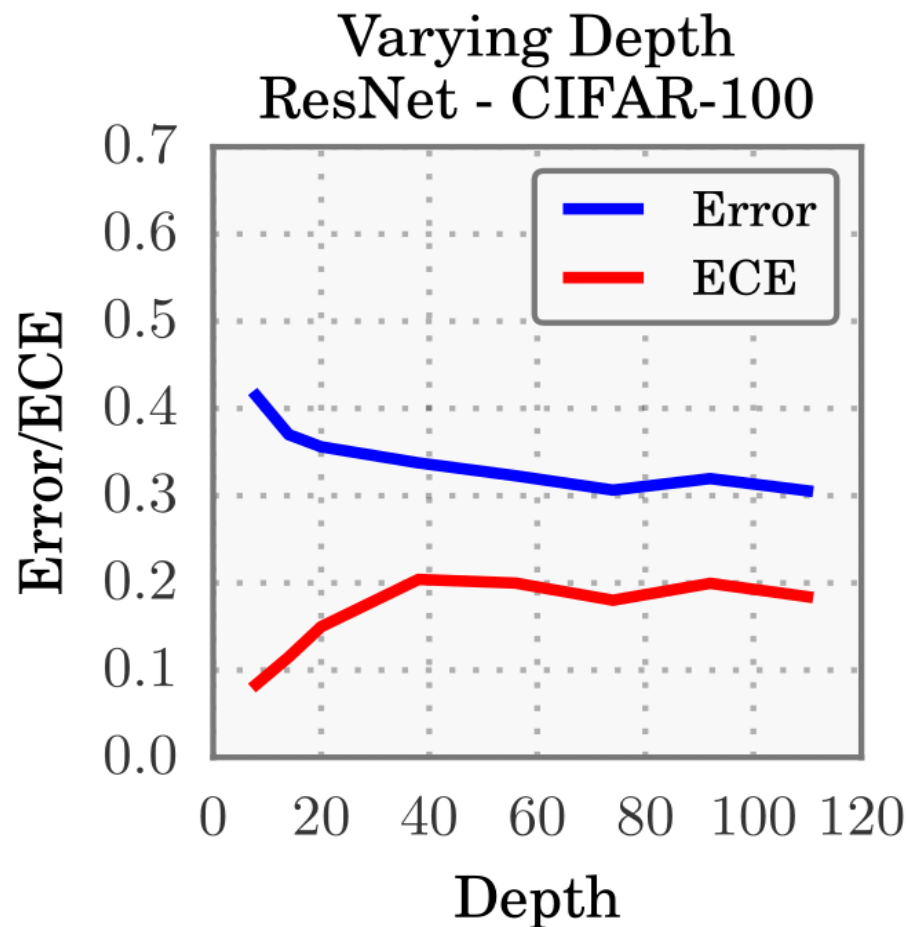
Overview

- **Introduction:** It's important for neural networks to be well-calibrated.
- **Definition:** How to measure model calibration?
 - ECE, MCE, Reliability diagrams
- **Problem:** Modern neural networks are no longer calibrated!
- **Analysis:** Which factors might influence model calibration?
- **Mitigation:** How to calibrate neural networks?
- **Experiments:** Which calibration methods perform best?

Factors influencing calibration

1. Model capacity: Depth & Width of network

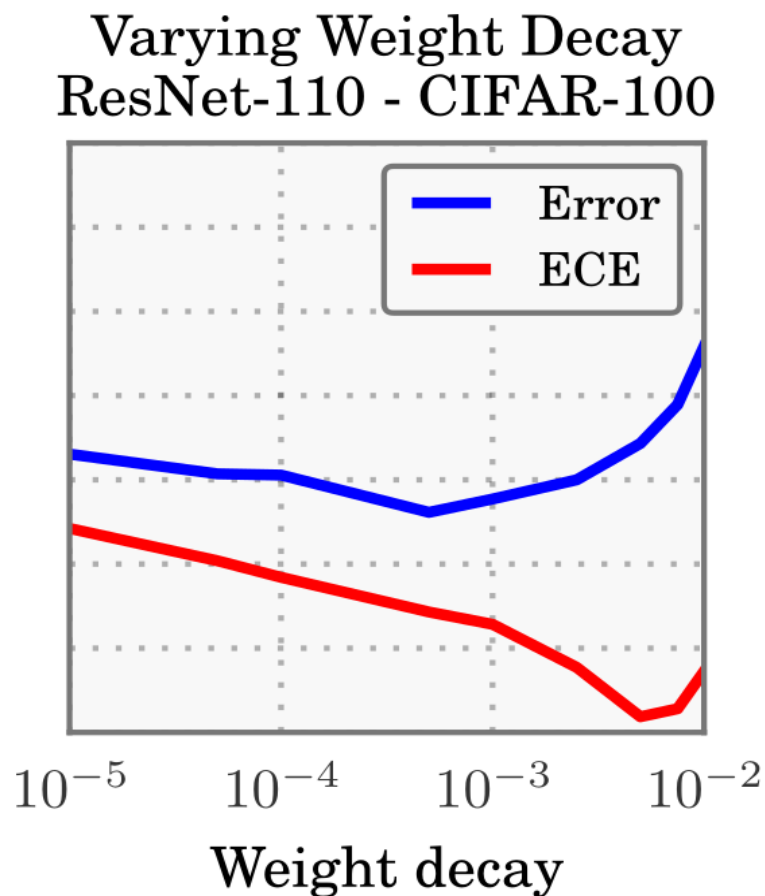
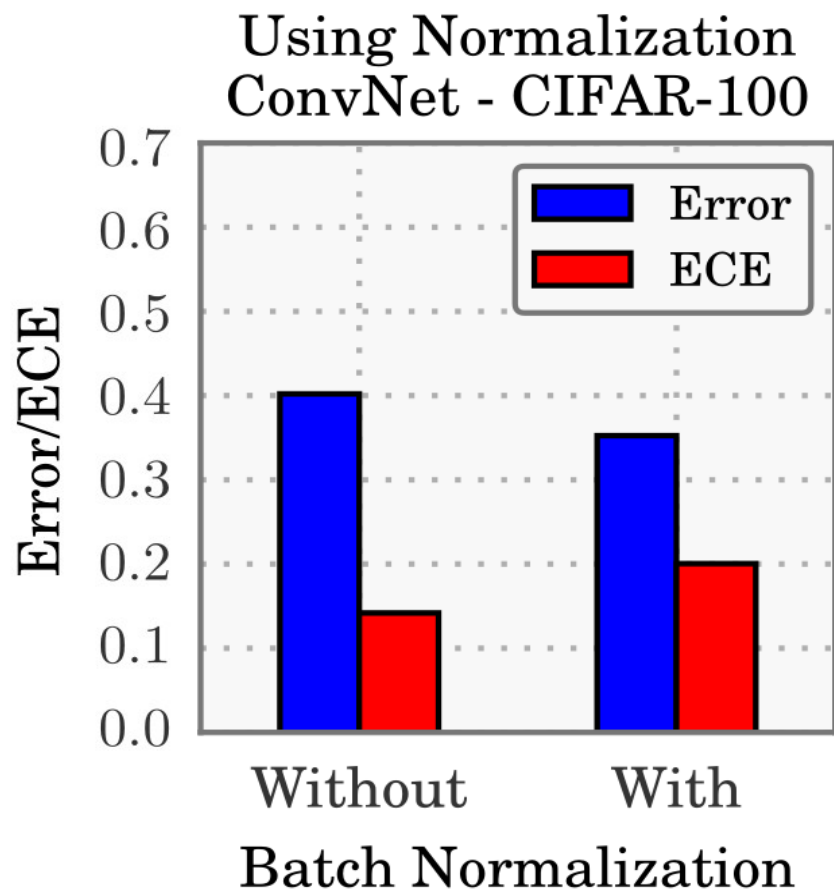
Factors influencing calibration



Factors influencing calibration

1. Model capacity: Depth & Width of network
2. Batch normalization
3. Weight decay

Factors influencing calibration

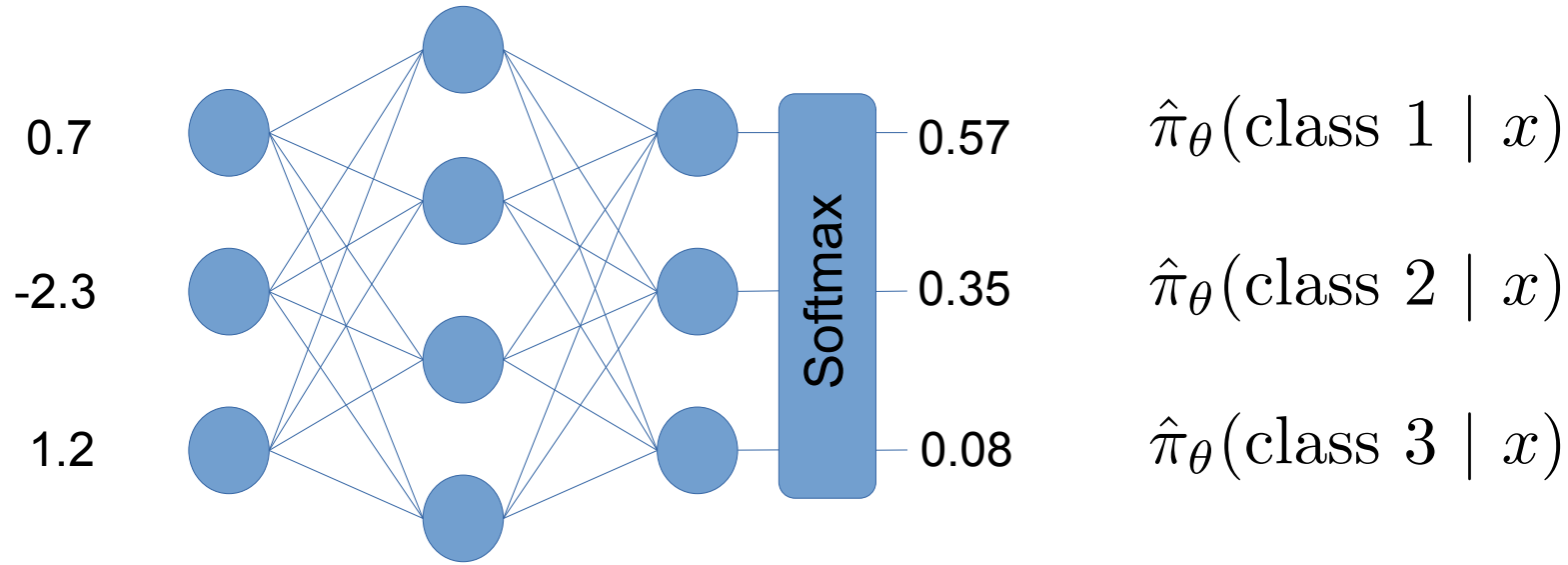


Factors influencing calibration

1. Model capacity: Depth & Width of network
2. Batch normalization
3. Weight decay
4. Training using negative log-likelihood / cross-entropy loss

Factors influencing calibration

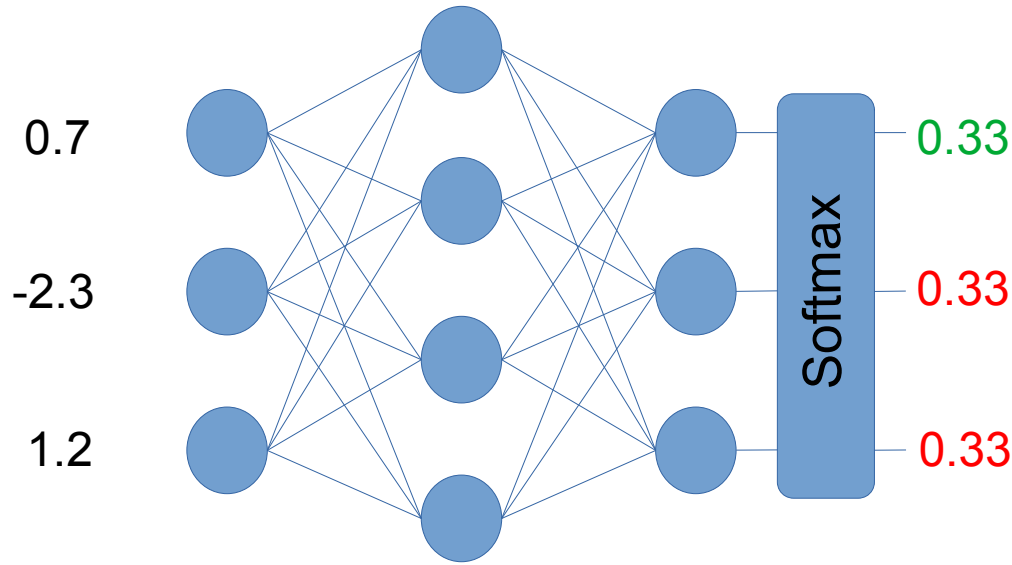
Training using negative log-likelihood / cross-entropy loss



$$NLL(\theta) = \arg \max_{\theta} \prod_{i=1}^n \hat{\pi}_{\theta}(y_i \mid x_i) = \arg \min_{\theta} - \sum_{i=1}^n \log(\hat{\pi}_{\theta}(y_i \mid x_i))$$

Factors influencing calibration

Training using negative log-likelihood / cross-entropy loss

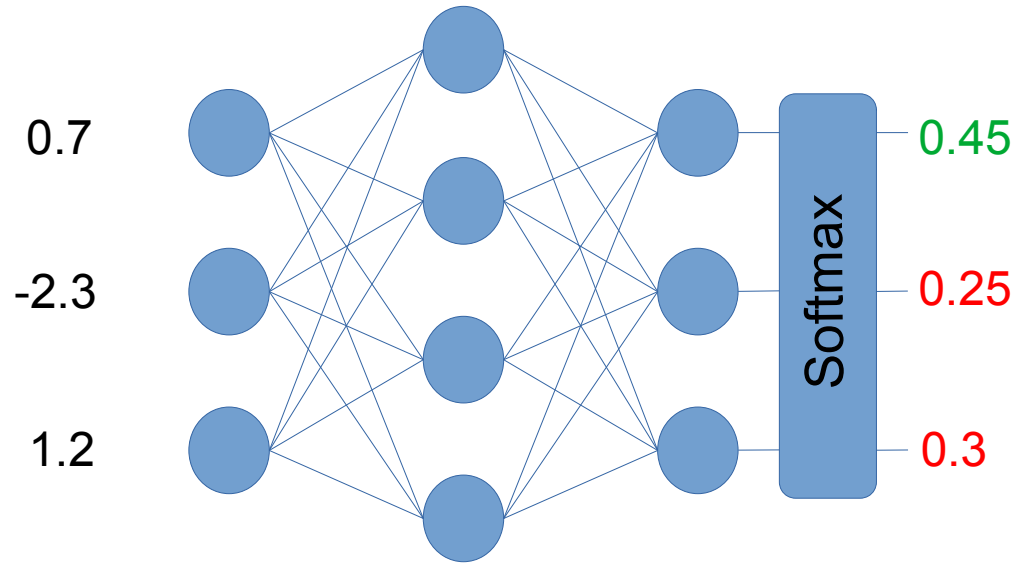


How to minimize NLL:

$$NLL(\theta) = \arg \max_{\theta} \prod_{i=1}^n \hat{\pi}_{\theta}(y_i | x_i) = \arg \min_{\theta} - \sum_{i=1}^n \log(\hat{\pi}_{\theta}(y_i | x_i))$$

Factors influencing calibration

Training using negative log-likelihood / cross-entropy loss



How to minimize NLL:

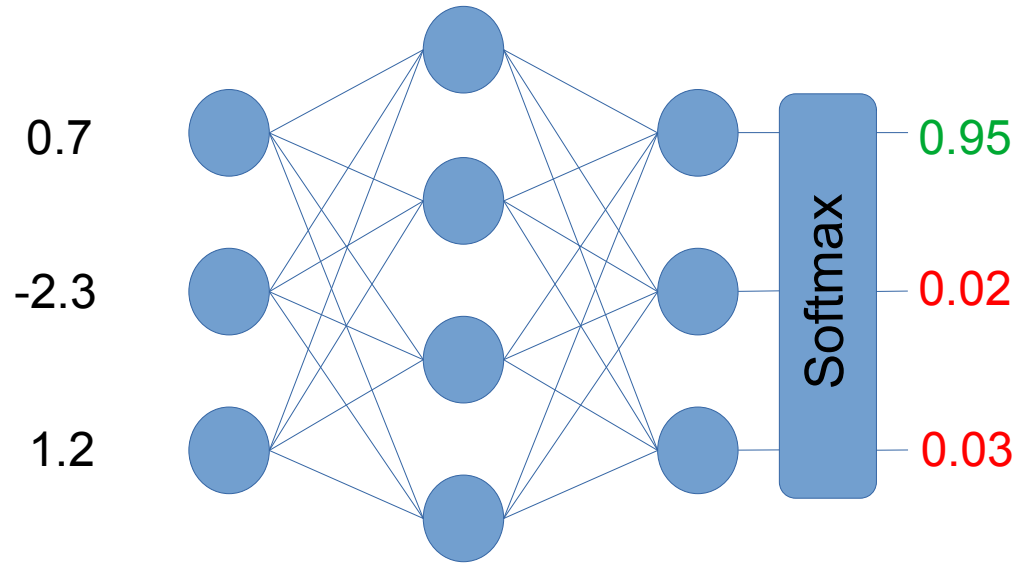
1) Predict the correct classes:

$$\hat{\pi}_{\theta}(y_i | x_i) \geq \hat{\pi}_{\theta}(y' | x_i) \quad \forall y' \in \mathcal{Y}$$

$$NLL(\theta) = \arg \max_{\theta} \prod_{i=1}^n \hat{\pi}_{\theta}(y_i | x_i) = \arg \min_{\theta} - \sum_{i=1}^n \log(\hat{\pi}_{\theta}(y_i | x_i))$$

Factors influencing calibration

Training using negative log-likelihood / cross-entropy loss



How to minimize NLL:

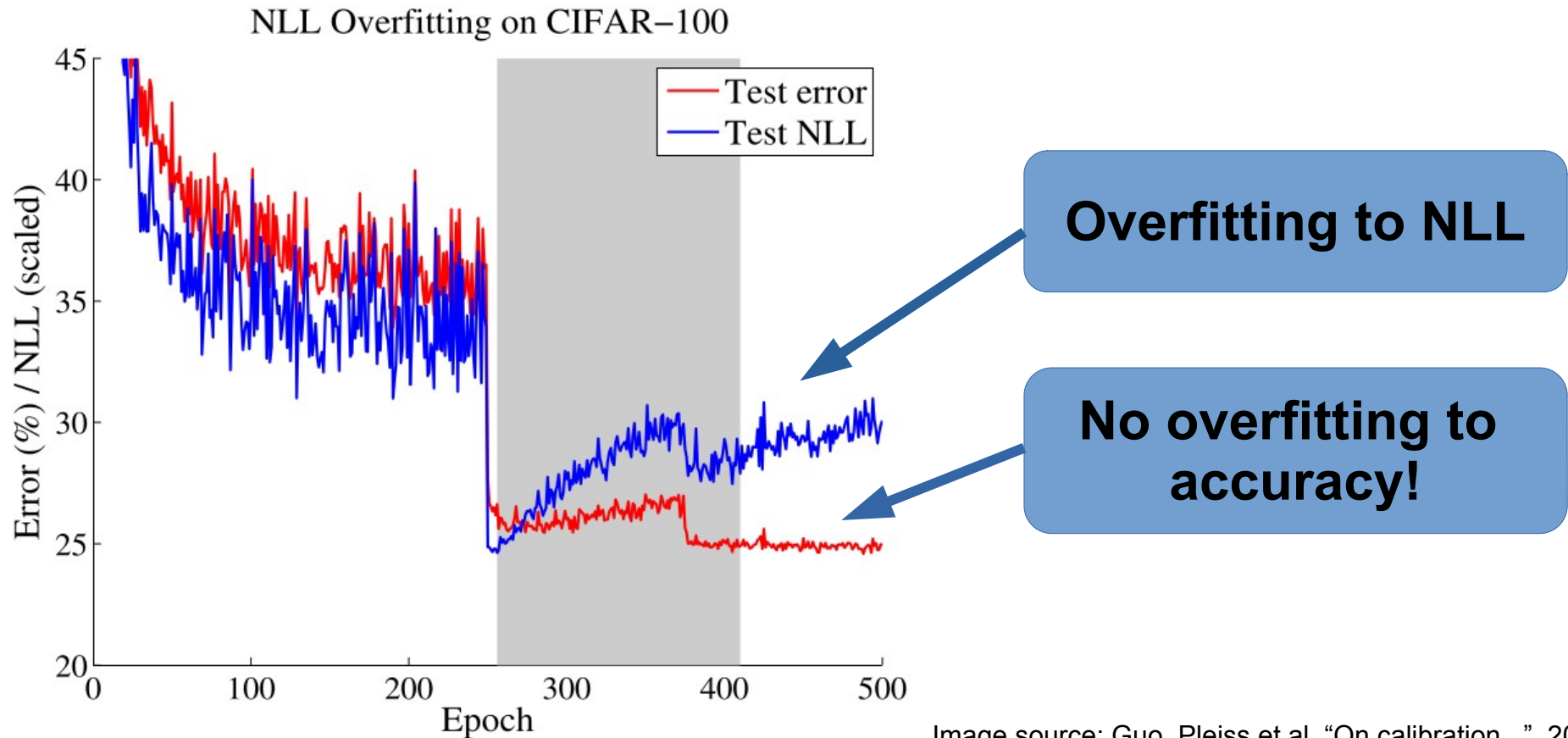
- 1) Predict the correct classes:
 $\hat{\pi}_{\theta}(y_i | x_i) \geq \hat{\pi}_{\theta}(y' | x_i) \quad \forall y' \in \mathcal{Y}$
- 2) Increase confidence in correct classes!

$$NLL(\theta) = \arg \max_{\theta} \prod_{i=1}^n \hat{\pi}_{\theta}(y_i | x_i) = \arg \max_{\theta} \sum_{i=1}^n \log \hat{\pi}_{\theta}(y_i | x_i)$$

Overfitting to NLL!

Factors influencing calibration

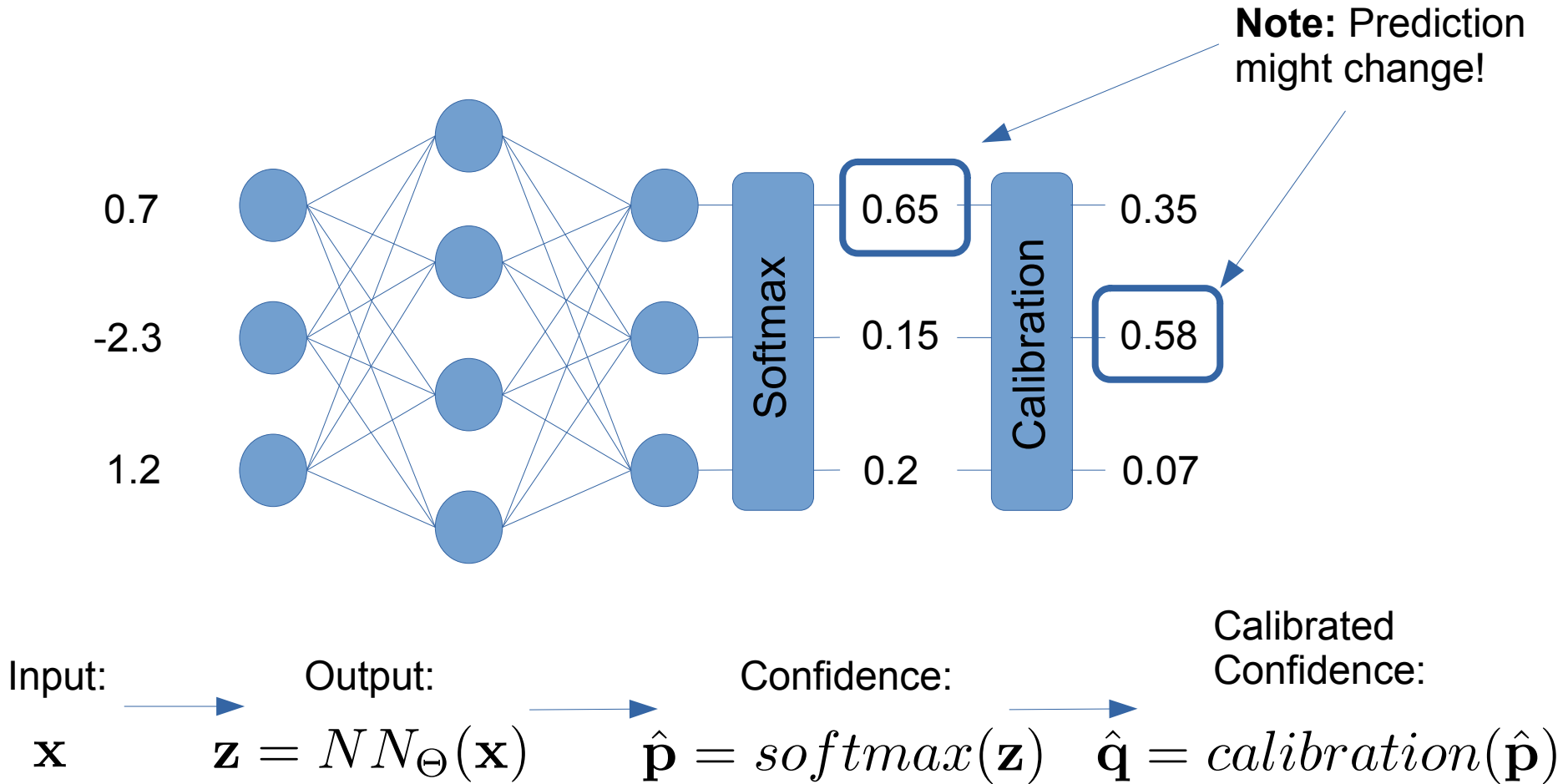
Training using negative log-likelihood / cross-entropy loss



Overview

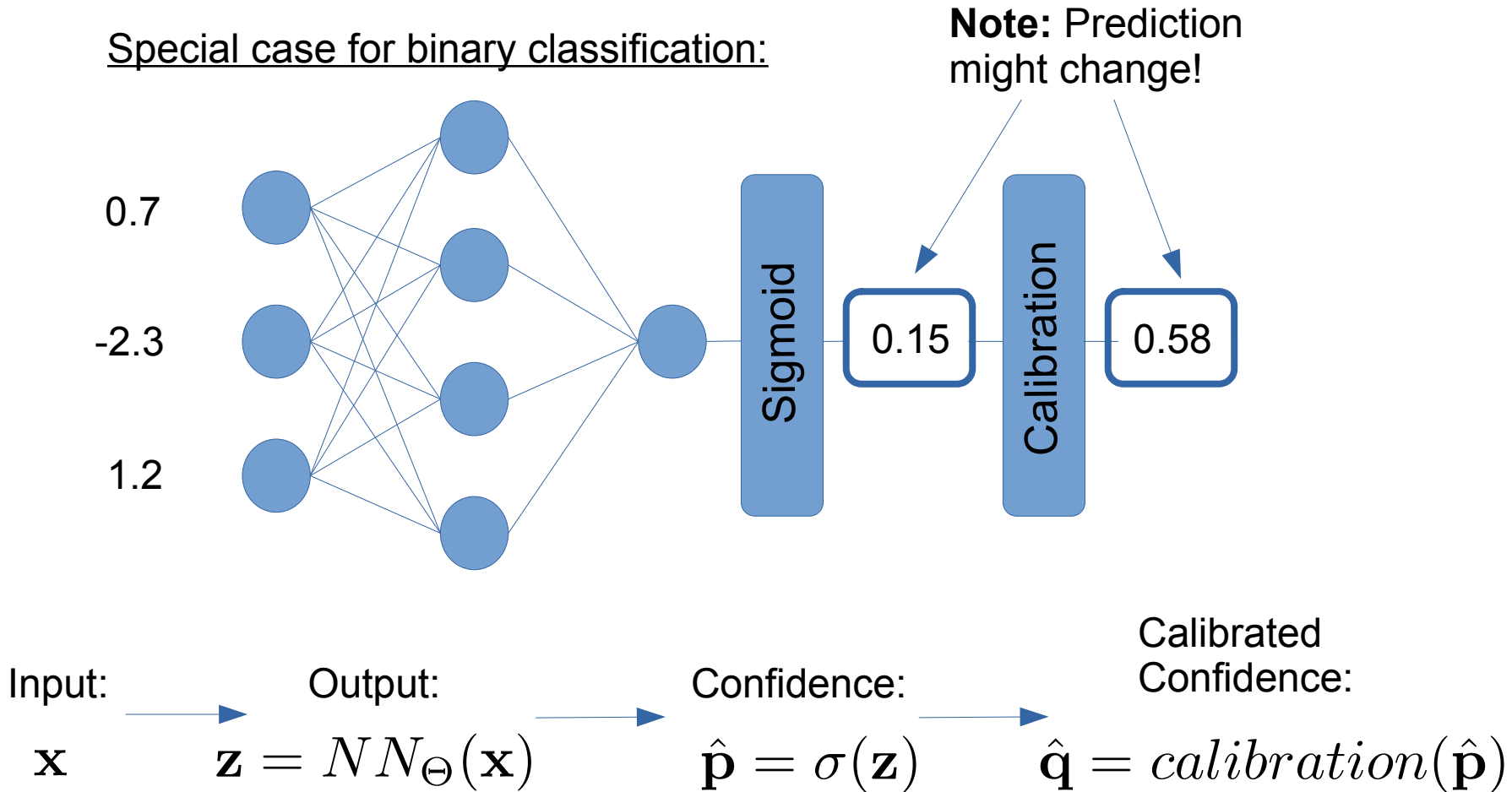
- **Introduction:** It's important for neural networks to be well-calibrated.
- **Definition:** How to measure model calibration?
 - ECE, MCE, Reliability diagrams
- **Problem:** Modern neural networks are no longer calibrated!
- **Analysis:** Which factors might influence model calibration?
 - Model capacity, Normalization, Regularization, NLL
- **Mitigation:** How to calibrate neural networks?
- **Experiments:** Which calibration methods perform best?

Calibration of neural networks



Calibration of neural networks

Special case for binary classification:



Histogram Binning

[Zadrozny et al. ICML 2001]

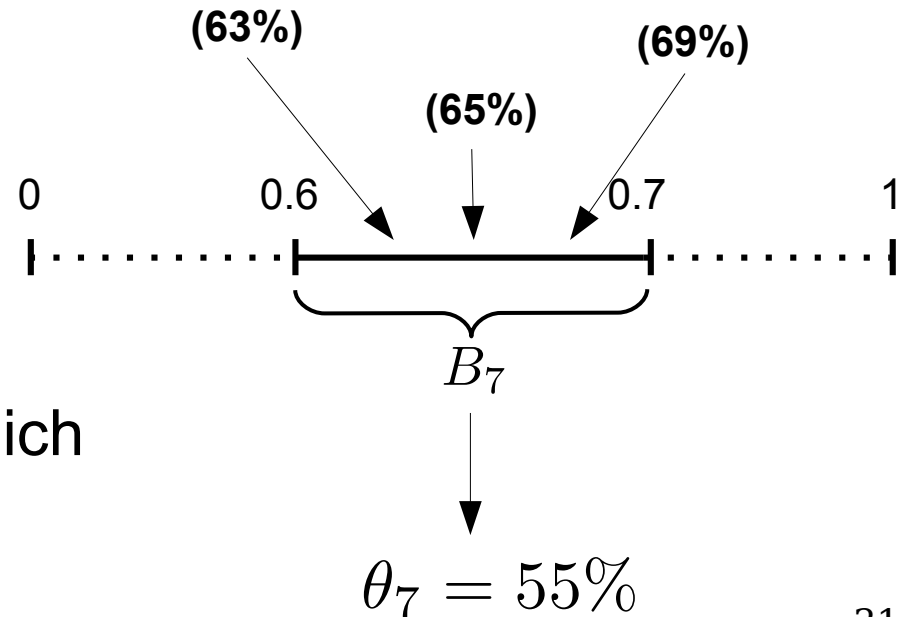
1. Group the predictions into M bins. Define bin B_m to be the set of all predictions (\hat{y}_i, \hat{p}_i) for which it holds that:

$$\hat{p}_i \in \left(\frac{m-1}{M}, \frac{m}{M} \right]$$

2. For all predictions in bin B_m output the probability θ_m

3. For each bin B_m find θ_m which minimizes

$$\sum_{y_i: \hat{p}_i \in B_m} (y_i - \theta_m)^2$$



Isotonic Regression

[Zadrozny et al. KDD 2002]

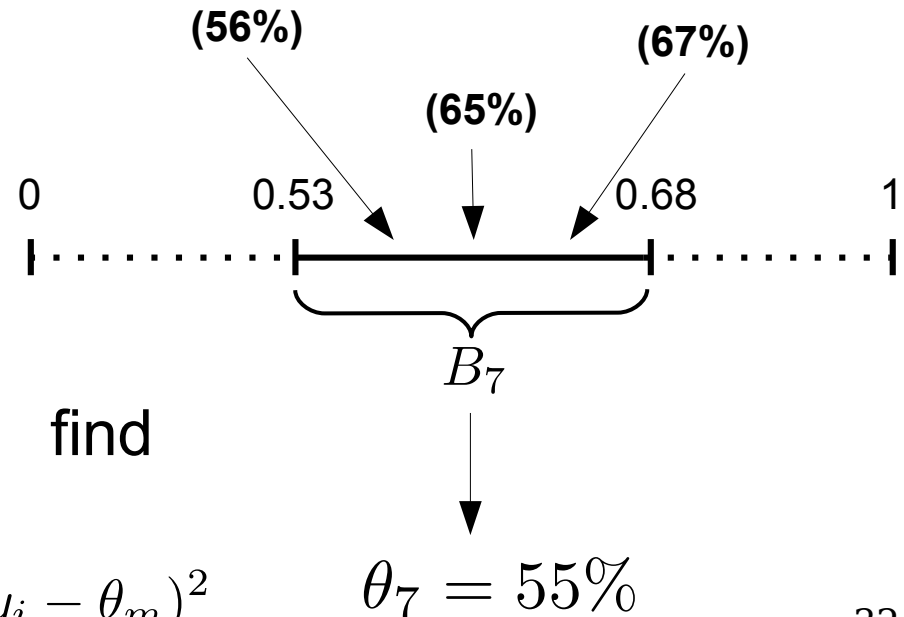
1. Group the predictions into M bins. Define bin B_m to be the set of all predictions (\hat{y}_i, \hat{p}_i) for which it holds that:

$$\hat{p}_i \in \left(\frac{m-1}{M}, \frac{m}{M} \right]$$

2. For all predictions in bin B_m output the probability θ_m

3. For each bin $B_m = (a_m, a_{m+1}]$ find (θ_m, a_m, a_{m+1}) which minimize

$$\sum_{y_i: \hat{p}_i \in B_m} (y_i - \theta_m)^2$$



Bayesian Binning into Quantiles (BBQ)

[Naeini et al. AAAI 2015]

- Look at all possible binning schemes at the same time!
- For a given validation set \mathcal{D} let \mathcal{S} be the set of all possible binning schemes for this data set.

- Previous models: Fix one binning scheme $s \in \mathcal{S}$ and compute optimal parameters θ for each s . Then predict $\hat{q}_i =$

- Bayesian Binning into Quantiles:

Prediction \hat{q}_i under the model s

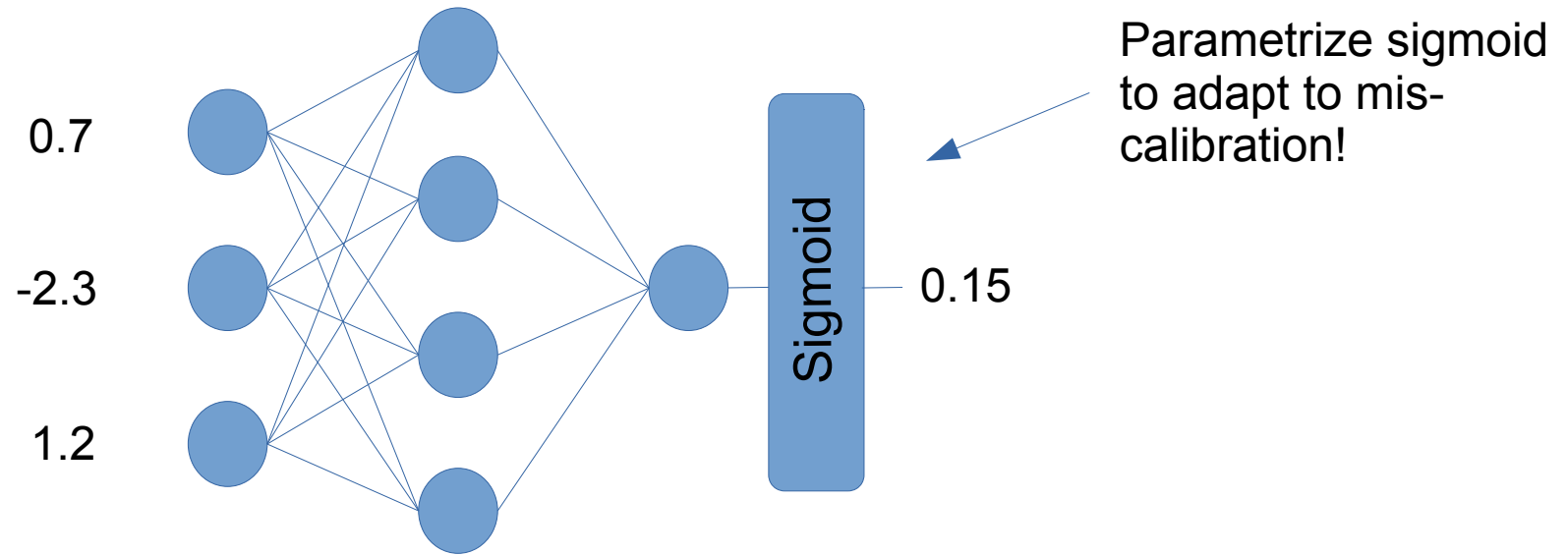
Sum over all binning schemes

How probable the model s is given the data \mathcal{D}

$$\hat{q}_i = \mathbb{P}(y_i = 1 \mid \hat{p}_i) = \sum_{s \in \mathcal{S}} \mathbb{P}(y_i = 1 \mid \hat{p}_i, S = s, \mathcal{D}) \cdot \mathbb{P}(S = s \mid \mathcal{D})$$

Platt scaling

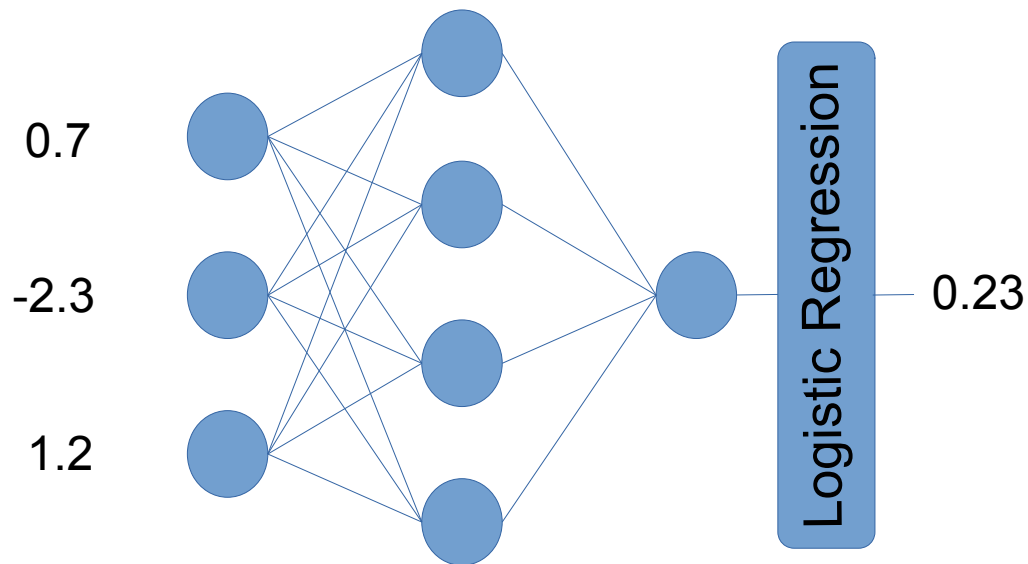
[Platt et al. Advances in large margin classifiers 1999]



Input: \mathbf{x} → Output: $\mathbf{z} = NN_{\Theta}(\mathbf{x})$ → Confidence: $\hat{\mathbf{p}} = \sigma(\mathbf{z})$

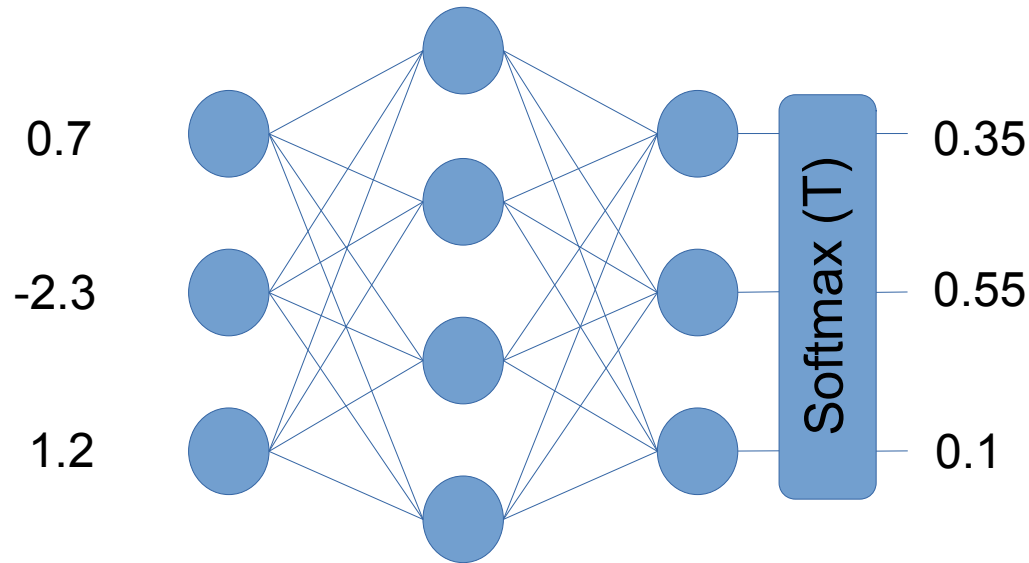
Platt scaling

[Platt et al. Advances in large margin classifiers 1999]



Input: \mathbf{x} → Output: $\mathbf{z} = NN_{\Theta}(\mathbf{x})$ → Confidence: $\hat{q} = \sigma(a \cdot \mathbf{z} + b) \quad a, b \in \mathbb{R}$

Temperature scaling

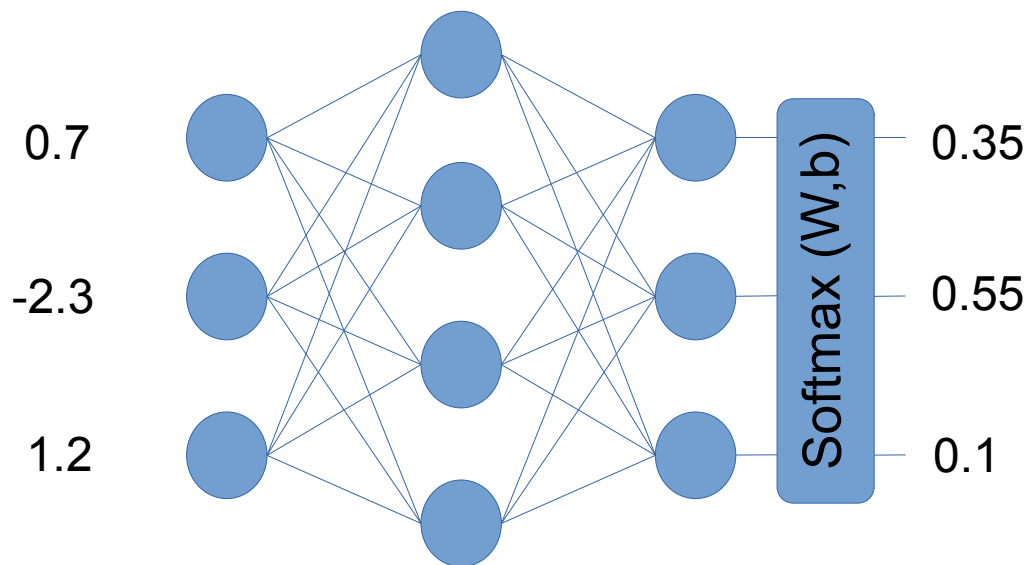


Temperature scaling

- Number of parameters is constant!
- This method doesn't change the predictions! \Rightarrow Accuracy stays the same
- Very easy to implement
- Fast to compute

Input: \mathbf{x} \longrightarrow Output: $\mathbf{z} = NN_{\Theta}(\mathbf{x})$ \longrightarrow Confidence: $\hat{\mathbf{q}} = softmax(\mathbf{z}/T)$ $T \in \mathbb{R}$

Matrix and Vector scaling

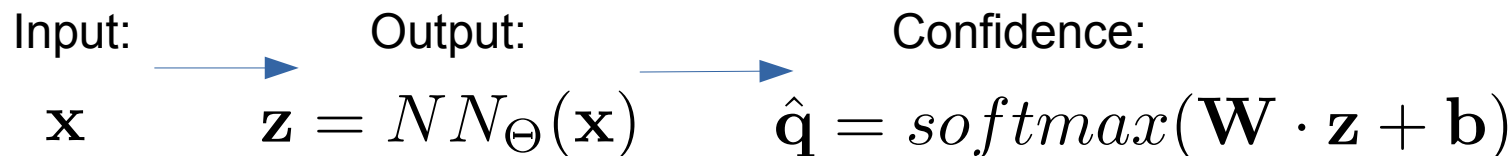


Matrix scaling

- No restrictions on W
- Number of parameters grows quadratically!

Vector scaling

- Restrict W to be a diagonal matrix
- Number of parameters grows linearly



$$\mathbf{W} \in \mathbb{R}^{k \times k}$$

$$\mathbf{b} \in \mathbb{R}^k$$

Overview

- **Introduction:** It's important for neural networks to be well-calibrated.
- **Definition:** How to measure model calibration?
 - ECE, MCE, Reliability diagrams
- **Problem:** Modern neural networks are no longer calibrated!
- **Analysis:** Which factors might influence model calibration?
 - Model capacity, Normalization, Regularization, NLL
- **Mitigation:** How to calibrate neural networks?
 - Binning, Platt- Matrix/Vector-, Temperature- scaling
- **Experiments:** Which calibration methods perform best?

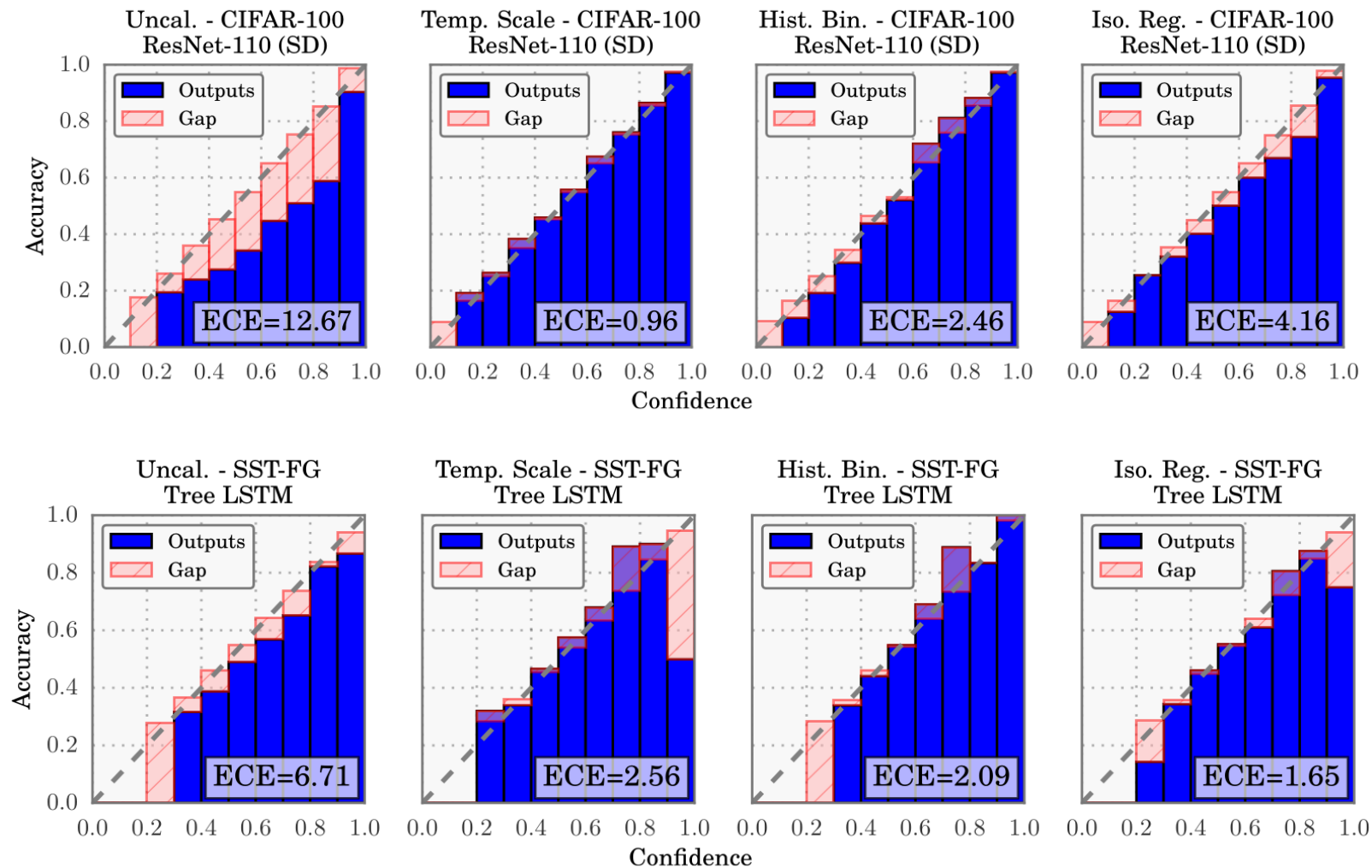
Experiments: Results ECE

Dataset	Model	Uncalibrated	Hist. Binning	Isotonic	BBQ	Temp. Scaling	Vector Scaling	Matrix Scaling
Birds	ResNet 50	9.19%	4.34%	5.22%	4.12%	1.85%	3.0%	21.13%
Cars	ResNet 50	4.3%	1.74%	4.29%	1.84%	2.35%	2.37%	10.5%
CIFAR-10	ResNet 110	4.6%	0.58%	0.81%	0.54%	0.83%	0.88%	1.0%
CIFAR-10	ResNet 110 (SD)	4.12%	0.67%	1.11%	0.9%	0.6%	0.64%	0.72%
CIFAR-10	Wide ResNet 32	4.52%	0.72%	1.08%	0.74%	0.54%	0.6%	0.72%
CIFAR-10	DenseNet 40	3.28%	0.44%	0.61%	0.81%	0.33%	0.41%	0.41%
CIFAR-10	LeNet 5	3.02%	1.56%	1.85%	1.59%	0.93%	1.15%	1.16%
CIFAR-100	ResNet 110	16.53%	2.66%	4.99%	5.46%	1.26%	1.32%	25.49%
CIFAR-100	ResNet 110 (SD)	12.67%	2.46%	4.16%	3.58%	0.96%	0.9%	20.09%
CIFAR-100	Wide ResNet 32	15.0%	3.01%	5.85%	5.77%	2.32%	2.57%	24.44%
CIFAR-100	DenseNet 40	10.37%	2.68%	4.51%	3.59%	1.18%	1.09%	21.87%
CIFAR-100	LeNet 5	4.85%	6.48%	2.35%	3.77%	2.02%	2.09%	13.24%
ImageNet	DenseNet 161	6.28%	4.52%	5.18%	3.51%	1.99%	2.24%	-
ImageNet	ResNet 152	5.48%	4.36%	4.77%	3.56%	1.86%	2.23%	-
SVHN	ResNet 152 (SD)	0.44%	0.14%	0.28%	0.22%	0.17%	0.27%	0.17%
20 News	DAN 3	8.02%	3.6%	5.52%	4.98%	4.11%	4.61%	9.1%
Reuters	DAN 3	0.85%	1.75%	1.15%	0.97%	0.91%	0.66%	1.58%
SST Binary	TreeLSTM	6.63%	1.93%	1.65%	2.27%	1.84%	1.84%	1.84%
SST Fine Grained	TreeLSTM	6.71%	2.09%	1.65%	2.61%	2.56%	2.98%	2.39%

Experiments: Results Error

Dataset	Model	Uncalibrated	Hist. Binning	Isotonic	BBQ	Temp. Scaling	Vector Scaling	Matrix Scaling
Birds	ResNet 50	22.54%	55.02%	23.37%	37.76%	22.54%	22.99%	29.51%
Cars	ResNet 50	14.28%	16.24%	14.9%	19.25%	14.28%	14.15%	17.98%
CIFAR-10	ResNet 110	6.21%	6.45%	6.36%	6.25%	6.21%	6.37%	6.42%
CIFAR-10	ResNet 110 (SD)	5.64%	5.59%	5.62%	5.55%	5.64%	5.62%	5.69%
CIFAR-10	Wide ResNet 32	6.96%	7.3%	7.01%	7.35%	6.96%	7.1%	7.27%
CIFAR-10	DenseNet 40	5.91%	6.12%	5.96%	6.0%	5.91%	5.96%	6.0%
CIFAR-10	LeNet 5	15.57%	15.63%	15.69%	15.64%	15.57%	15.53%	15.81%
CIFAR-100	ResNet 110	27.83%	34.78%	28.41%	28.56%	27.83%	27.82%	38.77%
CIFAR-100	ResNet 110 (SD)	24.91%	33.78%	25.42%	25.17%	24.91%	24.99%	35.09%
CIFAR-100	Wide ResNet 32	28.0%	34.29%	28.61%	29.08%	28.0%	28.45%	37.4%
CIFAR-100	DenseNet 40	26.45%	34.78%	26.73%	26.4%	26.45%	26.25%	36.14%
CIFAR-100	LeNet 5	44.92%	54.06%	45.77%	46.82%	44.92%	45.53%	52.44%
ImageNet	DenseNet 161	22.57%	48.32%	23.2%	47.58%	22.57%	22.54%	-
ImageNet	ResNet 152	22.31%	48.1%	22.94%	47.6%	22.31%	22.56%	-
SVHN	ResNet 152 (SD)	1.98%	2.06%	2.04%	2.04%	1.98%	2.0%	2.08%
20 News	DAN 3	20.06%	25.12%	20.29%	20.81%	20.06%	19.89%	22.0%
Reuters	DAN 3	2.97%	7.81%	3.52%	3.93%	2.97%	2.83%	3.52%
SST Binary	TreeLSTM	11.81%	12.08%	11.75%	11.26%	11.81%	11.81%	11.81%
SST Fine Grained	TreeLSTM	49.5%	49.91%	48.55%	49.86%	49.5%	49.77%	48.51%

Results



Overview

- **Introduction:** It's important for neural networks to be well-calibrated.
- **Definition:** How to measure model calibration?
 - ECE, MCE, Reliability diagrams
- **Problem:** Modern neural networks are no longer calibrated!
- **Analysis:** Which factors might influence model calibration?
 - Model capacity, Normalization, Regularization, NLL
- **Mitigation:** How to calibrate neural networks?
 - Binning, Platt- Matrix/Vector-, Temperature- scaling
- **Experiments:** Which calibration methods perform best?
 - Temperature scaling

My Take

- Interesting paper
- Well-written
- More data to show correlation between optimization techniques and ECE would have been appreciated

Takeaways

- **Fact:** Neural Networks are increasingly used in high risk decision making applications
- **Problem:** Modern neural networks are miscalibrated
- **Solution:** Performing Post-processing like for example temperature scaling to adjust confidence estimates helps to mitigate the problem

Appendix

Experiments: Datasets

Table	Description	# of classes	Train/Validation/Test
Caltech-UCSD	Bird images	200	5,994 / 2,897 / 2,897
Stanford Cars	Car images	196	8,041 / 4,020 / 4,020
ImageNet 2012	Natural scene images	1000	1.3M / 25,000 / 25,000
CIFAR-10/CIFAR-100	Color images	10 / 100	45,000 / 5,000 / 10,000
Street View House Numbers (SVHN)	House number images	10	598,388 / 6,000 / 26,032
20 News	News articles	20	9,034 / 2,259 / 7,528
Reuters	News articles	8	4,388 / 1,097 / 2,189
Stanford Sentiment Treebank	Movie reviews	2 / 5	6,920 / 872 / 1,821 544 / 1,101 / 2,210

Experiments: Networks

- Image classification tasks:
 - ResNets [He et al. CVPR 2016]
 - ResNets with stochastic depth [Huang et al. ECCV 2016]
 - Wide ResNets [Zagoruyko et al. BMVC 2016]
 - DenseNets [Huang et al. CVPR 2017]
- Document classification tasks:
 - Deep Averaging Networks [Iyyer et al. ACL 2015]
 - TreeLSTMs [Tai et al. ACL 2015]

Experiments: Results MCE

Dataset	Model	Uncalibrated	Hist. Binning	Isotonic	BBQ	Temp. Scaling	Vector Scaling	Matrix Scaling
Birds	ResNet 50	30.06%	25.35%	16.59%	11.72%	9.08%	9.81%	38.67%
Cars	ResNet 50	41.55%	5.16%	15.23%	9.31%	20.23%	8.59%	29.65%
CIFAR-10	ResNet 110	33.78%	26.87%	7.8%	72.64%	8.56%	27.39%	22.89%
CIFAR-10	ResNet 110 (SD)	34.52%	17.0%	16.45%	19.26%	15.45%	15.55%	10.74%
CIFAR-10	Wide ResNet 32	27.97%	12.19%	6.19%	9.22%	9.11%	4.43%	9.65%
CIFAR-10	DenseNet 40	22.44%	7.77%	19.54%	14.57%	4.58%	3.17%	4.36%
CIFAR-10	LeNet 5	8.02%	16.49%	18.34%	82.35%	5.14%	19.39%	16.89%
CIFAR-100	ResNet 110	35.5%	7.03%	10.36%	10.9%	4.74%	2.5%	45.62%
CIFAR-100	ResNet 110 (SD)	26.42%	9.12%	10.95%	9.12%	8.85%	8.85%	35.6%
CIFAR-100	Wide ResNet 32	33.11%	6.22%	14.87%	11.88%	5.33%	6.31%	44.73%
CIFAR-100	DenseNet 40	21.52%	9.36%	10.59%	8.67%	19.4%	8.82%	38.64%
CIFAR-100	LeNet 5	10.25%	18.61%	3.64%	9.96%	5.22%	8.65%	18.77%
ImageNet	DenseNet 161	14.07%	13.14%	11.57%	10.96%	12.29%	9.61%	-
ImageNet	ResNet 152	12.2%	14.57%	8.74%	8.85%	12.29%	9.61%	-
SVHN	ResNet 152 (SD)	19.36%	11.16%	18.67%	9.09%	18.05%	30.78%	18.76%
20 News	DAN 3	17.03%	10.47%	9.13%	6.28%	8.21%	8.24%	17.43%
Reuters	DAN 3	14.01%	16.78%	44.95%	36.18%	25.46%	18.88%	19.39%
SST Binary	TreeLSTM	21.66%	3.22%	13.91%	36.43%	6.03%	6.03%	6.03%
SST Fine Grained	TreeLSTM	27.85%	28.35%	19.0%	8.67%	44.75%	11.47%	11.78%

Paper Impact

On calibration of modern neural networks

[C Guo](#), [G Pleiss](#), [Y Sun](#)... - ... [Conference on Machine ...](#), 2017 - [proceedings.mlr.press](#)

Confidence calibration—the problem of predicting probability estimates representative of the true correctness likelihood—is important for classification models in many applications. We discover that modern neural networks, unlike those from a decade ago, are poorly calibrated. Through extensive experiments, we observe that depth, width, weight decay, and Batch Normalization are important factors influencing calibration. We evaluate the performance of various post-processing calibration methods on state-of-the-art architectures ...

☆ 77 Cited by 1220 Related articles All 7 versions >>

Paper impact

- **Confidence of out-of-distribution samples:**

- Enhancing the reliability of out-of-distribution image detection in NNs: <https://arxiv.org/pdf/1706.02690.pdf>
- Training Confidence-calibrated classifiers for detecting out-of-distribution samples: <https://arxiv.org/pdf/1711.09325.pdf>
- Learning Confidence for Out-of-Distribution Detection in Neural Networks: <https://arxiv.org/pdf/1802.04865.pdf>
- Deep anomaly detection with outlier exposure: <https://arxiv.org/pdf/1812.04606.pdf>
- Why ReLU networks yield high-confidence predictions far away from the training data and how to mitigate the problem: https://openaccess.thecvf.com/content_CVPR_2019/papers/Hein_Why_ReLU_Networks_Yield_High-Confidence_Predictions_Far_Away_From_the_CVPR_2019_paper.pdf

- **Application of paper:**

- A Clinically Applicable Approach to Continuous Prediction of Future Acute Kidney Injury: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6722431/>
- Deep k-Nearest Neighbors: Towards Confident, Interpretable and Robust Deep Learning: <https://arxiv.org/pdf/1803.04765.pdf?fbclid=IwAR2D5gqQf9SL0xRWBctEVrUCL9uUilf9IZrpPN83YZYbiCGdLAIMIhhaVns>

- **Comparison and Critique:**

- Can you trust your model's uncertainty? Evaluating predictive uncertainty under dataset shift: <https://arxiv.org/pdf/1906.02530.pdf>
- Measuring calibration in deep learning: https://openaccess.thecvf.com/content_CVPRW_2019/papers/Uncertainty%20and%20Robustness%20in%20Deep%20Visual%20Learning/Nixon_Measuring_Calibration_in_Deep_Learning_CVPRW_2019_paper.pdf

- **Calibration and fairness:**

- On fairness and calibration: <https://arxiv.org/pdf/1709.02012.pdf>