a) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$ ¿ f)

$$\nabla(\phi\psi) = \partial^i(\phi\psi)$$
$$= (\partial^i\phi)\psi + \phi(\partial^i\psi)$$
$$= (\nabla\phi)\psi + \phi(\nabla\psi)$$
$$\psi\nabla\phi + \phi\nabla\psi$$

f) $\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$

$$\nabla\times(\nabla\times a) = \varepsilon^{ijk}\partial_j(\nabla\times a)_k$$

$$= \varepsilon^{ijk}\partial_j\varepsilon^{kmn}\partial_m a_n$$

$$= \varepsilon^{ijk}\varepsilon^{kmn}\partial_j\partial_m a_n$$

$$= \varepsilon^{ijk}\varepsilon^{mnk}\partial_m\partial_j a_n$$

$$= (\delta^i_m\delta^j_n - \delta^i_n\delta^j_m)\partial_m\partial_j a_n$$

$$\delta^j_n\delta^i_m\partial_m\partial_j a_n - \delta^i_n\delta^j_m\partial_m\partial_j a_n$$

$$\partial_n\partial_i a_n - \delta^i_n\partial_m\partial_m a_n$$

$$\partial_i(\partial_n a_n) - \partial_m\partial_m a_n$$

$$\partial_i(\partial_n a_n) - \partial_m\partial_m a_n$$

$$\nabla(\nabla\cdot a) - \nabla^2 a.$$

d) $\nabla\cdot(\nabla\times a)$ ¿ ¿Qué se puede decir de $\nabla\times(\nabla\cdot a)$?

$$\partial_i(\nabla\times a)_i = \partial^i(\varepsilon^{ijk}\partial_j a_k) = 0$$

cte

No tiene sentido, el producto cruz está definido para vector y $\nabla\cdot a$ es un escalar.

Ejercicios /

a)

$\cos(3\alpha) = \cos^3\alpha - 3\cos\alpha\,sen^2(\alpha)$

$\cos(3\alpha) = \cos(3\alpha) + i\,sen(3\alpha) - i\,sen\,3\theta$

$[\cos\theta + i\,sen\,\theta]^3 - i\,sen^3\theta$

$\cos^3\theta + 3i\cos^2\theta\,sen\theta + 3\cos\theta(i\,sen\theta)^2 - i\,sen\,3\theta$

$\cos^3\theta + 3i\cos^2\theta\,sen\theta - 3\cos\theta\,sen^2\theta - i\,sen\,3\theta$

igualando las partes reales

$\cos(3\alpha) = \cos^3\alpha - 3\cos\alpha\,sen^2\alpha$.

Encuentre todas las raíces de las siguientes expresiones:

$re^{i\theta} = r\cos\theta + i\,r\,sen\,\theta$

(a) $\sqrt{2i} = (2i)^{1/2} = (2e^{i(\pi/2+2\pi k)})^{1/2} \longrightarrow$ si $k=0$

$\hookrightarrow |r| = 2$

$\theta = \pi/2 + 2\pi k$

$k = 0, 1$

si $k=0$

$\cdot \sqrt{2}\,e^{i(\pi/2)1/2} = \sqrt{2}\,e^{i\pi/4} \longrightarrow \sqrt{2}\cdot\cancel{\frac{\sqrt{2}}{2}} + i\sqrt{2}\cdot\frac{\sqrt{2}}{2}$

$1 + i$ $\cancel{//}$

si $k=1$

$= \sqrt{2}\,e^{i(\pi/2+2\pi)1/2}$

$= \sqrt{2}\,e^{i5\pi/4} \longrightarrow \sqrt{2}\left(\cancel{\frac{-\sqrt{2}}{2}}\right) + \sqrt{2}\left(\cancel{\frac{-\sqrt{2}}{2}}\right)$

$-1 - i$ $\cancel{//}$

(b) $\sqrt{1-\sqrt{3}i}$

$= (1-\sqrt{3}i)^{1/2} = \left(2e^{i(-\pi/3+2\pi k)}\right)^{1/2}$

$|r| = (1+3)^{1/2} = 2$

$\tan\theta = -\sqrt{3} \Rightarrow \theta = -\frac{\pi}{3}$

$k = 0, 1$

Casos: $k=0$

$= \sqrt{2}\, e^{i-\pi/6} \Rightarrow \sqrt{2}\left(\frac{\sqrt{3}}{2}\right) + i\sqrt{2}\left(\frac{-1}{2}\right) =$

$\frac{\sqrt{6}}{2} - i\frac{\sqrt{2}}{2}$

$k=1$

$\sqrt{2}\, e^{i\left(\frac{5\pi}{3}\right)\frac{1}{2}} = \sqrt{2}\, e^{i\frac{5\pi}{6}}$

$\frac{-\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$

—— // ——

c). $(-1)^{1/3}$

$= \left(e^{i(\pi+2\pi k)}\right)^{1/3}$

$|r| = 1$

$\theta = \pi$

$k = 0, 1$

Casos

$k=0$

$= e^{i\pi/3}$ $\cos\frac{\pi}{3} + i\operatorname{sen}\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$k=1$

$= e^{i\pi}$ $\cos\pi + i\operatorname{sen}\pi = -1$

$k=2$

$e^{i5\pi/3}$ $\cos\frac{5\pi}{3} + i\operatorname{sen}\frac{5\pi}{3}$

$\frac{1}{2} + i\frac{\sqrt{3}}{2}$

%

d) $8^{1/6}$

$|r| = 8$

$\theta = 0$

$k = 0, 1, 2, 3, 4, 5$

$(8e^{i2\pi k})^{1/6}$

Si $k=0$

$\to 8^{1/6}$

$\sqrt{2}$

$k=1$

$\sqrt{2}\, e^{i\pi/3} = \sqrt{2}\left(\frac{1}{2}\right) + i\sqrt{2}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

$k=2$

$\sqrt{2}\, e^{i2\pi/3}$

$\sqrt{2}\left(\frac{-1}{2}\right) + i\sqrt{2}\left(\frac{\sqrt{3}}{2}\right)$

$k=3$

$\sqrt{2}\, e^{i\pi}$

$= -\sqrt{2}$

$k=4$

$\sqrt{2}\left(-\frac{1}{2}\right) + i\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{2}}{2} - i\frac{\sqrt{3}}{2}$

$k=5$

$\frac{\sqrt{2}}{2} + i\sqrt{2}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{-\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

e) $\sqrt[4]{-8-8\sqrt{3}i} = 16 e^{i(4\pi/3 + 2\pi k)\, 1/4}$

$|r| = \sqrt{8^2+8^2+3^2}$

$\theta = \tan^{-1}(\sqrt{3}) + \pi$

$= 4\pi/3$

$k = 0, 1, 2, 3$

$k=0$

$2e^{i\pi/3}$

$2(1/2) + i2(\sqrt{3}/2)$

$1+\sqrt{3}i$

$k=1$

$2e^{i5\pi/6}$

$2\left(\frac{4\sqrt{3}}{2}\right) + i2\left(1/2\right)$

$-\sqrt{3}+1$

$k=2$

$2e^{i4\pi/3}$

$2\left(-1/2\right) + 2i\sqrt{-3/2}$

$k=3$

$2e^{i11\pi/6}$

$2\sqrt{3}/2 + i\left(-1/2\right)$

$\sqrt{3} - i$

Demuestra que:

(a). $Log(-ie) = 1 - \pi/2\, i$

$Log(-ie)$

$z = -ie$

$|z| = e$

$\theta = 3\pi/2$

$-\pi/2$

Por definición

$Log(z) = \ln|r| + i(\theta + 2\pi k)$

$= 1 - \pi/2\, i + 2\pi i k$

b) $Log(1-i) = 1/2 \ln(2) - \pi/4\, i$

$z = 1 - i$; $\ln(i-1) = \ln|\sqrt{2}| + i(-\pi/4)$

$|z| = \sqrt{2}$; $1/2 \ln(2) - \pi/4\, i$

$\tan\theta = 1$

(c) $\operatorname{Log}(e) = 1 + 2n\pi i$

$z = e$

$|z| = e$

$\theta = 0$

$\operatorname{Log}(e) = \ln|e| + i(0 + 2\pi n)$

$= 1 + 2n\pi i$

d. $\operatorname{Log}(i) = (2n + 1/2)\pi i$

$z = i$

$|z| = 1$

$\theta = \pi/2$

$\operatorname{Log}(i) = \ln(1) + i(\pi/2 + 2\pi n)$

$= \pi i\left(1/2 + 2n\right)$